Fall 2022 Graduate Course Descriptions

MATH 501. AIM Student Seminar	**Alben, Silas** **Alben, Silas**	**Fri 1:00 PM-2:00 PM** **Fri 3:00 PM-4:00 PM**

At least two 300 or above level math courses, and Graduate standing; Qualified undergraduates with permission of instructor only. (1). May be repeated for a maximum of 6 credits. Offered mandatory credit/no credit.

MATH 501 is an introductory and overview seminar course in the methods and applications of modern mathematics. The seminar has two key components: (1) participation in the Applied and Interdisciplinary Math Research Seminar; and (2) preparatory and post-seminar discussions based on these presentations. Topics vary by term.

MATH 520. Life Contingencies I	**TBD**	**T/Th 11:30 AM-1:00 PM**

MATH 424 and 425 with minimum grade of C-, plus declared Actuarial/Financial Mathematics Concentration. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of the Math 520-521 sequence is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. The main topics are (1) developing probability distributions for the future lifetime random variable, and (2) using those distributions to price life insurance and annuities.

Required Textbook: Actuarial Mathematics for Life Contingent Risks, by Dickson, David C.M./Hardy, Mary R./Waters, Howard R. ISBN-13: 978-1108478083

MATH 523. Loss Models I	**TBD**	**T/Th 10:00 AM-11:30 AM**

MATH/STATS 425. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Risk management and modeling of financial losses. Review of random variables (emphasizing parametric distributions), review of basic distributional quantities, continuous models for insurance claim severity, discrete models for insurance claim frequency, the effect of coverage modification on severity and frequency distributions, aggregate loss models, and simulation.

Textbook: Loss Models: From Data to Decisions, by Stuart A. Klugman, 9781118315323

MATH 525/STATS 525. Probability Theory	**Barvinok, Alexander** **Barvinok, Alexander** **TBD**	**T/Th 8:30 AM-10:00 AM** **T/Th 10:00 AM-11:30 AM** **M/W/F 8:00 AM-9:00 AM**

MATH 451 (strongly recommended). MATH 425/STATS 425 would be helpful. (3). (BS). May not be repeated for credit.

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

The following topics will be covered: sample space and events, random variables, concept and definition of probability and expectation, conditional probability and expectation, independence, moment generating functions, Law of large numbers, Central limit theorem, Markov chains, Poisson process and exponential distribution.

Required Textbook: Probability and Random Processes, by Grimmett, Geoffrey R. / Stirzaker, David R. (9780198572220) - 3RD 01

MATH 526/STATS 526. Discrete State Stochastic Processes

Cohen, Asaf	**T/Th 10:00 AM-11:30 AM**
Cohen, Asaf	**T/Th 8:30 AM-10:00 AM**
Kara, Ali	**T/Th 10:00 AM-11:30 AM**

MATH 525 or STATS 525 or EECS 501. (3). (BS). May not be repeated for credit.

This is a course on the theory and applications of stochastic processes, mostly on discrete state spaces. It is a second course in probability which should be of interest to students of mathematics and statistics as well as students from other disciplines in which stochastic processes have found significant applications.

The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include generating functions; recurrent events and the renewal theorem; random walks; Markov chains; branching processes; limit theorems; Markov chains in continuous time with emphasis on birth and death processes and queueing theory; an introduction to Brownian motion; stationary processes and martingales.

**Testbook: Essentials of Stochastic Processes, by Richard Durrett, 3rd Edition, 9783319456133

MATH 555 Introduction to Functions of a Complex Variable with Applications **TBD** **T/Th 1:00 PM-2:30 PM**

MATH 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

Intended primarily for students of engineering and of other cognate subjects. Doctoral students in mathematics elect Mathematics 596. Complex numbers, continuity, derivative, conformal representation, integration, Cauchy theorems, power series, singularities, and applications to engineering and mathematical physics.

