MATHEMATICS
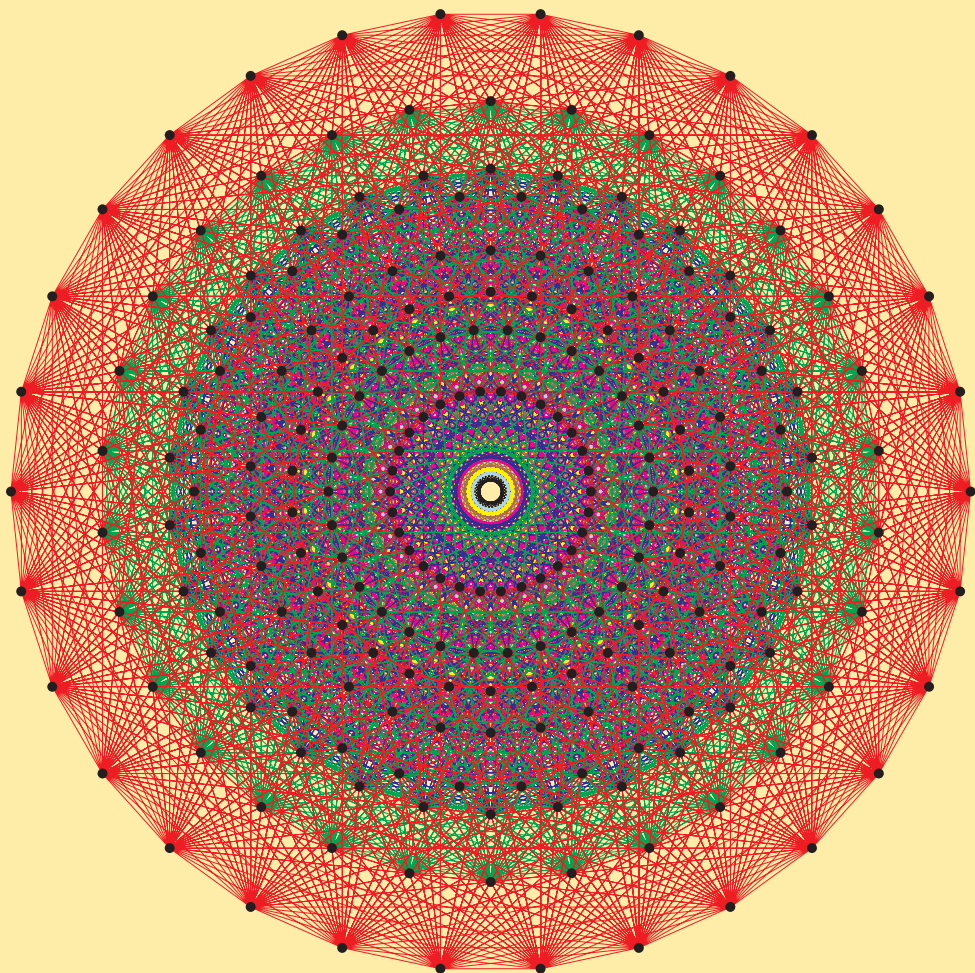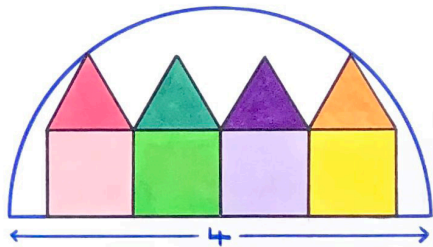## UNIVERSITY OF MICHIGAN

2022-2023
Undergraduate
Programs & Courses

Department of Mathematics
College of Literature, Science and the Arts

These houses are made from squares and equilateral triangles. What's their total area?

Contents

Introduction

Welcome to the Department of Mathematics! This booklet describes the wide variety of options open to potential mathematics majors. There are fourteen subprograms outlined in this booklet ranging from Honors Mathematics to Mathematical Biology to Actuarial Mathematics. Whether you are already committed to being a mathematics major or minor or are still deciding, we hope that this booklet will provide you with useful information. If you have any questions that are not answered here, the first place to turn is the Student Services Office, math-undergrad-office@umich.edu. This office is the focal point for all matters relating to the undergraduate mathematics degrees and provides a wealth of services including academic advising and career planning. All of the information in this brochure and more is available online at https://lsa.umich.edu/math/undergraduates.

The Department has approximately 150 teaching faculty, more than 160 graduate students, approximately 550 undergraduate majors, and about 250 students pursuing a math minor. All of our faculty teach both graduate and undergraduate courses and maintain research programs. Courses offered range from calculus to topics courses and seminars at the frontiers of current research in more than twenty specialty areas.

Our faculty take a strong interest in the undergraduate program and are available to students at all levels. Most of the junior/senior classes are small, often with fewer than 30 students. We encourage students to meet regularly with Mathematics Department advisors, all of whom are regular faculty members. Appointments may be scheduled online at https://lsa.umich.edu/math/undergraduates/advising.html

Within the mathematics major there are options to accommodate a multitude of different interests. The **Pure Mathematics** and **Honors Mathematics Programs** focus on the more theoretical side of mathematics, although there are also advanced courses devoted to a wide variety of applications, such as numerical analysis, modeling, cryptography, biomathematics, and financial mathematics. The **Mathematical Sciences Program** is oriented towards applications, although students are also exposed to theory. In the Mathematical Sciences Program you will choose one of eight different fields of study: Discrete and Algorithmic Methods, Numerical and Applied Analysis, Operations Research and Modeling, Probabilistic Methods, Mathematical Economics, Control Systems, Mathematical Physics,

or Mathematical Biology. The **Actuarial Mathematics Program** and the **Mathematics of Finance and Risk Management Program** provide preparation for employment in areas such as finance, pensions, and insurance. The **Secondary Mathematics Teaching Certificate Program** is coordinated with the School of Education to prepare students to teach mathematics in K-12 schools. A **Minor** in Mathematics is also an option for most students enrolled at the University of Michigan, no matter their school or college.

There are a variety of extracurricular activities available to our concentrators. The *Undergraduate Math Club* meets regularly to hear informal mathematical lectures and socialize (for current information see (https://lsa.umich.edu/math/undergraduates/extracurricular-activities/math-talks/math-club.html). The *Student Actuaries at Michigan* club (https://www.sam-umich.com/) helps prepare students for careers in the actuarial profession as well as fielding a number of intramural sports teams. The *Women in Mathematics* club (https://sites.google.com/umich.edu/wim-umich/home) provides social and educational support for women through study nights, social events, and mathematical lectures. The *STEM Society* (https://sites.google.com/a/umich.edu/stem-society/) works to improve math and science education in the local community and provide teaching opportunities to undergraduate students. The *Society of Undergraduate Math Students* facilitates friendships and community amongst those interested in mathematics.

Undergraduates may also participate in several mathematical competitions such as the *Putnam Exam*, the *Virginia Tech Regional Mathematics Contest*, and the *University of Michigan Undergraduate Mathematics Competition [(UM)$^2C^x$]*. More information on competitions can be found at https://lsa.umich.edu/math/undergraduates/extracurricular-activities/competitions.html.

Mathematics demands careful, rigorous, analytical reasoning. The intellectual development afforded by an undergraduate concentration in mathematics provides an excellent background for a wide variety of careers. Graduates of the University of Michigan Department of Mathematics have gone on to successful careers in law, medicine, politics, and business, as well as every aspect of science, computer science, economics, technology, and, of course, mathematics itself. Some have chosen to pursue further education, but many others have taken interesting and challenging positions in industry, business, and government directly after graduation. For more information on careers in mathematics please see https://lsa.umich.edu/math/undergraduates/research-and-career-opportunities.html.

The Intro Sequences

The ideal high school background for a student who intends to major in mathematics (or science or engineering) includes one and one-half years of algebra, one year of geometry, one-half year of trigonometry, and one year of pre-calculus and/or calculus. Inclusion of a high-school calculus course is not essential; courses 115 and 185 do not assume previous calculus experience. A student whose background is less thorough may need to begin college mathematics with Math 105 (Data, Functions and Graphs) and should expect that completion of a mathematics degree will require at least one additional semester.

Students seeking permission for honors majors may set up appointments with mathematics honors advisors to discuss their preparedness for these courses. First year students can seek advising during the Spring/Summer terms by going here: https://lsa.umich.edu/math/undergraduates/advising/first-year-orientation-advising.html.

Prerequisites for the major may be taken Pass/Fail.

Prospective students of mathematics may choose from a variety of entry-level sequences:

The sequence **Math 115-116-215** is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to major in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof. All sections are given two uniform midterms and a uniform final exam.

Math 175, 176, 285, and 286. Math 175 (Intro to Cryptology) requires permission from an honors mathematics placement advisor, but it is open to students not enrolled in the LSA Honors Program. The natural sequel to Math 175 is Math 176 (Explorations in Calculus); a course that is taught using inquiry based learning methods and is intended to prepare students for Math 285 (Honors Multivariable and Vector Calculus).

Math 156, 285, and 286 form the applied honors calculus sequence for science concentrators and engineering students. It is assumed that students are well-acquainted with the material of high school calculus (as evidenced by a score of 4 or 5 on the AB or BC Calculus

exam, for example). To provide students with the solid mathematical background they will need for subsequent coursework in mathematics, engineering, and science, the course gives equal time to applications and theory. In particular, while definitions are stated precisely and theorems are derived, technical details are often omitted.

The sequence **Math 185-186-285-286** is an introduction to calculus at the honors level. Math 185 (Honors Calculus I) presupposes a higher level of accomplishment in high school math courses and covers the theory behind the calculus. Most, though not all, students in Math 185 have completed a high school calculus course. Math 186 is usually followed by Math 285; after completion of Math 285, a prospective honors concentrator should take Math 286 followed by Math 217 and Math 451. Admission to Math 185 requires permission of the honors mathematics placement advisor, but it is open to students not enrolled in the LSA Honors Program.

Math 295 (Honors Mathematics I), together with the succeeding courses **Math 296-395-396**, provides an intensive introduction to theoretical mathematics. From the beginning, students are exposed to abstract concepts. These courses require a higher level of interest and commitment than the other honors introductory sequences. Most students in Math 295 have completed a high school calculus course. The student who completes Math 396 is prepared to explore the world of mathematics at the advanced undergraduate and graduate level. Admission to Math 295 requires permission from an honors mathematics placement advisor, but is open to students not enrolled in the LSA Honors Program.

Students who enter the University with a **score of 7 on the HL International Baccalaureate exam in Mathematics or 5 on the AB or BC Advanced Placement exam in Mathematics** have several additional options. A student should check with their College's general advisor or a Mathematics faculty advisor as to which mathematics course would be the most appropriate. Advanced Placement credit is granted in varying amounts. For a guide of AP credit, please visit https://admissions.umich.edu/apply/freshmen-applicants/ap-ib-credit.

Mechanics of the Major

If you are interested in a mathematics major, we urge you to learn about the requirements as early as possible, but no later than the end of your sophomore year. Inappropriate course selections may result in difficulties in fulfilling requirements in a timely manner. We recommend that you make an appointment with a major advisor to discuss the programs and to answer questions you may have. Appointments may be scheduled online at https://lsa.umich.edu/math/undergraduates/advising.html.

All prerequisite courses must be satisifed with a grade of C- or above. Students with lower grades in prerequisite courses must receive special permission of the instructor to enroll in subsequent courses.

Declaring the Major

The decision to major in mathematics should be made by the end of the sophomore year and officially registered by filling out a Major Declaration Form with a mathematics advisor. During your first advising session as a declared mathematics major, you should make a tentative decision about which major program you want to pursue and you should plan a possible sequence of courses to fulfill its requirements. Of course, as you progress through the program you may and will make many changes to this initial plan. Before you register for courses for each subsequent semester, you should make an advising appointment to review your progress and revise your plan for the remaining semesters. Regular advising is your best guarantee for completing the program in a timely manner.

Submajor Programs

There are six distinct major programs in mathematics: Pure Mathematics, Honors Mathematics, Mathematical Sciences, Actuarial Mathematics, Mathematics of Finance and Risk Management, and the Secondary Mathematics Teaching Certificate. The Mathematical Sciences Major is designed for the student interested in mathematics and its applications and splits up into eight subprograms depending on the particular applications. Although each of these programs has its own requirements and conditions, the prerequisites and basic

courses are the same throughout.

A student may pursue two (or more) Mathematics majors. However, unless precluded by requirements, for any pair of majors the set of courses used to satisfy conditions III and IV for either major must be distinct from the set of courses used to fulfill requirements I-IV of the other major (requirements listed later in this section). Exception: this rule does not apply if one of the pair is the Secondary Mathematics Teaching Certificate Program.

Students are urged to discuss their ultimate career goals with an advisor at an early stage to ensure that an appropriate program is planned.

All mathematics students are strongly encouraged to use Physics 140-240 as their Natural Science distribution requirement and to acquire a working knowledge of computers and their languages. Many of the careers open to mathematics majors involve heavy use of computing, and students preparing for such a career should take several computing courses, including EECS 280 and EECS 281.

Upper-level courses taken at another college or university can be used to satisfy major requirements only with written permission from the Director of Undergraduate Programs. Documentation (for example: syllabus, exams) is required to verify the equivalence of the external course. The text of the documentation must be officially translated into English. Any course that will count towards the degree must meet the math department's general standards for transfer credit. Students may submit a request to have their transfer credits evaluated to https://prep.math.lsa.umich.edu/transfercredit/extinstfind. Transfer courses do not count towards the student's GPA.

In particular, per LSA rules, students are required to complete at least 24 credit hours for the mathematics major (parts I-V). The Mathematics Department requires that a minimum of 18 mathematics credits should be taken in residency (at the University of Michigan, Ann Arbor campus). Six of these credits must come from the basic courses (part II) and nine from the core courses (part III) and/or cognate courses (part IV). The 18 credits for residency may include pre-requisites taken in Math.

All prerequisite courses must be satisfied with a grade of C- or above. Students with lower grades in prerequisite courses must receive permission of the instructor to enroll in subsequent courses.

Students who intend to major in mathematics and receive a grade of C- or lower in Math 217 should repeat this course before proceeding further. Math 217 students who wish to receive additional practice with reading and writing mathematical proofs should consider taking Math 201 concurrently with, or prior to, Math 217.

By LSA rules, to be awarded a degree in Mathematics a student must maintain a grade point average of at least 2.0 (a C average) in all mathematics courses and other courses used to fulfill concentration requirements. Moreover, the Pass/Fail option may not be used to meet major/minor program requirements. If a course was taken in residence and a grade of A+ through C, P, CR, or S was earned, then repetition of this course results in no additional credit or honor points. The course and grade appear on the transcript with the notation "NFC" or "Not for Credit."

A student repeating a course in which a C- through D- was previously earned will receive honor points but no additional credit towards a degree. The course appears on the transcript with the notation "Repetition." Repetition of a course in which an E, F, or U grade was originally earned produces both credit towards a degree and honor points for courses elected on the graded pattern; there is no special transcript notation. In all such cases, the first election and grade earned remain on the transcript. The grades earned by repetition of courses are not averaged and posted as a single entry, but they are posted as separate elections.

Pure Mathematics

The **Pure Mathematics Program** is designed to provide broad train-ing in basic modern mathematics including an introduction to the methods of rigorous mathematical proof and exposure to the major branches of mathematics: Algebra, Analysis, and Geometry/Topol-ogy.

I. Prerequisite to a major in **Pure Mathematics** is one of the pair of courses 215&217, 285&217, or 295&296. Note that Math 216 is not intended for Pure Mathematics majors.

All **Pure Mathematics** majors are also strongly encouraged to take Physics 140-141 and 240-241 and to acquire a working knowledge of a high-level computer language (e.g. _Fortran, C, or C++_) at a level equivalent to completion of EECS 183.

The major program must include at least nine courses: four basic courses **(II.)**, four elective courses **(III.)**, and one cognate course **(IV.)** as described below.

II. The basic courses consist of one from each of the following groups **completed with a grade of at least C-**:

1. **Modern Algebra**: Math 312, 412, or 493
2. **Differential Equations**: Math 286 or 316
3. **Analysis**: Math 351 or 451
4. **Geometry/Topology**: Math 431, 433, 490, or 590

More advanced students, such as those who have completed Math 396, may substitute higher level courses with the approval of an advisor.

Following Math 215 all students intending to concentrate in Pure Mathematics should elect Math 217 (Linear Algebra) rather than Math 216 (Introduction to Differential Equations). Math 216 is not intended for Pure Mathematics concentrators who generally take Math 316 (Differential Equations) after completing Math 217.

III. The four elective courses must be chosen in consultation with an advisor to provide a cohesive program that explores an area of mathematics in some depth. There is a good deal of freedom here, but a random selection of courses will not satisfy this requirement. The courses should be chosen from the following list or have a course number of 600 or above. Math 289 is a repeatable 1-credit course and can be used to satisfy the elective requirement only if taken for a total of 3 credits.

289	Problem Solving	526	Discrete State Stoch Process
310	Chance and Choice	537	Int. to Differentable Manifolds
354	Fourier Anal. and its Applic.	550	Intro. to Adaptive Systems
389	Explorations in Mathematics	551	Intro to Real Analysis
404	Inter. Diff. Equations	555	Intro. to Complex Variables
416	Theory of Algorithms	556	Methods of Applied Math I
420	Advanced Linear Algebra	557	Methods of Applied Math. II
423	Mathematics of Finance	558	Ordinary Diff. Equations
425	Introduction to Probability	559	Topics in Applied Math
431	Expl. in Euclidian Geometry	561	Linear Programming I
433	Intro. to Differential Geom.	562	Cont. Optimization Meth.
440	Lab of Geometry - LoG(M)	563	Advanced Mathematical Bio
450	Adv. Math for Engineers I	565	Combin. and Graph Theory
452	Advanced Calculus II	567	Intro. to Coding Theory
454	Bound. Val. Prob. for PDE	571	Num. Meth. for Sci. Comp. I
462	Mathematical Models	572	Num. Meth. for Sci. Comp II
463	Math Modeling in Biology	575	Intro. to Theory of Numbers
464	Inverse Problems	582	Introduction to Set Theory
465	Introduction to Combinatorics	590	An Intro. to Topology
471	Intro. to Numerical Methods	591	General & Diff. Topology
472	Numer. Methods w/Fin. Apps	592	Intro. to Algebraic Topology
475	Elementary Number Theory	593	Algebra I
481	Intro. to Mathematical Logic	594	Algebra II
490	Introduction to Topology	596	Analysis I (Complex)
498	Topics in Modern Math	597	Analysis II (Real)
525	Probability Theory		

IV. One cognate course should be chosen from some field other than mathematics. Almost any field is acceptable, but the course must be at the 300+ level and should have significant mathematical content, at least at the level of Math 215. A list of suggested courses is available online at https://lsa.umich.edu/math/undergraduates/advising/cognate-courses.html but in all cases **approval of a concentration advisor is required.**

Honors Mathematics

A student who is either in the LS&A Honors Program or is approved by the Departmental Honors Committee may declare an Honors major in mathematics. The **Honors Mathematics** major will acquire a greater command of abstractions and of the subtleties of mathematical rigor. Members of the Honors Committee of the Department of Mathematics counsel honors majors. Honors students who complete an honors major with distinction may receive on their diplomas the designations "with honors," "with high honors," or "with highest honors." However, these designations are not restricted to students officially enrolled in the Honors Program; any student whose course selection has followed the pattern of an honors major may ask the Chair of the Honors Committee to be considered for an honors designation. An honors citation will be awarded to any student who completes the honors major requirements with a major GPA of at least 3.25 and an LSA cumulative GPA of at least 3.4 at the time of graduation. Honors will automatically remove students without a 3.4 GPA. Citations of high and highest honors are awarded at the discretion of the Honors Committee on the basis of superior performance in advanced courses as attested by grades and individual faculty evaluations.

I. Students intending to pursue an **Honors Mathematics** major are advised to take one of the Honors introductory sequences 156-286, 175-286, 185-286, 295-296, 217-297, or some combination of these five. Please note that the sequence 295-396 is very theoretical. It is strongly recommended that students in the 156-286, 175-286, and 185-286 tracks also complete Math 217.

All **Honors Mathematics** majors are also strongly encouraged to take Physics 140-141 and 240-241 and to acquire a working knowledge of a high-level computer language (e.g. _Fortran, C,_ or _C++_) at a level equivalent to completion of EECS 183 and are encouraged to complete EECS 280 and EECS 281.

The **Honors** major program must include at least nine courses: four basic courses **(II.)**, four elective courses **(III.)**, and one cognate course **(IV.)** as described below.

II. The basic courses consist of one from each of groups 1, 2, 3 and 4 or groups 1, 3, 5 and 6 below **completed with a grade of at least C-:**

1. **Analysis:** Math 451
2. **Modern Algebra:** Math 493
3. **Linear Algebra:** Math 420 , 494, or 571
4. **Geometry/Topology:** Math 431, 433, 490, or 590
5. **Probability:** Math 525
6. **Differential Equations:** Math 404, 454, 556, 557, or 558

A student who has completed Math 295-296 or 217-297, with a grade of at least a C- is exempt from Math 451. A student who has completed Math 295-395 or 297-395, with a grade of at least a C- is exempt from Math 420.

III. Four elective courses must be chosen in consultation with an honors advisor to provide a cohesive program that explores an area of mathematics in some depth. There is a good deal of freedom allowed here, but a random selection of courses will not satisfy this requirement. The courses should be chosen from the following list or have a course number of 600 or above. Math 289 is a repeatable 1-credit course and can be used to satisfy the elective requirement only if taken three times. An honors counselor may approve another mathematics course or a course from another department with advanced mathematical content as one of these elective courses. The honors counselor may ask that the student arrange supplemental work in a given class not listed below to conform to expectations for an honors elective. **A student who completes the requirements for part II by choosing courses from groups 1, 3, 5 and 6 must complete a course in Complex Analysis.**

289 Problem Solving	556 Methods of Applied Math I
389 Explorations in Mathematics	557 Methods of Applied Math II
416 Theory of Algorithms	558 Ordinary Diff. Equations
431 Expl. in Euclidian Geometry	559 Topics in Applied Math
433 Intro. to Differential Geom.	561 Linear Programming I
440 Lab of Geometry - LoG(M)	563 Adv. Mathematical Biology
452 Advanced Calculus II	565 Combin. and Graph Theory
462 Mathematical Models	566 Combinatorial Theory
463 Math Modeling in Biology	567 Intro. to Coding Theory
464 Inverse Problems	571 Num. Meth. for Sci. Comp. I
465 Introduction to Combinatorics	572 Num. Meth. for Sci. Comp. II
471 Intro. to Numerical Methods	575 Intro. to Theory of Numbers
481 Intro. to Mathematical Logic	582 Introduction to Set Theory
490 Introduction to Topology	590 An Intro. to Topology
494 Honors Algebra II	591 General and Diff. Topology
525 Probability Theory	592 An Intro. to Algebraic Topology
526 Disc. Stochastic Processes	593 Algebra I
537 Differentiable Manifolds	594 Algebra II
551 Intro to Real Analysis	596 Analysis I (Complex)
555 Intro. to Complex Variables	597 Analysis II (Real)

IV. One cognate course should be chosen from some field other than mathematics. Almost any field is acceptable, but the course must be at the 300+ level and should have significant mathematical content, at least at the level of Math 215. A list of suggested courses is available online at https://lsa.umich.edu/math/undergraduates/advising/cognate-courses.html, but in all cases **approval of an advisor is required.**

Mathematical Sciences

The **Mathematical Sciences Program** is designed to provide broad training in basic mathematics together with some specialization in an area of application of mathematics. Each student must select one of the eight Program Options as a special area.

Because of the somewhat more specific requirements of the Program Options, careful planning and frequent consultation with your advisor are essential to ensure timely completion of the program.

I. Prerequisite to a major in **Mathematical Sciences** is one of the pair of courses 215&217, 285&217, or 295&296. In addition, students must complete EECS 183. Students are encouraged to take EECS 280 and EECS 281 as well. **For those not pursuing either a Physics concentration or a Physics minor, Physics 140-141 or Physics 139, and 240-241 or Physics 239 are required for the Numerical and Applied Analysis and Mathematical Physics options and strongly recommended for the other options.** Some of the options have additional requirements as noted below.

The major program must include at least nine courses: four basic courses **(II.)**, three courses from one of the Program Options **(III.)**, and two additional courses **(IV.)** as described below. At least two of the five optional courses must be mathematics courses.

II. The basic courses consist of one from each of the following groups **completed with a grade of at least C-**:

1. **Differential Equations**: Math 286 or 316
2. **Discrete Math/Modern Algebra**: Math 312, 412, 465, or 493
3. **Analysis**: Math 351, 354, 450, 451, or 454
4. **Probability**: Math 425 or 525

More advanced students, such as those who have completed Math 396, may substitute higher level courses with the approval of a major advisor. All students are strongly encouraged to include in their program one of the more theoretical courses: Math 412, 451, 493, or 525.

