Winter 2014 Graduate Course Descriptions

---

501 AIM Student Seminar Vershynin F 12:00-1:00pm & 3:00-4:00pm

Math 501 is a required course for all students enrolled in the Applied and Interdisciplinary Mathematics (AIM) MS and PhD graduate programs. In the Winter term, all first-year AIM students from both programs must sign up for this course. Due to the highly specialized content of the course, enrollment is available only for students in an AIM degree program.

The purpose of Math 501 is to address specific issues related to the process of studying applied mathematics in the AIM program and becoming an active member of the research community. The weekly meetings of the class will be divided among three types of sessions:

1. Focus on… "presentations." These are presentations on various topics, some of immediate practical significance for students and others of a farther-reaching nature. These discussions will include aspects of scholarly writing, research, and career development.
2. AIM Faculty Portraits. These are short presentations by faculty members in the Mathematics Department and other partner disciplines who are potential advisors or committee members for AIM students. The AIM faculty portraits provide a direct channel for students to discover what research is being done in various areas by current faculty, and to see what kind of preparation is required for participating in such research.
3. AIM Research Seminar Warm-up talks. One of the course requirements for Math 501 is weekly attendance of the AIM Research Seminar that takes place from 3-4 PM each Friday. The warm-up talks are presentations during the regular course meeting time by particularly dynamic speakers slated to speak in the AIM Research Seminar later the same day as a way to provide background material with the goal of making the AIM Research Seminar lecture more valuable for students.

Text: There is no regular textbook for Math 501.

Course Requirements Weekly attendance both of the course meeting and also of the AIM Research Seminar is required for Math 501. **If you are registering for Math 501 you must be available both during the regular class time of 12-1 on Fridays as well as during the AIM Research Seminar which runs 3-4 on Fridays. If you are teaching, you should keep both of these obligations in mind when you submit your class/seminar schedule prior to obtaining a teaching assignment.** Other requirements, including possible assignments related to topics discussed in the lectures, will be announced by the instructor in class.

Winter 2014 Graduate Course Descriptions

---

521 Life Contingencies II Marker TTh 10:00-11:30am
Prerequisites: Math 520 or permission of the instructor Moore TTh 11:30-1:00pm

Course Description: Background and Goals: This course extends the single decrement and single life ideas of Math 520 to multi-decrement and multiple-life applications directly related to life insurance. The sequence 520--521 covers covers the material for Examination 3L of the Casualty Actuarial Society and for Examination MLC of the Society of Actuaries.

Content: Topics include multiple life models--joint life, last survivor, contingent insurance; multiple decrement models---disability, withdrawal, retirement, etc.; and reserving models for life insurance. This corresponds to chapters 7--11 and 15 of Bowers et al.

Text: Actuarial Mathematics, Second Edition, Bowers et al. Publisher: Society of Actuaries; 2nd edition (May 1997) ISBN-10: 0938959468, ISBN-13: 978-0938959465

---

523 Risk Theory Wang TTh 11:30AM-1:00PM
Prerequisites: Math 425

Course Description: The goals of this course are to understand parametric distributions for the purpose of (1) modeling frequency, severity, and aggregate insurance losses, (2) analyzing the effects of insurance coverage modifications, (3) estimating parameters from insurance loss data, and (4) estimating future insurance losses via credibility theory.

---

525 Probability Theory Liechty TTh 10 - 11:30AM
Prerequisites: MATH 451 (strongly recommended). MATH 425/STATS 425.

Course Description: This is a fairly rigorous introduction to probability theory with some emphasis given to both theory and applications, although a knowledge of measure theory is not assumed. Topics covered are: probability spaces, discrete and continuous random variables, conditional probability, generating functions, Markov chains, limit theorems.

Text: Geoffrey Grimmett and David Stirzaker, Probability and Random Processes, Third Edition, Oxford University Press, 2008 . Optional complementary text: Introduction to Probability Models, 10th ed. S. Ross, Academic Press, 2010.

