Fall 2023 Graduate Course Descriptions

MATH 501. AIM Student Seminar — **Alben, Silas** — **Fri 1:00 PM-2:00 PM**
Alben, Silas — **Fri 3:00 PM-4:00 PM**

At least two 300 or above level math courses, and Graduate standing; Qualified undergraduates with permission of instructor only. (1). May be repeated for a maximum of 6 credits. Offered mandatory credit/no credit.

MATH 501 is an introductory and overview seminar course in the methods and applications of modern mathematics. The seminar has two key components: (1) participation in the Applied and Interdisciplinary Math Research Seminar; and (2) preparatory and post-seminar discussions based on these presentations. Topics vary by term.

MATH 520. Life Contingencies I — **Natarajan, Roger** — **T/Th 10:00 AM-11:30 AM**

MATH 424 and 425 with minimum grade of C-, plus declared Actuarial/Financial Mathematics Concentration. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of the Math 520-521 sequence is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. The main topics are (1) developing probability distributions for the future lifetime random variable, and (2) using those distributions to price life insurance and annuities.

Required Textbook: Actuarial Mathematics for Life Contingent Risks, by Dickson, David C.M./Hardy, Mary R./Waters, Howard R. ISBN-13: 978-1108478083

MATH 523. Loss Models I — **Young, Virginia** — **T/Th 8:30 AM-10:00 AM**

MATH/STATS 425. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Risk management and modeling of financial losses. Review of random variables (emphasizing parametric distributions), review of basic distributional quantities, continuous models for insurance claim severity, discrete models for insurance claim frequency, the effect of coverage modification on severity and frequency distributions, aggregate loss models, and simulation.

Textbook: *Loss Models: From Data to Decisions*, by Stuart A. Klugman, 9781118315323

MATH 525/STATS 525. Probability Theory — **TBD** — **T/Th 11:30 AM-1:00 PM**
TBD — **T/Th 1:30 PM-2:30 PM**
TBD — **M/W/F 8:00 AM-9:00 AM**

MATH 451 (strongly recommended). MATH 425/STATS 425 would be helpful. (3). (BS). May not be repeated for credit.

TBD

MATH 526/STATS 526. Discrete State Stochastic Processes — **Cohen, Asaf** — **T/Th 10:00 AM-11:30 AM**
Kolliopulos, Nikos — **T/Th 8:30 AM-10:00 AM**
Kara, Ali — **T/Th 11:30 PM-1:00 PM**

MATH 525 or STATS 525 or EECS 501. (3). (BS). May not be repeated for credit.

This is a course on the theory and applications of stochastic processes on discrete state spaces. Some specific topics include:
(1) Markov chains – Markov property,– recurrence and transience, – stationarity, – ergodicity, – coupling, – exit probabilities and expected exit times;
(2) Markov decision processes – optimal control, – Banach fixed point theorem;
(3) Exponential distribution and Poisson processes – memoryless property,– thinning and superposition, – compound Poisson processes;
(4) Markov chains in continuous time – generators and Kolmogorov equations, – embedded Markov chains,– stationary distributions and limit theorems, – exit probabilities and expected exit times,– Markov queues;
(5) Martingales – conditional expectations, – gambling (trading) with martingales, – optional sampling,
– applications to the computation of exit probabilities and expected exit times, – martingale convergence.

**Textbook: Essentials of Stochastic Processes, by Richard Durrett, 3rd Edition, 9783319456133

Math 538 Lie Algebras **Lam, Thomas** **WF 10:00 AM-11:30 AM**

Lie algebras arise naturally in mathematics and physics, and are fundamental from many perspectives. Lie algebras are fascinating in their own right, and the study of finite dimensional Lie algebras leads to interesting combinatorial structures, such as root systems, Dynkin diagrams, and Coxeter groups. This course should be valuable to those interested in representation theory and the study of algebraic and Lie groups, and should also be useful to those whose interests lie in areas such as combinatorics, geometry, and physics.