Textbook: Introductory Complex Analysis by Silverman, Richard A., 9780486646862

****MATH 556. Applied Functional Analysis** **Borcea, Liliana** **T/Th 10:00 AM-11:30 AM**

MATH 217, 419, or 420; MATH 451; and MATH 555. (3). (BS). May not be repeated for credit.

This is an introduction to methods of applied functional analysis. Students are expected to master both the proofs and applications of major results. The prerequisites include linear algebra, undergraduate analysis, advanced calculus and complex variables. This course is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program.

Required Textbook: Applied Analysis - by Hunter, John K. (9789812705433) – 01

MATH 558. Applied Nonlinear Dynamics **TBD** **T/Th 2:30 PM-4:00 PM**

MATH 451. (3). (BS). May not be repeated for credit.

Differential equations model systems throughout science and engineering and display rich dynamical behavior. This course emphasizes the qualitative and geometric ideas which characterize the post-Poincare era. The course surveys a broad range of topics with an emphasis on techniques, and results that are useful in applications. It is intended for students in mathematics, engineering, and the natural sciences and is a core course for the Applied and Interdisciplinary Mathematics graduate program. Proofs are given. Homework and exams concentrate on using rather than proving.

**Required Textbook: Nonlinear Ordinary Differential Equations by Jordan, Dominic / Smith, Peter (9780199208258) - 4TH 07

****MATH 565. Combinatorics and Graph Theory** **Lam, Thomas** **T/Th 10:00 AM-11:30 AM**

MATH 465. (3). (BS). May not be repeated for credit.

This is an introductory combinatorics class at the graduate level. The first half of the class is about graph theory, including topics such as extremal graph theory, Ramsey theory, chromatic numbers and polynomials. The second half of the class studies geometric combinatorics, including topics such as projective geometries, geometric lattices, matroids, and hyperplane arrangements.

Textbook: Course in Combinatorics, by Jacobus H. Vanlint & Richard M. Wilson, 9780521006019

MATH 568/BIOINF 568 Mathematical and Computational Neuroscience **Booth,Victoria** **M/W 10:00 AM-11:30 AM**

MATH 463 or 462 (for undergraduate students) or Graduate standing. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Computational neuroscience investigates the brain at many different levels, from single cell activity to small, local network computation to the dynamics of large neuronal populations. This course introduces modeling and quantitative techniques used to investigate neural activity at all these different levels. Topics covered include passive membrane properties, the Nernst potential, derivation of the Hodgkin-Huxley model, action potential generation, action potential propagation in cable and multi-compartmental models, reductions of the Hodgkin-Huxley model, phase plane analysis, linear stability and bifurcation analysis, synaptic currents, excitatory and inhibitory network dynamics.

MATH 571. Numerical Linear Algebra **Esedoglu, Selim** **T/Th 8:30 AM-10:00 AM**

MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit.

Math 571 is an introduction to numerical linear algebra, a core subject in scientific computing. Three types of problems are considered: (1) linear systems, (2) eigenvalues, (3) least-squares problems. These problems arise in many scientific applications and we'll study the accuracy, efficiency, and stability of the methods that have been developed for their solution. As an application, we'll consider finite-difference schemes for boundary value problems in 1D and 2D.

Textbook: Numerical Linear Algebra, by Lloyd N. Trefethen and David Bau. 9780898713619

MATH 573. Financial Mathematics I **Chen, Tao** **T/Th 1:00 PM-2:30 PM**

(3). (BS). May not be repeated for credit.

This is an introductory course in Financial Mathematics. This course starts with the basic version of the Mathematical Theory of Asset Pricing and Hedging (Fundamental Theorem of Asset Pricing in discrete time and discrete space). This theory is applied to problems of Pricing and Hedging of simple Financial Derivatives. Finally, the continuous-time version of the proposed methods is presented, culminating with the Black–Scholes model. A part of the course is devoted to the problems of Optimal Investment in discrete time (including Markowitz Theory and CAPM) and Risk Management (VaR and its extensions). This course shows how one can formulate and solve relevant problems of the financial industry via mathematical (in particular, probabilistic) methods. Although Math 526 is not a prerequisite for Math 573, it is strongly recommended that either these courses are taken in parallel, or Math 526 precedes Math 573.