---

526 Discrete Stochastic Processes — Nadtochiy TTh 10:00-11:30am; Yu TTh 11:30am-1pm

Prerequisites: *Required: Math 525 or basic probability theory including:probability measures, random variables, expectations, cumulative distribution and probability density functions, conditional probabilities and independence, forms of convergence of random variables (almost sure, in probability, in distribution), law of large numbers. Recommended: Good understanding of advanced calculus covering limits, series, the notions of continuity differentiation and integration; limit theorems for sums of random variables (e.g. central limit theorem); interchanging the limit and integration/expectation (monotone and dominated convergence theorems); linear algebra, including matrices, eigenvalues and eigenfunctions.*

Course Description: The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include: Markov chains (Markov property, recurrence and transience, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times, Markov queues); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales); time-permitting, introduction to stochastic integration and Ito's formula. Significant applications will be an important feature of the course.

Text: Required: Essentials of Stochastic Processes, 2nd ed. (Durrett). Whenever the lecture material goes beyond the scope of the book, the lecture notes will be posted in the Resources section of this website. Optional: Stochastic Processes (Ross), Introduction to Stochastic Processes (Cinlar), Introduction to Stochastic Processes (Lawler), Probability and Measure (Billingsley).

---

528 Topics in Property: Casual Insurance — Marker TTh 2:30-4:00pm

Prerequisites: *Math 215, Math 217, and Math 425, or equivalents. Or permission of instructor.*

Goal: Present fundamental ideas underlying Casualty Actuarial science

- What risks does a P-C insurance policy cover?
- How does the insured person or business can reduce its risk?
- How does an insurer set a price for covering risk?

• How much must an insurer set aside for its unpaid claims?

• How does an insurer reduce the uncertainty in its results?

The second part of the course will explore topics such as:

• Using statistical models to projecting claim amounts.

• Incorporating modern financial theory into actuarial projections.

• Topics in insurance company operations.

Much of this material is included in Casualty Actuarial Society (CAS) examinations 4 (same as SOA Exam C), 5, and 7.

Text: All materials are available online.

---

537 Introduction to Differentiable Manifolds Mustata MWF 11-12:00pm

This is an introductory course about differential manifolds and differential topology. This course is intended for students with a strong background in topology, linear algebra, and multivariable advanced calculus equivalent to the courses 590 and 594. Its goal is to introduce the basic concepts and results of differential topology.

Text: Differential Topology (Guillemin and Pollack). Occasionally, we will also draw material from Frank Warner's book, Foundations of differentiable manifolds and Lie groups.

Text: Biological Sequence Analysis (R. Durbin, et al.)

---

547 Probabilistic Modeling in Bioinformatics Burns MWF 9:00-10:00am

Prerequisites: *Background prerequisites are very flexible. This is meant to be an interdisciplinary course, and allowances will be made for diverse backgrounds. If you come from the quantitative side, basic probability (like Math/Stats 425) and perhaps some statistics would be enough; if you come from the biological or wet lab side, something at the level of MCDB 427 would be more than enough. In any case, the course will be liberally interspersed with background lectures for novices to one or another area we will touch upon. Guest lecturers will address topics in modeling DNA mechanics, on the experimental evidence and data, and on medical settings for some of the questions treated.*

Course Description: This course aims to review some of the more classical problems in “linear DNA analysis”, problems such as multiple sequence alignment, protein families and parsing the linear structure of protein coding gene sequences. These are studied via probabilistic models such as hidden markov models and more general graphical models. Transitional problems to be discussed are destabilization of the DNA double helix and its relation to DNA dynamics, and sequence prediction methods for the position of nucleosomes, the structures around which the 3D form of the DNA is built up. The latter half of the course will be directed towards recent work on the analysis of architecture and function of chromatin in 3D and 4D, based largely on *chromatin conformation capture* data, and time series thereof. Possible mechanical models of DNA global architecture will be considered. There has recently appeared very exciting work of Steven Smale on prediction of protein-DNA interactions and on the fundamental protein folding problem. The course is headed towards a series of open problems concerning his methods and whether they can be applied to the prediction of chromatin architecture. These methods come from probability and machine learning techniques.

