Unraveling the patterns of Painlevé zeros

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Project Description

In this project we plan to study zeros of solutions to Painlevé ODEs. There are six Painlevé ODEs which are second order nonlinear equations with applications in other fields of mathematics and physics. They admit families of rational solutions. As the degree of these solutions gets large, their complex valued zeros fill out certain shapes in a complex plane, for example triangles for Painlevé III and combinations of triangles and rectangles for Panlevé IV. It can happen that the solutions depend on extra parameters and it is intriguing to observe how zeros move as parameter changes.

We want to focus on the less studied family of rational solutions to the Painlevé VI equation. We expect that it depends on parameters. Our first goal is to compute and visualize its fascinating zeroes pattern.

Prerequisites:
Some experience in a programming language or Mathematica.
Math 216 or equivalent
An understanding of what a complex number is. MATH 555 would be nice but is not required.