Textbook: Stochastic Finance, by Hans Follmer & Alexander Schied, 9783110463446

****MATH 591. Differentiable Manifolds** **Spatzier, Ralf** **M/W/F 10:00 AM-11:00 AM**

MATH 451, 452 and 590. (3). (BS). May not be repeated for credit.

This is one of the basic courses for students beginning the PhD program in mathematics. The approach is rigorous and emphasizes abstract concepts and proofs.

Topics: Product and quotient topology, group actions, topological groups, topological manifolds, smooth manifolds, manifolds with boundary, smooth maps, partitions of unity, tangent vectors and differentials, the tangent bundle, submersions, immersions and embeddings, smooth submanifolds, Sard's Theorem, the Whitney Embedding Theorem, transversality, Lie groups, vector fields, Lie brackets, Lie algebras, multilinear algebra, vector bundles, differential forms, exterior derivatives, orientation, Stokes' Theorem, introduction to De Rham cohomology groups, homotopy invariance.

Optional Textbooks: Introduction to Smooth Manifolds(2nd edition), by John Lee; 978-1-4419-9981-8
An Introduction to Manifolds(2nd edition), by Loring W. Tu; 978-1-4419-7399-3

MATH 593. Algebra I **Snowden, Andrew** **M/W/F 2:00 PM-3:00 PM**

MATH 412, 420, and 451 or MATH 494. (3). (BS). May not be repeated for credit.

Topics include basics about rings and modules, including Euclidean rings, PIDs, UFDs, and basic constructions such as quotients, localizations and tensor products. We will cover the structure theory of modules over a PID, and standard matrix forms such as Smith normal form, Jordan canonical form and rational normal form. The course will also cover tensor, symmetric, and exterior algebras, and the classification of bilinear forms. Large portions of the class will involve solving and presenting solutions to problems.

****MATH 596. Analysis I** **Baik, Jinho** **T/Th 2:30 PM-4:00 PM**
Topic Title: Complex Analysis

MATH 451. (3). (BS). May not be repeated for credit. Students with credit for MATH 555 may elect MATH 596 for two credits only.

This is a theoretical and rigorous introductory course on complex analysis on the level of the first year math graduate students. Highly advanced math undergraduate students and graduate students from other disciplines may also take this course but they should expect that the workload is heavy and the pace is fast. Topics to be discussed include holomorphic functions, Cauchy's theorem, Cauchy's integral formula, power series, isolated singularities, meromorphic functions, Laurent series, conformal mappings, infinite product, and so on.

Textbook: Complex Analysis, by Lars Ahlfors; 9781470467678

****MATH 602. Real Analysis II** **Conlon, Joe** **T/Th 2:30 PM-4:00 PM**

MATH 590 and 597. (3). (BS). May not be repeated for credit.

Functional analysis is a core subject in mathematics. It has connections to probability and geometry, and is of fundamental importance to the development of analysis, differential equations, quantum mechanics and many other branches in mathematics, physics, engineering and theoretical computer science. The goal of this course is to introduce students to the basic concepts, methods and results in functional analysis. Topics to be covered include linear spaces, normed linear spaces, Banach spaces, Hilbert spaces, linear operators, dual operators, the Riesz representation theorem, the Hahn-Banach theorem, uniform boundedness theorem, open mapping theorem, closed graph theorem, compact operators, Fredholm Theory, reflexive Banach spaces, weak and weak* topologies, spectral theory, and applications to classical analysis and partial differential equations.

Optional Textbook: Functional Analysis, Peter D. Lax, ISBN-13: 978-0471556046, ISBN-10: 0471556041

****MATH 614. Commutative Algebra** **Mustata, Mircea** **T/Th 10:00 AM-11:30 AM**

MATH 593 and Graduate standing. (3). (BS). May not be repeated for credit.

