

THE UNIVERSITY OF MICHIGAN
DEPARTMENT OF MATHEMATICS

Qualifying Review examination in Algebraic Topology

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1. The *cone* CX on a topological space X is the quotient of $X \times [0, 1]$ by identifying all the points $(x, 1)$ to a single point $*$. Suppose that there exists an open neighborhood of the point $*$ in CX which is homeomorphic to \mathbb{R}^n . What are the possible sequences of the groups

$$H_0(X), H_1(X), H_2(X), \dots?$$

2. Let F_n be the free group on n generators. For which $n = 1, 2, \dots$ does there exist an injective homomorphism $h : F_n \rightarrow F_2$? For which $n = 1, 2, \dots$ can h be chosen so the image has finite index in F_2 ?
3. Let X be a path-connected space with basepoint $*$. Let $x \neq y \in X$ be points, and let $Y = X/(x \sim y)$.
 - (a) Is the homomorphism $p_* : \pi_1(X, *) \rightarrow \pi_1(Y, *)$ induced by the projection $p : X \rightarrow Y$ necessarily injective?
 - (b) Can p_* be an isomorphism?
4. Describe all the homotopy equivalence classes of CW-complexes which have exactly three cells of dimensions $0, 1, n$, respectively, for a given $n > 1$.
5. Compute the homology of the space $\mathbb{R}P^3/\mathbb{R}P^1$ (where $\mathbb{R}P^1$ is embedded in the standard way).