III. A student in the **Mathematical Sciences Program** must choose one of the eight options (a-h) below and complete at least three of the courses listed under that option. This requirement is designed to provide focus and depth to the program and can only be waived by a departmental advisor in favor of a program which provides this depth in some equivalent way. An acceptable program must include some of the more challenging courses. Advice should be sought from a departmental advisor before selecting an option. As an initial guide, we give a brief description of the options below.

IV. To complete the major program, each student should elect two additional advanced courses in mathematics or a related area. Every student must include, either here or elsewhere in their program, a cognate course numbered 300 or above taught outside the department which emphasizes the application of significant mathematical tools (at least at the level of Math 215) in another discipline. In all cases approval of a departmental advisor is required. This is a very flexible requirement designed to accomodate special interests and may be satisfied by a broad range of courses in other departments (generally numbered 300 or above) or by mathematics courses numbered 400 or above.

V. At least two of the courses in **III**. and **IV.** must be Math courses.

Program Options for Mathematical Sciences (a-h)

(a) **Discrete and Algorithmic Methods**
Discrete and algorithmic methods are concerned with the analysis of finite structures such as graphs, networks, codes, incidence structures, and combinatorial structures. The rapid growth of this area has been driven largely by its role as the mathematical core of computer science. Typical problems of this field involve optimization, emulation, or probabilistic estimation. Students choosing this option are strongly encouraged to take EECS 280 and 281.

416 Theory of Algorithms	575 Intro. to Theory of Numbers
420 Advanced Linear Algebra	EECS 376 Found. of Comp. Sci.
465 Intro. to Combinatorics	EECS 445 Intro to Machine Learning
475 Elementary Number Theory	EECS 477 Intro. to Algorithms
481 Intro. to Mathematical Logic	EECS 550 Information Theory
561 Linear Programming I	EECS 574 Comput. Complexity
565 Combin. and Graph Theory	EECS 586 Design/Anal. Algorith.
566 Combinatorial Theory	EECS 587 Parallel Computing
567 Intro. to Coding Theory	IOE 614 Integer Programming

(b) **Numerical and Applied Analysis**

As computers become more powerful, they are being used to solve increasingly complex problems in science and technology. Examples of such problems include developing high-temperature superconducting materials, determining the structure of a protein from its amino acid sequence, and creating methods to model global climate change. Industrial and government research laboratories require personnel who are trained in applying numerical and analytical techniques to solve such problems. Numerical techniques are algorithms for computer simulation, and analytical techniques may rely on series expansions such as the Taylor or Fourier series expansions. There is a close connection between numerical and analytical techniques. A new analytical technique often leads to more effective numerical algorithms; a good example is the development of wavelets and their applications in signal processing. Students wishing to enter this field must acquire a strong background in mathematics, science, and computing. **Students are encouraged to include EECS 402 and MATH 451 in their program, and to also consider doing a minor in another scientific discipline**.

Students in the Numerical and Applied Analysis Program may choose to pursue a second major in Data Science. This combination is a powerful recommendation to a prospective employer that the student can think quantitatively about information; collect, manage, analyze, and visualize massive datasets; and that the student has both the computational tools and the rigorous analytical methods to reason about information.

354 Fourier Anal. and its Applic.	471 Intro. to Numerical Methods
404 Intermediate Diff. Equations	550 Intro. to Adaptive Systems
420 Advanced Linear Algebra	555 Intro. to Complex Variables
423 Mathematics of Finance	AERO 225 Intro. to Gas Dynamics
451 Advanced Calculus I	ME 240 Dynam. and Vibrations
452 Advanced Calculus II	PHYS 340 Waves, Heat, Light
454 Bound. Val. Prob. for PDE	PHYS 401 Inter. Mechanics
462 Mathematical Models	STATS 406 Statistical Computing
463 Math Modeling in Biology	STATS 426 Intro. to Theor. Stat.
464 Inverse Problems	

(c) **Operations Research and Modeling**

Mathematical modeling refers generally to the representation of real-world problems in mathematical terms. In some sense this is necessary for any application of mathematics, but the term is used more often to refer to applications of mathematics to biological, mechanical, and human systems. Analysis of such systems involves complex

mathematical descriptions and leads to large problems which can be solved only by use of a computer. Operations Research studies integrated systems including health care, education, manufacturing processes, finance, and transportation. Because the emphasis is on the analysis and operation of systems, practitioners are also qualified to deal with managerial problems. Career opportunities are available in many parts of industry and government. **Most students should include Math 561 and Stats 426**.

420 Advanced Linear Algebra	IOE 515 Stochastic Processes
433 Intro to Differential Geom	IOE 543 Scheduling
462 Mathematical Models	IOE 610 Linear Programming II
463 Math Modeling in Biology	IOE 611 Nonlinear Programming
561 Linear Programming I	IOE 614 Integer Programming
CHE 510 Cont. Optimization Meth	STATS 426 Intro to Theory Stats

(d) Probabilistic Methods

Probability theory deals with the mathematics of randomness and its applications. It is the basis of mathematical statistics, where the goal is to draw inferences from samples. Non-statistical applications are found in many branches of the social, biological, and physical sciences, as well as in engineering. Because of the intimate connection between probability and statistics, students electing this option will usually include statistics courses in their program and **sometimes have a second major in Statistics. Students electing this option must complete Math 525.**

Students in the Probabilistic Methods Program may choose to pursue a second major in Data Science. This combination is a powerful recommendation to a prospective employer that the student can think quantitatively about information; collect, manage, analyze, and visualize massive datasets; and that the student has both the computational tools and the rigorous analytical methods to reason about information.

423 Mathematics of Finance	STATS 426 Intro. to Theor. Stat.
523 Risk Theory	STATS 430 Applied Probability
525 Probability Theory	STATS 466 Stat. Quality Control
526 Discrete State Stoch. Proc.	STATS 500 Applied Statistics I
547 Biological Sequence Analysis	STATS 501 Applied Statistics II
EECS 502 Stochastic Processes	STATS 550 Bayesian Decision
STATS 406 Intro. to Stat. Comp.	

(e) Mathematical Economics

One definition of economics is that it is the study of the optimal allocation of scarce resources. Several mathematical techniques are fundamental to this study: constrained optimization using Lagrange multipliers, n-dimensional calculus especially the Implicit Function Theorem (dependence of a solution on parameters), dynamics, probability and statistics to deal with inherent uncertainty, game theory to deal with decisions in which the actions of one agent affect the options of others, and proofs for understanding the derivation of economic principles. To ensure coverage of these topics, students choosing the Mathematical Economics option will usually choose Math 351 or 451 as their basic analysis course; Math 423, Stat 426, Econ 452, Econ 453, or Econ 454 as courses from the options list; and Econ 401 together with a mathematics course at the 400-level or above as their related courses. A student who completes this option should find opportunities available in business, government, and research organizations which collect, analyze, and model data having economic, social, and political parameters. Many students in this program pursue a second major in Economics; this combination is a powerful recommendation to a prospective employer that the student can think quantitatively, understand complex reasoning, and work with economic models.

420	Advanced Linear Algebra	561	Linear Programming I
423	Mathematics of Finance	562	Cont. Optimization Meth.
424	Compound Interest & Life Ins.	623	Computational Finance
452	Advanced Calculus II	ECON 452	Intro. to Econometrics
462	Mathematical Models	ECON 453	Advan. Stats and Econ I
471	Intro. to Numerical Methods	ECON 454	Advan. Stats and Econ II
472	Num. Meth. with Fin. App.	STATS 426	Intro. to Theor. Stat.
523	Risk Theory	ECON 409	Game Theory

(f) Control Systems

Control Systems is a fascinating field which draws upon knowledge in many areas of mathematics. It pervades industry, and its practitioners can be found in such diverse fields as automotive pollution control, avionics, and process control in manufacturing. A control designer will need to interface with the modeling group to develop a mathematical description of the system to be controlled, and perhaps with the testing group to characterize disturbances or other uncertainties affecting the system. The required performance of the system will then be ascertained from the intended use and translated into a set of mathematical specifications for a closed-loop system. At this stage the designer will select from an arsenal of tools for the

controller analysis and synthesis—this generally requires a solid foundation in linear algebra, differential equations, real analysis, and probability. Students planning to pursue graduate study in this area are recommended to include Math 451 and EECS 476 in their programs.

354 Fourier Anal. and its Applic.	EECS 376 Found. of Comp. Sci.
420 Advanced Linear Algebra	EECS 460 Control Sys. Anal./Des.
451 Advanced Calculus I	EECS 476 Data Mining
454 Bound. Val. Prob. for PDE	EECS 560 Linear Systems Theory
462 Mathematical Models	EECS 561 Digital Control Sys.
471 Intro. to Numerical Methods	EECS 562 Nonlinear Systems
555 Intro. to Complex Variables	EECS 565 Linear Feedback
561 Linear Programming I	EECS 567 Intro. to Robotics
562 Cont. Optimization Meth.	STATS 426 Intro. to Theor. Stat.

(g) Mathematical Physics

Among all of the disciplines which make significant use of mathematics, physics has the longest history. Indeed, several areas of mathematics were developed for the purpose of solving problems in physics. This option allows a student to pursue interests in physics which use undergraduate mathematics. It is designed to facilitate a concurrent major in Physics. **Every program must include at least two of the Physics courses from the list below**. Note that although Physics 401 is a prerequisite to several of these, it does not count as one of the option courses.

404 Intermediate Diff. Equations	PHYS 406 Stat. & Thermal Phys.
433 Intro. to Differential Geom.	PHYS 413 Intro. Nonlinear Dyn.
454 Bound. Val. Prob. for PDE	PHYS 435 Gravitational Phys.
471 Intro. to Numerical Methods	PHYS 452 Meth. of Theor. Phys. II
555 Intro. to Complex Variables	PHYS 453 Quantum Mechanics
PHYS 405 Intermed. Elec. & Mag.	

(h) Mathematical Biology

Ever since the advent of high-powered computing, it has become obvious that mathematics can contribute a great deal to biological and medical research. Indeed, in many cases mathematical approaches can answer questions that cannot be addressed by other means,

and thus mathematics is often an indispensable tool for biological research. Typical areas of application include such diverse areas as the topology of DNA, genetic algorithms, cell physiology, cancer biology and control strategies, micro-circulation and blood flow, the study of infectious diseases such as AIDS, the biology of popula-

tions, neuroscience and the study of the brain, developmental biology and embryology, the study of hormone secretion and endocrine control, and bioinformatics. The Mathematical Biology option will thus be appropriate for any student with an interest in biology or medicine and a desire to apply the mathematics they learn to current and important biological problems.

An additional prerequisite to this major is completion of the Introductory Biology sequence (Bio 171 and 172). *Students electing this option must complete Math 463 (Math Modeling in Biology), a second math course from the options list below and one advanced level (numbered over 300) course in biological sciences.* The options list below contains approved biological sciences courses but other courses in Biology, Physiology, Microbiology/ Immunology, Neuroscience, Bioinformatics, or Natural Resources and Environment can be accepted with approval of your mathematics advisor.

For the 2 advanced courses, one must come from the list of math courses below (so, a student will end up completing 463 and 2 additional Math courses from the list below).

Recommended cognate courses include STATS 426, STATS 510 and quantitative courses focused on biological processes such as BIO-PHYS 370, BIOPHYS/PHYS/CHEM 417, EEB/CMPLXSYS 430 and EEB/MATH 466.

404 Inter. Differential Equations
452 Advanced Calculus II
454 Boundary Val. Prob. for PDE
462 Mathematical Models
463 Math Modeling in Biology
471 Intro. to Numerical Methods
558 Applied Nonlinear Dynamics
559 Topics in Applied Mathematics

563 Adv Math Methods for Bio Sci
564 Topics in Math Biology
568/BIOINF 568 Math & Cmp Neurosci
BIO 305 Genetics
MCDB 310/BIOCHEM 415 Int Biochem
CHEM 351 Fundamen. of Biochem
MICROBIOL 301 Intro Microbiology

Actuarial Mathematics

The **Actuarial Mathematics** Program is designed to provide broad training in the basic mathematics underlying the operations of private and social insurance and employee benefit plans. The courses are organized to assist the student to prepare for several of the examinations of the Casualty Actuarial Society and the Society of Actuaries.

Non-credit review classes for some of the professional actuarial examinations are organized each term; ask your actuarial advisor about the time and place of these classes. It is strongly recommended that some of these exams be passed before graduation. Summer internships are an important component of the educational program, and students are strongly encouraged to seek an internship no later than the conclusion of their junior year. Students are encouraged to take either Math 422 to satisfy their upper-level writing requirement.

I. Prerequisite to major in **Actuarial Mathematics** is one of the pair of courses 215&217, 285&217, or 295&296. In addition, each student must complete two introductory courses in Economics (Econ 101 and 102), and acquire a working knowledge of a high-level computer language (e.g. Fortran, C, or C++) at a level equivalent to completion of EECS 183.

The major program must include at least eleven courses: 4 basic courses **(II.)**, 5 special Actuarial Mathematics courses **(III.)**, and 2 related courses as described below **(IV.)**.

II. Basic courses consist of one from each of the following groups **completed with a grade of at least C**-:
1. **Differential Equations**: Math 286 or 316
2. **Probability**: Math 425
3. **Statistics**: Stats 426
4. **Analysis:** Math 351 or 451

More advanced students, such as those who have completed Math 396, may substitute higher level courses with the approval of a departmental advisor.

III. The special **Actuarial Mathematics** courses must include Math 423, Math 424, Math 520, Math 523, and at least one of Math 521 or Math 524. Note that Math 520 and 523 are offered only in the Fall terms. Similarly, Math 521 and 524 are offered only in the Winter terms. Moreover, these courses have important prerequisites; thus, careful planning is essential. Students are strongly enrouraged to meet with an advisor to plan their path through the major.

IV. To complete the major each student should elect two additional intermediate or advanced courses related to **Actuarial Mathematics**. Some, but not all, of the courses numbered 300 and above offered by Accounting, Computer Science, Economics, Finance, Industrial and Operations Engineering, and Statistics are appropriate here. Some specific approved courses are:

422 Risk Mgmt & Insurance	FIN 302 Financial Decisions
462 Mathematical Models	FIN 317 Corp Financing Decisions
623 Computational Finance	FIN 408 AND 409 (1.5 Credits Each)
ECON 401 Inter. Micro Thry	Capital Markets AND Fixed Inc. Sec.
ECON 402 Inter. Macro	IOE 310 Intro Optim. Methods
ECON 409 Game Theory	IOE 452 Corporate Finance
ECON 452 Intro to Econometrics	

All Accounting courses 300 and above, except 471
Some Statistics courses numbered above 400

Actuarial Mathematics students interested in receiving VEE credits from the Society of Actuaries are encouraged to elect ACC 300 & FIN 302 for their related courses.

Other courses may sometimes be appropriate and in all cases **approval of an advisor in Actuarial Mathematics is required.**

Mathematics of Finance and Risk Management

The program in **Mathematics of Finance and Risk Management** (or **Mathematics of Finance** for short) is designed to provide a broad education in the quantitative aspects of risk management and finance. Today's financial instruments require sophisticated mathematical techniques for their valuation. These techniques come from the fields of probability, statistics, and differential equations.

I. Prerequisite to major in **Mathematics of Finance** is one of the pair of courses 215&217, 285&217, or 295&296. In addition, each student must complete two introductory courses in Economics (Econ 101 and 102), and acquire a working knowledge of a high-level computer language (e.g. *Fortran*, *C* or *C++*) at a level equivalent to completion of EECS 183.

The major program must include at least ten courses: four basic courses **(II.)**, four special **Mathematics of Finance** courses **(III.)**, and two related courses **(IV.)** as described below.

II. The basic courses consist of one from each of the following groups **completed with a grade of at least C-**:
   1. **Differential Equations**: Math 286 or 316
   2. **Probability**: Math 525
   3. **Statistics**: Stats 426
   4. **Analysis**: Math 351 or 451

More advanced students, such as those who have completed Math 396, may substitute higher level courses with the approval of a departmental advisor.

III. The special **Mathematics of Finance** courses must include Math 423, 474, 472, and 526. Careful planning is essential.

IV. To complete the major each student should elect two additional intermediate or advanced courses related to **Mathematics of Finance**. Some, but not all, of the courses numbered 300 and above offered by Accounting, Computer Science, Economics, Finance, Industrial and Operations Engineering, and Statistics are appropriate here. Some specific approved courses are:

422 Risk Mgmt & Insurance   ECON 452 Intro to Econometrics

427 Retire. Plans & Other Empl Ben   FIN 302 Financial Decisions
462 Mathematical Models   FIN 317 Corp Financng Decsns
623 Computational Finance   FIN 408 & FIN 409 (1.5 credits each)
ECON 401 Inter. Micro Theory   Capital Markets & Fixed Inc. Sec
ECON 402 Inter. Macro   IOE 310 Intro Optim. Methods
ECON 409 Game Theory   IOE 452 Corporate Finance

All Accounting courses 300 and above, except 471
Some Statistics courses numbered above 400

Other courses may sometimes be appropriate and in all cases **approval of the advisor in Mathematics of Finance is required.**

Secondary Mathematics Teaching Certificate

The **Secondary Mathematics Teaching Certificate** program is designed to provide the broad training in mathematics necessary to be a successful teacher of mathematics at the secondary level, grades 6-12. LSA degree candidates must earn at least 104 LSA credits and at least 30 Education credits. Please note that the LSA B.S. degree requires 60 credits in physical and natural science and mathematics; students with less than 60 credits are eligible for an A.B. degree. Appointments with a Mathematics Department teaching certificate advisor may be scheduled online at https://lsa.umich.edu/math/undergraduates/advising.html. For information specific to the School of Education, students should contact the SoE Office of Student Services, 1228 SEB, (734) 615-1528, https://soe.umich.edu/academics-admissions/degrees/certification/undergraduate-secondary-teacher-education. *It is essential that students planning to obtain a teaching certificate consult a teaching certificate advisor, either in Mathematics or Education, prior to beginning their concentration program.*

The School of Education application deadline for fall term admission to the certification program is February 1. Applications received after February 1 are reviewed on a rolling basis through May 1. Students are encouraged to apply during their sophomore year. The School of Education will also accept winter term applications received by October 1 of a student's junior year.

I. The math prerequisites for the teaching certificate program are Math 215&217, 285&217, or 295&296 and a working knowledge of a high-level computer language (e.g. *Fortran, C* or *C++*) at a level equivalent to completion of EECS 183.

II. The basic courses for a candidate for a teaching certificate consist of one course from each of the following five groups (chosen with the approval of a teaching certificate advisor), **completed with a grade of at least a C-:**

1. **Modern Algebra:** Math 312, 412, or 493
2. **Geometry**: Math 431
3. **Probability**: Math 425 or 525
4. **Analysis:** Math 351 or 451
5. **Secondary Mathematics**: Math 486

III. The program requires ten specific Education courses, listed below, totaling 30 credits. These are elected in the junior and senior years in a specified order. **Consult with the School of Education Office of Student Services for the order and timing of these courses.**

> **Methods of Teaching Math (or minor field):** EDUC 413 (3 credits)
> **Practicum in Teaching Methods:** EDUC 307I and 307II (4 credits)
> **Educational Psychology:** EDUC 391 (3 credits)
> **Literacy in the Content Area:** EDUC 402 (3 credits)
> **Education in a Multi-cultural Society:** EDUC 392 (3 credits)
> **Directed Teaching:** EDUC 302 (10 credits)
> **Problems and Principles:** EDUC 304 (2 credits)
> **Teaching with Technology:** EDUC 446 (1 credit)
> **Teaching Students with Exceptionalities:** EDUC 445 (1 credit)

The last four of these are to be elected concurrently. **We repeat: students are encouraged to apply for admission to the certification program during their sophomore year.** Forms are available at the School of Education, Office of Student Services (1228 SEB).

IV. Additionally, every student must successfully complete: an introductory course in psychology (should be taken before Ed 391) and Michigan's licensure requirements.

V. Every Teaching Certificate student must also obtain a major or minor (SoE, not LSA) in another academic field. This normally requires 20-24 credits in a structured program in an area other than mathematics. Consult the _Bulletin_ of the School of Education for acceptable programs.

Minor in Mathematics

The **Minor in Mathematics** is designed to enable a student with a significant interest in Mathematics to deepen their knowledge while pursuing a major in another field. While the major will often be in a field which makes significant use of mathematics, such as a science or a quantitative social science, it may be in any area of study. Students from outside LSA, for example those from the College of Engineering, may also pursue a Minor in Mathematics. LSA regulations allow Advanced Placement credits and prerequisites for a student's major to count also as prerequisites for the student's minor. For students enrolled in LSA, only one course may be shared between the requirements of a minor and the requirements of a major. This rule does not apply to students enrolled outside of LSA. Courses used to meet the requirements of a minor may not be taken pass/fail. **All courses for the minor program must be completed with a grade of at least a C-.**

The prerequisite to a **Minor in Mathematics** is one of the sequences Math 115-116, 175-176, 185-186, or 295-296; 217-297; or Math 156. These all provide a thorough grounding in the calculus of functions of one variable. Advanced Placement credits in Math 120 and 121 also fulfill the prerequisite requirement.

The **Minor** consists of courses from the following two lists:

Basic courses:
Multivariable Calculus:	Math 215 or 285
Linear Algebra:	Math 214, 217, 417, or 419
Differential Equations:	Math 216 or 286

Upper-level courses:
Analysis/Diff. Equations:	Math 316, 351, 354, 404, 450, 451, 452, 454, 555
Algebra/Number Theory:	Math 312, 389, 412, 420, 471, 475, 493, 494, 561, 571, 575
Geometry/Topology:	Math 431, 433, 440, 490
Applied Mathematics:	Math 354, 371, 404, 423, 425, 450, 454, 462, 463, 471, 472, 550, 561,563, 571
Discrete Mathematics:	Math 310, 312, 389, 403, 412, 416, 420, 425, 465, 475, 481, 493, 561,566,567, 582
Financial/Actuarial :	Math 423, 424, 520, 523, 524

A student must select at least 5 courses consisting of at least 1 basic course and at least 2 upper-level courses. Course elections must follow these additional rules:

- For basic courses, a total of no more than 3 courses may be elected. Students may elect only one course from each available area (e.g. a student **can elect** both 215 & 217, but **may not elect** both 217 & 417).
- For upper level courses a student may elect between 2 and 4 courses. The upper level courses are not restricted, such that a student may elect multiple courses from the same area (e.g. electing both Math 433 and Math 490 in the Geometry & Topology area **is** acceptable).
- Engineering students are encouraged to include a linear algebra course in their minor selections.

All courses carry 3 or 4 credit hours and the total number of required credit hours is between 15 and 18. **Per LSA rule, LSA students must take at least 9 credits in-residence.** All in-residence credits must be taken from the Mathematics Department.

In all cases, more advanced courses may be substituted with the approval of a math advisor. *In particular, students who have satisfied the prerequisite with the honors sequence Math 295-296 or 217-297 will need to consult an advisor for the proper selection of courses.*

Other modifications can also be made with the approval of a math advisor. Finally, classes offered outside of Mathematics **cannot** be used to satisfy the requirements of the minor.

Graduation Requirements

The Departmental requirements for each of the major and minor programs are described in detail above. The final certification that you have satisfied the requirements for your program is provided to the Senior Auditors on the Release Form. This form is to be requested from your departmental advisor **before** the beginning of your final semester in the program. It will list your current and future courses and declare that if these courses are completed satisfactorily, the major requirements will be satisfied. Early submission of this form is very important to allow time for any required adjustments.

At the same time you complete the release form you should use Wolverine Access to Apply for Graduation electronically. The College of LSA has a number of further requirements which must be satisfied before you can graduate. These are described in the LSA Bulletin. General Counselors in the LSA Academic Advising Office (https:// lsa.umich.edu/advising/), (734) 764-0332, are trained in the administration of these regulations and should be consulted regularly to ensure that all requirements will be satisfied by the time you expect to graduate.