In this course, we will study the basic theory of Lie algebras, with the majority of our focus on the complex semisimple case. We intend to cover most of the content of Humphrey's book (Introduction to Lie Algebras and Representation Theory), including structure theorems for Lie algebras, classifications of root systems, and highest weight representation theory.

This course will be taught at the advanced undergraduate, or introductory graduate level, and should be accessible to students with experience in abstract algebra and linear algebra.

Required Text: Introduction to Lie Algebras and Representation Theory PAPERBACK by Humphreys, J.E. (9780387900520) - 7TH 72 SPRINGER – Free version available online through UM Library

MATH 555 Introduction to Functions of a Complex Variable with Applications **Burns, D.** **T/Th 1:00 PM-2:30 PM**

MATH 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

Intended primarily for students of engineering and of other cognate subjects, as well as students in the Applied and Interdisciplinary Mathematics (AIM) graduate program. Doctoral students in mathematics should elect Mathematics 596. Complex numbers, continuity, derivative, conformal representation, integration, Cauchy theorems, power series, singularities, and applications to engineering and mathematical physics, asymptotics and matrix analysis. This is a core course for the AIM graduate program.

Textbook: *Introductory Complex Analysis by Silverman*, Richard A., 9780486646862

MATH 556. Applied Functional Analysis **Borcea, Liliana** **T/Th 10:00 AM-11:30 AM**

MATH 217, 419, or 420; MATH 451; and MATH 555. (3). (BS). May not be repeated for credit.

This is an introduction to methods of applied functional analysis. Students are expected to master both the proofs and applications of major results. The prerequisites include linear algebra, undergraduate analysis, advanced calculus and complex variables. This course is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program.

Required Textbook: *Applied Analysis* - by Hunter, John K. (9789812705433) – 01

MATH 558. Applied Nonlinear Dynamics **Wu, Sijue** **T/Th 2:30 PM-4:00 PM**
Topic: Advanced Ordinary Differential Equations

MATH 216, 217, and 451/452. (3). (BS). May not be repeated for credit.

Differential equations model systems throughout science and engineering and display rich dynamical behavior. This course emphasizes the qualitative and geometric ideas which characterize the post Poincare era. The course surveys a broad range of topics with emphasis on techniques, and results that are useful in applications. It is intended for students in mathematics, engineering, and the natural sciences and is a core course for the Applied and Interdisciplinary Mathematics graduate program.

Course material will be taken from Chapters 1-10, and Chapter 15 of the text.

There will be weekly homeworks, midterm and final exams.

Required Textbook: M. Hirsh, S. Smale, and R. *Devaney, Differential Equations, Dynamical Systems, and an Introduction to Chaos*, 3rd ed., Elsevier.

MATH 565. Combinatorics and Graph Theory **TBD** **T/Th 8:30 AM-10:00 AM**

MATH 465. (3). (BS). May not be repeated for credit.

TBD

MATH 568/BIOINF 568. Mathematical and Computational Neuroscience **Booth, Victoria** **M/W 10:00 AM-11:30 AM**

MATH 463 or 462 (for undergraduate students) or Graduate standing. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Computational neuroscience investigates the brain at many different levels, from single cell activity to small, local network computation to the dynamics of large neuronal populations. This course introduces modeling and quantitative techniques used to investigate neural activity at all these different levels. Topics covered include passive membrane properties, the Nernst potential, derivation of the Hodgkin-Huxley model, action potential generation, action potential propagation in cable and multi-compartmental models, reductions of the Hodgkin-Huxley model, phase plane analysis, linear stability and bifurcation analysis, synaptic currents, excitatory and inhibitory network dynamics.

MATH 571. Numerical Linear Algebra **TBD** **T/Th 8:30 AM-10:00 AM**

MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit.

TBD

MATH 573. Financial Mathematics I **Bayraktar, Erhan** **T/Th 1:00 PM-2:30 PM**

(3). (BS). May not be repeated for credit.