Text: Readings will be provided, mainly papers from the literature, but some detailed references and notes provided.

---

555 Introduction to Complex Variables — Doering — MW 8:30-10:00am

Prerequisites: *MATH 451 (strongly recommended). MATH 425/STATS 425.*

Course Description: This course is an introduction to the theory of complex valued functions of a complex variable with substantial attention to applications in science and engineering. Concepts, calculations, and the ability to apply principles to physical problems are emphasized over proofs, but arguments are rigorous. Material in Math 555 is prerequisite to many advanced courses in applied mathematics, science and engineering. Content: Differentiation and integration of complex valued functions of a complex variable, series, mappings, residues, with applications including Fourier and Laplace transforms and inversions. We will also consider evaluation of improper real integrals and, as time permits, applications to electrostatics, thermostatics, and fluid dynamics.

Text: Complex Variables and Applications, 8th ed. (Churchill and Brown)

---

557 Applied Asymptotic Analysis: — Borcea — TTh 1:00pm - 2:30pm

Prerequisites: *MATH 217, 419 or 420; MATH 451 and 555*

Course Description: This class is a course on asymptotic methods. It surveys the important topics: asymptotic approximation of integrals using Laplace's method, stationary phase and the method of steepest descent; asymptotic methods for differential equations including WKB expansions, multiple scales and singular perturbation theory. The course introduces the asymptotic techniques and shows how to obtain solid error estimates of the approximations using rigorous analysis.

Text: Applied Asymptotic Analysis , Miller, Peter 0-8218-4078-9 Required

---

563 Advanced Mathematical Methods for the Biological Sciences — Jackson — TTh 11:30-1:00pm

Prerequsities: *Math 404, 451, 454*

Course Description: This course will focus on the derivation, analysis, and simulation of partial differential equations (PDEs) that model specific phenomena in molecular, cellular, and population biology. A goal of this course is to understand how the underlying spatial variability in natural systems influences motion and behavior. Mathematical topics covered include derivation of relevant PDEs from first principles; reduction of PDEs to ODEs under steady state, quasi-steady state, and traveling wave assumptions; solution techniques for PDEs, birfucation analysis and concepts of spatial stability and instability. These topics will be introduced within the setting of classical and current problems in biology and the biomedical sciences such as cell motion, transport of biological substances, tumor growth, immunology, and biological pattern formation. Above all, this course aims to enhance the interdisciplinary training of advanced undergraduate and graduate students from mathematics and other disciplines by introducing fundamental properties of partial differential equations in the context of interesting biological phenomena.

---

566 Combinatorial Theory — Lam — MWF 11:00-12:00pm

Prerequisites: *Previous exposure to abstract algebra and (proof-based) linear algebra.*

Course Description: This course will be an introduction to algebraic combinatorics. Some of the topics I intend to discuss are: generating functions, posets and Mobius functions, Young tableaux, and symmetric functions.

---

567 Introduction to Coding and Information Theory — Lagarias — TTh 1:00-2:30pm

Prerequisites: *A working knowledge of linear algebra is needed. Students should know some basic probability theory, as in Math 425, but will review it.*