Department Faculty

For an up to date list of all department faculty, please visit https://lsa.umich.edu/math/people

Professors
Silas Alben, *Applied Mathematics, Mathematical Biology*
Jinho Baik, *Analysis, Probability Theory*
David Barrett, *Complex Analysis*
Alexander Barvinok, *Combinatorics*
Hyman Bass, *Algebra/Algebraic Geometry, Mathematics Education*
Erhan Bayraktar, *Financial and Actuarial*
Bhargav Bhatt, *Algebra/Algebraic Geometry*
Lydia Bieri, *Analysis, Applied Mathematics, Differential Equations, Geometry and Topology*
Andreas Blass, *Logic and Foundations, Combinatorics, Computer Science*
Anthony Bloch, *Analysis, Applied Mathematics, Differential Equations*
Victoria Booth, *Applied Mathematics, Mathematical Biology*
Liliana Borcea, *Applied Mathematics*
Daniel Burns, *Algebra/Algebraic Geometry, Analysis, Geometry and Topology*
Richard Canary, *Geometry and Topology*
Dmitry Chelkak, *Probability Theory, Mathematical Physics, Analysis*
Joseph Conlon, *Mathematical Physics, Applied Mathematics*
Stephen DeBacker, *Algebra/Algebraic Geometry*
Selim Esedoglu, *Applied Mathematics*
Sergey Fomin, *Combinatorics*
Daniel Forger, *Mathematical Biology*
Robert Griess, *Algebra/Algebraic Geometry*
Zaher Hani, *Analysis, Differential Equations*
Mel Hochster, *Algebra/Algebraic Geometry*
Trachette Jackson, *Mathematical Biology, Applied Mathematics*
Lizhen Ji, Analysis, *Geometry and Topology*
Mattias Jonsson, *Algebra/Algebraic Geometry, Analysis, Financial and Actuarial*
Tasho Kaletha*, Number Theory*
Smadar Karni, *Applied Mathematics*
Sarah Koch*, Analysis, Geometry and Topology*
Robert Krasny, *Applied Mathematics*
Igor Kriz, *Geometry and Topology*
Jeffrey Lagarias, *Combinatorics, Computer Science, Geometry and Topology, Number Theory*

Thomas Lam, *Combinatorics*
Robert Megginson, *Analysis, Mathematics Education*
Peter Miller, *Applied Mathematics*
Mircea Musţată, *Algebra/Algebraic Geometry*
Kartik Prasanna, *Number Theory*
Mark Rudelson, *Analysis, Probability Theory*
Karen Smith, *Algebra/Algebraic Geometry*
Andrew Snowden, *Algebra/Algebraic Geometry, Number Theory*
Ralf Spatzier, *Geometry and Topology*
David Speyer, *Differential Equations, Combinatorics*
Martin Strauss, *Computer Science*
Alejandro Uribe, *Analysis, Geometry and Topology, Mathematical Physics*
Shravan Veerapaneni, *Applied Mathematics*
Divakar Viswanath, *Applied Mathematics*
Sijue Wu, *Differential Equations*
Virginia Young, *Financial and Actuarial*
Michael Zieve, *Algebra/Algebraic Geometry, Number Theory*

Associate Professors
Wei Ho, *Algebra/Algebraic Geometry, Number Theory*
Kristen Moore, *Applied Mathematics, Financial and Actuarial*
Alex Wright, *Geometry and Topology*

Assistant Professors
Charlotte Chan, *Number Theory*
Asaf Cohen, *Financial and Actuarial*
Sarah Peluse, *Mathematics*
Alexander Perry, *Algebra/Algebraic Geometry*
Aaron Pixton, *Algebra/Algebraic Geometry*
Linh Truong, *Geometry/Topology*
Jenny Wilson, *Algebra/Algebraic Geometry*

Lecturers
Irina Arakelian, *Mathematics Education*
Hanna Bennett, *Geometry and Topology, Mathematics Education*
Patrick Boland, *Geometry and Topology*
Robert Cochrane, *Mathematics*
Jeff Dunworth, *Applied Mathematics*
James Heffers, *Mathematics*
Elmas Irmak, *Geometry and Topology*
Paul Kessenich, *Analysis, Differential Equations, Mathematics Education*
Angela Kubena, *Geometry and Topology, Mathematics Education*

P. Gavin LaRose, *Applied Mathematics, Mathematics Education*
Roger Natarajan, *Financial and Actuarial*
Ali Nazari, *Financial and Actuarial*
Chaya Norton, *Geometry/Topology*
Scott Schneider, *Logic and Foundations*
Amy Shell-Gellasch, *Mathematics*
Ike Tan, *Mathematics*
Michael Weiss, *Mathematics*
Nina White, *Mathematics Education*
Elizabeth Wolf, *Computer Science*

Postdoctoral Assistants
Zhongshan An, *Analysis*
Thomas Anderson, *Applied Mathematics*
Philip Arathoon, *Applied Mathematics*
Ahmad Barhoumi, *Analysis, Applied Mathematics*
Daniel Bergman, *Applied Mathematics*
Alexander Bertoloni Meli, *Number Theory*
Elliot Blackstone, *Applied Mathematics*
Pierre-Louis Blayac, *Dynamical Systems*
Martin Bobb, *Analysis*
Viktor Burghardt, *Algebraic Geometry*
Robert Cass, *Number Theory*
Stephanie Chan, *Number Theory*
Zhen Chao, *Number Theory*
Qianyu Chen, *Algebraic Geometry*
Ruiyuan Chen, *Logic*
Tao Chen, *Financial and Actuarial*
Patrick Daniels, *Number Theory*
Purba Das, *Probability Theory*
Shreyasi Datta, *Number Theory*
Christopher DiScenza, *Mathematics*
Alexander Dobner, *Number Theory*
Qi Feng, *Mathematics of Finance*
Khashayar Filom, *Geometry and Topology*
Nir Gadish, *Combinatorics*
Yibo Gao*, Statistics*
Terrence George, *Combinatorics*
Roozbeh Gharakhloo, *Mathematics*
Maria Han Veiga, *Applied Mathematics*
Nate Harman, *Combinatorics*
Matthew Harrison-Trainor, *Logic*
Zhachary Himes, *Algebraic Geometry*
Jonathan Husson, *Probability Theory*

Lena Ji, *Algebraic Geometry*
Ali Kara, *Financial Mathematics*
Asaf Katz, *Geometry and Topology, Dynamical Systems*
Jennifer Kenkel, *Algebra/Algebraic Geometry*
Dae Wook Kim, *Applied Mathematics*
Donghan Kim, *Financial Mathematics*
Ruby Kim, *Mathematical Biology*
Peter Koymans, *Number Theory*
Matthew Kvalheim, *Mathematics*
Robert Laudone, *Algebra/Algebraic Geometry, Combinatorics*
Hai Le, *Probability Theory*
Guchuan Li, *Geometry and Topology*
Liying Li, *Probability Theory*
Wenyuan Li, *Dynamical Systems*
Aida Maraj, *Algebraic Geometry*
Nicholas McCleerey, *Geometry/Topology*
Dao Nguyen, *GProbability Theory*
Dominyka Norgilas, *Financial Mathematics*
Stephen Pietromonaco, *Algebraic Geometry*
Andrei Prokhorov, *Analysis*
Nick Rome, *Number Theory*
George Seelinger, *Combinatorics*
Melissa Sherman-Bennett, *Mathematics*
Austyn Simpson, *Commutative Algebra*
Daniel Smolkin, *Algebra/Algebraic Geometry*
David Stapleton, *lgebraic Geometry*
Nattalie Tamam, *Dynamical Systems*
Amir Vig, *Analysis*
Xiaojia Wang, *Mathematics*
Zhenhua Wang, *Financial and Actuarial*
Theodore Weisman, *Manifolds*
Lei Xue, *Combinatorics*
Wei Yan, *Financial and Actuarial*
Wijit Yangjit, *Mathematics*
Giorgio Young, *Operator Theory*
Claudia Yun, *Combinatorics*
Elad Zelingher, *Number Theory*
Hui Zhu, *Applied Mathematics*
Foling Zhu, *Analysis*
Jialiang Zou, *Number Theory*

Academic Advising

You should consult with a departmental advisor before registering for courses each semester and before you make any important changes in your program. Appointments and counseling sessions are held in the Student Services Office (2082 East Hall). To make an appointment online please see https://lsa.umich.edu/math/undergraduates/advising.html. For advice on general LSA regulations (distribution, languages, etc.) and reassurance that you understand the rules, you should see a General Counselor in the LSA Academic Advising Office (1255 Angell Hall), (734) 764-0332. Course information is available from the course guide online at the LSA website: https://www.lsa.umich.edu/cg.

Transfer credit for courses taken elsewhere is administered by the Credit Evaluation division of the Undergraduate Admissions Office (1220 SAB), (734) 763-1074, https://admissions.umich.edu/apply/transfer-applicants/transfer-credit. If Math courses are not listed, materials can be submitted for evaluation by following the instructions on the department website, https://lsa.umich.edu/math/undergraduates/transfer-credit.html.

For women seeking careers in science or mathematics, the Women in Science and Engineering (WISE) Program offers academic and career counseling, workshops on combining careers with various lifestyles, contact with female role models, lists of scholarships and awards, and a resource center listing opportunities for women in science and engineering; visit https://www.wise.umich.edu/ for more information, including locations and contact numbers.

Academic Help

Your first source for help with course work is the instructor of the course. For Introductory courses, including Math 105, 115, and 116, the Math Lab (B860 East Hall), (734) 936-0160, https://lsa.umich.edu/math/undergraduates/course-resources/math-lab.html, offers drop-in help during the day and many evening and weekend hours. The Department also maintains a private tutor list of advanced students willing (for pay) to assist with many undergraduate courses: https://lsa.umich.edu/math/undergraduates/course-resources/tutor-list.html. Additional information can be found at https://www.lsa.umich.edu/advising/

The national engineering honor society Tau Beta Pi (1228 EECS), (734) 615-4187 also offers tutoring in a range of science, engineering, and mathematics courses.

The Sweetland Center for Writing (1310 North Quad), (734) 764-0429 offers free individual assistance on a specific writing project or with general writing skills.

Around the Mathematics Department

Aside from the Undergraduate Office, there are several other departmental offices which are relevant to undergraduate students. Questions, complaints, praise, and comments about graduate students, graders, and Graduate Student Instructors should be addressed to the Graduate Program Staff (2082 East Hall), math-grad-office@umich.edu, (734) 764-7436. Similar messages concerning faculty may be addressed to the Associate Chair for Education (2084 East Hall), math-acue@umich.edu, (734) 763-4223. For other administrative resources, undergraduate students may contact the Chair's Office (2074 East Hall), (734) 764-0335.

A great deal of information about the Department is available on-line—start with the Mathematics website at https://lsa.umich.edu/math and follow the desired links.

Mathematics Library

The Mathematics Library (3175 Shapiro Library), (734) 936-2327, https://www.lib.umich.edu/shapiro-undergraduate-library is an important resource for students and faculty alike. The Library houses one of the best mathematics collections in the world including most of the major periodicals, monographs, and textbooks. Course reserve books are kept here and there are terminals for accessing the on-line general library catalog MIRLYN. Free access is also available to the American Mathematical Society's Mathematical Reviews Database (MathSciNet) online . It contains bibliographic data and reviews of mathematical research literature from articles published in almost all mathematical journals in the world. You can access MathSciNet and other resources by navigating to https://guides.lib.umich.edu/c.php?g=282871.

Computing Facilities

The Department has arranged for several software packages of special interest to students of mathematics to be available at many of the University's public computing stations. These include *Matlab* and *Mathematica*. The Department has 4 computing lab rooms equipped for use by students of Math courses that require labs. They are available to all faculty and graduate students and undergraduates, and also to students in the REU program (https://lsa.umich.edu/math/undergraduates/research-and-career-opportunities/research/research-experience-for-undergraduates--reu-.html).

Career Planning

One of the main goals of the Student Services Office is to acquaint students with the many career options that the study of mathematics affords. The office collects and makes available information on graduate study, careers in the industrial and service sectors, and mathematics education, as well as opportunities for summer employment in mathematical fields. To help graduating students secure entry-level positions in private companies, government agencies, and non-profit organizations, the Undergraduate Office provides notice of employment opportunities and on-campus recruiting. In the Fall semester, the Department invites Mathematics Alumni/ae as well as outside companies to participate in a Career Conference. Students will have the opportunity to speak with companies hiring as well as learn about what types of careers past students have gone on to in industry, government, and mathematics education. We also maintain an extensive web page on career options: https://lsa.umich.edu/math/undergraduates/research-and-career-opportunities/career.html.

Because summer employment in a mathematical field is one of the best ways to decide whether a career in mathematics is what you want, the Undergraduate Office assists students in finding meaningful summer experiences that will enhance their mathematical skills and give them a taste of using mathematics in the "real world." Information on opportunities for summer employment in business or industry, and summer research positions are sent to students in the weekly "math missive." In addition, there are many programs which support undergraduate students doing mathematical research during the summer, both at U-M and across the country. Details are available in the Student Services Office (math-undergrad-office@umich.edu).

Personal Counseling

Counseling and Psychological Services (CAPS, 3100 Michigan Union, (734) 764-8312), provides individual and group counseling on a wide range of concerns. They also offer workshops which focus on relaxation techniques, strategies for managing study time, and methods for coping with test-taking anxiety. Counseling is free to students on a walk-in basis or by appointment: https://caps.umich.edu/.

Complaints and Problems

We hope you won't have any, but, just in case, here are a few strategies. General problems concerning instructors that cannot be resolved privately should be directed to the Student Services Office (math-undergrad-office@umich.edu), (734) 763-4223. In particular, any case of unprofessional attitudes or actions by an instructor or grader should be reported immediately. In many cases the Student Services Office can serve as an intermediary to help resolve the problem quickly. Problems that cannot be resolved in the Department might be directed to the Office of Student Academic Affairs (1255 Angell Hall), (734)764-7297, the Office of Institutional Equity (734) 763-0235, or the Ombudsman (6015 Fleming Administration), (734) 763-3545.

Extracurricular Activities

Your undergraduate experience in mathematics will be greatly enriched by participating in some of the math-related extracurricular activities sponsored by the department. Interaction with your fellow majors is an integral part of a well-rounded undergraduate education.

The **Undergraduate Math Club** is run for and by undergraduate math majors, with the assistance of a faculty moderator. It is an informal organization which sponsors social events and talks by faculty and students. A typical meeting begins with free pizza and drinks, followed by a 45 minute talk on an interesting mathematical problem, application, or idea (or all three!). The selected topic is something which isn't usually seen in the standard curriculum. Some of these topics lead to important concepts in theoretical or applied research, while others explain a clever solution to an interesting problem. Everything is formulated to be accessible to students whose technical background consists only of calculus and some exposure to the methods of proof. A few of the meetings will also feature information regarding graduate schools, internships, and career planning. For more information see the web page:https://lsa.umich.edu/math/un-

dergraduates/extracurricular-activities/math-talks/math-club.html.

The **Student Actuaries at Michigan** (SAM) club is an organization for undergraduates and graduates interested in the field of actuarial and financial mathematics. There are monthly meetings on topics of interest, sometimes featuring speakers from industry. The group organizes study groups for the professional examinations and co-ordinates visits to campus by industry recruiters. It also sponsors a variety of athletic and social activities including an end-of-year picnic. For more information see the web page: https://www.sam-umich.com/.

The **Women in Mathematics** (WIM) club seeks to provide social and educational support for women studying mathematics and statistics. The club provides opportunities to hear from female speakers in mathematics, allows members to meet others with similar academic interests, and provides peer support for classes and career options. WIM holds study nights, brings in speakers, and hosts group social events. For more information see the web page: https://umwomenin-math.weebly.com/.

The **STEM Society** is a student group that aims to expose K-12 students, especially those in lower socioeconomic areas, to inquiry-based learning in the fields of science and math. They hope to disprove common stereotypes that students may have about science and math as well as to increase their awareness of the diverse op-portunities available in these fields. In addition, they strive to provide undergraduates with an interest in teaching or sharing their passion for science and math with opportunities to do so. For more informa-tion see their webpage: https://sites.google.com/a/umich.edu/stem-society/.

The **Research Experience for Undergraduates (REU)** is designed to provide undergraduates with the opportunity to pursue research under the tutelage of experienced faculty members. Typically students work with a faculty member on a project of mutual interest for 6-8 weeks during the summer. The projects range anywhere from mathematical modeling in the sciences and engineering to solving abstract and conceptual problems, depending on the background and the interests of student and faculty member. Each student will be required to write a report about the project, which is due at the project's completion. Each student who participates in the REU program is paid a stipend of approximately $4,500 during the summer. Students have the chance to expand their knowledge, learn new material, and combine professional development with close contact between faculty and student peers. If you are interested in participating and would like to learn more, please contact the Undergraduate Office, (math-undergraduate-office@umich. edu). For summer REU positions, applications will be available online in late November and are due in early February. You can apply here when the application is available: https://lsa.umich.edu/math/undergraduates/research-and-career-opportunities/research/research-experience-for-undergraduates--reu-.html

The **William Lowell Putnam** competition is a nationwide mathematics competition sponsored annually by the Mathematical Association of America (MAA). All full-time undergraduate students may participate and each year about 30 Michigan students compete. The competition consists of an examination on math topics common to the undergraduate curriculum and is given in early December. The emphasis is on ingenuity rather than knowledge. There are both team and individual prizes, including a graduate fellowship. In 2011, the University of Michigan team placed 8th overall, receiving an Honorable Mention. As preparation for the Putnam exam, the Department offers a Problem-Solving seminar which meets weekly during the Fall and Winter terms. Students may receive one hour of course credit for this seminar by enrolling in Math 289. In this seminar students are exposed to interesting mathematical material not found in other courses. They also develop their problem-solving abilities. The seminar is open to undergraduate students at all levels.

As additional preparation, students are encouraged to participate in the **Virginia Tech Mathematical Competition** administered within the department each November.

The Department also sponsors a local competition. The **University of Michigan Undergraduate Mathematics Competition [(UM)^{2}C^x]** consists of an examination given towards the end of the winter semester each year. It is open to all full-time undergraduates on the Ann Arbor campus and offers substantial cash prizes which vary from year to year depending on financing. The Problem-Solving seminar is also recommended as preparation for the [(UM)^{2}C^x]. Information on competitions can be found here: https://lsa.umich.edu/math/undergraduates/extracurricular-activities/competitions.html

Also listed in the **weekly departmental bulletin** are a variety of other special events. Many are specialized seminars that are not at a level accessible to undergraduates, but some events include more general colloquium talks, special lectures, and mathematical movies. It is a good idea to check the bulletin every Monday morning. It is posted in the hallway near the main Department Office, 2074 East Hall, and on the web https://www.math.lsa.umich.edu/seminars_events/.

An excellent way to improve your mastery of mathematics is to get involved in teaching it. There are positions available to undergraduates as **graders** and **tutors**. Graders correct and grade homework assignments in the larger courses. A student is eligible to grade any course he/she has passed with a grade of B or better, but the positions are competitive. A call for applications will be sent to majors from the Graduate Program Office (2082 East Hall). Tutors in the Mathethematics Lab are available for drop-in help in introductory courses through Math 217. For more information on tutoring positions, go to https://lsa.umich.edu/math/undergraduates/extracurricular-activities/teaching-experiences.html. Irina Arakelian (3859 East Hall), (734) 936-1775, arakel@umich.edu is the current Mathematics Lab coordinator.

Wear your Math T-shirt with pride. Math T-shirts are cheaper than clothing and may be purchased in the Student Services Office (2082 East Hall). Please be sure to contribute a photograph of yourself wearing the T-Shirt to the department's "Where Has Your Math T-Shirt Been" album by sending a copy to math-undergrad-office@umich.edu.

From Our Graduates

Math is among the pinnacles of human knowledge, and it is increasingly relevant in our world. All the modern technology you use (computers, smartphones, GPS, etc.) is based on complex mathematical theories. Even if you don't become a "mathematician", the jobs that require some degree of mathematical or logical thinking are growing to dominate the modern workplace. If you can do math, you can probably do those other things as well, because math puts them all into a larger, well-developed context. And lastly, if you like problem solving, math is a ton of fun.

(Ibrahim T., B.S. 2005)

I work in the development of a large-scale, distributed storage system for a local business. I use my math degree almost every day. Not only do subjects I learned explicitly, like number theory and abstract and linear algebra, come into constant use when trying to do novel things on very large scales, but the background and rigor that the U of M math department taught me gave me a good foundation for teaching myself whatever new math I need to solve a problem.

(Adam E., B.S. 2009)

Majoring in math was one of the best decisions I have ever made. The critical thinking and logical reasoning skills that I developed studying math provided the fundamentals that I needed to succeed in the real world (even though I have never ever had to usestochastic processes, Ito's formula, and the other 70% of material I learned), and when I do run into real world math problem, it is very comforting to know that I used to know the stuff. Also, the best aspect of being a math major is that when we tell other people we studied math in college, they automatically stereotype us as smart, which is definitely a good stereotype to have. The free math T-shirt was also pretty sweet. Go blue!

(Austin B., B.S. 2011)

No matter what you major in college, you are bound to worry about what to do upon graduation. But if you major in math, you will have versatile skills sets that will set you apart from others. Whether it be business, medical, legal or any other field you can think of, you will have the necessary background to do virtually anything.

(Young A., B.S. 2005)

I started a job as a Data Manager in a healthcare analytics company right out of college. I had no experience when I started, not even an internship to put on my resume. Five years and four promotions later, I am now an Operations Manager with 27 people reporting up through me. My friends and family ask me all the time whether or not I use my math degree in my job; they assume that because I majored in math that I should be sitting around crunching numbers all day. The truth is that I don't do complex calculations very often, and I have probably forgotten almost all of the theorems I once knew, but a math education is so much more than calculations and theorems. My math curriculum exposed me to concepts of logic and reasoning that are applicable in both programming and conflict resolution. It taught me to relate variables and data sets which helped me understand databases on a deeper level. I can quickly analyze large volumes of data and draw conclusions based on summarized results or determine the best course of action given several alternatives. I see details that others miss and ask the right questions to identify root causes of issues. Et cetera, et cetera. In short, math taught me problem-solving and critical thinking. For a long time, I took this for granted thinking that everyone was equipped with comparable abilities. After five years in the real world, I realize that it was actually these problem-solving and critical thinking skills that set me apart from my peers and allowed me to excel in a field I originally knew nothing about, and it was my math education that taught me those skills. Math was once a discipline reserved for professors and rocket scientists, but I can say from experience that the possibilities for math majors in the modern workforce are endless. You can't go wrong with a math degree.

(Nicole S., B.S. 2007)

I majored in Financial Math, Economics, and Statistics and will be pursuing a PhD in Finance at the University of Chicago. Majoring in the mathematics program definitely prepared me well for graduate level work, from building robustness and precision in defining variables of interest to studying specific algorithms for numerical approximation. The major includes a wide variety of courses that are readily applicable in the "real world" (that is in fact, quite "complex") by providing a fundamental understanding of the financial devices from the elegant world of mathematics. I also developed a solid background in probability theory needed for anyone going into quantitative finance. The program definitely gave me useful tools with which I hope to create a positive drift towards success in the random walk of life.

(Ben C., B.S. 2012)

Courses

Attention Graduate Students: Due to University rules, graduate students cannot receive credit for courses numbered under 400. Please check the Rackham <u>Program Guide for more details.</u>

Math 105 - Data, Functions, and Graphs

Prerequisites: 3-4 years HS math including trigonometry
Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)
Student Body: First-year students.
Credit: 4 Credits. No credit granted to those who have completed any Mathematics course numbered 110 or higher. A maximum of four credits may be earned in Math 105 and 110.

Background and Goals:
Math 105 serves both as a preparatory class to the calculus sequences and as a class for students who are interested in strengthening their math skills. Students who successfully complete 105 are fully prepared for Math 115.

Content:
This course presents concepts from four points of view: geometric (graphs), numeric (tables), symbolic (formulas), and written (verbal descriptions). The emphasis is on the mathematical modeling of real-life problems using linear, polynomial, exponential, logarithmic, and trigonometric functions. Students develop their reading, writing, and questioning skills in an interactive class-room setting.

Alternatives:
None

Subsequent Courses:
The course prepares students for Math 115 (Calc I).

Math 110 - Pre Calculus (Self-Paced)

Prerequisites: 3-4 years HS math
Frequency: Fall (I)
Student Body: First-year students.
Credit: 2 Credits. No credit granted to those who have completed any Mathematics course numbered 110 or higher. A maximum of four credits may be earned in Math 105 and 110.

Background and Goals:
Math 110 is a condensed, half-term version of Math 105 designed specifi-cally to prepare students for Math 115. It is open only to students who have enrolled in Math 115 and whose performance on the first uniform examina-tion indicates that they will have difficulty completing that course success-fully. This self-study course begins shortly after the first uniform examination in Math 115 and is completed under the guidance of an instructor without regular classroom meetings. Students must receive permission from the Math 115 Course Director or other designated representative to enroll in the course, and they must visit the Math Lab as soon as possible after enrolling to receive printed course information. Enrollment opens the day after the first Math 115 uniform examination, and enrollment must be completed by the Friday of the following week.

Content:
The course is a condensed, half-term version of Math 105 designed for stu-dents who appear to be prepared to handle calculus but are not able to suc-cessfully complete Math 115. Students may enroll in Math 110 only on the recommendation of a mathematics instructor after the third week of classes in the Fall and must visit the Math Lab to complete paperwork and receive course materials. The course covers data analysis by means of functions and graphs.

Alternatives:
Math 105 covers the same material in a traditional classroom setting.

Subsequent Courses:
The course prepares students for Math 115 (Calc I).

Math 115 - Calculus I

Prerequisites: 3-4 years HS math including trigonometry
Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)
Student Body: First-year students.
Credit: 4 Credits. No credit for students who have
                  completed any course numbered 111 or higher .

Background and Goals:
The sequence Math 115-116-215 is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to major in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof. All sections are given two uniform midterms and a final exam.