This is an introductory course in Financial Mathematics. This course starts with the basic version of the Mathematical Theory of Asset Pricing and Hedging (Fundamental Theorem of Asset Pricing in discrete time and discrete space). This theory is applied to problems of Pricing and Hedging of simple Financial Derivatives. Finally, the continuous-time version of the proposed methods is presented, culminating with the Black–Scholes model. A part of the course is devoted to the problems of Optimal Investment in discrete time (including Markowitz Theory and CAPM) and Risk Management (VaR and its extensions). This course shows how one can formulate and solve relevant problems of the financial industry via mathematical (in particular, probabilistic) methods. Although Math 526 is not a prerequisite for Math 573, it is strongly recommended that either these courses are taken in parallel, or Math 526 precedes Math 573.

Required Textbook: Stochastic Finance: An Introduction in Discrete Time; Hans Föllmer and Alexander Schied, 4th edition; 978-3110463446

Optional Textbook: Economics and Mathematics of Financial Markets by Jaksa Cvitanic and Fernando Zapatero

MATH 591. Differentiable Manifolds **Spatzier, Ralf** **M/W/F 10:00 AM-11:00 AM**

MATH 451, 452 and 590. (3). (BS). May not be repeated for credit.

This is one of the basic courses for students beginning the PhD program in mathematics. The approach is rigorous and emphasizes abstract concepts and proofs.

Topics: Product and quotient topology, group actions, topological groups, topological manifolds, smooth manifolds, manifolds with boundary, smooth maps, partitions of unity, tangent vectors and differentials, the tangent bundle, submersions, immersions and embeddings, smooth submanifolds, Sard's Theorem, the Whitney Embedding Theorem, transversality, Lie groups, vector fields, Lie brackets, Lie algebras, multilinear algebra, vector bundles, differential forms, exterior derivatives, orientation, Stokes' Theorem, introduction to De Rham cohomology groups, homotopy invariance.

Optional Textbooks: Introduction to Smooth Manifolds(2nd edition), by John Lee; 978-1-4419-9981-8
An Introduction to Manifolds(2nd edition), by Loring W. Tu; 978-1-4419-7399-3

MATH 593. Algebra I **Mustata, Mircea** **M/W/F 2:00 PM-3:00 PM**

MATH 412, 420, and 451 or MATH 494. (3). (BS). May not be repeated for credit.

The course will cover basic results about rings and modules, an introduction to homological algebra, the structure of modules over PIDs, an introduction to categories and functors, and multilinear algebra. I will assume that students have had (at least) a full year long sequence in algebra at the advanced undergraduate level.

In addition to the regular class time, we will meet weekly one more time (on a day and time to be decided) to work on problems, examples, etc.

There is no textbook required for this course.

MATH 596. Analysis I **Baik, Jinho** **T/Th 2:30 PM-4:00 PM**
Topic Title: Complex Analysis

MATH 451. (3). (BS). May not be repeated for credit. Students with credit for MATH 555 may elect MATH 596 for two credits only.

This is a theoretical and rigorous introductory course on complex analysis on the level of the first year math graduate students. Highly advanced math undergraduate students and graduate students from other disciplines may also take this course but they should expect that the workload is heavy and the pace is fast. Topics to be discussed include holomorphic functions, Cauchy's theorem, Cauchy's integral formula, power series, isolated singularities, meromorphic functions, Laurent series, conformal mappings, infinite product, and so on.

Textbook: Complex Analysis, by Lars Ahlfors; 9781470467678

MATH 602. Real Analysis II **Chelkak, Dmitry** **T/Th 1:00 PM-2:30 PM**

Topic: Functional Analysis

MATH 590 and 597. (3). (BS). May not be repeated for credit.

Functional analysis is a core subject in mathematics. It has connections to probability and geometry, and is of fundamental importance to the development of analysis, differential equations, quantum mechanics and many other branches in mathematics, physics, engineering and theoretical computer science. The goal of this course is to introduce students to the basic concepts, methods and results in functional analysis. Topics to be covered include linear spaces, normed linear spaces, Banach spaces, Hilbert spaces, linear operators, dual operators, the Riesz representation theorem, the Hahn-Banach theorem, uniform boundedness theorem, open mapping theorem, closed graph theorem, compact operators, Fredholm Theory, reflexive Banach spaces, weak and weak* topologies, spectral theory, and applications to classical analysis and partial differential equations.

