# LAB 1: CANCER, SERIES, AND ODE SOLUTIONS, PART B

*winter 2021*


## 1. Matlab

*MATLAB* commands we use in this lab are `ode45`, and `plot` to plot solutions. These are (re)introduced below. The lab assignment follows this section.

1.1. **ode45.** Finds a numerical approximation to a differential equation.

```
>> [tsol,ysol] = ode45( func_handle, [tmin,tmax], initial_v );
```

1.2. **plot.** Plot one vector against another.

```
>> plot( t_values, y_values );
```

1.3. **axis.** Set the $x$ and $y$ domains of a plot:

```
>> axis( [xmin xmax ymin ymax] );
```

1.4. **hold.** Plot later plots on the same axes (`hold on`), or replace the current plot (`hold off`):

```
>> hold on;
>> hold off;
```

1.5. **title, xlabel, ylabel.** Set the title, $x$- and $y$-axis labels for an existing plot:

```
>> title( 'title text' );
>> xlabel( 'x-axis label text' );
>> ylabel( 'y-axis label text' );
```

1.6. **legend.** Set the legend for the plot:

```
>> legend( 'label 1', 'label 2', 'label 3'...);
```

## 2. Background

In this lab, we are studying the Gompertz equation, a first-order ordinary differential equation which models the growth of cancerous tumors,

$$\frac{dy}{dt} = ry \ln(K/y). \tag{1}$$

The constants $r$ and $K$ in this equation are positive, and we consider $r = 0.1$ and $K = 10$. The function $y(t)$ gives the volume of the tumor at time $t$. The initial condition, $y(0) = y_0$, must be positive (that is, greater than zero), and we will in general take $y(0) = 1$.

You will complete a lab report as described in section 6 with your partner. This is due at the beginning of the next lab period.

## 3. Exercise 1

Find the exact solution to the Gompertz equation. *(Hint: you can do this by separating variables and noting that* $\ln(K/y) = \ln(K) - \ln(y)$*.)* Check that you and group members agree on the solution. Also find the exact solution to the $n = 1$ (linear) approximation to the differential equation that is valid near $y = K$.

Take $r = 0.1$, $K = 10$, and $y(0) = 1$. Plot the exact solution to the Gompertz equation, (1), with the exact solution to the linear approximation (about $K$—you found this in Exercise 3 of the Prelab) and a numerical solution (generated with `ode45`) of the quadratic ($n = 2$) approximation.

How do the different solutions differ? How and where are they similar? Compare with your work in Part A, Exercise 3; does your work here give you confidence in the accuracy of the approximations to solutions of a differential equation that are generated by `ode45`?

## 4. Exercise 2

For all of the preceding work we have taken $y(0) = 1$. Would you expect the approximations to the Gompertz equation by expanding it around $y = K$ to be good approximations when $y = 1$?

Let's consider some values close to the expansion point. Find the exact solution to the Gompertz equation and the linear approximation when $y(0) = 8$ (you may take $r = 0.1$ and $K = 10$ still). Find numerical approximations using `ode45` to the approximations with $n = 2$ and $n = 3$. Plot all of these solutions together. Do the solutions to the approximate equations look similar to the exact solution? Do the different approximations behave as you expect?

Can you find an exact solution for the equation obtained by the $n = 2$ truncation of the Taylor series? This is a *logistic equation*, which we consider in [BB, §2.5]. It's a good challenge practice problem to find the exact solution!

## 5. Exercise 3

Finally, recall in the Prelab we looked at the expansion of the differential equation around the point $y = 1$. Check with your partners that you agree on the form of the expansion in this case. Then use `ode45` to generate (approximate) solutions to the Gompertz equation and the $n = 1$, $n = 2$, and $n = 3$ Taylor approximations to the equation (you may need to pick carefully the time interval on which you are generating the solution) and plot them. How good are the approximations? What happens to the agreement between the solutions to the approximate equations and the solution to the Gompertz equation as time goes on? Be sure you can explain why this makes sense given what your work in Exercise 2 suggests about how well the solutions to the approximate equations give insight on the behavior of the original.

## 6. Lab Report

Imagine that you are a biomedical engineering consultant, and that you are responding to the following request. Your lab writeup should respond to this, and be collaboratively produced by you and your partners. *All of you will submit a copy of the same writeup paper.* Note that you will need to include figures from the work that you did in the course of Parts A and B of the lab to produce a good writeup[1], and that you will need to include the equations and mathematical work underlies your conclusions.

The researcher's request is as follows:

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Dear Ima Consultant:

I am studying the development of cancer tumors, and need your assistance in understanding the predictions and validity of the Gompertz model of cancer tumor size. In particular, I need you to determine what the model predicts for the behavior of the tumor in certain treatment regimes, and when different approximations to the model may be appropriate (or not).

Specifically:

- What do the parameters in the model represent in the context of the modeled tumor?
- If a treatment reduces the rate of tumor growth, will that have a significant impact on the long-term outcome of the cancer? If instead the initial tumor size was changed, e.g., by a surgical intervention that removed most of the tumor, would that have a significant impact on the long-term outcome?
- If we linearize the Gompertz model, what assumptions are we making? What type of behavior is predicted for the tumor? Is the linearization adequate to predict the behavior of the tumor, and are there circumstances in which the simplification would be significantly better or worse?
- If the model is simplified by assuming a small tumor size, what is the resulting equation? What can we determine from the simplified equation? To what extent does this change if we consider a nonlinear simplification?

The technical requirements for the report, dictated by our lab, are that it should have the following format:

I. **Introduction:** The introduction should summarize the purpose and contents of your report in 3–6 complete sentences. You should include the Gompertz equation and the other equations you consider, with a note on how you analyze them, but otherwise keep the technical notation to a minimum.

II. **Body:** The body of the report should answer the questions of interest, by describing how the different parts of your analysis allow you to do this. You should include the mathematics that underlies your work, including how Taylor expansions are used to simplify the Gompertz equation, and the equations that you obtain and solve.

III. **Conclusion:** Provide a short, several paragraph, summary of your results that ties together the work you have described in the body.

[1] How do you get figures from *MATLAB*? You can do this by exporting them: in the figure window, select File → Export Setup. You can accept the defaults, and click Export. Select a file type, e.g., JPEG, give the export a name, and click Save.

Finally, as a 2 or 3 sentence appendix, include a discussion of how your work in math 216 is connected to the analysis you have done in the report. Note that *this should not be a reference list of texts*: instead, indicate how the mathematical analysis that you did in the lab is connected with your other work in the course.

I very much look forward to receiving your report,
Dr. A. Tonifa Chi
Biomedical Solutions, Inc.

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## References

[BB] Brannan, James R, and William E Boyce. Differential Equations: an Introduction to Modern Methods And Applications. Third edition. Hoboken, NJ: Wiley, 2015.