Content:
The course presents the concepts of calculus from four points of view: geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing, and questioning skills, as well as their ability to work cooperatively. Topics include functions and graphs, derivatives and their applications to real-life problems in various fields, and an introduction to integration. The classroom atmosphere is interactive and cooperative. Both individual and team homework is assigned.

Alternatives:
Math 185 (Honors Calc I) is a more theoretical course which covers some of the same material. Math 175 (Intro to Cryptology) is a non-calculus alternative for students with a good command of first-semester calculus. Math 295 (Honors Math I) is a much more intensive and rigorous course. A student whose preparation is insufficient for Math 115 should take Math 105 (Data, Functions & Graphs).

Subsequent Courses:
Math 116 (Calc II) is the natural sequel. A student who has done very well in this course could enter the honors sequence at this point by taking Math 156 (Applied Honors Calc II).

Math 116 - Calculus II

Prerequisites: Math 115 or AP credit
Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)
Student Body: First-year students.
Credit: 4 Credits. No credit for students who have completed or are enrolled in Math 156, 176, 186, 215, 216, 285, or 286.

Background and Goals:
The sequence Math 115-116-215 is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to major in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof. All sections are given two uniform midterms and a final exam.

Content:
The course presents the concepts of calculus from four points of view: geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing and questioning skills, as well as their ability to work cooperatively. Topics include techniques of integration, applications of integration, Taylor series, an introduction to differential equations, infinite series, and parametric equations and polar coordinates. The classroom atmosphere is interactive and cooperative. Both individual and team homework is assigned.

Alternatives:
Math 186 (Honors Calc II) is a more theoretical course which covers much of the same material. Math 156 (Applied Honors Calc II) also covers much of the same material emphasizing applications to science and engineering.

Subsequent Courses:
Math 215 (Calc III) is the natural sequel. A student who has done very well in this course could enter the honors sequence at this point by taking Math 285 (Honors Calc III).

Math 117 - Calculus II

Prerequisites: Math 116 or BC Exam score of 5 in 2020
Frequency: Fall (I)
Student Body: First-year students.
Credit: 2 Credits.

Background and Goals:
Math 117 is intended for students who have received a 5 on the BC Calculus AP exam in 2020. This course was created to bridge the knowledge gap that many of these students will have due to the pandemic, before attending more advanced U(M) mathematics courses. It is a self-paced, online course and does not formally meet.

The course is 2 credits, and students have the choice to officially enroll in the class for credit/no credit (if their program requires proof of completion), or students may elect to take the course unofficially and be given access to the course materials ahead of time. It is completely self paced and students may complete the material on their own time.

Content:
This is a calculus course. Topics covered include calculus in polar coordinates; calculus in parametric coordinates; sequences and series; convergence tests for series; power series; Taylor polynomials; Taylor series; applications of Taylor polynomials and Taylor series.

Subsequent Courses:
Math 215 (Calc III) is the natural sequel.

Math 128 - Explorations in Number Theory

Prerequisites: 3 years HS math
Frequency: Sporadic
Student Body: First-year students and sophomores.
Credit: 4 Credits. No credit after completing any 200+ level math course, except 385, 485, 489, and 497.
Area: Number Theory

Background and Goals:
This course is intended for students who want to engage in mathematical reasoning without having to take calculus first. It is particularly well-suited for non-science majors or those who are thoroughly undecided. Students will make use of software to conduct numerical experiments and to make empirical discoveries. Students will formulate precise conjectures and in many cases prove them. Thus the students will, as a group, generate a logical development of the subject.

Content:
After studying factorizations and greatest common divisors, emphasis will shift to the patterns that emerge when the integers are classified according to the remainder produced upon division by some fixed number (congruences). Once some basic tools have been established, applications will be made in several directions. For example, students may derive a precise parameterization of Pythagorean triples.

Alternatives:
None

Subsequent Courses:
This course does not provide preparation for any further study of mathematics.

Math 145 - Houghton Scholars Calculus Workshop I

Prerequisites: Concurrent enrollment in Math 115 and permission of instructor
Frequency: Fall (I)
Student Body: First year students.
Credit: 2 Credits.

Background and Goals:
This course is an intensive supplemental workshop experience for students in the Douglas Houghton Scholars Program. In a small-class setting, students work in small groups on problems more challenging than those in the regular section of Math 115. The goal is to develop the students' problem-solving capabilities and promote their interest in challenging problems.

Content:
The workshop follows the structure of Math 115. Content includes geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing, and questioning skills, as well as their ability to work cooperatively. Topics include functions and graphs, derivatives and their applications to real-life problems in various fields, and an introduction to integration.

Alternatives:
None

Subsequent Courses:
Math 146 (Houghton Scholars Calc Workshop II)

Math 146 - Houghton Scholars Calculus Workshop II

Prerequisites: Concurrent enrollment in Math 116 and permission of instructor
Frequency: Winter (II)
Student Body: First year students.
Credit: 2 Credits.

Background and Goals:
This course is an intensive supplemental workshop experience for students in the Douglas Houghton Scholars Program. In a small-class setting, students work in small groups on problems more challenging than those in the regular section of Math 116. The goal is to develop the students' problem-solving capabilities and promote their interest in challenging problems.

Content:
The workshop follows the structure of Math 116 (Calc II). Content includes geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing and questioning skills, as well as their ability to work cooperatively. Topics include techniques of integration, applications of integration, Taylor series, an introduction to differential equations, and infinite series.

Alternatives:
None

Subsequent Courses:
Math 215 (Calc III), Math 216 (Intro to Diff. Equations)

Math 147 - Introduction to Interest Theory

Prerequisites: Math 115
Frequency: Fall (I), Winter (II)
Student Body: First and second year students.
Credit: 3 Credits. No credit after completing any 200+ level math course, except 385, 485, 489, or 497.
Area: Actuarial & Financial

Background and Goals:
This course is designed for students who seek an introduction to the mathematical concepts and techniques employed by financial institutions such as banks, insurance companies, and pension funds. Actuarial students, and other mathematics majors, should elect Math 424 which covers the same topics but on a more rigorous basis requiring considerable use of calculus. The course is not part of a sequence. Students should possess financial calculators.

Content:
Topics covered include: various rates of simple and compound interest, present and accumulated values based on these; annuity functions and their application to amortization, sinking funds, and bond values; depreciation methods; introduction to life tables, life annuity, and life insurance values.

Alternatives:
Math 424 (Compound Interest & Life Insurance) covers the same material in greater depth and with a higher level of mathematical content.

Subsequent Courses:
None

Math 156 - Applied Honors Calculus II

Prerequisites: Score of 4 or 5 on Advanced Placement AB or BC
Calculus exam
Frequency: Fall (I)
Student Body: First-year students.
Credit: 4 Credits. Credit is granted for either 116 or 156,
but not both.

Background and Goals:
Math 156 is part of the applied honors calculus sequence for engineering
and science majors. The course is an alternative to Math 116 for students
with strong mathematical ability. Applications and concepts receive equal
treatment. Theorems are stated precisely and are derived, but technical de-
tails are omitted. Examples are given to illustrate the theory. Critical think-
ing and class participation are encouraged. The goal is to provide students
with the solid background needed for subsequent courses in mathematics,
engineering, and science.

Content:
Riemann sums, definite integral, fundamental theorem of calculus, applica-
tions of integral calculus (e.g. arclength, surface area, work, center of mass,
probability density functions), improper integrals, infinite sequences and
series, geometric series, alternating series, power series, Taylor series, dif-
ferential equations, complex numbers. Students are introduced to MAPLE.

Alternatives:
Math 116 (Calc II).

Subsequent Courses:
Math 285 (Honors Calc III) is the natural sequel.

Math 175 - Introduction to Cryptology

Prerequisites: Permission of honors advisor
Frequency: Fall (I)
Student Body: First-year students.
Credit: 4 Credits. No credit granted to those who have completed a 200-level or higher Mathematics course other than 310, 385, 489, or 497.

Background and Goals:
The goal of this course is to ultimately learn about the modern public key encryption method called RSA. In order to do this, the course focuses on introducing a number of mathematical ideas involved in the development and analysis of codes. Mathematical topics include enumeration and elementary coding used in ancient times, but the bulk of the course is devoted to elementary number theory. Students also work throughout the course on effectively communicating mathematics, both written and orally. In particular, there is a strong emphasis on writing and class presentations. The primary goal of the course is to not only understand how various cryptosystems work, but why.

Content:
This course is an alternative to Math 185 as an entry to the honors calculus sequence. The course stresses discovery as a vehicle for learning. This is an IBL course, where students will be required to work in groups and experiment in class. The course has two different aspects, namely class time (twice a week) and lab time (once a week). Grades will be based on worksheets, homework and class participation, with a strong emphasis on homework. Personal computers will be a valuable experimental tool in this course and students will be asked to learn to program in one of MATHEMATICA or SAGE. Moreover, for the write-up of the worksheets, the use of LaTeX is highly recommended, but not required.

Alternatives:
Math 115 (Calc I), Math 185 (Honors Calc I), or Math 295 (Honors Math I).

Subsequent Courses:
Math 176 (Explorations in Calc) or Math 116 (Calc II).

Math 176 - Explorations in Calculus

Prerequisites: Background in calculus is suggested; highly recommended for students who have taken Math 175
Frequency: Winter (II)
Student Body: First-year students.
Credit: 4 Credits. Credit is granted for either Math 176 or 186, but not both

Background and Goals:
This course is an Inquiry-Based version of Honors Calculus I and II (such as Math 185/186) and provides the necessary preparation for Multivariable Calculus (Math 215 or the honors version, Math 285). A student who has had some exposure to calculus (e.g., AB or BC in high school, or Math 115) will be well-prepared for this course. The main aspects of the course are worksheets and homework. The worksheets will be discussed and solved in groups during class time and individually written up later. The use of LaTeX is required for the write-up. The majority of class time will be spent working in groups and presenting ideas and solutions to problems.

Content:
This is a calculus course from a theoretical perspective. The main emphasis is on learning how to write rigorous proofs for topics the students are probably already familiar with. The main mathematical topics include sequences and series of real numbers, properties of the real line, continuity and limits of real functions, differentiation and integration.

Alternatives:
Math 186 (Honors Calc II)

Subsequent Courses:
Math 285 (Honors Calc III)

Math 185 - Honors Calculus I

Prerequisites: Permission of honors advisor during orientation
Frequency: Fall (I)
Student Body: First-year students.
Credit: 4 Credits.

Background and Goals:
Most students take calculus in high school, and it may seem that there isn't much new to learn. The goal of this course is to develop the familiar concepts of calculus using a more rigorous and theoretical approach. In particular, with its emphasis on how to use appropriate mathematical language, this course lays a solid foundation for future math courses, and is suitable for students intending to pursue a major in mathematics, science, or engineering who desire a more complete understanding of the underpinnings of calculus. This sequence is not restricted to students enrolled in the LSA Honors Program.

Content:
Topics covered include functions, graphs, continuity, limits, derivatives, and integrals. Approximately one meeting per week will be devoted to introducing linear algebra.

Alternatives:
Math 115 (Calc I) is a less theoretical course which covers much of the same material. Math 295 (Honors Math I) gives a much more theoretical treatment of much of the same material.

Subsequent Courses:
Math 186 (Honors Calc II) is the natural sequel.

Math 186 - Honors Calculus II

Prerequisites: Permission of honors advisor or Math 185
Frequency: Winter (II)
Student Body: First-year students.
Credit: 4 Credits. Credit is granted for either Math 176 or 186, but not both.

Background and Goals:
Most students take calculus in high school, and it may seem that there isn't much new to learn. The goal of this course is to develop the familiar concepts of calculus using a more rigorous and theoretical approach. In particular, with its emphasis on how to use appropriate mathematical language, this course lays a solid foundation for future math courses, and is suitable for students intending to pursue a major in mathematics, science, or engineering who desire a more complete understanding of the underpinnings of calculus. This sequence is not restricted to students enrolled in the LSA Honors Program. This course is a continuation of Math 185.

Content:
Topics include integral calculus, transcendental functions, infinite sequences and series (including Taylor's series), and – time permitting – some simple applications to elementary differential equations.

Alternatives:
Math 116 (Calc II) is a less theoretical course which covers much of the same material. Math 156 (Applied Honors Calc II) is more application based, but covers much of the same material.

Subsequent Courses:
Math 285 (Honors Calc III) is the natural sequel.

Math 201 - Introduction to Mathematical Writing

Prerequisites: Math 115 and intellectual curiosity
Frequency: Fall (I), Winter (II)
Student Body: Prospective mathematics concentrators.
Credit: 1 Credit. Course is designed to be taken concurrently with Math 217. Repeatable.

Background and Goals:
This is a course about the nuts and bolts of mathematical writing. The course introduces the fundamentals of mathematical communication (e.g., sets, functions, quantifiers) and explores various proof techniques (e.g., contrapositive, contradiction, induction). Most importantly, it provides guided practice in rudimentary proof writing.

Content:
Topics include elementary set theory, functions, existential and universal quantifiers, and proof techniques.

Alternatives:
None

Subsequent Courses:
217, 295, 185

Math 205 - Calculus of Several Variables

Prerequisites: Math 116, 156, 176, 186, or 296
Frequency: Winter (II)
Student Body: First and Second-year students
Credit: 4 Credits. Credit exclusion: 0 credits granted after 215 or 285
Recent Texts: None

Background and Goals:
This is a multivariable calculus course that is an alternative to Math 215 for students intending to concentrate in Math, Stats or the Social Sciences. Topics covered include graphs, limits, continuity, and partial derivatives of functions of several variables; vectors; optimization including Lagrange multipliers; line and volume integrals; and Green's Theorem.

Content:
Topics covered include vector algebra and vector functions; basic matrix operations; basic analytic geometry of planes, surfaces, and solids; functions of several variables, including partial differentiation, gradients, differentials, two variable Taylor polynomials of degrees one and two, and constrained and unconstrained optimization; epsilon-delta definitions of limits, continuity, and differentiability of functions of several variables; line and volume integrals and applications; vector fields; and Green's Theorem. The course includes regular use of mathematical software for visualization. The focus in general is on concepts and solving problems rather than theory and proof. Connections to familiar concepts from single variable calculus and multivariable applications to probability and economics are emphasized.

Alternatives:
Math 215 (Multivariable and Vector Calculus) and Math 285 (Honors Multivariable and Vector Calculus)

Subsequent Courses:
Natural sequels are Math 217 (Linear Algebra) or Math 425 (Introduction to Probability)

Math 214 - Applied Linear Algebra

Prerequisites: Math 116, 156, 176, 186, or 296
Frequency: Fall (I), Winter (II)
Student Body: Engineering students, particularly in Industrial and Operations Engineering or Computer Science.
Credit: 4 Credits. No credit granted to those who have completed or are enrolled in Math 217, 417, 419, or 420. Math 214 may not be used for the Statistics major

Background and Goals:
An introduction to matrices and linear algebra. This course covers the basics needed to understand a wide variety of applications that use the ideas of linear algebra, from linear programming to mathematical economics. The emphasis is on concepts and problem solving. The sequence 214-215 is not for math majors. It is designed as an alternate to the sequence 215-216 for engineering students who need more linear algebra and less differential equations background.

Content:
An introduction to the main concepts of linear algebra… matrix operations, echelon form, solution of systems of linear equations, Euclidean vector spaces, linear combinations, independence and spans of sets of vectors in Euclidean space, eigenvectors and eigenvalues, similarity theory. There are applications to discrete Markov processes, linear programming, and solutions of linear differential equations with constant coefficients.

Alternatives:
Math 217 (Linear Algebra) is a more theoretical course which covers much of the material of Math 214 at a deeper level. Math 419 Linear Spaces & Matrix Theory) has a more theoretical emphasis. Mathematics majors are required to take Math 217.

Subsequent Courses:
Math 561 (Linear Programming I), Math 462 (Math Models), Math 571 (Numerical Linear Algebra) , Math 420 (Advanced Linear Algebra).

Math 215 - Multivariable & Vector Calculus

Prerequisites: Math 116, 156, 176, 186, or 296
Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)
Student Body: Sophomores.
Credit: 4 Credits. Credit is granted for only one course among Math 215 and 285. Credit Exclusion: 2 credits granted after Math 205.

Background and Goals:
The sequence Math 115-116-215 is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof.

Content:
Topics include vector algebra and vector functions; analytic geometry of planes, surfaces, and solids; functions of several variables, partial differentiation, and optimization techniques including the method of Lagrange multipliers; line, surface, and volume integrals and applications; vector fields and integration; Green's Theorem, Stokes' Theorem, and Gauss' Theorem. There is a weekly computer lab.

Alternatives:
Math 285 (Honors Calc III) is a somewhat more theoretical course which covers the same material.

Subsequent Courses:
For students intending to concentrate in mathematics or who have some interest in the theory of mathematics as well as its applications, the appropriate sequel is Math 217 (Linear Algebra). Students who intend to take only one further mathematics course and who need a linear algebra course should take Math 214; those who need differential equations should take Math 216.

Math 216 - Introduction to Differential Equations

Prerequisites: Math 116, 156, 176, 186, or 296
Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)
Student Body: Sophomore engineering students.
Credit: 4 Credits. Credit is granted for only one course among Math 216, 286, and 316.

Background and Goals:
For a student who has completed the calculus sequence, there are two sequences which deal with linear algebra and differential equations: 216&417 (or 419) and 217&316. The sequence 216&417 emphasizes problem-solving and applications and is intended for students of engineering and the sciences. Mathematics majors and other students who have some interest in the theory of mathematics should elect the sequence 217&316.

Content:
Math 216 is a basic course on differential equations, intended for engineers and other scientists who need to apply the techniques in their work. The lectures are accompanied by a computer lab and recitation section where students have the opportunity to discuss problems and work through computer experiments to further develop their understanding of the concepts of the class. Topics covered include some material on complex numbers and matrix algebra, first and second order linear and non-linear systems with applications, introductory numerical methods, and elementary Laplace transform techniques.

Alternatives:
Math 286 (Honors Diff. Equations) covers much of the same material, but with much more emphasis on theory. The sequence Math 217&316 covers all of this material and substantially more at greater depth and with greater emphasis on the theory.

Subsequent Courses:
Math 404 (Intermediate Diff. Equations) covers further material on differential equations. Math 217 (Linar Algebra) and Math 417 (Matrix Algebra I) cover further material on linear algebra. Math 371/Engin 303 (Numerical Methods) and Math 471/472 (Intro to Numerical Methods/Numerical Methods with Financial Applications) cover additional material on numerical methods.

Math 217 - Linear Algebra

Prerequisites: Math 205, 215 or 285
Frequency: Fall (I), Winter (II), Spring (IIIa)
Student Body: Sophomore prospective mathematics concentrators.
Credit: 4 Credits. No credit granted to those who have completed or are enrolled in Math 419 or 420. 2 credits granted to those who have completed Math 214 or 417.

Background and Goals:
For a student who has completed the calculus sequence, there are two sequences which deal with linear algebra and differential equations: Math 216&417 (or 419) and Math 217&316. The sequence 216&417 emphasizes problem-solving and applications and is intended for students of Engineering and the sciences. Mathematics majors and other students who have some interest in the theory of mathematics should elect the sequence 217&316. These courses are explicitly designed to introduce the student to both the concepts and applications of their subjects and to the methods by which the results are proved.

Content:
This course is taught in the Inquiry Based Learning style. The topics covered include: systems of linear equations; matrix algebra; vectors, vector spaces, and subspaces; geometry of $\mathbf{R}^n$; linear dependence, bases, and dimension; linear transformations; Eigenvalues and Eigenvectors; diagonalization; inner products; the spectral theorem. Throughout there will be an emphasis on the concepts, logic, and methods of theoretical mathematics.

Alternatives:
Math 214 (Applied Linear Algebra), 417 (Matrix Algebra I), and 419 (Linear Spaces & Matrix Theory) cover similar material with more emphasis on computation and applications and less emphasis on proofs.

Subsequent Courses:
The intended course to follow Math 217 is Math 316 (Diff. Equations). Math 217 is also prerequisite for Math 312 (Applied Modern Algebra), Math 412 (Intro to Modern Algebra), and all of the more advanced courses in mathematics.

Special Note: The course Math 201 is designed to be taken concurrently with, or prior to, Math 217.

Math 285 - Honors Multivariable & Vector Calculus

Prerequisites: Math 156, 176, 186, or permission of instructor
Frequency: Fall (I), Winter (II)
Student Body: Sophomores and first-year students with suitable background.
Credit: 4 Credits. Credit is granted for only one course among Math 215 and 285. Credit Exclusion: 2 credits granted after Math 205.

Background and Goals:
The sequence Math 185-186-285-286 is an introduction to calculus at the honors level. It is taken by students intending to major in mathematics, science, or engineering as well as students heading for many other fields who want a somewhat more theoretical approach. Although much attention is paid to concepts and solving problems, the underlying theory and proofs of important results are also included. This sequence is not restricted to students enrolled in the LS&A Honors Program.

Content:
Topics include vector algebra and vector functions; analytic geometry of planes, surfaces, and solids; functions of several variables and partial differentiation, maximum-minimum problems; line, surface, and volume integrals and applications; vector fields and integration; curl, divergence, and gradient; Green's Theorem and Stokes' Theorem. Additional topics may be added at the discretion of the instructor.

Alternatives:
Math 215 (Calc III) is a less theoretical course which covers the same material.

Subsequent Courses:
Math 216 (Intro to Diff. Equations), Math 286 (Honors Diff. Equations), or Math 217 (Linear Algebra).

Math 286 - Honors Differential Equations

Prerequisites: Math 285 or permission of instructor
Frequency: Fall (I), Winter (II)
Student Body: Sophomores and first year students with suitable background.
Credit: 3 Credits. Credit is granted for only one course among Math 216, 286, and 316.

Background and Goals:
The sequence Math 185-186-285-286 is an introduction to calculus at the honors level. It is taken by students intending to major in mathematics, science, or engineering as well as students heading for many other fields who want a somewhat more theoretical approach. Although much attention is paid to concepts and solving problems, the underlying theory and proofs of important results are also included. This sequence is not restricted to students enrolled in the LS&A Honors Program.

Content:
Topics include first-order differential equations, higher-order linear differential equations with constant coefficients, an introduction to linear algebra, linear systems, the Laplace Transform, series solutions, and other numerical methods (Euler, Runge-Kutta). If time permits, Picard's Theorem will be proved.

Alternatives:
Math 216 (Intro to Diff. Equations) and Math 316 (Diff. Equations) cover much of the same material.

Subsequent Courses:
Math 471 (Intro to Numerical Methods) and/or Math 572 (Numerical Methods for Diff. Equations) are natural sequels in the area of differential equations.

Math 289 - Problem Solving

Prerequisites: Permission
Frequency: Fall (I), Winter (II)
Student Body: Undergraduate students insterested in learning to solve problems.
Credit: 1 Credit. May be repeated for credit

Background and Goals:
One of the better ways to develop mathematical abilities is by solving problems using a variety of methods. Familiarity with numerous methods is a great asset to the developing student of mathematics. Methods learned in attacking a specific problem frequently find application in many other areas of mathematics. In many instances an interest in and appreciation of mathematics is better developed by solving problems than by hearing formal lectures on specific topics. This course is intended for superior students who have exhibited both ability and interest in doing mathematics, but it is not restricted to honors students. This course is excellent preparation for the Putnam competition.

Content:
Students and one or more faculty and graduate student assistants will meet in small groups to explore problems in many different areas of mathematics. Problems will be selected according to the interests and background of the students.

Alternatives:
None

Subsequent Courses:
This course may be repeated for credit.

Math 295 - Honors Mathematics I

Prerequisites: Permission of honors advisor
Frequency: Fall (I)
Student Body: First-year students.
Credit: 4 Credits.

Background and Goals:
Math 295-296-395-396 is the most theoretical and demanding honors math sequence. The emphasis is on concepts, problem solving, as well as the underlying theory and proofs of important results. It provides an excellent background for advanced courses in mathematics. The expected background is high school trigonometry and algebra (previous calculus is not required, but is helpful.) This sequence is **not** restricted to students enrolled in the LS&A Honors program. Math 295 and 296 may be substituted for any Math 451 requirement. Math 296 and 395 may be substituted for any Math 217 requirement.

Content:
Axioms of the real numbers, completeness and connectedness in the real line. Functions of a real variable, limits and continuity, uniform continuity, extreme and intermediate value theorems, differentiation, integration, the fundamental theorem of calculus, Taylor's theorem with remainder.

Alternatives:
Math 156 (Applied Honors Calc II), Math 175 (Intro to Cryptology), and Math 185 (Honors Calc I) are alternative honors courses.

Subsequent Courses:
Math 296 (Honors Math II)

Math 296 - Honors Mathematics II

Prerequisites: Math 295
Frequency: Winter (II)
Student Body: First-year students.
Credit: 4 Credits.

Background and Goals:
Math 295-296-395-396 is the most theoretical and demanding honors calculus sequence. The emphasis is on concepts, problem solving, as well as the underlying theory and proofs of important results. It provides an excellent background for advanced courses in mathematics. The expected background is high school trigonometry and algebra (previous calculus is not required, but is helpful.) This sequence is **not** restricted to students enrolled in the LS&A Honors program. Math 295 and 296 may be substituted for any Math 451 requirement. Math 296 and 395 may be substituted for any Math 217 requirement.