Optional Textbook: Functional Analysis, Peter D. Lax, ISBN-13: 978-0471556046, ISBN-10: 0471556041

MATH 614. Commutative Algebra **Snowden, Andrew** **T/Th 10:00 AM-11:30 AM**

MATH 593 and Graduate standing. (3). (BS). May not be repeated for credit.

Commutative algebra is a field that interacts strongly with many other areas of mathematics, including algebraic geometry, algebraic combinatorics, algebraic number theory, and several complex variables. This course is an introduction that will include material on the uses of the prime spectrum, behavior of primes under integral extensions of rings, Noetherian rings and modules, Noether normalization, the Hilbert basis theorem, an introduction to affine algebraic geometry, primary decomposition, normal rings, discrete valuation rings, Dedekind domains, Artinian rings, flatness, completion, and dimension theory, including the Krull height theorem. Some basic material from category theory will also be introduced.

No textbook is required for this course.

MATH 623/IOE 623. Computational Finance Kim, Donghan T/Th 10:00 AM-11:30 AM

MATH 316 and MATH 425 or 525. (3). (BS). May not be repeated for credit.

This is a course in computational methods in finance and financial modeling. Particular emphasis will be put on interest rate models and interest rate derivatives. Specific topics include Black-Scholes theory, no-arbitrage and complete markets theory, term structure models, Hull and White models, Heath-Jarrow-Morton models, the stochastic differential equations and martingale approach, multinomial tree and Monte Carlo methods, the partial differential equations approach, finite difference methods.

Required Texts: Monte Carlo methods in financial engineering / Paul Glasserman/ 0387004513

Optional Texts: The Mathematics Of Financial Derivatives A Student Introduction, Paul Wilmott, Sam Howison, Jeff Dewynne, 9780511812545

****MATH 625/STATS 625. Probability and Random Processes I Rudelson, Mark T/Th 10:00 AM-11:30 AM**

Topic: Probability Theory

Math 597 or permission of instructor. (3). (BS). May not be repeated for credit.

This is the first part of a graduate level probability course.

Topics include: Kolmogorov's extension theorem, sums of independent random variables, the Law of Large Numbers, convergence of random variables, characteristic functions, Central Limit Theorems, filtrations, conditional expectation, discrete time martingales and their convergence, optional stopping theorem, elements of measure concentration. If time permits, we will also cover elements of large deviation theory and/or information theory.

Textbooks: no textbook is required.
Recommended textbook: Probability: Theory and Examples, by Rick Durrett; 978-1108473682.

MATH 629. Machine Learning for Finance II Nazari, Ali F 1:00 PM- 2:30PM / S 10AM-2PM

This is a graduate level course intended for students in the Master's Program in Quantitative Finance and Risk Management (Quant Program).

The aim of the course is to prepare students in the Quant Program to meet the needs of finance industry employers by providing the students with a theoretical understanding of and practical experience in applying data science concepts as they pertain to financial mathematics. In addition, the topics will include practical implementation of the techniques in Python on financial data or other sample data when financial data not available. The course will focus on mathematical foundations, practical programming exercises, domain expertise, and technical communication and will be divided into the following content areas:

I. Classical Statistical Learning (Classification, Regression, Support Vector Machine, Nearest Neighbors)
II. Ensemble Learning, Dimensionality Reduction
III. Neural Networks, Deep Networks
IV. Model Interpretability, Feature Importance, Feature Reduction

Course content will be taught across two terms (two credits each term) and will culminate in students' completion of a final project at the end of the second semester.

****MATH 631. Introduction to Algebraic Geometry Pixton, Aaron T/Th 11:30 AM-1:00 P**M

MATH 594 or permission of instructor. Graduate standing. Previous knowledge: General topology. Familiarity with the language of category theory. Commutative algebra is recommended but not essential; you should have a solid grasp of localizations (of rings/modules) and tensor products though. (3). (BS). May not be repeated for credit.