Commutative algebra is a field that interacts strongly with many other areas of mathematics, including algebraic geometry, algebraic combinatorics, algebraic number theory, and several complex variables. This course is an introduction that will include material on the uses of the prime spectrum, behavior of primes under integral extensions of rings, Noetherian rings and modules, Noether normalization, the Hilbert basis theorem, an introduction to affine algebraic geometry, primary decomposition, normal rings, discrete valuation rings, Dedekind domains, Artinian rings, flatness, completion, and dimension theory, including the Krull height theorem. Some basic material from category theory will also be introduced. There is no text: lecture notes will be provided.

MATH 623/IOE 623. Computational Finance TBD T/Th 10:00 AM-11:30 AM

MATH 316 and MATH 425 or 525. (3). (BS). May not be repeated for credit.

This is a course in computational methods in finance and financial modeling. Particular emphasis will be put on interest rate models and interest rate derivatives. Specific topics include Black-Scholes theory, no-arbitrage and complete markets theory, term structure models, Hull and White models, Heath-Jarrow-Morton models, the stochastic differential equations and martingale approach, multinomial tree and Monte Carlo methods, the partial differential equations approach, finite difference methods.

Textsbooks: Monte Carlo methods in financial engineering / Paul Glasserman/ 0387004513
The Mathematics Of Financial Derivatives A Student Introduction, Paul Wilmott, Sam Howison, Jeff Dewynne, 9780511812545

****MATH 625/STATS 625. Probability and Random Processes I Bayraktar, Erhan T/Th 10:00 AM-11:30 AM**

This is for Math/AIM Ph.D. students only, or requires instructor approval. (3). (BS). May not be repeated for credit.

A graduate level course on probability theory and the theory of martingales in discrete time.

Topics include measure theory and integration; characteristic functions; convergence concepts; limit theorems; conditional expectation; martingales (uniform integrability, martingale convergence theorems, optional sampling theorem)

Textbooks: Probability with Martingale, by David Williams; 978-0521406055
Probability: Theory and Examples, by Rick Durrett; 978-1108473682

MATH 629. Machine Learning for Finance II Nazari, Ali F 12PM- 1:30PM / S 10AM-2PM

This is a graduate level course intended for students in the Master's Program in Quantitative Finance and Risk Management (Quant Program).

The aim of the course is to prepare students in the Quant Program to meet the needs of finance industry employers by providing the students with a theoretical understanding of and practical experience in applying data science concepts as they pertain to financial mathematics. In addition, the topics will include practical implementation of the techniques in Python on financial data or other sample data when financial data not available. The course will focus on mathematical foundations, practical programming exercises, domain expertise, and technical communication and will be divided into the following content areas:

I. Classical Statistical Learning (Classification, Regression, Support Vector Machine, Nearest Neighbors)
II. Ensemble Learning, Dimensionality Reduction
III. Neural Networks, Deep Networks
IV. Model Interpretability, Feature Importance, Feature Reduction

Course content will be taught across two terms (two credits each term) and will culminate in students' completion of a final project at the end of the second semester.

****MATH 631. Introduction to Algebraic Geometry Pixton, Aaron T/Th 11:30 AM-1:00 PM**

MATH 594 or permission of instructor. Graduate standing. Previous knowledge: General topology. Familiarity with the language of category theory. Commutative algebra is recommended but not essential; you should have a solid grasp of localizations (of rings/modules) and tensor products though. (3). (BS). May not be repeated for credit.

This is the first half of a year-long sequence in algebraic geometry. In the first semester, we will introduce the basic notions and objects of modern algebraic geometry - sheaves and schemes. We will be loosely following Ravi Vakil's notes "Foundations of Algebraic Geometry".