Content:
Sequences and series of functions, power series, uniform convergence, real analytic functions. Vector spaces, bases, linear transformations, dual spaces, determinants, traces, eigenvalues, inner-product spaces, spectral theory. Limits and continuity in Euclidean space, derivative as a linear map, Chain Rule, inverse/implicit function theorems.

Alternatives:
None

Subsequent Courses:
Math 395 (Honors Analysis I)

Math 297 - Introduction to Analysis

Prerequisites: A in Math 217 or permission of the Undergraduate
Program Director
Frequency: Winter (II)
Student Body: First and Second-year students.
Credit: 4 Credits.

Background and Goals:
This is a course in analysis for students who know how to write rigorous
mathematical arguments and possess a firm understanding of the standard
concepts of linear algebra. It is specifically designed for students who ex-
celled in Math 217, love mathematics, and wish to transition into the Honors
Analysis Sequence.

Content:
This is a course in real analysis for students possessing both a firm under-
standing of how to read and write rigorous mathematical arguments and a
solid understanding of the standard concepts of linear algebra at the level
of Math 217. Topics covered include: axioms of the real numbers; complete-
ness, compactness, and connectedness for finite dimensional inner product
spaces; sequences, series, and limits in inner product spaces; continuity and
uniform continuity for functions of finite dimensional inner product spaces;
the extreme and intermediate value theorems, differentiation, integration,
the fundamental theorem of calculus, and Taylor's theorem with remainder
for functions of one real variable. The emphasis is on concepts, problem
solving, and the underlying theory and proofs. It provides an excellent back-
ground for advanced courses in mathematics.

Alternatives:
None

Subsequent Courses:
Math 395 (Honors Analysis I)

Math 310 - Chance and Choice

Prerequisites: Sophomore standing and one previous university math course
Frequency: Winter (II)
Student Body: For students who want a conceptual and analytical introduction to probability & its basic notions.
Credit: 3 Credits.

Background and Goals:
The course content is accessible to students with a strong precalculus background. However, we recommend some calculus background. With its few prerequisites and broad interest, it is also an ideal course for students wanting to explore mathematical thinking at a higher level.

Content:
The course is a hands-on introduction to various topics in probability. These include basic probability and combinatorics, conditional probability, expectations, random walks, Poisson distributions and Markov chains. The fundamental, ancient, and deep concepts underlying these are random-ness, fairness, coincidence, and bias. These are all important in theoretical probability, statistics and real world applications, and the course pursues these ideas from both conceptual and applied points of view.

The class is run in an IBL (inquiry based learning) format where students discuss and discover ideas and concepts by working in groups on handouts. There will be no lectures, no exams, and no textbook.

Alternatives:
None

Subsequent Courses:
None

Math 312 - Applied Modern Algebra

Prerequisites: Math 217
Frequency: Fall (I)
Student Body: Sophomore and Junior mathematics and computer science majors.
Credit: 3 Credits. No credit for those who have completed or are enrolled in Math 412.

Background and Goals:
One of the main goals of the course (along with every course in the algebra sequence) is to expose students to rigorous, proof-oriented mathematics. Students are required to have taken Math 217, which should provide a first exposure to this style of mathematics. A distinguishing feature of this course is that the abstract concepts are not studied in isolation. Instead, each topic is studied with the ultimate goal being a real-world application.

Content:
Sets and functions, relations and graphs, rings, Boolean algebras, semi-groups, groups, and lattices. Applications from areas such as switching, automata, and coding theory, and may include finite and minimal state machines, algebraic decompositions of logic circuits, semigroup machines, binary codes, and series and parallel decomposition of machines.

Alternatives:
Math 412 (Intro to Modern Algebra) is a more abstract and proof-oriented course with less emphasis on applications and is better preparation for most subsequent mathematics courses.

Subsequent Courses:
Math 312 is one of the alternative prerequisites for Math 416 (Theory of Al-gorithms), and several advanced EECS courses make substantial use of the material of Math 312. Another good follow-up course is Math 475 (Elementary Number Theory).

Math 316 - Differential Equations

Prerequisites: Math 205, 215 or 285; and 217
Frequency: Fall (I), Winter (II)
Student Body: A mix of undergraduate mathematics, science, and
                  economics majors.
Credit: 3 Credits. Credit is granted for only one course
                  among Math 216, 286, and 316.

Background and Goals:
This is an introduction to differential equations for students who have studied linear algebra (Math 217). It treats techniques of solution (exact and approximate), existence and uniqueness theorems, some qualitative theory, and many applications. Proofs are given in class; homework problems include both computational and more conceptually oriented problems.

Content:
First-order equations: solutions, existence and uniqueness, and numerical techniques; linear systems: eigenvector-eigenvalue solutions of constant coefficient systems, fundamental matrix solutions, nonhomogeneous systems; higher-order equations, reduction of order, variation of parameters, series solutions; qualitative behavior of systems, equilibrium points, stability. Applications to physical problems are considered throughout.

Alternatives:
Math 216 (Intro to Diff. Equations) covers somewhat less material without presupposing linear algebra and with less emphasis on theory. Math 286 (Honors Diff. Equations) is the honors version of Math 316.

Subsequent Courses:
Math 471 (Intro to Numerical Methods) and/or Math 572 (Numerical Methods for Diff. Equations) are natural sequels in the area of differential equations, but Math 316 is also preparation for more theoretical courses such as Math 451 (Advanced Calc I).

Math 351 - Principles of Analysis

Prerequisites: Math 205, 215, or 285; and 217 or permission of instructor.
Frequency: Fall (I), Winter (II)
Student Body: Sophomores and Juniors.
Credit: 3 Credits. No credit granted to those who have completed or are enrolled in Math 451.
Area: Analysis

Background and Goals:
This course has two goals: 1) a rigorous development of the ideas underlying calculus and 2) to enhance the student's ability to understand and write mathematical proofs. The course content is similar to that of Math 451, but the course is taught in an inquiry-based manner.

Content:
Logic and techniques of proof, sequences, continuous functions, uniform continuity, differentiation, integration, and the Fundamental Theorem of Calculus.

Alternatives:
Math 451 (Advanced Calc I) covers similar topics while assuming more background than 351.

Subsequent Courses:
None

Math 354 - Fourier Analysis and its Applications

Prerequisites: Math 216, 286, or 316
Frequency: Fall (I), Winter (II)
Student Body: Junior and Senior math and non-math majors.
Credit: 3 Credits. No credit granted to those who have completed or are enrolled in Math 450 or 454.
Area: Analysis

Background and Goals:
This course is an introduction to Fourier analysis with emphasis on applications. The course also can be viewed as a way of deepening one's understanding of the 100-and 200-level material by applying it in interesting ways.

Content:
This is an introduction to Fourier Analysis geared towards advanced undergraduate students from both pure and applied areas. It should be particularly suitable for majors in the sciences and engineering. Topics will include properties of complex numbers, the Discrete Fourier Transform, Fourier series, the Dirichlet and Fejer kernals, convolutions, approximations by trigonometric polynomials, uniqueness of Fourier coefficients, Parseval's identity, properties of trigonometric polynomials, absolutely convergent Fourier series, convergence of Fourier series, applications of Fourier series, and the Fourier transform, including the Poisson summation formula and Plancherel's identity. While the main effort will be to establish the foundations of the subject, applications will include the Fast Fourier Transform, the heat equation, the wave equation, sampling, and signal processing.

Alternatives:
Math 454 covers some of the same material with more emphasis on partial differential equations.

Subsequent Courses:
This course is good preparation for Math 451 (Advanced Calc I), which covers the theory of calculus in a mathematically rigorous way.

Math 371 (Engin 371) - Numerical Methods

Prerequisites: Engin 101; and one of Math 216, 286, or 316; and one of Math 214, 217, 417, or 419.
Frequency: Fall (I), Winter (II)
Student Body: Sophomore, Junior, and Senior engineering students.
Credit: 3 Credits. No credit after Math 471 or Math 472.
Area: Applied/NA

Background and Goals:
This is a survey course of the basic numerical methods which are used to solve practical scientific problems. Important concepts such as accuracy, stability, and efficiency are discussed. The course provides an introduction to MATLAB, an interactive program for numerical linear algebra, and may provide practice in FORTRAN programming and the use of software library subroutines. Convergence theorems are discussed and applied, but the proofs are not emphasized.

Content:
Floating point arithmetic, Gaussian elimination, polynomial interpolation, spline approximations, numerical integration and differentiation, solutions to non-linear equations, ordinary differential equations, polynomial approximations. Other topics may include discrete Fourier transforms, two-point boundary-value problems, and Monte-Carlo methods.

Alternatives:
Math 471 (Numerical Analysis) provides a more in-depth study of the same topics, with a greater emphasis on analyzing the accuracy and stability of the numerical methods. Math 571 (Numerical Linear Algebra) is a detailed study of the solution of systems of linear equations and eigenvalue problems, with some emphasis on large-scale problems. Math 572 (Numerical Methods for Diff. Equations) covers numerical methods for both ordinary and partial differential equations. Math 571 and 572 can be taken in either order.

Subsequent Courses:
This course is basic for many later courses in science and engineering. It is good background for 571-572.

Math 389 - Explorations in Mathematics

Prerequisites: Reasonable familiarity with proofs at the level of
Math 217 or 295 expected
Frequency: Winter (II)
Student Body: Undergraduates.
Credit: 3 Credits.

Background and Goals:
The course is designed to show how new mathematics is actually created:
how to take a problem, make models and experiment with them, and search
for underlying structure. The format involves little formal lecturing, much
laboratory work, and student presentations discussing partial results and ap-
proaches. Course website: https://www.math.lsa.umich.edu/courses/389/

Content:
Problems for projects are drawn from a wide variety of mathematical areas,
pure and applied. Problems are chosen to be accessible to undergraduates.

Alternatives:
None

Subsequent Courses:
 After this course students should be ready for a variety of courses and
research experiences.

Math 395 - Honors Analysis I

Prerequisites: Math 296 or 297
Frequency: Fall (I)
Student Body: First-year students and sophomores.
Credit: 4 Credits.

Background and Goals:
This course is a continuation of the sequence Math 295-296 and has the same theoretical emphasis. Students are expected to understand and construct proofs.

Content:
Inverse/implicit function theorems, immersion/submersion theorems. Quotient and dual spaces, inner product spaces, spectral theory. Metric spaces, basic point-set topology. Integration in Euclidean space, Fubini's theorem, change of variables formula. Topics in linear algebra: tensor products, exterior and symmetric powers, Jordan and rational canonical forms.

Alternatives:
None

Subsequent Courses:
Math 396 (Honors Analysis II).

Math 396 - Honors Analysis II

Prerequisites: Math 395
Frequency: Winter (II)
Student Body: Sophomores.
Credit: 4 Credits.

Background and Goals:
This course is a continuation of Math 395 and has the same theoretical emphasis. Students are expected to understand and construct proofs.

Content:
Submanifolds (with or without corners) of Euclidean space, abstract manifolds, tangent and cotangent spaces, immersion/submersion theorems. Partitions of unity, vector fields and differential forms on manifolds, exterior differentiation, integration of differential forms. Stokes' theorem. deRham cohomology, Riemannian metrics, Hodge star operator and the standard vector calculus versions of Stokes' theorem.

Alternatives:
None

Subsequent Courses:
Students who have successfully completed the sequence Math 295-396 are generally prepared to take a range of advanced undergraduate and graduate courses such as Math 493 (Honors Algebra I), Math 525 (Probability Theory), Math 590 (Intro to Topology), and many others.

Math 403 (DATASCI 403) - Intro to Discrete Mathematics

Prerequisites: Linear Algebra (one of MATH 214, 217, 286, 296, 417, or 419) or instructor permission.
Frequency: Fall (I), Winter (II)
Student Body: Data Science Masters students and undergrad students who are not math majors
Credit: 3 Credits. No credit granted to those who have completed or are enrolled in MATH 465, 565, or 566.

Background and Goals:

This course is intended for students in the Data Science Masters program, or who are undergraduates that are not mathematics majors. This course will not count towards the Mathematics Major.

Content:

Propositional logic; quantifiers; basic logical deduction rules; fundamental properties of natural numbers, especially mathematical induction and well-ordering; sets; set operations and their algebraic properties; functions, including properties like injectivity and surjectivity; relations; partial and total orders; equivalence relations and partitions; elementary combinatorics, including permutations, binomial coefficients, and inclusion-exclusion; graphs; and discrete probability, including conditional probability, Bayes's theorem, independence, expectation, variance, and standard deviation.

Alternatives:

Mathematics majors should elect Math 465.

Math 404 - Intermediate Differential Equations

Prerequisites: Math 216, 286, or 316
Frequency: Fall (I), Winter (II)
Student Body: Undergraduate and graduate students from engineering and LS&A.
Credit: 3 Credits.
Area: Applied/NA

Background and Goals:
This course is an introduction to the modern qualitative theory of ordinary differential equations with emphasis on geometric techniques and visualization. Much of the motivation for this approach comes from applications. Examples of applications of differential equations to science and engineering are a significant part of the course. There are relatively few proofs.

Content:
Geometric representation of solutions, autonomous systems, flows and evolution, linear systems and phase portraits, nonlinear systems, local and global behavior, linearization, stability, conservation laws, periodic orbits. Applications: free and forced oscillations, resonance, relaxation oscillations, competing species, Zeeman's models of heartbeat and nerve impulse, chaotic orbits, strange attractors.

Alternatives:
Math 558 (Applied Nonlinear Dynamics) covers some of the same material at a faster pace and includes additional topics.

Subsequent Courses:
Math 454 (Boundary Value Problems for Partial Diff. Equations) is a natural sequel.

Math 412 - Introduction to Modern Algebra

Prerequisites: Math 205, 215 or 285; and Math 217
Frequency: Fall (I), Winter (II)
Student Body: Mainly undergraduate mathematics concentrators with some graduate students from other departments.
Credit: 3 Credits. 1 credit after Math 312
Area: Algebra

Background and Goals:
This course is designed to serve as an introduction to the methods and concepts of abstract mathematics. A typical student entering this course has substantial experience in using complex mathematical (calculus) calculations to solve physical or geometrical problems, but is inexperienced at analyzing carefully the content of definitions and the logical flow of ideas which underlie and justify these calculations. Although the topics discussed here are quite distinct from those of calculus, an important goal of the course is to introduce the student to this type of analysis. Much of the reading, homework exercises, and exams consists of theorems (propositions, lemmas, etc.) and their proofs. Math 217, or equivalent, required as background.

Content:
The initial topics include ones common to every branch of mathematics: sets, functions (mappings), relations, and the common number systems (integers, rational numbers, real numbers, complex numbers). These are then applied to the study of particular types of mathematical structures: groups, rings, and fields. These structures are presented as abstractions from many examples such as the common number systems together with the operations of addition or multiplication, permutations of finite and infinite sets with function composition, sets of motions of geometric figures, and polynomials. Notions such as generator, subgroup, direct product, isomorphism, and homomorphism are defined and studied.

Alternatives:
Math 312 (Applied Modern Algebra) is a less abstract course which replaces some of the material on rings and fields of Math 412 with additional applications to areas such as switching and coding theory.

Subsequent Courses:
A student who successfully completes this course will be prepared to take a number of other courses in abstract mathematics: Math 416 (Theory of Algorithms), Math 451 (Advanced Calc I), Math 475 (Elem. Number Theory), Math 575 (Intro to Theory of Numbers), Math 493/494 (Honors Algebra I/II), Math 481 (Intro to Math Logic), and Math 582 (Intro to Set Theory). All of these courses will extend and deepen the student's grasp of modern abstract mathematics.

Math 416 - Theory of Algorithms

Prerequisites: Math 312, 412; or EECS 280 and Math 465; or
permission of instructor
Frequency: Sporadically
Student Body: Largely computer science majors with a few
graduate students from other fields.
Credit: 3 Credits.

Background and Goals:
Many common problems from mathematics and computer science may
be solved by applying one or more algorithms — well-defined procedures
that accept input data specifying a particular instance of the problem and
produce a solution. Students entering Math 416 typically have encountered
some of these problems and their algorithmic solutions in a programming
course. The goal here is to develop the mathematical tools necessary to
analyze such algorithms with respect to their efficiency (running time) and
correctness. Different instructors will put varying degrees of emphasis on
mathematical proofs and computer implementation of these ideas.

Content:
Typical problems considered are: sorting, searching, matrix multiplication,
graph problems (flows, traveling salesman), and primality and pseudo-
primality testing (in connection with coding questions). Algorithm types such
as divide-and-conquer, backtracking, greedy, and dynamic programming are
analyzed using mathematical tools such as generating functions, recurrence
relations, induction and recursion, graphs and trees, and permutations. The
course often includes a section on abstract complexity theory including NP
completeness.

Alternatives:
This course has substantial overlap with EECS 586 (Design & Analysis of
Algorithms) — more or less depending on the instructors. In general, Math
416 will put more emphasis on the analysis aspect in contrast to the design
of algorithms aspect.

Subsequent Courses:
EECS 574 (Computational Complexity) and 575 (Advanced Cryptography)
include some topics which follow those of this course.

Math 417 - Matrix Algebra I

Prerequisites: Three mathematics courses beyond Math 110
Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)
Student Body: Largely engineering and science students, both
                  undergraduate and graduate.
Credit: 3 Credits. No credit granted to those who have
                  completed or are enrolled in Math 214, 217, 419, or
                  420. Math 417 and Math 419 may **not** be used as
                  electives in the Statistics major.
Area: Algebra

Background and Goals:
Many problems in science, engineering, and mathematics are best formulat-
ed in terms of matrices — rectangular arrays of numbers. This course is an
introduction to the properties of and operations on matrices with a wide vari-
ety of applications. The main emphasis is on concepts and problem-solving,
but students are responsible for some of the underlying theory. Diversity
rather than depth of applications is stressed. This course is not intended
for mathematics majors, who should elect Math 217, and/or Math 493-494 if
pursuing the honors major.

Content:
Topics include matrix operations, echelon form, general solutions of systems
of linear equations, vector spaces and subspaces, linear independence and
bases, linear transformations, determinants, orthogonality, characteristic
polynomials, eigenvalues and eigenvectors, and similarity theory. Applica-
tions include linear networks, least squares method (regression), discrete
Markov processes, linear programming, and differential equations.

Alternatives:
Math 419 (Linear Spaces & Matrix Theory) is an enriched version of Math
417 with a somewhat more theoretical emphasis. Math 217 (Linear Algebra)
is also a more theoretical course which covers much of the material of 417
at a deeper level (despite its lower number).

Subsequent Courses:
This course serves as prerequisite to several courses: Math 452 (Advanced
Calc II), Math 462 (Math Models), Math 420 (Advanced Linear Algebra, and
Math 571 (Numerical Linear Algebra).

Math 419 - Linear Spaces and Matrix Theory

Prerequisites: 4 courses beyond Math 110
Frequency: Fall (I), Winter (II)
Student Body: Largely engineering graduate students and under
graduates; some mathematics undergraduates.
Credit: 3 Credits. Credit is granted for only one course
among Math 214, 217, 417, and 419. No
credit granted to those who have completed or are
enrolled in Math 420. Math 417 and Math 419 may
not be used as electives in the Statistics major.
Area: Algebra

Background and Goals:
Math 419 covers much of the same ground as Math 417 but presents the
material in a somewhat more abstract way in terms of vector spaces and
linear transformations instead of matrices. There is a mix of proofs, calcu-
lations, and applications with the emphasis depending somewhat on the
instructor. A previous proof-oriented course is helpful but by no means
necessary.

Content:
Basic notions of vector spaces and linear transformations: spanning, linear
independence, bases, dimension, matrix representation of linear transfor-
mations; determinants; eigenvalues, eigenvectors, Jordan canonical form,
inner-product spaces; unitary, self-adjoint, and orthogonal operators and
matrices, applications to differential and difference equations.

Alternatives:
Math 417 (Matrix Algebra I) is less rigorous and theoretical and more ori-
ented to applications. Math 217 (Linear Algebra) is similar to Math 419 but
more proof-oriented. Math 420 (Advanced Linear Algebra) is much more
abstract and sophisticated.

Subsequent Courses:
This course serves as prerequisite to several courses: Math 420 (Advanced
Linear Algebra), Math 452 (Advanced Calc II), Math 462 (Math Models),
Math 561 (Linear Programming I), and Math 571 (Numerical Linear Algebra).

Math 420 - Advanced Linear Algebra

Prerequisites: Math 214, 217, 417, or 419 and one of 296, 412, or 451
Frequency: Fall (I)
Student Body: Graduate students who pursue, or may pursue, the Mathematics masters program and undergraduate mathematics majors. This is a required course for the MS degree in mathematics.
Credit: 3 Credits.
Area: Algebra

Background and Goals:
This is an introduction to the formal theory of abstract vector spaces and linear transformations. It is expected that students have completed at least one prior linear algebra course. The emphasis is on concepts and proofs with some calculations to illustrate the theory. Students should have significant mathematical maturity, at the level of Math 412 or 451. In particular, students should expect to work with and be tested on formal proofs.

Content:
Topics are selected from: vector spaces over arbitrary fields (including finite fields); linear transformations, bases, and matrices; inner product spaces, duals and spaces of linear transformations, theory of determinants, eigenvalues and eigenvectors; applications to linear differential equations; bilinear and quadratic forms; spectral theorem; Jordan Canonical Form, least squares, singular value theory.

Alternatives:
None.

Subsequent Courses:
The natural sequel to Math 420 is Math 593 (Algebra I).

Math 422 (BE 440) - Risk Management and Insurance

Prerequisites: Math 115, Junior standing, and permission of instructor
Frequency: Sporadically
Student Body: Junior and Senior mathematics majors; some business undergraduates.
Credit: 3 Credits. Satisfies the Upper Level Writing Requirement (URLW).
Area: Actuarial & Financial

Background and Goals:
This course is designed to allow students to explore the insurance mechanism as a means of replacing uncertainty with certainty. A main goal of the course is to explain, using mathematical models from the theory of interest, risk theory, credibility theory, and ruin theory, how mathematics underlies many important individual and societal problems.

Content:
We will explore how much insurance affects the lives of students (automobile insurance, social security, health insurance, theft insurance) as well as the lives of other family members (retirements, life insurance, group insurance). While the mathematical models are important, an ability to articulate why the insurance options exist and how they satisfy the consumer's needs are equally important. In addition, there are different options available (e.g., in social insurance programs) that offer the opportunity of discussing alternative approaches. This course may be used to satisfy the LS&A upper-level writing requirement.

Alternatives:
None

Subsequent Courses:
None

Math 423 - Mathematics of Finance

Prerequisites: Math 217, Math 425 OR Math 525, and EECS 183
or equivalent
Frequency: Fall (I), Winter (II), Spring (IIIa)
Student Body: Junior and Senior mathematics majors; some
business undergraduates.
Credit: 3 Credits.
Area: Actuarial & Financial

Background and Goals:
This course is an introduction to the mathematical models used in finance and economics with particular emphasis on models for pricing derivative instruments such as options and futures. The goal is to understand how the models derive from basic principles of economics and to provide the necessary mathematical tools for their analysis. A solid background in basic probability theory is necessary.

Content:
Topics include risk and return theory, portfolio theory, the capital asset pricing model, the random walk model, stochastic processes, Black-Scholes Analysis, numerical methods, and interest rate models.

Alternatives:
None

Subsequent Courses:
Math 523 (Loss Models I) and Math 623 (Computational Finance).

Math 424 - Math Interest Theory

Prerequisites: Math 205, 215 or 285
Frequency: Fall (I), Winter (II)
Student Body: Undergraduate and graduate students in the
                 Actuarial Mathematics Program, or students
                 interested in exploring the concepts underlying the
                 theory of interest.
Credit: 3 Credits.
Area: Actuarial & Financial

Background and Goals:
This course explores the concepts underlying the theory of interest and then
applies them to concrete problems. The course also includes applications
of spreadsheet software. The course is a prerequisite to advanced actuarial
courses. It also helps students prepare for some of the professional actu-
arial exams.

Content:
This course covers interest theory and its application to valuation of mon-
etary deposits, annuities, loans, and bonds. Interest compounding and pay-
ments may be discrete or continuous. Other topics include yield rates, spot
rates, forward rates, duration, convexity, immunization, and asset-liability
matching.

Alternatives:
Math 424 is required for students concentrating in actuarial mathematics;
others may take Math 147 (Intro to Interest Theory), which deals with the
same techniques but with less emphasis on continuous growth situations.

Subsequent Courses:
Math 520 (Life Contingencies I) applies the concepts of Math 424 and
probability theory to the valuation of life contingencies (death benefits and
pensions).