Course Description: Coding theory was invented to facilitate reliable transmission of information over noisy communication networks. The basic algebraic and geometric ideas behind it are important in a range of pure and applied questions. Course objectives: To cover basic information theory and the foundations of theory of error correcting codes. For math students it introduces to an important area of applications of linear algebra and algebraic structures. For EE/CS students it provides a mathematical basis for study of communications technology. Course Description: Introduction to Shannon's theorem and channel capacity. Entropy. Review of tools from linear algebra and abstract algebra. Finite fields and polynomials over finite fields. Basic examples of codes: Golay, Hamming, BCH, Reed-Muller, Reed-Solomon codes. New codes from old codes. Linear and cyclic codes. Introduction to decoding including syndrome decoding, weight enumerators. Fundamental limits on coding efficiency. If time permits, I hope to cover one additional topic: most likely LDPC codes or turbo codes. [Some modifications to the above topics might occur depending on the composition of the class.]The course grade will be based on problem sets (at most 8), a midterm exam (probably in class) and a final (possibly take-home).

Text: Stephen Roman, Coding and Information Theory, Graduate Texts in Math. No. 134, Springer-Verlag, 1992.

Another reference; van Lint, Intro. coding theory(third edition) Springer 1999). (This text is NOT the same as S. Roman, Introduction to Coding and Information Theory, Springer 1997.) The text may be supplemented with other materials.

---

571 Numerical Linear Algebra Krasny TTh 10:00-11:30am

Prerequisites: *A course in linear algebra (e.g. Math 217, 417, 419, or equivalent) and some familiarity with Matlab*

Course Description: Math 571 is an introduction to numerical linear algebra, a core subject in scientific computing. Three types of problems are considered: (1) linear systems, (2) eigenvalues, (3) least squares problems. These problems often arise in scientific applications, and many algorithms have been developed for their solution. However, standard approaches may fail if the problem size is too large or if the problem is ill-conditioned. We'll investigate these issues and study some of the algorithms that provide accurate and efficient numerical results. As an application, we'll consider finite-difference schemes for boundary value problems. The course grade will be based on homework, a midterm exam, and a final exam. Some homework problems will require Matlab computing.

Text: Numerical Linear Algebra by Trefethen and Bau (published by SIAM)

---

572 Numerical Methods for Scientific Computing II Karni TTh 11:30-1:00pm

Prerequisites: *Solid background in advanced calculus, linear algebra and working knowledge of a computing programming language (such as C, C++ or Fortran) or a computing environment (such as Matlab or Python).*

Course Description: This course is an introduction to numerical methods for boundary-value and initial-value problems. The course will cover numerical methods for ordinary differential equations and for elliptic, parabolic and hyperbolic partial differential equations. Nonlinear hyperbolic partial differential equations may also be discussed, if time permits.

The course will focus on the derivation of methods, on their accuracy, stability and convergence properties with brief comments on practical aspects of efficient implementation of the methods. The course should be useful to students in mathematics, physics and engineering.

טופיצס: Finite difference approximations; boundary-value and initial-value ODEs, consistency, stability and convergence, Lax equivalence theorem, Gaussian elimination, Gauss-Seidel, Jacobi, and SOR, Runge-Kutta and multistep methods, methods for stiff ODEs, elliptic equations, diffusion equation, Crank-Nicolson, stability analysis by Fourier and energy methods, maximum principle, ADI, linear advection equation, CFL condition, upwind, Lax-Wendroff, Lax Friedrichs, scheme. Numerical boundaries for the wave equation. Conservation laws and shocks. Finite volume methods (time permitting)

Text: Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems by R.J. LeVeque, SIAM, 2007.

---

582 Introduction to Set Theory Blass MWF 12:00 -1:00pm

Prerequisites: *The official prerequisite, "Math 412 or 451 or equivalent experience with abstract mathematics," means that students should be comfortable with writing mathematical proofs. No specific knowledge of set theory will be presupposed.*

Course Description: This is an introductory course in axiomatic set theory.

Topics include:

- The intuitive concept of set; paradoxes.
- Type theory and the cumulative hierarchy of sets.
- The Zermelo-Fraenkel axioms for set theory and basic set-theoretic constructions.
- Set-theoretic representation of the fundamental concepts of mathematics (e.g., function, number) and proofs of basic properties of these concepts (e.g., mathematical induction).

