Computations of Steady Solutions in Rayleigh-Bénard Convection

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Introduction — The objective of this project is to utilize new advanced computational tools to discern the fundamental mechanisms governing the global transport of heat and momentum in an outstanding problem in fluid dynamics, i.e., Rayleigh-Bénard convection, the buoyancy-driven flow in a fluid layer heated from below and cooled from above. Specifically, we seek to elucidate the flow structure and heat transport properties of steady albeit *dynamically unstable* solutions in the strongly nonlinear regime. In order to push the computations to sufficiently large/small parameter regimes to discover the asymptotic transport behavior, **the participants are expected to modify the existing Matlab code to a C/C++ code.**

Background and significance — Convection is buoyancy-driven fluid flow resulting from density variations in the presence of a gravitational field. Beyond its role in myriad engineering applications, convection underlies many of nature's dynamical designs on larger-than-human scales, including the atmospheric and oceanic motions central to meteorology and climate science. A key feature of convection is heat transport, and predicting transport for large applied temperature gradients in the strongly nonlinear regime remains a major challenge for the field. Since the 1960s two distinct scaling theories have contended to quantitatively characterize the strongly nonlinear regime, yet no clear winner has emerged. Our recent investigations of steady roll solutions [1, 2] offer new evidence for one of these theories. Interestingly (and perhaps somewhat surprisingly), the preliminary work reveals that aspect-ratio-optimized steady roll solutions transport *more* heat than turbulent experiments or simulations at comparable parameters. This study has the potential to resolve the 60-year conundrum concerning asymptotic transport in turbulent convection.

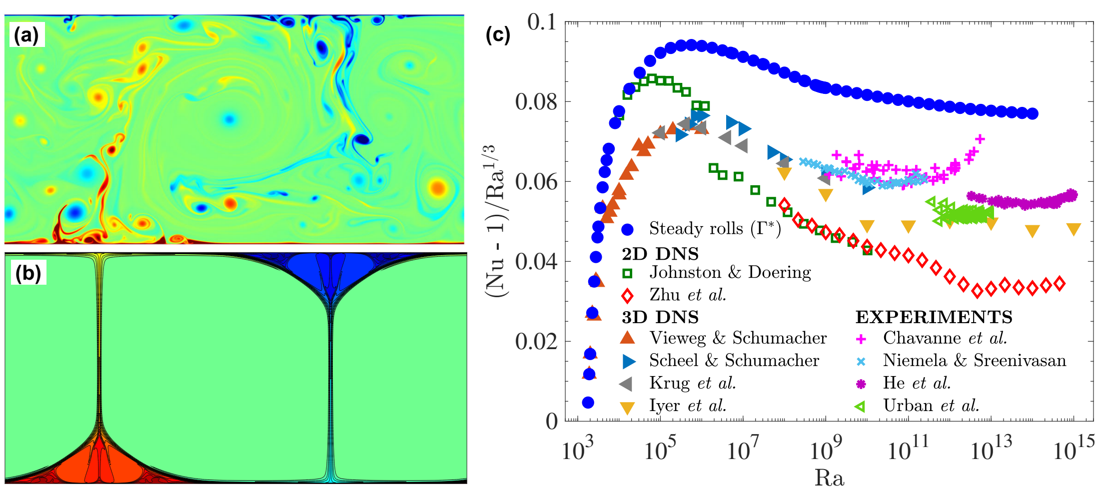

Figure 1: (a,b): Temperature fields for (a) direct numerical simulations of turbulent 2D convection and (b) the corresponding fully resolved unstable steady solution at similar parameters. Steady rolls comprise the backbone of turbulent convection. (c): Compensated heat flux for aspect-ratio-optimized steady rolls and turbulent 2D & 3D direct numerical simulations and experiments.

Prerequisites — Linear Algebra (Math 214) and Numerical Methods (Math 371 or 471 or 571). Coding skills in Matlab, C/C++ or Fortran.

References

[1] B. Wen, D. Goluskin, M. LeDuc, G. P. Chini, C. R. Doering. 2020 Steady Rayleigh–Bénard convection between stress-free boundaries, J. Fluid Mech. 905 (R4).

[2] B. Wen, D. Goluskin, C. R. Doering. 2021 Steady Rayleigh–Bénard convection between no-slip boundaries, in press in J. Fluid Mech. (arXiv:2008.08752).