Math 425 (Stats 425) - Introduction to Probability

Prerequisites: Math 205, 215 or 285
Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)
Student Body: About 80% undergraduate mathematics, engineering, and computer science majors with a few graduate students.
Credit: 3 Credits.
Area: Analysis; Actuarial & Financial

Background and Goals:
This course introduces students to both useful and interesting ideas from the mathematical theory of probability and to a number of applications of probability to a variety of fields including genetics, economics, geology, business, and engineering. The theory developed together with other mathematical tools such as combinatorics and calculus are applied to everyday problems. Concepts, calculations, and derivations are emphasized. The course will make essential use of the material of Math 116 and 215.

Content:

Topics include the basic results and methods of both discrete and continuous probability theory: conditional probability, independent events, random variables, joint distributions, expectations, variances, and covariances. The culminating results are the Law of Large Numbers and the Central Limit Theorem. Beyond this, different instructors may add additional topics of interest.

Alternatives:

Math 525 (Probability Theory) is a similar course at a faster pace and with deeper coverage. A stronger mathematical background is helpful for Math 525.

Subsequent Courses:
Stats 426 (Intro to Theoretical Statistics) is a natural sequel for students. Math 423 (Math of Finance) and Math 523 (Loss Models I) include many applications of probability theory.

Math 427 - Retirement Plans and Other Employee Benefits

Prerequisites: Math 115, Junior standing or permission of instructor
Frequency: Sporadically
Student Body: Mainly Actuarial Math, some non-math students.
Credit: 3 Credits. Satisfies the Upper Level Writing Requirement (URLW).
Area: Actuarial & Financial

Background and Goals:
An overview of the range of employee benefit plans, the considerations (actuarial and others) which influence plan design and implementation practices, and the role of actuaries and other benefit plan professionals and their relation to decision makers in management and unions. This course is certified for satisfaction of the upper-level writing requirement.

Content:
Particular attention will be given to government programs which provide the framework, and establish requirements, for privately operated benefit plans. Relevant mathematical techniques will be reviewed, but are not the exclusive focus of the course.

Alternatives:
None

Subsequent Courses:
Math 521 (Life Contingencies II) and/or Math 522 (Actuarial Theory of Pensions and Social Security), which can be taken independently of each other, provide more in depth examination of the actuarial techniques used in employee benefit plans.

Math 429 - Internship

Prerequisites: Major in Mathematics
Frequency: Winter (II)
Student Body: International Mathematics Students with CTP
requirements for internships.
Credit: 1 Credit (Credit/No Credit grading scheme)
Recent Texts: None

Background and Goals:
Credit is granted for a full-time internship of at least eight weeks that is used to enrich a student's academic experience and/or allows the student to explore careers related to their academic studies. Internship credit is not retroactive and must be prearranged.

Content:
Course content is determined by student's internship.

Alternatives:
None

Subsequent Courses:
None

Math 431 - Explorations in Euclidian Geometry

Prerequisites: Math 205, 215, or 285; and Math 217
Frequency: Fall (I)
Student Body: Teaching certificate candidates and mathematics and engineering students.
Credit: 3 Credits.
Area: Teaching

Background and Goals:
This course deepens students' understanding of the structure and practice of mathematics through an axiomatic development of Euclidean geometry. Throughout the course, students will write proofs, make conjectures, communicate mathematics verbally and in writing, and critique mathematical arguments. At instructor discretion, examples of non-Euclidean geometries may be studied for comparison.

Content:
Topics selected depend heavily on the instructor but may include classification of isometries of the Euclidean plane; similarities; rosette, frieze, and wallpaper symmetry groups; tessellations; triangle groups; finite, hyperbolic, and taxicab non-Euclidean geometries.

Alternatives:
An alternative geometry course at this level is Math 433 (Intro to Diff. Geometry).

Subsequent Courses:
None

Math 433 - Introduction to Differential Geometry

Prerequisites: Math 205, 215 or 285; and Math 217
Frequency: Winter (II)
Student Body: Half undergraduate math majors, half graduate students from EECS and physics.
Credit: 3 Credits.
Area: Geometry/Topology

Background and Goals:
This course is about the analysis of curves and surfaces in 2- and 3-space using the tools of calculus and linear algebra. There will be many examples discussed, including some which arise in engineering and physics applications. Emphasis will be placed on developing intuition and learning to use calculations to verify and prove theorems. Students need a good background in multivariable calculus (Math 215) and linear algebra (preferably Math 217). Some exposure to differential equations (Math 216 or Math 316) is helpful but not absolutely necessary.

Content:
Curves and surfaces in three-space using calculus. Curvature and torsion of curves. Curvature, covariant differentiation, parallelism, isometry, geodesics, and area on surfaces. Gauss-Bonnet Theorem. Minimal surfaces.

Alternatives:
None

Subsequent Courses:
Math 635 (Diff. Geometry) and Math 636 (Topics in Diff. Geometry) further study Riemannian manifolds and their topological and analytic properties. Physics courses in general relativity and gauge theory will use some of the material of this course.

Math 440 - LoG(M) | Laboratory of Geometry

Prerequisites: Math 205, 215 or 285; and Math 217
Frequency: Fall (I), Winter (II)
Student Body: Undergraduate math majors and minors
Credit: 3 Credits.

Background and Goals:
The LoG-M course aims to provide undergraduates with mathematics research-type projects which focus on methods in computation and visualization. Projects differ each semester and are designed and led by experienced research faculty in the department. Undergrads are assigned to teams which work through one research problem throughout the semester.

*Until January 2021: Each of the first five undergraduates to report this sentence to the Undergraduate Math Office will receive a free math t-shirt.*

Content:
Varies, centered on geometry & visualization.

Alternatives:
Math 389 also looks at research-type projects but in a much more structured way.

Subsequent Courses:
None

Math 450 - Advanced Mathematics for Engineers I

Prerequisites: Math 216, 286, or 316
Frequency: Fall (I), Winter (II), Summer (IIIb)
Student Body: Mathematics and engineering students; engineering graduate students.
Credit: 4 Credits. No credit after Math 354 or Math 454.
Area: Analysis

Background and Goals:
This course is an introduction to some of the main mathematical techniques in engineering and physics. It is intended to provide some background for courses in those disciplines with a mathematical requirement that goes beyond calculus. Model problems in mathematical physics are studied in detail. Applications are emphasized throughout.

Content:
Topics covered include: Fourier series and integrals; the classical partial differential equations (the heat, wave and Laplace's equations) solved by separation of variables; an introduction to complex variables and conformal mapping with applications to potential theory. A review of series and series solutions of ODEs will be included as needed. A variety of basic diffusion, oscillation, and fluid flow problems will be discussed.

Alternatives:
This course overlaps with Math 454 (Boundary Variable Problems for Partial Diff. Equations) and, to a much lesser extent, with Math 555 (Intro to Complex Variables). The coverage of PDEs in Math 450 is not as in-depth as Math 454; for example, in 450 coverage of special functions is reduced to the simplest Bessel functions. Those students needing a more thorough discussion of PDEs and boundary-value problems should take 454. On the other hand, 450 provides a broader introduction to applied methods.

Subsequent Courses:
Math 555 (Intro to Complex Variables) and Math 556 (Applied Functional Analysis) are graduate-level courses that further develop both the theory and applications covered in 450.

Math 451 - Advanced Calculus I

Prerequisites: A thorough understanding of calculus and one of 217, 312, 412, or permission of instructor
Frequency: Fall (I), Winter (II), Spring (IIIa)
Student Body: Half undergraduate mathematics majors, half science and engineering graduate students.
Credit: 3 Credits. No credit after 351.
Area: Analysis

Background and Goals:
This course has two complementary goals: (1) a rigorous development of the fundamental ideas of calculus, and (2) a further development of the student's ability to deal with abstract mathematics and mathematical proofs. The key words here are "rigor" and "proof;" almost all of the material of the course is geared toward understanding and constructing definitions, theorems (propositions, lemmas, etc.), and proofs. This is considered one of the more difficult among the undergraduate mathematics courses, and students should be prepared to make a strong commitment to the course. In particular, it is strongly recommended that some course which requires proofs (such as Math 412) be taken before Math 451.

Content:
Topics covered include: logic and techniques of proofs; sets, functions, and relations; cardinality; the real number system and its topology; infinite sequences, limits, and continuity; differentiation; integration, the Fundamental Theorem of Calculus, infinite series; sequences and series of functions.

Alternatives:
Although Math 450 (Advanced Math for Engineers I) is an alternative prerequisite for some courses, the emphasis of the two courses is quite distinct. Math 351 (Principles of Analysis) covers similar topics.

Subsequent Courses:
The natural sequel to Math 451 is 452 (Advanced Calc II), which extends the ideas considered here to functions of several variables. In a sense, Math 451 treats the theory behind Math 115/116 (Calc I/II), while Math 452 does the same for Math 215 (Calc III) and a part of Math 216 (Intro to Diff. Equations). Math 451 is also a prerequisite for several other courses: Math 575 (Intro to Theory of Numbers), Math 590 (Intro to Topology), Math 596 (Analysis I - Complex), and Math 597 (Analysis II - Real).

Math 452 - Advanced Calculus II

Prerequisites: Math 217, 417, 419 or 420 (may be concurrent) and Math 351 or 451
Frequency: Winter (II)
Student Body: Mathematics undergraduates with some non-mathematics graduate students.
Credit: 3 Credits.
Area: Analysis

Background and Goals:
This course gives a rigorous development of multivariable calculus and elementary function theory with some view towards generalizations. Concepts and proofs are stressed. This is a relatively difficult course, but the stated prerequisites provide adequate preparation.

Content:
Topics include: (1) partial derivatives and differentiability; (2) gradients, directional derivatives, and the chain rule; (3) implicit function theorem; (4) surfaces, tangent planes; (5) max-min theory; (6) multiple integration, change of variable, etc.; (7) Greens' and Stokes' theorems, differential forms, exterior derivatives.

Alternatives:
None

Subsequent Courses:
Math 452 is good general background for any of the more advanced courses in analysis - Math 596 (Analysis I - Complex) , 597 (Analysis II - Real) or differential geometry or topology - Math 537 (Intro to Diff. Manifolds), 635 (Diff. Geometry).

Math 454 - Boundary Value Problems for Partial Differential Equations

Prerequisites: Math 205, 215 or 285; and Math 216, 286, or 316
Frequency: Fall (I), Winter (II), Spring (IIIa)
Student Body: Some mathematics undergraduates, but more non-mathematics graduate students.
Credit: 3 Credits. 1 credit after Math 354. No credit granted to those who have completed or are enrolled in Math 450.
Area: Applied/Numerical Analysis

Background and Goals:
This course is devoted to the use of Fourier series and other orthogonal expansions in the solution of initial-value and boundary-value problems for second-order linear partial differential equations. Emphasis is on concepts and calculation. The official prerequisite is ample mathematical preparation.

Content:
Classical representation and convergence theorems for Fourier series; method of separation of variables for the solution of the one-dimensional heat and wave equation; the heat and wave equations in higher dimensions; eigenfunction expansions; spherical and cylindrical Bessel functions; Legendre polynomials; methods for evaluating asymptotic integrals (Laplace's method, steepest descent); Laplace's equation and harmonic functions, including the maximum principle. As time permits, additional topics will be selected from: Fourier and Laplace transforms; applications to linear input-output systems, analysis of data smoothing and filtering, signal processing, time-series analysis, and spectral analysis; dispersive wave equations; the method of stationary phase; the method of characteristics.

Alternatives:
None

Subsequent Courses:
Math 454 is prerequisite to Math 571 (Numerical Linear Algebra) and Math 572 (Numerical Methods for Diff. Equations). Although it is not a formal prerequisite, it is good background for Math 556 (Applied Functional Analysis).

Math 462 - Mathematical Models

Prerequisites: Math 216, 286, or 316; and Math 217, 417, or 419
Frequency: Winter (II)
Student Body: Mainly Junior and Senior math majors; students from engineering, biology, physics, & medicine
Credit: 3 Credits. 1-3 credits after 463 depending on overlap
Area: Applied/Numerical Analysis

Background and Goals:
The focus of this course is the application of a variety of mathematical techniques to solve real-world problems. Students will learn how to model a problem in mathematical terms and use mathematics to gain insight and eventually solve the problem. Concepts and calculations, using applied analysis and numerical simulations, are emphasized.

Content:
Construction and analysis of mathematical models in physics, engineering, economics, biology, medicine, and social sciences. Content varies considerably with instructor. Recent versions: Use and theory of dynamical systems (chaotic dynamics, ecological and biological models, classical mechanics), and mathematical models in physiology and population biology.

Alternatives:
Students who are particularly interested in biology should consider Math 463 (Math Modeling in Biology).

Subsequent Courses:
Any higher-level course in differential equations.

Math 463 (Bioinf 463/Biophys 463) - Math Modeling in Biology

Prerequisites: Math 214, 217, 417, or 419; and 216, 286, or 316
Frequency: Fall (I)
Student Body: Juniors, Seniors, and first year graduate students (half engineering and half LS&A).
Credit: 3 Credits.
Area: Applied/Interdisciplinary

Background and Goals:

The complexities of the biological sciences make interdisciplinary involvement essential and the increasing use of mathematics in biology is inevitable as biology becomes more quantitative. Mathematical biology is a fast growing and exciting modern application of mathematics that has gained worldwide recognition. In this course, mathematical models that suggest possible mechanisms that may underlie specific biological processes are developed and analyzed. Another major emphasis of the course is illustrating how these models can be used to predict what may follow under currently untested conditions. The course moves from classical to contemporary models at the population, organ, cellular, and molecular levels. The goals of this course are: (i) Critical understanding of the use of differential equation methods in mathematical biology and (ii) Exposure to specialized mathematical and computational techniques which are required to study ordinary differential equations that arise in mathematical biology. By the end of this course students will be able to derive, interpret, solve, understand, discuss, and critique discrete and differential equation models of biological systems.

Content:

This course provides an introduction to the use of continuous and discrete differential equations in the biological sciences. Biological topics may include single species and interacting population dynamics, modeling infectious and dynamic diseases, regulation of cell function, molecular interactions and receptor-ligand binding, biological oscillators, and an introduction to biological pattern formation. There will also be discussions of current topics of interest such as Tumor Growth and Angiogenesis, HIV and AIDS, and Control of the Mitotic Clock. Mathematical tools such as phase portraits, bifurcation diagrams, perturbation theory, and parameter estimation techniques that are necessary to analyze and interpret biological models will also be covered. Approximately one class period each week will be held in the mathematics computer laboratory where numerical techniques for finding and visualizing solutions of differential and discrete systems will be discussed.

Alternatives:

None

Subsequent Courses:

Math 563 (Advanced Math Methods for the Biological Sciences).

Math 464 - Inverse Problems

Prerequisites: Math 217, 417, or 419 and Math 216, 286, or 316
Frequency: Sporadically
Student Body: Upper-level undergraduates and graduate students in mathematics, science, and engineering.
Credit: 3 Credits.
Recent Texts: None
Area: Applied/Numerical Analysis

Background and Goals:
Solution of an inverse problem is a central component of fields such as medical tomography, geophysics, non-destructive testing, and control theory. The solution of any practical inverse problem is an interdisciplinary task. Each such problem requires a blending of mathematical constructs and physical realities. Thus, each problem has its own unique components; on the other hand, there is a common mathematical framework for these problems and their solutions. This framework is the primary content of the course. This course will allow students interested in the above-named fields to have an opportunity to study mathematical tools related to the mathematical foundations of said fields.

Content:
The course content is motivated by a particular inverse problem from a field such as medical tomography (transmission, emission), geophysics (remote sensing, inverse scattering, tomography), or non-destructive testing. Mathematical topics include ill-posedness (existence, uniqueness, stability), regularization (e.g., Tikhonov, least squares, modified least squares, variation, mollification), pseudoinverses, transforms (e.g., k-plane, Radon, X-ray, Hilbert), special functions, and singular-value decomposition. Physical aspects of particular inverse problems will be introduced as needed, but the emphasis of the course is investigation of the mathematical concepts related to analysis and solution of inverse problems.

Alternatives:
None

Subsequent Courses:
None

Math 465 - Introduction to Combinatorics

Prerequisites: Linear Algebra (one of Math 214, 217, 296, 417, or 419) or permission of instructor

Frequency: Fall (I), Winter (II)

Student Body: Upper-level undergraduates and graduate students in mathematics, science, and engineering.

Credit: 3 Credits. No credit granted to those who have completed or are enrolled in Math 565 or 566.

Area: Discrete Mathematics

Background and Goals:
Combinatorics is the study of finite mathematical objects, including their enumeration, structural properties, design, and optimization. Combinatorics plays an increasingly important role in various branches of mathematics and in numerous applications, including computer science, statistics and statistical physics, operations research, bioinformatics, and electrical engineering. This course provides an elementary introduction to the fundamental notions, techniques, and theorems of enumerative combinatorics and graph theory.

Content:
An introduction to combinatorics, covering basic counting techniques (inclusion-exclusion, permutations and combinations, generating functions) and fundamentals of graph theory (paths and cycles, trees, graph coloring). Additional topics may include partially ordered sets, recurrence relations, partitions, matching theory, and combinatorial algorithms.

Alternatives:
Math 565 (Combinatorics & Graph Theory) is significantly more demanding and proof-oriented. Math 566 (Combinatorial Theory) is even more advanced.

Subsequent Courses:
Math 565 (Combinatorics & Graph Theory) and 566 (Combinatorial Theory).

Math 466 (EEB 466) - Mathematical Ecology

Prerequisites: Math 217, 417, or 419; Math 286 or 316; and Math 450 or 451

Frequency: Sporadically

Student Body: The course is intended for graduate students and advanced undergrads interested in the mathematical analysis and modeling of ecological systems.

Credit: 3 Credits.

Area: Applied

Background and Goals:
This course gives an overview of mathematical approaches to questions in the science of ecology. Topics include: formulation of deterministic and stochastic population models; dynamics of single-species populations; and dynamics of interacting populations (predation, competition, and mutualism), structured populations, and epidemiology. Emphasis is placed on model formulation and techniques of analysis.

Content:
Why do some diseases become pandemic? Why does the introduction of certain species result in widespread invasions? Why do some populations grow while others decline and still others cycle rhythmically? How are all of these phenomena influenced by climate change? These and many other fundamental questions in the science of ecology are intrinsically quantitative and concern highly complex systems. To answer them, ecologists formulate and study mathematical models. This course is intended to provide an overview of the principal ecological models and the mathematical techniques available for their analysis. Emphasis is placed on model formulation and techniques of analysis. Although the focus is on ecological dynamics, the methods we discuss are readily applicable across the sciences. The course presumes mathematical maturity at the level of advanced calculus with prior exposure to ordinary differential equations, linear algebra, and probability.

Alternatives:
None

Subsequent Courses:
None

Math 471 - Introduction to Numerical Methods

Prerequisites: Math 216, 286, or 316; Math 214, 217, 417, or 419; and a working knowledge of one high-level computer language
Frequency: Fall (I), Winter (II), Summer (IIIb)
Student Body: Juniors, Seniors, and Master's level (half Engineering, half LS&A).
Credit: 3 Credits. No credit granted to those who have completed or are enrolled in Math 371 or 472.
Area: Applied/Numerical Analysis

Background and Goals:
This is a survey of the basic numerical methods which are used to solve scientific problems. The emphasis is evenly divided between the analysis of the methods and their practical applications. Some convergence theorems and error bounds are proved. The course also provides an introduction to MATLAB, an interactive program for numerical linear algebra, as well as practice in computer programming. One goal of the course is to show how calculus and linear algebra are used in numerical analysis.

Content:
Topics may include computer arithmetic, Newton's method for non-linear equations, polynomial interpolation, numerical integration, systems of linear equations, initial value problems for ordinary differential equations, quadrature, partial pivoting, spline approximations, partial differential equations, Monte Carlo methods, 2-point boundary value problems, Dirichlet problem for the Laplace equation.

Alternatives:
Math 371/Engin 303 (Numerical Methods) is a less sophisticated version intended principally for Sophomore and Junior engineering students; the sequence Math 571/572 (Numerical Linear Algebra/Numerical Methods for Diff. Equations) is mainly taken by graduate students, but should be considered by strong undergraduates.

Subsequent Courses:
Math 471 is good preparation for Math 571 (Numerical Linear Algebra) and Math 572 (Numerical Methods for Diff. Equations), although it is not prerequisite for these courses.

Math 472 - Numerical Methods with Financial Applications

Prerequisites: Math 216, 286, or 316; Math 214, 217, 417, or 419; and a working knowledge of one high-level computer language. Math 425 OR 525 is recommended.
Frequency: Fall (I)
Student Body: Majors in the Actuarial and Financial Math programs.
Credit: 3 Credits. No credit granted to those who have completed or are enrolled in Math 371 or 471.
Area: Actuarial & Financial

Background and Goals:
This is a survey of the basic numerical methods which are used to solve scientific problems. The goals of the course are similar to those of Math 471, but the applications are chosen to be of interest to students in the Actuarial Mathematics and Financial Mathematics programs.

Content:
Topics may include: Newton's method for non-linear equations, systems of linear equations, numerical integration, interpolation and polynomial approximation, ordinary differential equations, partial differential equations—in particular the Black-Scholes equation, Monte Carlo simulation, and numerical modeling.

Alternatives:
Math 371/Engin 303 (Numerical Methods) is a less sophisticated version intended principally for Sophomore and Junior engineering students.

Subsequent Courses:
None

Math 474 - Intro to Stochastic Analysis for Finance

Prerequisites: Math 525 and 423; or advisor permission
Frequency: Fall (I), Winter (II)
Student Body: Majors in the Financial Math program.
Credit: 3 Credits.
Area: Finance

Background and Goals:
This is an undergraduate level course in stochastic analysis and applications to quantitative finance. The aim of this course is to teach the probabilistic techniques and concepts from the theory of continuous-time stochastic processes and their applications to modern mathematical finance. It is a continuation of Math 423.

Content:
The course starts with the basic theory of diffusion processes. Specifically, it covers the topics: stochastic integrals, continuous-time martingales, stochastic calculus, and stochastic differential equations. It introduces the students Ito's formula and geometric Brownian motion, which are fundamental concepts in the theory of mathematical finance. Afterwards, the course focuses on mathematical finance models in continuous-time. First, basic definitions and models are being introduced: portfolio dynamics, arbitrage theory (including the celebrated Black-Scholes' equation and formula), and hedging. Then, the course covers more advanced models, using the martingale approach to arbitrage theory. This includes martingale pricing, stochastic discounting, Girsanov's theorem, revisiting the Black-Scholes model. Finally, the course introduces multidimensional models and the concepts of complete and incomplete markets.

Alternatives:
None.

Subsequent Courses:
None

Math 475 - Elementary Number Theory

Prerequisites: No specific prerequisite
Frequency: Fall (I), Winter (II)
Student Body: Mainly mathematics undergraduates; some non-mathematics undergraduates and graduate students.
Credit: 3 Credits.
Area: Number Theory

Background and Goals:
This is an elementary introduction to number theory, especially congruence arithmetic. Number Theory is one of the few areas of mathematics in which problems easily describable to a layman (is every even number the sum of two primes?) have remained unsolved for centuries. Recently some of these fascinating but seemingly useless questions have come to be of central importance in the design of codes and ciphers. In addition to strictly number-theoretic questions, concrete examples of structures such as rings and fields from abstract algebra are discussed. Concepts and proofs are emphasized, but there is some discussion of algorithms which permit efficient calculation. Students are expected to do simple proofs and may be asked to perform computer experiments. Although there are no special prerequisites and the course is essentially self-contained, most students have some experience in abstract mathematics and problem solving and are interested in learning proofs. At least three semesters of college mathematics are recommended. A Computational Laboratory (Math 476, 1 credit) will usually be offered as an optional supplement to this course.

Content:
Topics usually include the Euclidean algorithm, primes and unique factorization, congruences, Chinese Remainder Theorem, Hensel's Lemma, Diophantine equations, arithmetic in polynomial rings, primitive roots, quadratic reciprocity, and quadratic fields. This material corresponds to Chapters 1-3 and selected parts of Chapter 5 of Niven, Zuckerman, and Montgomery.

Alternatives:
Math 575 (Intro to Theory of Numbers) moves much faster, covers more material, and requires more difficult exercises. There is some overlap with Math 412 (Intro to Modern Algebra).

Subsequent Courses:
Math 475 may be followed by Math 575 (Intro to Theory of Numbers) and is good preparation for Math 412 (Intro to Modern Algebra).

Math 476 - Computational Laboratory in Number Theory

Prerequisites: Prior or concurrent enrollment in Math 475 or 575
Frequency: Fall (I), Winter (II), Sporadically
Student Body: Undergraduate mathematics majors.
Credit: 1 Credit.
Area: Number Theory

Background and Goals:
Intended as a companion course to Math 475 (Elem. Number Theory) or 575 (Intro to Theory of Numbers) Participation should boost the student's performance in either of those classes. Students in the Lab will see mathematics as an exploratory science (as mathematicians do).

Content:
Students will be provided with software with which to conduct numerical explorations. No programming necessary, but students interested in programming will have the opportunity to embark on their own projects. Students will gain a knowledge of algorithms which have been developed for number theoretic purposes, e.g., for factoring.