Infinite cardinal and ordinal numbers and their arithmetic.
The axiom of choice and equivalent axioms (e.g., Zorn's Lemma).

Additional topics may be discussed if time permits.

Text: Elements of Set Theory ,either edition is OK, Herbert B. Enderton Required

---

590 An Introduction to Topology Scott MWF 12:00 - 1:00pm

Prerequisites: *Math 451 or an equivalent real analysis course.*

Course Description: This is an introduction to topology with the emphasis on the set-theoretic aspects of the subject.

Basic topics to be covered include metric spaces, topological spaces, continuous functions, homeomorphisms, compactness and connectedness. Many examples will be discussed including surfaces and manifolds. If time permits, some more advanced topics will also be covered. These could include quotient spaces, group actions, and a topological proof of the fundamental theorem of algebra.

Text: Topology, Second edition, James R. Munkres 0-13-181629-2 Required

---

592 Introductory Algebraic Topology Kriz MWF 10:00-11:00am

Prerequisites: *Math 591 or a similar point set topology course.*

Course Description: This is a beginning course in algebraic topology, suitable for first year graduate students or other students at a similar level. We will cover the fundamental group and covering spaces and singular homology theory. Some interesting enrichment topics and geometric applications will also be presented.

Text: A Concise Course in Algebraic Topology , J.P. May 978-0226511832 Optional
Elements of Algebraic Topology J.R. Munkres 978-0201627282 Optional

---

594 Algebra II Griess TTh 8:30 - 10:00am

Prerequisites: *Mathematical Maturity. Strong linear algebra background. Previous coursework in abstract algebra would be helpful but not necessary. Students will be expected to write rigorous proofs in the course. Previous experience in writing proofs is recommended.*

Course Description: This is the alpha course on groups and fields. We study general theory of groups (subgroup structure, centralizers, normalizers, homomorphisms, etc.), group actions (point stabilizers, multiple transitivity, primitivity) and applications to Sylow theory, decompositions of groups. There will be analysis of familiar groups, such as symmetric groups, alternating groups and linear groups, and group presentations (by generators and relations). In field theory, we shall develop the theory of algebraic field extensions and its relationship to solving polynomial equations. Study of groups acting as field automorphisms will accompany the study of field extensions. The Fundamental Theorem of Galois Theory will be fully discussed and many types of field extensions analyzed, including transcendental. Supplementary topics on groups and fields will be included as time and interest permit.

Text: Algebra, a Graduate Course Author. I. Martin Isaacs available at www.ams.org

[33] Algebra: A Graduate Course - I. Martin Isaacs, University of Wisconsin - AMS, 1994, 516 pp., Hardcover, ISBN-10: 0-8218-4799-6, ISBN-13: 978-0-8218-4799-2, List: US$79, All AMS Members: US$63.20, GSM/100

---

597 Analysis II (Real Analysis) Wu MWF 2:00 - 3:00pm

Prerequisites: *Math 451, 452, Math 513*

Course Description: References: Royden: Real Analysis. Natanson: Theory of Functions of a Real Variable. Dym and Mckean: Fourier Analysis. Stein and Weiss: Introduction to Fourier Analysis on Euclidean Spaces. Folland: Real Analysis. Background and Goals: This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.Content: Topics include Lebesgue measure on the real line and R^n; measurable functions and integration; differentiation theory, fundamental theorem of calculus; function spaces, L^p(R), C(K), Holder and Minkowski inequalities, duality; general measure spaces, product measures, Fubini's Theorem; Radon-Nikodym Theorem, signed measures, introduction to Fourier analysis. Grades will be based on weekly homework and a midterm and a final exam.