****MATH 636. Topics in Differential Geometry** **Ji, Lizhen** **M/W/F 3:00 PM-4:00 PM**
Topic: Discrete subgroups of Lie groups

MATH 635 and Graduate standing. (3). (BS). May not be repeated for credit.

One important duality in mathematics is Discrete versus Continuous. This started with the Greek mathematics and has been extensively studied as one of the main themes of mathematics until now, and will be continued into the foreseeable future. One important example is the multifaceted fruitful interactions between discrete subgroups and continuous groups, or rather Lie groups. This includes the abelian case such as Z contained in R, more generally Z^n in R^n, and the non-abelian case of SL(n, Z) in SL(n, R) and its generalizations to semisimple Lie groups.

In this course, we will give an introduction to the rich subject of discrete subgroups of Lie groups from the following perspectives: (1) Geometry: locally symmetric spaces and their rigidity properties such as the Mostow strong rigidity and the Margulis superrigidity. (2) Topology: geometric group theory and some basic properties such as Kazhdan property T, and cohomological dimension and duality property. (3) Analysis: the spectral theory of automorphic forms and the Selberg trace formula.

We will start from basics and emphasize foundational theories such as the construction of discrete subgroups through arithmetic subgroups (generalizations of SL(n, Z)), and the reduction theory for arithmetic subgroups. Besides cofinite volume discrete subgroups, i.e., lattices, we will also discuss some non-cofinite ones, such as thin groups. Some historical perspectives on the development of the subjects will also be explained.

We will make use of many references. Some of the basic ones include the following:

1. A. Borel, Introduction to arithmetic groups. Translated from the 1969 French original by Lam Laurent Pham. Edited and with a preface by Dave Witte Morris. American Mathematical Society, 2019.
2. D.W. Morris, Introduction to arithmetic groups. 2015. xii+475 pp.
3. M.S. Raghunathan, Discrete subgroups of Lie groups. Springer, 1972.
4. R. Zimmer, Ergodic theory and semisimple groups. Birkhäuser, 1984.
5. Y. Benoist, Five lectures on lattices in semisimple Lie groups. Géométries à courbure négative ou nulle, groupes discrets et rigidités, 117–176, Sémin. Congr., 18, Soc. Math. France, 2009.
6. L. Ji, Arithmetic groups and their generalizations. What, why, and how. American Mathematical Society; International Press, 2008.
7. N. Bergeron, The spectrum of hyperbolic surfaces. Translated from the 2011 French original by Brumley. Universitext. Springer, 2016.
8. 8. M. Einsiedler, T. Ward, Ergodic theory with a view towards number theory. Springer, 2011.

****MATH 650. Fourier Analysis** **Wu, Sijue** **T/Th 1:00 PM-2:30PM**

Math 596, 597, 602

Harmonic analysis is a beautiful and core subject in mathematics, and an important tool in the modern study of partial differential equations, probability, analytic number theory and many other subjects. The goal of this course is to introduce students to some basic methods and tools in treating linear and nonlinear operators. Topics covered include the Fourier transform, the Hardy-Littlewood Maximal function, singular integrals, A_p weights, Hardy spaces, BMO, Carleson measure, para-products, the T1 Theorem and the Cauchy integral of Calderon.

Course materials will be taken from the following references.

- Javier Duoandikoetxea: Fourier Analysis (available online at lib.umich.edu)
- Jean-Lin Journe: Caldeon-Zygmund operators, pseudo-differential operators, and the Cauchy integral of Calderon (available online at lib.umich.edu)
- Elias M. Stein: Harmonic analysis: real-variable methods, orthogonality, and oscillatory integrals.
- Stein & Weiss: Introduction to Fourier analysis on Euclidean spaces

Grading policy: grades will be based a few assignments, attendance and participation.

MATH 651. Topics in Applied Mathematics **TBD** **M/W 10:00 AM-11:30AM**
Topic: Modeling and Mechanics

Math 450 or 454. (3). (BS).

