

Van Eenam Lecture Series

February 11, 12 & 13, 2025

University of Michigan
Department of Mathematics



René Carmona
Professor of Engineering
and Finance
Princeton University

Mean Field Games and Mean
Field Controls: Where Do We
Stand Twenty Years Later?

TUES., FEB. 11, 2025 | 4:00PM | 1360 EH

Optimal Control of Conditional
Processes

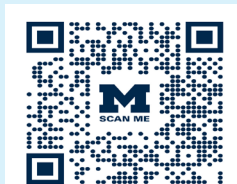
WED., FEB. 12, 2025 | 4:00PM | 1360 EH

From Nash Equilibrium to
Social Optimum and Back: A
Mean Field Perspective

THURS., FEB. 13, 2025 | 4:00PM | 4448 EH

EAST HALL
530 CHURCH STREET, ANN ARBOR

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René Carmona, formerly the chair of the department of Operations Research and Financial Engineering, is an associate member of the Department of Mathematics, a member of the Program in Applied and Computational Mathematics and the Andlinger Center for Energy and the Environment. For over twenty years, he was Director of Graduate Studies of the Bendheim Center for Finance where he oversaw the Master in Finance program. He obtained a Ph.D. in probability from Marseille University where he held his first academic job. After time spent at Cornell and Princeton, he moved to the University of California-Irvine in 1981, and eventually back to Princeton University in 1995. Professor Carmona is a Fellow of the Institute of Mathematical Statistics, of the Society for Industrial and Applied Mathematics, and of the American Mathematical Society. He is the founding chair of the SIAM Activity Group on Financial Mathematics and Engineering, a founding editor of the Electronic Journal of Probability and Electronic Communications in Probability, and the SIAM Journal on Financial Mathematics. His two-volume book, co-authored with F. Delarue, was the recipient of the J.L. Doob Prize awarded every three years by the American Mathematical Society.

Van Eenam Lecture Series

February 11, 12 & 13, 2025

René Carmona

Professor of Engineering and Finance
at Princeton University

Lecture I: Mean Field Games and Mean Field Controls: Where Do We Stand Twenty Years Later?

TUESDAY, FEB. 11, 2025 | 4:00 PM | 1360 EAST HALL

The first part of this talk will review the early developments in the mathematical theory that began with the introduction of Mean Field Games twenty years ago. We will briefly discuss several applications to highlight the need for an asymptotic analysis in the search for Nash equilibria and social optima in competitive and cooperative large populations. Additionally, we will explain why and how a probabilistic approach was introduced to complement and enhance the original partial differential equation approach to these problems. Next, we will introduce the concept of the master equation and its role in renewing interest in optimal transportation and the analysis of partial differential equations in the Wasserstein space of probability measures. Finally we will discuss some of the new challenges currently investigated.

Lecture II: Optimal Control of Conditional Processes

WEDNESDAY, FEB. 12, 2025 | 4:00 PM | 1360 EAST HALL

In this talk, we consider the conditional control problem introduced by P.L. Lions in his lectures at the College de France in November 2016. As originally stated, the problem does not fit in the usual categories of control problems considered in the literature, so its solution requires new ideas, if not new technology. In his lectures, Lions emphasized some of the major differences with the analysis of classical stochastic optimal control problems and in so doing, raised the question of the possible differences between the value functions resulting from optimization over the class of Markovian controls as opposed to the general family of open loop controls. While the equality of these values is accepted as a "folk theorem" in the classical theory of stochastic control, optimizing an objective function whose values strongly depend upon the past history of the controlled trajectories of the system is a strong argument in favor of differences between the optimization results over these two different classes of control processes. The goal of the talk is to elucidate this quandary and provide elements of response to Lions' original conjecture. We shall report on the solution of the "soft killing" case published in a joint work with M. Lauriere and P.L.Lions in the Illinois Journal of Mathematics, a recent "conditional mimicking" theorem proven with D. Lacker, and further investigations currently worked out with S. Daudin.

Lecture III: From Nash Equilibrium to Social Optimum and back: A Mean Field Perspective

THURSDAY, FEB. 13, 2025 | 4:00 PM | 4448 EAST HALL

Mean field games (MFG) and mean field control (MFC) problems have been introduced to study large populations of strategic players. They correspond respectively to non-cooperative or cooperative scenarios, and the goals of their analyses are to find the Nash equilibriums and social optimums. These frameworks provide approximate solutions to situations with a finite number of players and have found a wide range of applications, from economics to biology and machine learning. In this paper, we study how the players can transition from a non-cooperative to a cooperative regime, and back. The first direction is reminiscent of mechanism design, in which the game's definition is modified so that non-cooperative players reach an outcome similar to a cooperative scenario. To better understand the second direction we introduce the "price of instability" and study how players that are initially cooperative gradually deviate from a social optimum to reach a Nash equilibrium when they decide to optimize their individual cost very much in the spirit of the "free rider" phenomenon. To formalize these connections, we introduce two new classes of games which lie between MFG and MFC: λ -interpolated mean field games, in which the cost of an individual player is a λ -interpolation of the MFG and the MFC costs, and ρ -partial mean field games, in which a proportion ρ of the population deviates from the social optimum by playing the game non-cooperatively. We shall conclude with algorithm for myopic players to learn a ρ -partial mean field equilibrium, and we illustrate it on a stylized model.

Joint work with G. Dayanikli, M. Lauriere and F. Delarue