---

615 Commutative Algebra II: Topics in Commutative Algebra — Hochster — MWF 2:00-3:00pm

Prerequisites: *Math 614*

Course Description: The course will deal with integral closure of ideals, including Rees valuations, the Lipman-Sathaye Jacobian theorem and its application to the

Briancon-Skoda theorem, the relationship with tight closure theory (which will be developed as needed) and other closure operations on ideals, as well as other related topics, including the behavior of symbolic powers of ideals. Many open questions will be discussed.

There is no required textbook. Eisenbud's "Commutative Algebra with a view towards Algebraic Geometry" is the best overall reference.

Text: Lecture notes will be provided.

---

626 Random Processes — Rudelson — TTh 10:00-11:30am

Prerequisites: *Math 625 or instructor's approval.*

Course Description: The course will focus on discrete time Markov chains and ergodic theory. After covering the basics of Markov chain theory, we will concentrate on mixing in finite chains. Mixing time characterizes how fast a Markov chain approaches the stationary distribution. This theory has seen a rapid progress in the last 20 years. Mixing in Markov chains plays a key role in many sampling and approximate counting algorithms in computer science.

Text: There is no required textbook. Recommended textbooks:

Durrett, Rick Probability: theory and examples. Fourth edition. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, Cambridge, 2010. x+428 pp. ISBN: 978-0-521-76539-8

Levin, David A.; Peres, Yuval; Wilmer, Elizabeth L. Markov chains and mixing times. With a chapter by James G. Propp and David B. Wilson. American Mathematical Society, Providence, RI, 2009. xviii+371 pp. ISBN: 978-0-8218-4739-8

---

632 Algebraic Geometry II Fulton TTh 10:00-11:30am

Prerequisites: *Math 631 and a good background in commutative algebra.*

Course Description: This course will develop some of the basic tools of algebraic geometry, with emphasis on examples and applications. Here are some of the topics we hope to discuss:

Sheaves, abstract algebraic varieties. Cohomology of sheaves. Coherent sheaves on projective varieties. Riemann-Roch for curves and surfaces. Blowups, tangent and normal cones. Introduction to intersection multiplicities and intersection theory. Picard groups, class groups. Grothendieck groups of bundles and sheaves. Grassmann and flag bundles. Representable functors. Schemes. Other topics, as time permits: Toric varieties. Hilbert schemes. Grothendieck Riemann-Roch Theorem. Mastering these tools requires a good deal of heavy lifting, much of which needs to be done individually. Signing up for this course includes an agreement that any paper passed in contains only the work of those whose names are on it.

---

635 Differential Geometry Spatzier MWF 11:00-12:00pm

Prerequisites: *This course requires a solid understanding of point set topology and manifolds, e.g. MATH 591 or MATH 437.*

Course Description: This is an introduction to Riemannian geometry which has close links with the theory of Lie groups, dynamical systems, complex and algebraic geometry.

We will cover the basic notions: affine structure, Riemannian metrics, geodesics, curvature, first and second variation formulas, Jacobi fields, spaces of constant curvature, geometry of sub manifolds. Global geometry, that is linking the underlying geometry with topological properties of the space has been of great interest and will form the second part of the course. Examples are the sphere theorem which tells us about metrics of positive curvature strictly between 1 and 4 can only be realized on spheres. Or that manifolds of nonpositive curvature have Euclidean space as universal cover. In all this, we will emphasize the geometric ideas, and keep the formalism within reason.

Text: Do Carmo: Riemannian Geometry

---

636 Topics in DIfferential Geometry: Riemann Surfaces and Complex Dynamics — Koch — TTh 11:30-1:00pm

Prerequisites: *A solid background in complex analysis, and in covering space theory.*

Course Description: One aspect of complex dynamical systems concerns the study of iterating rational maps on the Riemann sphere. A wealth of complicated and deep behavior can emerge when a rational map is iterated; this behavior is governed by the orbits of the critical points of the map under iteration. The rational map is said to be `critically finite' if every critical point is eventually periodic. Critically finite maps are particularly nice to study. Loosely speaking, in the space of all rational maps, those which are critically finite play a role akin to the rational numbers in the reals (we will make this notion precise in the course!).