The course will develop mathematical modeling methods that are useful for solving problems in continuum mechanics. We will discuss some applications including fluid-structure interactions and the mechanics of organisms. Many of the modeling methods are useful in other areas of applied mathematics, physics, engineering, and biology. The course is aimed at graduate students in applied mathematics, engineering, and the sciences.

No textbook for this course

****MATH 654. Introduction to Fluid Dynamics** **Silas Alben** **T/Th 2:30 PM-4:00 PM**

Vector calculus. Complex analysis and differential equations at a senior undergraduate level (Math 450). Elementary physics (mechanics). Please contact instructor if you are unsure about the pre-requisites.

The course is a broad introduction to the physics and mathematics of fluid dynamics. We'll use classical applied mathematical tools to gain physical insight into fluid phenomena. No knowledge of fluid mechanics is assumed, but some upper-level undergraduate mathematics is (see pre-reqs).

Topics: Continua and conservation laws. Inviscid flow. Irrotational flow. Vorticity. Complex variable methods. Water waves. Airfoil theory and conformal transformations. Viscosity and the Navier-Stokes equations. Phenomena at zero Reynolds number. Boundary layers. Flow instabilities. Possible special topics: fluid-structure interactions and biological propulsion.

Textbook: Elementary Fluid Dynamics, by D. J. Acheson; 978-0198596790

MATH 656. Introduction to Partial and Differential Equations **Hani, Zaher** **T/Th 2:30 PM-4:00 PM**

MATH 558, 596 and 597 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

Partial differential equations are at the core of models in science, technology, economics and related fields. These equations and their solutions have interesting structures that are studied by methods of analysis, geometry, probability and other mathematical fields.

This course will introduce students in pure and applied mathematics to concepts and methods, that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations.

Topics to be covered will include nonlinear first order equations, linear elliptic, hyperbolic and parabolic equations. The method of characteristics, energy methods, maximum principle, Fourier transform and Sobolev spaces.

Required: Fritz, John: "Partial Differential Equations", Springer, 4th Ed. 978-0387906096

****MATH 658. Topics in Ordinary Differential Equations** **Bloch, Anthony** **T/Th 10:00 AM-11:30AM**
Topic: Nonlinear Dynamics and Geometric Mechanics

A senior undergraduate or graduate course in differential equations. (3). (BS).

This course will discuss geometric aspects of the modern theory of ordinary differential equations and dynamical systems, with applications to various mechanical and physical systems. Topics will include: the qualitative theory of ODE's on manifolds, symplectic and Poisson geometry, nonlinear stability theory, Lagrangian and Hamiltonian mechanics, integrable systems, reduction and symmetries, mechanical systems with constraints and controls, and optimal control.

Optional Textbooks: Nonholonomic Mechanics and Control(2nd edtion), by Anthony Bloch; 978-1-4939-3016-6
Mathematical Methods of Classical Mechanics, by V. Arnold; 978-1-4757-1695-5

****MATH 668. Advanced Combinatorics** **Speyer, David** **M/W/F 11:00 AM-12:00 PM**

MATH 664 or equivalent. Graduate standing. (3). (BS). May not be repeated for credit.

This course will develop the representation theory of the general linear group over the complex numbers and its connection to symmetric function theory. Topics which will definitely appear include symmetric functions, schur polynomials, Young tableaux, schur functors, highest weight theory and the Littlewood Richardson rule. Other topics that are likely to appear include webs, connections to representation theory of the symmetric group, the RSK correspondence, honeycombs, hives, the saturation conjecture and crystals. The expected background is abstract linear algebra, including tensors, and comfort with combinatorial arguments.

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****MATH 678. Modular Forms** **Bertoloni Meli, Alexander** **M/W 2:30 PM-4:00 PM**

This course will be an introduction to modular forms following the book by Diamond and Shurman and culminating in the theorem of Deligne that attaches a 2-dimensional Galois representation to certain eigenforms. Along the way, we'll study modular curves, Eisenstein series, Hecke operators, eigenforms, and the Eichler--Shimura relation.

