Michigan Mathematics Rainich Lectures

March 20- March 22, 2018

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Titles: Arithmetic And Locally Symmetric Spaces, Galois Theory And Locally Symmetric Spaces
Abstract: Langlands proposed an extraordinary correspondence between representations of Galois groups and automorphic forms, which has deep, and completely unexpected, implications for the study of both objects. The simplest special case is Gauss' law of quadratic reciprocity. In the so called `regular, self-dual' case much progress has been made in the roughly 40 years since Langlands made these conjectures. In these talks I will discuss recent progress in regular, but non-self-dual case. In this case the automorphic forms in question can be realized as cohomology classes for arithmetic locally symmetric spaces, i.e., quotients of symmetric spaces by discrete groups. Thus instead of the Langlands correspondence being a relationship between algebra and analysis, it can be thought of as a relationship between algebra and topology. This realization of the Langlands correspondence is in many ways more concrete. It also admits to generalizations not envisioned by Langlands, for instance relating mod p Galois representations with mod p cohomology classes.

In these talks I will describe the expected Langlands correspondence in the special cases of modular curves (an example of the `self-dual' case) and arithmetic hyperbolic 3-manifolds (an example of the `non-self-dual' case). I will try both to present the general picture and to give numerical examples. I will also describe various recent theorems in the latter case due to Lan, Harris, Thorne and myself; to Peter Scholze; and to Allen, Calegari, Caraiani, Gee, Helm, Le Hung, Newton, Scholze, Thorne and myself.

Title: Modularity Lifting Theorems In The Setting Of Locally Symmetric Spaces
Abstract Calegari and Geraghty described an approach to modularity lifting theorems in the setting of locally symmetric spaces, where the basic numerology of the Taylor-Wiles method seemed to break down. A group of 10 mathematicians (Allen, Calegari, Caraiani, Gee, Helm, Le Hung, Newton, Scholze, Thorne and myself) were recently able to get this approach to work, the key ingredient being to systematically work in a derived framework. As applications we were able to prove the meromorphic continuation and functional equation of the L-series of elliptic curves over CM fields and to prove the Ramanujan conjecture for the action of Hecke operators on the cohomology of arithmetic hyperbolic three manifolds. I will describe these results and give an outline of the proof.

https://lsa.umich.edu/math/seminars/rainich-lectures.html

All Lectures @ 4:10 PM Room 1360 East Hall