

# Distinguished RTG Lecture Series

## Professor Jared Weinstein Boston University April 3, 4, 6 & 7, 2023



### Talk 1: Higher Modularity of Elliptic Curves

Monday, April 3, 4:30 pm, 4088 East Hall

**Abstract:** Elliptic curves  $E$  over the rational numbers are modular: this means there is a nonconstant map from a modular curve to  $E$ . When instead the coefficients of  $E$  belong to a function field, it still makes sense to talk about the modularity of  $E$  (and this is known), but one can also extend the idea further and ask whether  $E$  is ' $r$ -modular' for  $r=2,3,\dots$ . To define this generalization, the modular curve gets replaced with Drinfeld's concept of a 'shtuka space'. The  $r$ -modularity of  $E$  is predicted by Tate's conjecture. In joint work with Adam Logan, we give some classes of elliptic curves  $E$  which are 2- and 3-modular.

### Talk 2: Local Shimura Varieties

Tuesday, April 4, 4:00 pm, 1360 East Hall

**Abstract:** In the classification of unitary representations of the Lie group  $SL_2(\mathbb{R})$ , the "discrete series" representations can be modeled on a space of  $L^2$  functions on the upper half plane, and there's essentially one for each integer. What if  $\mathbb{R}$  is replaced with the field  $\mathbb{Q}_p$  of  $p$ -adic numbers? The group  $SL_2(\mathbb{Q}_p)$  still has a discrete series, but it is much richer than that of  $SL_2(\mathbb{R})$ . To study it, one can consider a  $p$ -adic version of the upper half plane. But strangely, the  $p$ -adic upper half plane isn't simply connected, and to unlock the full discrete series, one needs to study some of its finite covering spaces (the Lubin-Tate tower).

And for  $p$ -adic groups other than  $SL_2(\mathbb{Q}_p)$ ? This talk will be an invitation to the chain of beautiful ideas leading from the Lubin-Tate tower to a much more general notion of "local Shimura varieties" which are implicated in the Langlands program for  $p$ -adic fields.

### Talks 3 & 4: Geometrization of the Local Langlands Program

Thursday, April 6, 4:00 pm, 3088 East Hall & Friday, April 7, 4:00 pm, 1084 East Hall

**Abstract:** In geometric Langlands, you start with a curve  $X$ , and try to put into correspondence two very different-looking entities: Local systems of rank  $n$  on  $X$  on the one hand, and Hecke eigensheaves on  $\text{Bun}_n(X)$  on the other. In the recent work of Fargues-Scholze, there is a beautiful conjectural extension of this story to the  $p$ -adic setting. In that setting, the role of  $X$  is played by the Fargues-Fontaine curve, and the local systems are Galois representations.

The conjecture of Fargues-Scholze has impressive explanatory power when it comes to the local Langlands program. For instance, it implies all cases of the Kottwitz conjecture on the cohomology of local shtuka spaces. We only know certain cases of that conjecture, notably for the Lubin-Tate tower (Harris-Taylor). We will discuss work with Hansen and Kaletha detailing partial results on the Kottwitz conjecture.

*Professor Weinstein earned his PhD at the University of California, Berkeley and was a NSF Postdoctoral Fellow at the University of California, Los Angeles (UCLA) and a member of the Institute for Advanced Study (IAS). He is the recipient of the Simons Foundation and Sloan Fellowships.*