Textbook: A First Course in Modular Forms, by Diamond and Shurman; 0-387-23229-X

MATH 684. Recursion Theory **Harrison-Trainor, Matthew** **T/Th 1:00 PM-2:30 PM**

Math 681 or equivalent. (3). (BS). May not be repeated for credit

Recursion Theory, also known as Computability Theory, is the study of the theory of computation and definability. The basic notion is that of a computable function, namely, that there is a computer program or algorithm which, given an input, can compute the output of that function. A simple counting argument observes that there are countably many computer programs but uncountably many functions, and so some functions must not be computable; in particular, a classical observation of Turing is that the Halting problem is non-computable. Other interesting mathematical problems are known to be non-computable, such as determining whether a polynomial with integer coefficients has an integer solution, or determining whether two words in a finitely presented group are equal. This course will explore the world of the non-computable.

We will develop the basic theory of computability theory, such as the Halting problem, computably enumerable sets, and the Recursion Theorem. A central notion is that of relative computability. Given access to a (non-computable) function f, we ask whether we can compute another function g. If so, we say that g is computable relative to f and think of g, and we think of g as being closer to computable, or simpler, than g is. (This is analogous to the polynomial-time reductions in complexity theory.) The ordering of functions under relative computability is incredibly rich and we will prove many interesting results. For example, we will prove the Friedberg Muchnik Theorem which says that there are two non-computable computably enumerable sets which are incomparable in the sense that neither computes the other.

One application of computability that we will cover is Gödel's Incompleteness Theorem, which says that one cannot deduce all mathematical truths from a computable set of true axioms. We will certainly outline the proof of this, though the level of depth will depend on the interests of the students enrolled.

Math 481 or Math 681 would be helpful for this course, but no prerequisites are required other than mathematical maturity appropriate for a 600-level course. Students unsure whether they are adequately prepared for the course are encouraged to write to the instructor.

****MATH 695. Algebraic Topology I** **Kriz, Igor** **M/W/F 9:00 AM-10:00 AM**

MATH 591 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

The main objective of this course is to answer the question "What is algebraic topology?" In other words, what does the subject study and what techniques does it use? We will explore the specific type of information about a topological space (the "weak homotopy type"), which the familiar tools of fundamental group and homology approach. In the process, we will refine those tools (by introducing, for example, homology and cohomology with coefficients, higher homotopy groups, spectral sequences, generalized homology and cohomology, and duality). We will use these techniques to learn more about topological spaces, with examples and applications, and also learn ways to extend the methods of algebraic topology to other areas of mathematics. The concepts which most immediately come up are Tor and Ext groups in homological algebra, but we will also set up the more general mechanism of derived categories and derived functors, which can be used to apply methods of algebraic topology in algebraic geometry, analysis, arithmetic, and representation theory.

There will be no exams or quizzes, homework will be collected on a weekly basis.

Optional Texts: A Concise Course in Algebraic Topology, Peter May, ISBN 13: 9780226511825

****MATH 697. Topics in Topology** **Pixton, Aaron** **M/W/F 12:00 PM-1:00 PM**
Topic: Moduli of Curves

Graduate standing. (3). (BS). May be repeated for credit. Previous knowledge: Cohomology (singular and/or de Rham), along with some idea of either what an algebraic curve of genus g looks like or what a Riemann surface of genus g looks like.

The moduli space of curves is a fundamental object in geometry; depending on your perspective, points in it either correspond to algebraic curves or to compact Riemann surfaces. In this course, we will survey various features of the moduli space of curves, with particular emphasis on combinatorial aspects. We will switch between algebraic and topological perspectives at different parts in the course, but in either case we will omit some of the more technical details and focus more on describing/using results and building intuition.