Alternatives:
None

Subsequent Courses:
None

Math 481 - Introduction to Mathematical Logic

Prerequisites: Math 412 or 451 or equivalent experience with
abstract mathematics
Frequency: Fall (I)
Student Body: Undergraduate math, philosophy, and computer
science majors; a few non-math graduate students.
Credit: 3 Credits.
Area: Logic

Background and Goals:
All of modern mathematics involves logical relationships among mathemati-
cal concepts. In this course we focus on these relationships themselves
rather than the ideas they relate. Inevitably this leads to a study of the (for-
mal) languages suitable for expressing mathematical ideas. The explicit goal
of the course is the study of propositional and first-order logic; the implicit
goal is an improved understanding of the logical structure of mathematics.
Students should have some previous experience with abstract mathematics
and proofs, both because the course is largely concerned with theorems and
proofs and because the formal logical concepts will be much more meaning-
ful to a student who has already encountered these concepts informally. No
previous course in logic is prerequisite.

Content:
In the first third of the course the notion of a formal language is introduced
and propositional connectives ('and,' 'or,' 'not,' 'implies'), tautologies, and
tautological consequence are studied. The heart of the course is the study
of first-order predicate languages and their models. The new elements here
are quantifiers ('there exists' and 'for all'). The study of the notions of truth,
logical consequence, and probability leads to the completeness and com-
pactness theorems The final topics include some applications of these theo-
rems, usually including non-standard analysis. This material corresponds to
Chapter 1 and sections 2.0-2.5 and 2.8 of Enderton.

Alternatives:
Math 681 (Math Logic), the graduate introductory logic course, also has no
specific logic prerequisite but does presuppose a much higher general level
of mathematical sophistication. Philosophy 414 (Logic & Artificial Intel-
ligence) may cover much of the same material with a less mathematical
orientation.

Subsequent Courses:
Math 481 is not explicitly prerequisite for any later course, but the ideas
developed have application to every branch of mathematics.

Math 486 - Concepts Basic to Secondary School Math

Prerequisites: Math 205, 215, or 285; and 217; or permission of the instructor. Prerequisites must be completed with a minimum grade of C- or better.

Frequency: Winter (II)

Student Body: Undergraduate majors in the Teaching Certificate Program and minors in other teaching programs

Credit: 3 Credits.

Area: Teaching

Background and Goals:
This course is designed for students who intend to teach middle or high school mathematics. Concepts and proofs are emphasized over calculation. The syllabus consists of high-school mathematics from an advanced per-spective. The course is conducted in a discussion (Inquiry Based Learning) format. Class participation is essential and constitutes a significant part of the course grade.

Content:
Topics covered have included number systems and their axiomatics; number theory, particularly a study of divisibility, primes, and prime factorizations; the abstract theory of sets, operators, and functions; and the epsilon-delta underpinnings of limits and derivatives.

Alternatives:
There is no real alternative, but the Teaching Certificate requirement of Math 486 may be waived for strong students who intend to do graduate work in mathematics.

Subsequent Courses:
Prior completion of Math 486 may be of use for some students planning to take Math 312 (Applied Modern Algebra), 412 (Intro to Modern Algebra), or 425 (Intro to Probability).

Math 489 - Math for Elementary and Middle School Teachers

Prerequisites: Math 385 or permission of instructor
Frequency: Winter (II)
Student Body: Undergraduates in the Elementary Teaching Certificate Program.
Credit: 3 Credits.
Area: Teaching

Background and Goals:
This course, together with its predecessor, Math 385, provides a coherent overview of the mathematics underlying the elementary and middle school curriculum. It is required of all students intending to earn an elementary teaching certificate and is taken almost exclusively by such students. Concepts are heavily emphasized with some attention given to calculation and proof. The course is conducted using a discussion format. Class participation is expected and constitutes a significant part of the course grade. Enrollment is limited to 30 students per section. Although only two years of high school mathematics is required, a more complete background including pre-calculus or calculus is desirable.

Content:
Topics covered include fractions and rational numbers, decimals and real numbers, probability and statistics, geometric figures, and measurement. Algebraic techniques and problem-solving strategies are used throughout the course.

Alternatives:
There is no alternative course.

Math 490 - Introduction to Topology

Prerequisites: Math 351, 451; or previous exposure to real analysis; or permission of instructor

Frequency: Fall (I)
Student Body: Mathematics, math-education, science and engineering.
Credit: 3 Credits.
Area: Geometry/Topology

Background and Goals:
Topology is a fundamental area of mathematics that provides a foundation for analysis and geometry. Once a set has a topology (so it becomes a "topological space"), we can start to build on it. For example, the notion of a continuous function makes sense on a topological space, and in fact, this is the most general setting where the idea of a continuous function makes sense. The goal of this course is to introduce you to the world of topology, with emphasis placed on careful reasoning, understanding and constructing proofs. Math 490 is an Inquiry Based Learning course. This means that the students work in groups, guided by worksheets, to explore and develop new material with minimal guidance from the instructor. Depending on the instructor, students may also be asked to regularly present material in class.

Content:
We will generalize important concepts like continuity and compactness from the setting of real analysis into the more general setting of topological spaces. We draw motivation from two major theorems from real analysis: 1) the Extreme Value Theorem, and 2) the Intermediate Value Theorem. Another thing we will do in this course is rigorously explore what it means for two topological spaces to be "the same". To this end, we will develop tools that help to distinguish topological spaces from each other. Topics include, but are not limited to: metric spaces, compactness, connectedness, product spaces, quotient spaces.

Alternatives:
Math 590 (Intro to Topology) is a deeper and more difficult presentation of much of the same material. Math 433 (Intro to Diff. Geometry) is a related course at about the same level.

Subsequent Courses:
Math 490 is not prerequisite for any later course but provides good background for Math 591 (General & Diff. Topology) or any of the other courses in geometry or topology.

Math 493 - Honors Algebra I

Prerequisites: Math 296 or 297(enforced) or permission of instructor
Frequency: Fall (I)
Student Body: Mainly undergraduate mathematics majors with a few graduate students from other fields.
Credit: 3 Credits.
Area: Algebra

Background and Goals:
Math 493-494 is one of the more abstract and challenging sequences in the undergraduate program. Its goal is to introduce students to the basic structures of modern abstract algebra (groups, rings, fields, and modules) in a rigorous way.

Emphasis is on concepts and proofs; calculations are used to illustrate the general theory. Exercises tend to be quite challenging. Students must have some previous exposure to rigorous proof-oriented mathematics and be prepared to work hard.

Content:
The course covers basic definitions and properties of groups, fields, and vector spaces including homomorphisms, isomorphisms, subgroups, and bilinear forms.

Further topics are selected from: Sylow theorems; structure theorem for finitely-generated abelian groups; permutation representation; the symmetric and alternating groups; vector spaces over arbitrary fields; spectral theorem; and linear groups.

Alternatives:
Math 412 (Intro to Modern Algebra) covers about half of the material of Math 493.

Subsequent Courses:
Math 494 (Honors Algebra II).

Math 494 - Honors Algebra II

Prerequisites: Math 493 (enforced)
Frequency: Winter (II)
Student Body: Mainly undergraduate mathematics majors with a few graduate students from other fields.
Credit: 3 Credits.
Area: Algebra

Background and Goals:
Math 493-494 is one of the more abstract and challenging sequences in the undergraduate program. Its goal is to introduce students to the basic structures of modern abstract algebra (groups, rings, fields, and modules) in a rigorous way.

Emphasis is on concepts and proofs; calculations are used to illustrate the general theory. Exercises tend to be quite challenging. Students must have some previous exposure to rigorous proof-oriented mathematics and be prepared to work hard.

Content:
Basic definitions and properties of rings and modules: quotients, ideals, factorization, field extensions. Further topics selected from representation theory, structure theory of modules over a PID, Jordan canonical form, Galois theory, Nullstellensatz, finite fields, Euclidean, Prinicipal ideals & unique factorization domains, polynomial rings in one and several variables and algebraic varieties.

Alternatives:
None

Subsequent Courses:
The natural sequel to Math 494 is Math 593 (Algebra I).

Math 497 - Topics in Elementary Mathematics

Prerequisites: Math 489 or permission of instructor
Frequency: Fall (I)
Student Body: Undergraduates in the Elementary Teaching Certificate Program.
Credit: 3 Credits.
Area: Teaching

Background and Goals:
This is a required course for elementary teaching certificate candidates that extends and deepens the coverage of mathematics begun in the required two-course pair Math 385&489. Topics are chosen from geometry, algebra, computer programming, logic, and combinatorics. Applications and problem-solving are emphasized. The class meets three times per week in recitation sections. Grades are based on class participation, two one-hour exams, and a final exam.

Content:
Selected topics in geometry, algebra, computer programming, logic, and combinatorics for prospective and in-service elementary, middle, or junior-high school teachers. Content will vary from term to term.

Alternatives:
None

Subsequent Courses:
None

Math 498 - Topics in Modern Mathematics

Prerequisites: Junior or Senior standing
Frequency: Sporadically
Student Body: Junior and Senior students from mathematics and other fields.
Credit: 3 Credits.
Area:

Background and Goals:
As a topics course, this course will vary greatly from term to term. In one recent offering, the aim of the course was tdscuss the mathematics underlying machine learning.

Content:
varies

Alternatives:
None

Subsequent Courses:
None

Math 520 - Life Contingencies I

Prerequisites: Math 424, and 425 0or 525, both with a minimum
grade of C-, and permission of instructor
Frequency: Fall (I)
Student Body: Undergraduate students of actuarial mathematics
Credit: 3 Credits.
Area: Actuarial & Financial

Background and Goals:
Quantifying the financial impact of uncertain events is the central challenge
of actuarial mathematics. The goal of this course is to teach the basic ac-
tuarial theory of mathematical models for financial uncertainties, mainly the
time of death. Math 520 and 521 also help students prepare for some of the
professional actuarial exams. In addition to actuarial students, this course
is appropriate for anyone interested in mathematical modeling outside of the
sciences.

Content:
The main topics are the development of (1) probability distributions for the
future lifetime random variable; (2) probabilistic methods for financial pay-
ments on death or survival; and (3) mathematical models of actuarial reserv-
ing.

Alternatives:
Math 523 (Loss Models I) is a complementary course covering the applica-
tion of stochastic process models.

Subsequent Courses:
Math 520 is prerequisite to all succeeding actuarial courses. Math 521 (Life
Contingencies II) extends the single decrement and single life ideas of Math
520 to multi-decrement and multiple-life applications directly related to life
insurance and pensions.

Math 521 - Life Contingencies II

Prerequisites: Math 520, with a minimum grade of C-, or
permission of instructor
Frequency: Winter (II)
Student Body: Undergraduate students of actuarial mathematics
Credit: 3 Credits.
Area: Actuarial & Financial

Background and Goals:
This course extends the single decrement and single life ideas of Math 520
to multi-decrement and multiple-life applications directly related to life insur-
ance. Math 520 and 521 also help students prepare for some of the profes-
sional actuarial exams.

Content:
Topics include multiple life models—joint life, last survivor, contingent insur-
ance; multiple decrement models—disability, withdrawal, retirement, etc.;
and reserving models for life insurance.

Alternatives:
None

Subsequent Courses:
None

Math 523 - Loss Models I

Prerequisites: Math 425 or 525 with a minimum grade of C-
Frequency: Fall (I)
Student Body: Undergraduate students in actuarial mathematics
Credit: 3 Credits.
Area: Actuarial

Background and Goals:
Risk management is of major concern to all financial institutions especially casualty insurance companies. This course is relevant for students in insurance and provides background for the professional examinations in Actuarial Modeling offered by the Society of Actuaries (Exam C) and the Casualty Actuary Society (Exam 3). Students should have a basic knowledge of common probability distributions (Poisson, exponential, gamma, binomial, etc) and have at least Junior standing.

Content:
Review of random variables (emphasizing parametric distributions), review of basic distributional quantities, continuous models for insurance claim severity, discrete models for insurance claim frequency, the effect of coverage modification on severity and frequency distributions, aggregate loss models, and credibility.

Alternatives:
None

Subsequent Courses:
Math 524 (Loss Models II)

Math 524 - Loss Models II

Prerequisites: Math 523, Stats 426 with a minimum grade of C-
Frequency: Winter only (II)
Student Body: Undergraduate students in actuarial mathematics
Credit: 3 Credits.
Area: Actuarial

Background and Goals:
Risk management is of major concern to all financial institutions especially casualty insurance companies. This course is relevant for students in insurance and provides background for the professional examinations in Actuarial Modeling offered by the Society of Actuaries (Exam C) and the Casualty Actuary Society (Exam 3). Students should have a basic knowledge of common probability distributions (Poisson, exponential, gamma, binomial, etc) and have at least Junior standing.

Content:
Frequentist and Bayesian estimation of probability distributions, model selection, empirical Bayes estimation in credibility, simulation, and other topics in casualty insurance.

Alternatives:
None

Subsequent Courses:
None

Math 525 (Stats 525) - Probability Theory

Prerequisites: One semester of multivariable calculus (205, 215, 285, or equivalent) and one semester of analysis (351, 451, or equivalent).
Frequency: Fall (I), Winter (II)
Student Body: A mix of undergraduate and graduate students, from mathematics, statistics, and engineering
Credit: 3 Credits.
Area: Analysis

Background and Goals:
This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

Content:
The following topics will be covered: sample space and events, random variables, concept and definition of probability and expectation, conditional probability and expectation, independence, moment generating functions, Law of large numbers, Central limit theorem, Markov chains, Poisson process and exponential distribution.

Alternatives:

EECS 501 (Probability & Random Processes) also covers some of the same material with less emphasis on mathematical rigor. Math/Stats 425 (Intro to Probability) treats similar topics, but is accessible with less mathematical background.

Subsequent Courses:
Math 526 (Discrete State Stochastic Processes.), Stats 426 (Intro to Theoretical Statistics), and the sequence Stats 510/511 (Mathematical Statistics I /II) are natural sequels.

Math 526 (Stats 526) - Discrete State Stochastic Processes

Prerequisites: Math 525 or EECS 501
Frequency: Fall (I), Winter (II)
Student Body: Half undergraduates, half graduate students from mathematics and many other fields
Credit: 3 Credits.
Area: Analysis

Background and Goals:
This is a course on the theory and applications of stochastic processes, mostly on discrete state spaces. It is a second course in probability which should be of interest to students of mathematics and statistics as well as students from other disciplines in which stochastic processes have found significant applications.

Content:
The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include generating functions; recurrent events and the renewal theorem; random walks; Markov chains; branching processes; limit theorems; Markov chains in continuous time with emphasis on birth and death processes and queueing theory; an introduction to Brownian motion; stationary processes and martingales.

Alternatives:
This course is similar to EECS 502 (Stochastic Processes) and IOE 515 (Stochastic Processes), although the latter course tends to be somewhat more oriented to applications. It is cross-listed as Statistics 526.

Subsequent Courses:
The next courses in probability are Math 625 (Probabiility & Random Processes I) and 626 (Probability & Random Processes II), which presuppose substantial additional background (e.g., Math 597).

Math 528 - Topics in Casualty Insurance

Prerequisites: Math 217, 417, 419, or permission of instructor
Frequency: Winter (II), generally in even years
Student Body: Undergraduate students of actuarial mathematics and insurance majors in business
Credit: 3 Credits.
Area: Actuarial & Financial

Background and Goals:
Historically the Actuarial Program has emphasized life, health, and pension topics. This course will provide background in casualty topics for the many students who take employment in this field. Guest lecturers from the industry will provide some of the instruction. Students are encouraged to take the Casualty Actuarial Society's Course 3 examination at the completion of the course.

Content:
The insurance policy is a contract describing the services and protection which the insurance company provides to the insured. This course will develop an understanding of the nature of the coverages provided and the bases of exposure and principles of the underwriting function, how products are designed and modified, and the different marketing systems. It will also look at how claims are settled, since this determines losses which are key components for insurance ratemaking and reserving. Finally, the course will explore basic ratemaking principles and concepts of loss reserving.

Alternatives:
None

Subsequent Courses:
None

Math 547 - Biological Sequence Analysis

Prerequisites: Flexible. Math/Stats 425 or 525, or Biology 427, or BioChem 451, or basic programming skills desirable; or permission of instructor.
Frequency: Sporadically
Student Body: Mainly math, stats, biostats, & bioinfo, biology, biomedical and engin students.
Credit: 3 Credits.
Area: Analysis

Background and Goals: This course covers topics in Biological Sequence Analysis.

Content:
Probabilistic models of proteins and nucleic acids. Analysis of DNA/RNA and protein sequence data. Algorithms for sequence alignment, statistical analysis of similarity scores, hidden markov models, neural networks, training, gene finding, protein family profiles, multiple sequence alignment, sequence comparison, and structure prediction. Analysis of expression array data.

Alternatives:
Bioinformatics 526

Subsequent Courses:
Bioinformatics 551 (Proteome Informatics)

Math 550 (CMPLXSYS 510) - Introduction to Adaptive Systems

Prerequisites: Math 205, 215, or 285; Math 217; and Math 425 or Math 525
Frequency: Sporadically
Student Body: Graduate and undergraduate students from many disciplines
Credit: 3 Credits.

Background and Goals:
This course centers on the construction and use of agent-based adaptive models to study phenomena which are prototypical in the social, biological, and decision sciences. These models are "agent-based" or "bottom-up" in that the structure is placed at the level of the individuals as basic components; they are "adaptive" in that individuals often adapt to their environment through evolution or learning. The goal of these models is to understand how the structure at the individual or micro level leads to emergent behavior at the aggregate or macro level. Often the individuals are grouped into sub-populations or interesting hierarchies, and the researcher may want to understand how the structure or development of these populations affects macroscopic outcomes.

Content:
The course will start with classical differential equation and game theory approaches. It will then focus on the theory and application of particular models of adaptive systems such as models of neural systems, genetic algorithms, classifier systems, and cellular automata. Time permitting, we will discuss more recent developments such as sugarscape and echo.

Alternatives:
Cross-listed as Complex Systems 510.

Subsequent Courses:
None

Math 551 - Intro to Real Analysis

Prerequisites: Math 451 & 452, or MATH 295 & 296; and abstract linear algebra such as Math 217 .
Frequency: Fall (I), Winter (II)
Student Body: Math undergraduates & masters students, non-math graduates & non-math undergraduates.
Credit: 3 Credits. No credit granted to students who have taken or are enrolled in Math 597
Area: Analysis

Background and Goals:
The objective of this course is to develop Lebesgue measure theory and to study abstract spaces such as metric spaces and topological spaces.

Content:
This is an introductory course on real analysis on the level between MATH 451 (Advanced Calculus) and MATH 597 (Real Analysis). If time permits, we may also consider abstract measure theory.

Lebesgue measure, measurable functions, Lebegue integral, convergence theorems, metric spaces, topological spaces, Hilbert and Banach spaces

Alternatives:
There are many overlaps with Math 597, but we will introduce hte subjects at a slower pace.

Subsequent Courses:
None

Math 555 - Introduction to Complex Variables

Prerequisites: Math 450 or 351 or 451. Students who had 450 (or equivalent) but not 451 are encouraged to take 451 simultaneously with 555.
Frequency: Fall (I), Winter (II), Spring (IIIa)
Student Body: Graduate students in mathematics, science, and engineering, and some advanced mathematics, science, and engineering undergraduates
Credit: 3 Credits.
Area: Applied/Numerical Analysis

Background and Goals:
This course is an introduction to the theory of complex-valued functions of a complex variable with substantial attention to applications in science and engineering. Concepts, calculations, and the ability to apply principles to physical problems are emphasized over proofs, but arguments are rigorous. The prerequisite of a course in advanced calculus is essential. This is a core course for the AIM graduate program.

Content:
Differentiation and integration of complex-valued functions of a complex variable, series, mappings, residues, applications. Evaluation of improper real integrals, applications in ideal fluid dynamics. This corresponds to Chapters 1-9 of Churchill & Brown.

Alternatives:
Math 596 (Analysis I -Complex) covers the theoretical material of Math 555 with an emphasis on proofs rather than applications.

Subsequent Courses:
Math 555 is prerequisite to many advanced courses in applied mathematics, including the sequence Math 556 (Applied Functional Analysis) and 557 (Applied Asymptotic Analysis), and in science and engineering.

Math 556 - Applied Functional Analysis

Prerequisites: Math 217, 419, or 420; Math 451; and Math 555
Frequency: Fall (I)
Student Body: Graduate students in mathematics, science, and engineering
Credit: 3 Credits.
Area: Applied/Numerical Analysis

Background and Goals:
This is an introduction to methods of applied analysis with emphasis on Fourier analysis and partial differential equations. Students are expected to master both the proofs and applications of major results. The prerequisites include linear algebra, advanced calculus, and complex variables.

Content:
Topics in functional analysis that are used in the analysis of ordinary and partial differential equations. Metric and normed linear spaces, Banach spaces and the contraction mapping theorem, Hilbert spaces and spectral theory of compact operators, distributions and Fourier transforms, Sobolev spaces and applications to elliptic PDEs.

Alternatives:
Math 454 (Boundary Value Problems for Partial Diff. Equations) is an undergraduate course on the same topics.

Subsequent Courses:
Math 557 (Applied Asymptotic Analysis), Math 558 (Ordinary Differential Equations), Math 656 (Partial Diff. Equations), and Math 658 (Ordinary Diff. Equations).

Math 557 - Applied Asymptotic Analysis

Prerequisites: Math 217, 419, or 420; Math 451; and Math 555
Frequency: Winter (II)
Student Body: Graduate students in mathematics, science, and
                 engineering
Credit: 3 Credits.
Area: Applied/Numerical Analysis

Background and Goals:
This is an introduction to methods of asymptotic analysis including asymp-
totic expansions for integrals and solutions of ordinary and partial differential
equations. The prerequisites include linear algebra, advanced calculus, and
complex variables. Math 556 is not a prerequisite.

Content:
Topics include stationary phase, steepest descent, characterization of sin-
gularities in terms of the Fourier transform, regular and singular perturbation
problems, boundary layers, multiple scales, WKB method. Additional topics
depend on the instructor but may include non-linear stability theory, bifurca-
tions, applications in fluid dynamics (Rayleigh-Benard convection), combus-
tion (flame speed).

Alternatives:
None

Subsequent Courses:
Math 656 (Partial Diff. Equations) and 658 (Ordinary Diff. Equations).

Math 558 - Applied Nonlinear Dynamics

Prerequisites: Math 451
Frequency: Fall (I)
Student Body: Graduate students in mathematics, science, and engineering
Credit: 3 Credits.
Area: Applied/Numerical Analysis

Background and Goals:
This course is an introduction to Ordinary Differential Equations and Dynamical Systems with emphasis on qualitative analysis.

Content:
The basic results on qualitative behavior, centered on themes of stability and phase plane analysis will be presented in a context that includes applications to a variety of classic examples. The proofs of the fundamental facts will be presented, along with discussions of examples.

Alternatives:
Math 404 (Intermediate Diff. Equations) is an undergraduate course on similar topics.

Subsequent Courses:
Math 658 (Ordinary Diff. Equations)

Math 559 - Topics in Applied Mathematics

Prerequisites: Vary by topic, check with instructor
Frequency: Sporadically
Student Body: Undergraduate and graduate students in mathematics or science
Credit: 3 Credits.
Recent Texts: Varies
Area: Applied/Numerical Analysis

Background and Goals:
This is an advanced topics course intended for students with strong interests in the intersection of mathematics and the sciences, but not necessarily experience with both applied mathematics and the application field. Mathematical concepts, as well as intuitions arising from the field of application, will be stressed.

Content:
This course will focus on particular topics in emerging areas of applied mathematics for which the application field has been strongly influenced by mathematical ideas. It is intended for students with interests in mathematical, computational, and/or modeling aspects of interdisciplinary science, and the course will develop the intuitions of the field of application as well as the mathematical concepts. The applications considered will vary with the instructor and may come from physics, biology, economics, electrical engineering, and other fields. Recent examples have been: Nonlinear Waves, Mathematical Ecology, and Computational Neuroscience.

Alternatives:
None

Subsequent Courses:
Other courses in applied mathematics.

Math 561 (TO 518, IOE 510) - Linear Programming I

Prerequisites: Math 214, 217, 417, or 419
Frequency: Fall (I), Winter (II)
Student Body: Graduate and undergraduate students from many fields
Credit: 3 Credits.

Background and Goals:
The allocation of constrained resources such as funds among investment possibilities or personnel among production facilities is a fundamental problem which is very well-suited to mathematical analysis. Each such problem has as its goal the maximization of some positive objective such as investment return or the minimization of some negative objective such as cost or risk. Such problems are called Optimization Problems. Linear Programming deals with optimization problems in which both the objective and constraint functions are linear (the word "programming" means "planning" rather than computer programming). In practice, such problems involve thousands of decision variables and constraints, so a primary focus is the development and implementation of efficient algorithms. However, the subject also has deep connections with higher-dimensional convex geometry. A recent survey showed that most Fortune 500 companies regularly use linear programming in their decision making. This course will present both the classical and modern approaches to the subject and discuss numerous applications of current interest.

