AIM Qualifying Review Exam in Probability and Discrete Mathematics

August 2025

There are five (5) problems in this examination.

There should be sufficient room in this booklet for all your work. But if you use other sheets of paper, be sure to mark them clearly and staple them to the booklet. No credit will be given for answers without supporting work and/or reasoning.

Problem 1 Suppose $0 \leq x, y, z \leq 1$ are random variables that have the joint density function

$$\frac{1}{8}xyz.$$

(a) Find the joint density of x and y subject to the constraint $x + y + z = 1$.

(b) Find the probability that $x \leq \frac{1}{2}$ subject to the constraint $x + y + z = 1$.

Problem 1

Problem 1

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Problem 2 In any set of 101 distinct integers chosen from $\{1, 2, 3, \ldots, 200\}$, prove that there must be some integer that divides another.

Problem 2

Problem 2

Problem 2

Problem 3

How many numbers in the set $\{1, 2, 3, \ldots, 1000\}$ are divisible by 2, 3, or 5?

Problem 3

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Problem 3

Problem 4

Suppose $x_1, \ldots, x_m$ and $y_1, \ldots, y_n$ are two sequences of numbers. A sequence such as a, b, c is a common subsequence if it occurs as a subsequence (not necessarily in contiguous positions) in both sequences. For example, $1, 2, 3$ is a subsequence of $7, 1, 8, 8, 2, 4, 3, 9$.

(a) Devise an efficient algorithm to find the length of the longest common subsequence.

(b) Devise an efficient algorithm to find the longest common subsequence.

Problem 4

Problem 4

Problem 4

Problem 5

You are required to sort a sequence of numbers $x_1, \ldots, x_n$. However, you are only allowed to use k-flips, with $k \in \{1, 2, \ldots, n\}$. A k flip will take the first k numbers in the sequence and reverse their order. Describe an $O(n^2)$ algorithm to sort using k-flips assuming each k-flip to cost k operations.

Problem 5

Problem 5

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