Rough list of topics:

1. constructing the moduli space of curves: hyperbolic geometry, Teichmuller theory, algebraic geometry
2. the cohomology of the moduli space of curves
3. the Deligne-Mumford compactification
4. tropical curves Prerequisites: Cohomology (singular and/or de Rham), along with some idea of either what an algebraic curve of genus g looks like or what a Riemann surface of genus g looks like.

****MATH 709. Topics in Analysis** **Bieri, Lydia** **T/Th 8:30 AM-10:00 AM**
Topic: The Mathematics of General Relativity Theory

Graduate standing, knowledge in partial differential equations and differential geometry. (3). (BS).

Partial differential equations (PDE) on manifolds with rich geometrical features are studied in pure mathematics to unravel the structures of their solutions and the spaces they live in. PDE describe phenomena in the real world including physics, medicine, biology or economics. They have become essential to science, technology and to modern life. In general relativity (GR) the Einstein equations describe the laws of the Universe. GR unifies space, time and gravitation. A spacetime in GR is a Lorentzian manifold where the metric solves the Einstein equations. They can be written as a system of nonlinear, second-order, hyperbolic PDE. The unknown is the metric.

Typical physical questions are formulated as initial value problems for the Einstein equations under specific conditions. The solution will lay open the geometry of the resulting spacetime. Today, the methods of geometric analysis have proven to be most effective to investigate these structures. In this course, we introduce some of these methods which are universal and can be applied to other PDE outside GR.

First, we will introduce the spacetime as a solution of the Einstein equations. Then we will discuss topics from linear and nonlinear wave equations on flat and on curved backgrounds. Along the way, the role of curvature in GR will be given special attention. We will study the initial value problem in GR, and introduce the concept of black holes. In view of the latter, we shall prove Penrose's incompleteness theorem and study the extensions of this result by Hawking and Penrose. Those results are better known as the `singularity theorems'. The most important breakthrough along this way is certainly Christodoulou's result on the formation of black holes, showing that a closed trapped surface will form through the focusing of gravitational waves.

Finally, we will address questions in modern research on gravitational waves and their geometric-analytic structures. These are produced during extreme events in our Universe like supernovae and when binary black holes merge. These waves were detected for the first time in 2015 by LIGO. This marks the beginning of a new era where we `decode' information transported by the spacetime itself from distant parts of the Universe.

****MATH 731. Topics in Algebraic Geometry** **Smith, Karen** **M/W 1:00 PM-2:30 PM**
Topic: F-Singularities in Commutative Algebra and Algebraic Geometry

Graduate standing. Basic Algebraic Geometry (e.g. Math 631). (3). (BS). May be repeated for credit.

The course is an introduction to Frobenius splitting and related techniques in prime characteristic for understanding and measuring singularities of algebraic varieties (or local commutative rings) and proving vanishing theorems for projective varieties. Topics covered include Frobenius splitting (or F-purity), F-regularity, F-rationality, test elements and test ideals, compatible splitting, and generalizations to the Cartier Algebra, as well as prime characteristics singularity invariants such as F-pure threshold, Hilbert-Kunz Multiplicity and F-signature. If time permits and student interest dictates, we will also discuss the technique of ''reduction to characteristic p" for applying and comparing our results to topics in complex birational geometry including rational singularities, log-canonicity, klt singularities, the multiplier ideal and various vanishing theorems.

****MATH 738. Topics in Representation Theory** **Harman, Nate** **M/W 10:00 AM-11:30 AM**
Topic: Category Theory for Representation Theorists

Graduate standing. Math 593 & 594. (3). (BS). May be repeated for credit.

The language and tools of category theory have dramatically changed the field of representation theory over the last several decades. This course will be a tour of some of the categorical methods and ideas that have reshaped modern representation theory.

Topics include: basic category theory, Morita equivalence, tensor categories, Tannakian formalism, braidings and knot invariants, combinatorial and diagrammatic descriptions of representation theoretic categories, categorification, highest weight theory, and categorical actions of lie algebras.

No textbook is required.