This is the first half of a year-long sequence in algebraic geometry. In the first semester, we will introduce the basic notions and objects of modern algebraic geometry - sheaves and schemes. We will be loosely following Ravi Vakil's notes "Foundations of Algebraic Geometry".

MATH 636. Topics in Differential Geometry **Wright, Alexander** **M/W/F 3:00 PM-4:00 PM**
Topic: Outer Automorphism Groups of Free Groups

Math 592

This course will be an introduction to outer automorphism groups of free groups. These groups are a central topic in geometric group theory with rich analogies to mapping class groups and arithmetic groups. The course will include related topics such as automorphisms of free groups, metric graphs, Outer space, the sphere complex, etc.

The students will work together to produce notes on the lectures. See http://www-personal.umich.edu/~alexmw/Math797Notes.pdf for an example.

There is no required textbook for this course.

MATH 650. Fourier Analysis **Rudelson, Mark** **T/Th 1:00 PM-2:30PM**

Math 597 or permission of instructor

Fourier analysis is one of the most powerful tools in PDEs, probability, analytic number theory, as well as signal processing and computer science. This course will cover the basics of Fourier analysis starting from Fourier series, Dirichlet and Fejer kernel and Fejer-Lebesgue theorem. We will continue with Fourier integral in one and multiple dimensions, Schwartz functions, theory of distributions and its applications, Sobolev spaces and embedding theorems. At a more advanced stage of the course we will discuss Fourier transform in complex domains leading to boundary behavior of harmonic and analytic functions and Paley-Wiener theory for functions and distributions and applications of the Paley-Wiener theory to PDEs. Time permitting, we will discuss applications of Fourier analysis to probability, convex geometry, and number theory.

No textbook for this course

MATH 655. Topics in Fluid Dynamics **Veerapaneni, Shravan** **T/Th 2:30 PM-4:00 PM**

Numerical analysis, basic theory of ordinary and partial differential equations, experience with programming (e.g., MATLAB).

Topic: Mathematics of Microhydrodynamics

This course will cover the mathematical theory of particulate and multiphase flows encountered in various engineering and natural processes. Computational techniques for simulating complex fluids will be reviewed. We will examine a wide range of applications, from flows in thin films, porous media, and microfluidic chips to flows around suspended particles. Topics covered will include:
• Linearity and reversibility in Stokes flow, uniqueness, reciprocal theorem
• Constitutive laws for simple and complex interfaces, both passive and active
• Boundary integral equations
• Differential geometry in 2D, simulation of bubbles, vesicles and active particles
• Spherical harmonics, BIE analysis on spheres
• Small deformation theory and application
No textbook is required.

MATH 656. Introduction to Partial and Differential Equations **Miller, Peter** **T/Th 2:30 PM-4:00 PM**

MATH 558, 596 and 597 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

Partial differential equations are at the core of models in science, technology, economics and related fields. These equations and their solutions have interesting structures that are studied by methods of analysis, geometry, probability and other mathematical fields.

This course will introduce students in pure and applied mathematics to concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations.

Topics that may be covered include nonlinear first order equations; linear elliptic, hyperbolic and parabolic equations; the method of characteristics, energy methods, maximum principles, Fourier transforms and Sobolev spaces.

Required textbook: Fritz, John: *Partial Differential Equations*, Springer, 4th Ed. 978-0387906096

MATH 668. Advanced Combinatorics **Fomin, Sergey** **T/Th 11:30 AM-1:00 PM**

MATH 565 or 566 or equivalent. Graduate standing. (3). (BS). May not be repeated for credit.

This introductory course in combinatorial matrix theory will focus on connections between linear algebra and algebraic combinatorics.

Tentative list of topics: Combinatorial techniques in linear algebra. Basic algebraic graph theory. Determinantal identities. Matroids and projective geometry. Grassmannians and Schubert cells. Canonical forms and factorizations. Polynomials with real roots. Totally positive matrices.