Content:
Formulation of problems from the private and public sectors using the mathematical model of linear programming. Development of the simplex algorithm; duality theory and economic interpretations. Postoptimality (sensitivity) analysis; algorithmic complexity; the ellipsoid method; scaling algorithms; applications and interpretations. Introduction to transportation and assignment problems; special purpose algorithms and advanced computational techniques. Students have opportunities to formulate and solve models developed from more complex case studies and to use various computer programs.

Subsequent Courses:
IOE 610 (Linear Programming II) and IOE 611 (Nonlinear Programming)

Math 562 (IOE 511) - Continuous Optimization Methods

Prerequisites: Math 214, 217, 417, or 419
Frequency: Fall (I)
Student Body: Graduate students in engineering and LSA. Occasionaly it attracts an undergraduate Math or Engineering student.
Credit: 3 Credits.

Background and Goals:
Optimization is widely used in engineering and science models. The goal of this course is to give a rigorous background to the field. Examples are drawn from engineering and science.

Content:
Survey of continuous optimization problems. Unconstrained optimization problems: unidirectional search techniques, gradient, conjugate direction, quasi-Newtonian methods; introduction to constrained optimization using techniques of unconstrained optimization through penalty transformation, augmented Lagrangians, and others; discussion of computer programs for various algorithms.

Alternatives:
Cross-listed as IOE 511.

Subsequent Courses:
This is not a prerequisite for any other course.

Math 563 - Advanced Mathematical Methods for the Biological Sciences

Prerequisites: Diff Eq: 216 or 316 (or above), Linear algebra: 214, 217 (or above). Undergrads also need Math 463 or 462 for undergraduates and proficiency in programming (preferably MATLAB)
Frequency: Fall (I)
Student Body: Upper-level math majors and graduate students in engineering, bioinfo, biophysand applied math
Credit: 3 Credits.

Background and Goals:
Biological systems behave in a way that reflects underlying spatial heterogeneities. Programming experience, particularly with MATLAB, is required.

Content:
This course will focus on mathematical modeling of how biological objects, such as molecules, cells or whole organisms, move and interact in both time and space. Depending on the biological process, time scales for interaction can vary widely, from milliseconds to days, and spatial scales can vary from microns to miles, however we can use similar mathematical techniques to capture behavior of these diverse systems. These techniques stem from the theory of random walks and involve partial differential equations (PDEs). The course will cover analytical and numerical techniques for solving random walk systems and associated PDE in the context of modeling diverse biological processes that operate that the molecular, cellular and population levels. Agent-based modeling will also be introduced to model highly complex and variable interactions among biological objects. Grades will be based on biweekly homework sets, consisting of analytical and numerical problems, and a modeling project.

Alternatives:
None

Subsequent Courses:
None

Math 564 - Topics in Mathematical Biology

Prerequisites: Variable, permission of instructor
Frequency: Winter (II), not every year
Student Body: Juniors, Seniors, but mainly graduate students
Credit: 3 credits
Recent Texts: None

Background and Goals:
This is an advanced course on further topics in mathematical biology. Topic will vary according to the instructor. Possible topics include modeling infectious diseases, cancer modeling, mathematical neurosciences or biological oscillators, among others. The sample description below is for a course in biological oscillators from Winter 2006.

Content:
From sleeping patterns, heartbeats, locomotion, and firefly flashing to the treatment of cancer, diabetes, and neurological disorders, oscillations are of great importance in biology and medicine. Mathematical modeling and analysis are needed to understand what causes these oscillations to emerge, properties of their period and amplitude, and how they synchronize to signals from other oscillators or from the external world.   The goal of this course will be to teach students how to take real biological data, convert it to a system of equations and simulate and/or analyze these equations. Models will typically use ordinary differential equations. Mathematical techniques introduced in this course include 1) the method of averaging, 2) harmonic balance, 3) Fourier techniques, 4) entrainment and coupling of oscillators, 4) phase plane analysis, and 5) various techniques from the theory of dynamical systems. Emphasis will be placed on primary sources (papers from the literature) particularly those in the biological sciences. Consideration will be given in the problem sets and course project to interdisciplinary student backgrounds.

Alternatives:
None

Subsequent Courses:
None

Math 565 - Combinatorics and Graph Theory

Prerequisites: Math 412, 451, or equivalent experience with
abstract mathematics
Frequency: Fall (I)
Student Body: Largely math and EECS grad students with a few
mathematics undergraduates
Credit: 3 Credits
Area: Combinatorics

Background and Goals:
This course has two somewhat distinct halves devoted to (1) graph theory
and (2) topics in the theory of finite partially ordered sets. Students should
have taken at least one proof-oriented course.

Content:
The first part of this course will be devoted to graph theory. A graph (in the
combinatorial sense) is a finite set of points and a specification of which pairs
of these points are deemed "adjacent." Despite the simplicity of the con-
cept, it leads to numerous interesting theorems, problems, and applications.
Topics in the graph theory part of the course include (if time permits) trees, k-
connectivity, Eulerian and Hamiltonian graphs, tournaments, graph coloring,
planar graphs, Euler's formula, the 5-Color theorem, Kuratowski's theorem,
and the matrix-tree theorem.

The second part of the course will deal with topics in the theory of finite par-
tially ordered sets. This will include material about Möbius functions, lattices,
simplicial complexes, and matroids.

Alternatives:
There is small overlap with Math 566 (Combinatorial Theory). Math 416
(Theory of Algorithms) is somewhat related but much more concerned with
algorithms.

Subsequent Courses:
Math 566 (Combinatorial Theory)

Math 566 - Combinatorial Theory

Prerequisites: Math 493, or equivalent experience with abstract algebra
Frequency: Winter (II)
Student Body: Undergraduates and graduate students from math stats, engineering, & other natural & social sciences
Credit: 3 Credits.
Area: Combinatorics

Background and Goals:
This course is a rigorous introduction to classical combinatorial theory. Concepts and proofs are the foundation, but there are copious applications to modern industrial problem-solving.

Content:
Permutations, combinations, generating functions, and recurrence relations. The existence and enumeration of finite discrete configurations. Systems of representatives, Ramsey's Theorem, and extremal problems. Construction of combinatorial designs.

Alternatives:
There is no real alternative, although there is some overlap with Math 565 (Combinatorics and Graph Theory).

Subsequent Courses:
Sequels are Math 664/665 (Combinatorial Theory I/II) and Math 669. (Topics in Combinatorial Theory).

Math 567 - Introduction to Coding Theory

Prerequisites: Math 214, 217, 417, 419, or 420
Frequency: Winter (II)
Student Body: Undergraduate mathematics majors and EECS
graduate students
Credit: 3 Credits.
Area: Algebra

Background and Goals:
This course is designed to introduce mathematics majors to an important
area of applications in the communications industry. Using linear algebra it
will cover the foundations of the theory of error-correcting codes and prepare
a student to take further EECS courses or gain employment in this area. For
EECS students it will provide a mathematical setting for their study of com-
munications technology.

Content:
Introduction to coding theory focusing on the mathematical background
for error-correcting codes. Topic include: Shannon's Theorem and channel
capacity; review of tools from linear algebra and an introduction to abstract
algebra and finite fields; basic examples of codes such and Hamming, BCH,
cyclic, Melas, Reed-Muller, and Reed-Solomon; introduction to decoding
starting with syndrome decoding and covering weight enumerator polynomi-
als and the Mac-Williams Sloane identity. Further topics range from asymp-
totic parameters and bounds to a discussion of algebraic geometric codes in
their simplest form.

Alternatives:
None

Subsequent Courses:
Math 565 (Combinatorics and Graph Theory) and Math 556 (Applied
Functional Analysis) are natural sequels or predecessors. This course also
complements Math 312 (Applied Modern Algebra) by presenting direct ap-
plications of finite fields and linear algebra.

Math 568 (BIOINF 568) - Mathematical & Computational Neuroscience

Prerequisites: Math 463 or 462 (for undergrad students) or graduate standing. Advisory: Math 214, 217, 417 or 419; and Math 216, 286, or 316
Frequency: Fall (I), Winter (II)
Student Body: Grad & undergrad students in applied & interdisci plinary math, phys, applied phys, neurosci, biophys, bioinfo, biomed engineering & other engin programs w/research in neural engineering.
Credit: 3 Credits.
Area: Applied/Interdisciplinary

Background and Goals:
In this course, students will derive, interpret and solve mathematical models of neural systems. The neural models consist of ordinary and partial differential equations and students are required to analytically and numerically solve the equations. Additional mathematical analysis techniques of phase plane analysis, linear stability of equilibria and bifurcation analysis will also be covered.

Content:
Computational neuroscience provides a set of quantitative approaches to investigate the biophysical mechanisms and computational principles underlying the function of the nervous system. This course introduces students to mathematical modeling and quantitative techniques used to investigate neural systems at many different scales, from single neuron activity to the dynamics of large neuronal networks.

Alternatives:
None

Subsequent Courses:
None

Math 571 - Numerical Linear Algebra

Prerequisites: Math 214, 217, 417, 419, or 420 and one of Math 450, 451, or 454; or permission of instructor
Frequency: Fall (I), Winter (II)
Student Body: Mathematics and engineering graduate students, strong undergraduates
Credit: 3 Credits.
Area: Applied/Numerical Analysis

Background and Goals:
This course is an introduction to numerical linear algebra, which is at the foundation of much of scientific computing. Numerical linear algebra deals with (1) the solution of linear systems of equations, (2) computation of eigenvalues and eigenvectors, and (3) least squares problems. We will study accurate, efficient, and stable algorithms for matrices that could be dense, or large and sparse, or even highly ill-conditioned. The course will emphasize both theory and practical implementation.

Content:
This course is an introduction to numerical linear algebra, a core subject in scientific computing. Three types of problems are considered: (1) linear systems, (2) eigenvalues and eigenvectors, and (3) least squares problems. Topics include: Gram-Schmidt orthogonalization, QR factorization, singular value decomposition (SVD), normal equations, vector and matrix norms, condition number, backward error analysis, LU factorization, Cholesky factorization, reduction to Hessenberg and tridiagonal form, power method, inverse iteration, Rayleigh quotient iteration, QR algorithm, Krylov subspace methods, Arnoldi iteration, GMRES, steepest descent, conjugate gradient method, preconditioning, applications to image compression, finite-difference schemes for two-point boundary value problems, Dirichlet problem for the Laplace equation, least squares data fitting.

Alternatives:
Math 471 (Intro. to Numerical Methods) is a survey course in numerical methods at a more elementary level.

Subsequent Courses:
Math 572 (Numer. Meth. for Diff Eq) covers initial value problems for ordinary and partial differential equations. Math 571 and 572 may be taken in either order. Math 671 (Analysis of Numerical Methods I) is an advanced course in numerical analysis with varying topics chosen by the instructor.

Math 572 - Numerical Methods for Differential Equations

Prerequisites: Math 214, 217, 417, 419, or 420 and one of Math 450, 451, or 454; or permission of instructor
Frequency: Winter (II)
Student Body: Mathematics and engineering graduate students, strong undergraduates
Credit: 3 Credits.
Area: Applied/Numerical Analysis

Background and Goals:
This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. Graduate students from engineering and science departments and strong undergraduates are also welcome. The course is an introduction to numerical methods for solving ordinary differential equations and hyperbolic and parabolic partial differential equations. Fundamental concepts and methods of analysis are emphasized. Students should have a strong background in linear algebra and analysis, and some experience with computer programming.

Content:
Content varies somewhat with the instructor. Numerical methods for ordinary differential equations; Lax's equivalence theorem; finite difference and spectral methods for linear time dependent PDEs: diffusion equations, scalar first order hyperbolic equations, symmetric hyperbolic systems.

Alternatives:
There is no real alternative; Math 471 (Intro to Numerical Methods) covers a small part of the same material at a lower level. Math 571 and 572 may be taken in either order.

Subsequent Courses:
Math 671 (Analysis of Numerical Methods I) is an advanced course in numerical analysis with varying topics chosen by the instructor.

Math 575 - Introduction to Theory of Numbers

Prerequisites: Math 451 and one of Math 420 or 494
Frequency: Winter (II)
Student Body: Roughly half honors mathematics undergraduates
                and half graduate students
Credit: 3 Credits. 1 credit after Math 475
Area: Number Theory

Background and Goals:

Many of the results of algebra and analysis were invented to solve problems in number theory. This field has long been admired for its beauty and elegance and recently has turned out to be extremely applicable to coding problems. This course is a survey of the basic techniques and results of elementary number theory. Students should have significant experience in writing proofs at the level of Math 451 and should have a basic understanding of groups, rings, and fields, at least at the level of Math 412 and preferably Math 493. Proofs are emphasized, but they are often pleasantly short.

Content:

This is a first course in number theory. Topics covered include divisibility and prime numbers, congruences, quadratic reciprocity, quadratic forms, arithmetic functions, and Diophantine equations. Other topics may be covered as time permits or by request.

Alternatives:

Math 475 (Elem. Number Theory) is a version of Math 575 which puts much more emphasis on computation and less on proof. Only the standard topics above are covered, the pace is slower, and the exercises are easier.

Subsequent Courses:

All of the advanced number theory courses Math 675 (Analytic Theory of Numbers), 676 (Theory of Algebraic Numbers), 677 (Diphantine Problems), 678 (Modular), and 679 (Elliptic Curves) presuppose the material of Math 575. Each of these courses is devoted to a special subarea of number theory.

Math 582 - Introduction to Set Theory

Prerequisites: Math 412 or 451 or equivalent experience with abstract mathematics
Frequency: Winter (II)
Student Body: Undergraduate mathematics (often honors) majors and some graduate students
Credit: 3 Credits.
Area: Logic

Background and Goals:
One of the great discoveries of modern mathematics was that essentially every mathematical concept may be defined in terms of sets and membership. Thus Set Theory plays a special role as a foundation for the whole of mathematics. One of the goals of this course is to develop some understanding of how Set Theory plays this role. The analysis of common mathematical concepts (e.g., function, ordering, infinity) in set-theoretic terms leads to a deeper understanding of these concepts. At the same time, the student will be introduced to many new concepts (e.g., transfinite ordinal and cardinal numbers, the Axiom of Choice) which play a major role in many branches of mathematics. The development of Set Theory will be largely axiomatic with the emphasis on proving the main results from the axioms. Students should have substantial experience with theorem-proof mathematics; the listed prerequisites are minimal and stronger preparation is recommended. No course in mathematical logic is presupposed.

Content:
The main topics covered are set algebra (union, intersection), relations and functions, orderings (partial, linear, well), the natural numbers, finite and denumerable sets, the Axiom of Choice, and ordinal and cardinal numbers.

Alternatives:
Some elementary set theory is typically covered in a number of advanced courses, but Math 582 is the only course which presents a thorough development of the subject.

Subsequent Courses:
Math 582 is not an explicit prerequisite for any later course, but it is excellent background for many of the advanced courses numbered 590 and above.

Math 583 - Probabilistic and Interactive Proofs

Prerequisites: Math 412, 451 or instructor permission
Frequency: Winter (II)
Student Body: Undergraduate mathematics (often honors) majors and some graduate students
Credit: 3 Credits.

Background and Goals:
Can we be convinced that a proof is correct, even if we only check it in three places? Can a proof convince us that a statement is true, while giving us no aid in convincing anyone else that the statement is true? The answer to both is affirmative. How? Using randomness and interaction, two elements missing from traditional deductive proofs. Why? Checking a proof in just a few places is useful for checking computer-generated proofs that are too long to read (for example, a big data algorithm); there are also surprising connections to showing that certain functions cannot be computed or even approximatedefficiently. A "zero-knowledge proof" might be used, for example, for a purchaser Peggy to prove to a vender Victor that Peggy is the rightful owner of a credit card, without giving Victor any ability to prove (fraudulently) that Victor is the owner of that credit card.

Content:
Probabilistically-checkable proofs, zero-knowledge proofs, and other interactive proofs are studied and their computational and other advantages discussed. Appropriate background material in mathematics and computer science is presented.

Math 590 - An Introduction to Topology

Prerequisites: Math 451
Frequency: Winter (II)
Student Body: Mathematics graduate students, some non-math graduate students, and mathematics undergraduates
Credit: 3 Credits.
Area: Geometry/Topology

Background and Goals:
The purpose of this course is to introduce basic concepts of topology. Most of the course will be devoted to the fundamentals of general (point set) topology.

Content:
Topics include metric spaces, topological spaces, continuous functions and homeomorphisms, separation axioms, quotient and product topology, compactness, and connectedness. We will also cover a bit of algebraic topology (e.g., fundamental groups) as time permits.

Alternatives:
Math 490 (Intro to Topology) is a more gentle introduction that is more concrete, somewhat less rigorous, and covers parts of both Math 591 (General and Diff. Topology) and Math 592 (Intro to Algebraic Topology). Math 591 is a more rigorous course covering much of this material and more. Combinatorial and algebraic aspects of the subject are emphasized over the geometrical.

Subsequent Courses:
Students who take Math 590 will be well prepared to continue with Math 591 (General and Diff. Topology) and/or Math 537 (Diff. Manifolds).

Math 591 - General and Differential Topology

Prerequisites: Math 451, 452, 590
Frequency: Fall (I)
Student Body: Mainly mathematics graduate students, a few mathematics undergraduates and non-math graduate students
Credit: 3 Credits.
Area: Geometry/Topology

Background and Goals:
This is one of the basic courses for students beginning the PhD program in mathematics. The approach is rigorous and emphasizes abstract concepts and proofs. The first 2-3 weeks of the course will be devoted to general topology, and the remainder of the course will be devoted to differential topology.

Content:
Topics include: Product and quotient topology, CW-complexes, group actions, topological groups, topological manifolds, smooth manifolds, manifolds with boundary, smooth maps, partitions of unity, tangent vectors and differentials, the tangent bundle, submersions, immersions and embeddings, smooth submanifolds, Sard's Theorem, the Whitney embedding theorem, transversality, Lie groups, vector fields, Lie brackets, Lie algebra, multilinear algebra, vector bundles, differential forms, exterior derivatives, orientation, De Rham cohomology groups, homotopy invariance, degree theory.

Alternatives:
None

Subsequent Courses:
Math 592 (Intro to Algebraic Topology) is the natural sequel.

Alternatives:
None

Subsequent Courses:
Math 592 (Intro to Algebraic Topology) is the natural sequel.

Math 592 - An Introduction to Algebraic Topology

Prerequisites: Math 591
Frequency: Winter (II)
Student Body: Largely mathematics graduate students
Credit: 3 Credits.
Area: Geometry/Topology

Background and Goals:
This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

Content:
Fundamental group, covering spaces, simplicial complexes, graphs and trees, applications to group theory, singular and simplicial homology, Eilenberg-Steenrod axioms, Brouwer's and Lefschetz' fixed-point theorems, and other topics.

Alternatives:
None

Subsequent Courses:
Math 695 (Algebraic Topology I).

Math 593 - Algebra I

Prerequisites: Math 494; or Math 412, 420, and 451
Frequency: Fall (I)
Student Body: Largely mathematics graduate students
Credit: 3 Credits.
Area: Algebra

Background and Goals:
This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs. This course, together with Math 594, offer excellent preparation for the PhD Qualifying exam in algebra. Students should have had a previous course equivalent to Math 493 (Honors Algebra I).

Content:
Topics include basics about rings and modules, including Euclidean rings, PIDs, UFDs. The structure theory of modules over a PID will be an important topic, with applications to the classification of finite abelian groups and to Jordan and rational canonical forms of matrices. The course will also cover tensor, symmetric, and exterior algebras, and the classification of bilinear forms with some emphasis on the field case.

Alternatives:
None

Subsequent Courses:
Math 594 (Algebra II) and Math 614 (Commutative Algebra I).

Math 594 - Algebra II

Prerequisites: Math 593
Frequency: Winter (II)
Student Body: Largely mathematics graduate students
Credit: 3 Credits.
Area: Algebra

Background and Goals:
This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

Content:
Topics include group theory, permutation representations, simplicity of alter nating groups for n>4, Sylow theorems, series in groups, solvable and nilpo-tent groups, Jordan-Hölder Theorem for groups with operators, free groups and presentations, fields and field extensions, norm and trace, algebraic losure, Galois theory, and transcendence degree.

Alternatives:
None

Subsequent Courses:
Math 612 (Algebra III), Math 613 (Homological Algebra), Math 614 (Commu-tative Algebra I), and various topics courses in algebra.

Math 596 - Analysis I (Complex)

Prerequisites: Math 451
Frequency: Fall (I)
Student Body: Largely mathematics graduate students
Credit: 3 Credits. 2 credits after Math 555
Area: Analysis

Background and Goals:
This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

Content:
Review of analysis in $\mathbf{R}^2$ including metric spaces, differentiable maps, Jacobians; analytic functions, Cauchy-Riemann equations, conformal mappings, linear fractional transformations; Cauchy's theorem, Cauchy integral formula; power series and Laurent expansions, residue theorem and applications, maximum modulus theorem, argument principle; harmonic functions; global properties of analytic functions; analytic continuation; normal families, Riemann mapping theorem.

Alternatives:
Math 555 (Intro to Complex Variables) covers some of the same material with greater emphasis on applications and less attention to proofs.

Subsequent Courses:
Math 597 (Analysis II -Real), Math 604 (Complex Analysis II), and Math 605 (Several Complex Variables).

Math 597 - Analysis II (Real)

Prerequisites: Math 451 and 420; or Math 395
Frequency: Winter (II)
Student Body: Largely mathematics graduate students
Credit: 3 Credits. Reduced credits (2) granted to students who have taken or are enrolled in Math 551
Area: Analysis

Background and Goals:
This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

Content:
Topics include: Lebesgue measure on the real line; measurable functions and integration on **R**; differentiation theory, fundamental theorem of calculus; function spaces, $L^p(\mathbf{R})$, C(K), Hölder and Minkowski inequalities, duality; general measure spaces, product measures, Fubini's Theorem; Radon-Nikodym Theorem, conditional expectation, signed measures, introduction to Fourier transforms.

Alternatives:
None

Subsequent Courses:
Math 602 (Real Analysis II).

Historical Courses Offerings.

This is a *historical* listing of math courses by semester.

Fall: 105, 115, 116, 117, 145, 147, 156, 175, 185, 201, 214, 215, 216, 217, 285, 286, 295, 312, 316, 351, 371, 395, 403, 404, 412, 417, 419, 420, 423, 424, 425, 427, 431, 440, 450, 451, 454, 463, 465, 471, 472, 474, 481, 490, 493, 497, 498, 501, 520, 523, 525, 526, 543, 550, 555, 556, 558, 559, 561, 565, 571, 591, 593, 596

Winter: 105, 115, 116, 146, 147, 176, 186, 201, 214, 215, 216, 217, 285, 286, 296, 297, 310, 316, 351, 354, 371, 389, 396, 403, 404, 412, 416, 417, 419, 422, 423, 424, 425, 433, 440, 450, 451, 452, 454, 462, 465, 471, 472, 474, 475, 486, 489, 494, 498, 521, 524, 525, 526, 542, 551, 555, 557, 559, 561, 562, 564, 566, 567, 571, 572, 575, 582, 583, 590, 592, 594, 597

Spring: 105, 115, 116, 215, 216, 217, 417, 423, 425, 451, 454

Summer: 105, 115, 116, 215, 216, 417, 423, 425, 450, 471

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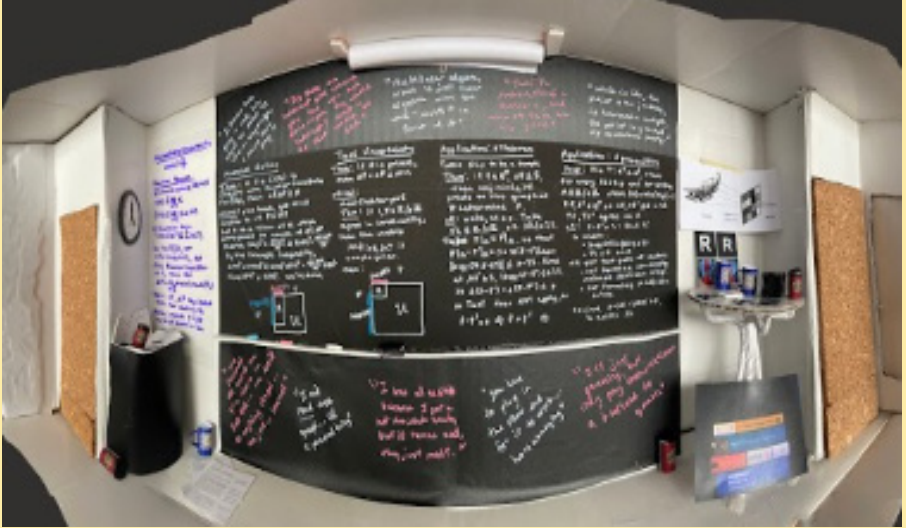
Model of the Nesbitt Room created by Maiya Yu

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