There is no required textbook for this course.

MATH 671. Numerical Methods I **Veerapaneni, Shravan** **T/Th 1:00 PM – 2:30 PM**

Emerging topics in large-scale scientific computing

This course will survey emerging paradigms in solving large-scale scientific computing problems. Emphasis will be given on techniques that can be used for discretization and computational solution of partial differential equations (PDEs). Broadly, the course will cover selected topics from (i) Classical fast algorithms, (ii) ML for PDE solution, (iii) Tensor network methods and (iv) Variational quantum algorithms.

MATH 679. Eliptic Curves **Zieve, Michael** **M/W 11:30 AM-1:00 PM**

Familiarity with algebra at the level of Math 493/494 or Math 593/594

This course is an introduction to elliptic curves, focusing on the arithmetic side of the theory. It develops the theory of elliptic curves over various types of fields, and in particular over algebraic number fields.

Textbook: *Arithmetic of Elliptic Curves* by Joseph H. Silverman; 9780387094946

MATH 682. Recursion Theory **Chen, Ruiyuan** **T/Th 1:00 PM-2:30 PM**

This course will cover Descriptive Set Theory (DST), a branch of mathematical logic with many connections to other areas. DST originated in real analysis with the work of Borel, Baire, and Lebesgue in understanding the precise nature of sets and functions on the real line, which led to the development of integration and measure theory. Nowadays, depending on who you ask, DST may mean various things: the general study of "explicitly definable" sets and functions, and how hard it is to define them; the study of "computable" sets and functions relative to arbitrary oracles; "measure theory without measures"; infinitary groups, rings, and other algebraic structures; infinitary propositional/first-order logic; and the "classification of classification problems" throughout mathematics.

We will develop the basics of DST from a logician's perspective, including Boolean (sigma-)algebras and spaces, Polish and quasi-Polish spaces, the Borel and projective hierarchies, Baire category, and Lebesgue measure. Other topics we may cover, depending on the interests of the class: Polish groups and their actions (the groups R and R^2 are isomorphic; can you think of an isomorphism?), infinite two-player games and the Borel determinacy theorem, infinite Borel combinatorics and Ramsey theory, countable model theory and classification problems (are there "more" groups or graphs?), point-free topology and DST, and effective DST.

There are no hard prerequisites, other than mathematical maturity appropriate for a 600-level course. Some familiarity with basic topology, set theory (especially ordinals), and/or first-order logic would be helpful, but is not required; we will review these topics as they arise.

MATH 695. Algebraic Topology I **Kriz, Igor** **M/W/F 9:00 AM-10:00 AM**

MATH 591 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

There are few areas of mathematics today where one does not encounter the term "cohomology." While the term may refer to vastly different concepts, a modern mathematician understands what they have in common. The purpose of this course is to explore the methods cohomology arises from, the underlying field of homotopy theory, and the many contexts its techniques apply to. Starting from the concrete cases of spaces and chain complexes, we will then move on to frameworks such as derived categories, calculational tools such as spectral sequences, as well as general phenomena such as duality. Examples of generalized cohomology theories, such as K-theory, will also be discussed.

There will be no exams or quizzes, homework will be collected on a weekly basis.

Optional Texts: A Concise Course in Algebraic Topology, Peter May, ISBN 13: 9780226511825

MATH 697. Topics in Topology **Truong, Linh** **T/Th 2:30 PM-4:00 PM**
Topic: Heegaard Floer Homology

This course will assume a basic understanding of smooth manifolds (smooth maps, derivatives, differential forms) and algebraic topology (homology, cohomology).

Since its introduction in the late 1980s, Floer homology has become an important tool in low-dimensional topology. This course will introduce a version of Floer homology called Heegaard Floer homology, an invariant for knots, three-manifolds, and four-manifolds. As applications we may discuss minimal genus problems, detecting exotic smooth structures on four-manifolds and finding topological properties of knots.

There is no required textbook for this course.