In the 1980s, William Thurston established a topological characterization of critically finite rational maps: he proved that critically finite rational maps are determined by topological invariants, and he completely characterized which invariants arise. His theorem is one of the most important results in the field of complex dynamics. It has extensive applications to understanding not only the dynamics of rational maps, but also to understanding the geometry of parameter spaces.

The central goal of this course will be to provide a proof of Thurston's theorem. The proof invokes a fundamental theme in much of Thurston's work: iterating a holomorphic map on a Teichmueller space and looking for a fixed point. In this course, we will begin by establishing the necessary background from Teichmueller theory and hyperbolic geometry. We will then take a detour through the world of iterating complex polynomials and studying the associated parameter spaces, illustrating some of the powerful applications of Thurston's theorem. Along the way, we will explore the geometry of the Mandelbrot set and see how Thurston's theorem can be used to navigate through it.

Text: References/notes will be provided. Teichmuller theory, volume I, by John H. Hubbard (optional)

---

637 Lie Groups — Prasad — TTh 11:30 -1:00pm

Prerequisites: linear algebra and differential topology.

Course Description: This is a basic introduction to Lie groups. We will begin with the definition of Lie groups, and give many examples. We will define the Lie algebra of a Lie group, and connect them via left-invariant vector fields and the exponential map (prior knowledge of Lie algebras is not assumed). We will establish some structure theorems in Lie theory.

Towards the end of the course we will discuss representation theory, with an emphasis towards representations of compact groups.

---

650 Fourier Analysis Rudelson TTh 11:30 - 1:00pm

Prerequisites: *Measure theory and complex analysis (at the level of Math 596, Math 597), basic Hilbert space notions.*

Course Description: This is a graduate level course in harmonic analysis. Harmonic analysis provides a natural tool for studying shift-invariant operators. This is a large class of linear operators, including differential operators, convolution-type integral operators etc.

The topics include:

Fourier series: Dirichlet and Fejer kernels, Fejer-Lebesgue theorem, L_2 convergence.

Fourier transform:

Plancherel theorem, inversion formula, Fourier transform in Schwartz space and in the space of the tempered distributions, Poisson summation formula.

Connections to complex analysis:

Paley--Wiener theory, Hardy spaces, F. and M. Riesz theorem.

Optional topics (depending on time and the interests of the audience):

Fourier analysis on finite abelian groups. Radon transform and applications to convex geometry.

Text: There is no required textbook. Recommended textbooks:

Stein, Elias M.; Shakarchi, Rami Fourier analysis. An introduction. Princeton Lectures in Analysis, 1. Princeton University Press, Princeton, NJ, 2003. xvi+311 pp. ISBN: 0-691-11384-X

Katznelson, Yitzhak An introduction to harmonic analysis. Third edition. Cambridge Mathematical Library. Cambridge University Press, Cambridge, 2004. xviii+314 pp. ISBN: 0-521-83829-0; 0-521-54359-2

---

654 Introduction to Mathematical Fluid Dynamics — Alben — TTh 2:30 - 4:00pm

Prerequisites: *Vector calculus. Complex analysis and differential equations at a senior undergraduate level (Math 450). Elementary physics (mechanics).*

Course Description: The course is a broad introduction to fluid dynamics. The course studies physical phenomena using a wide range of classical applied mathematical tools. Topics: Continua and conservation laws. Inviscid flow. Irrotational flow. Vorticity. Complex variable methods. Airfoil theory and conformal transformations. Viscosity and the Reynolds number. The limit of zero Reynolds number. Boundary layers. Instabilities.

Text: Elementary Fluid Dynamics by D. J. Acheson (Oxford University Press). ISBN: 0198596790.

---

669 Combinatorial Theory: Interger Points and Polytopes — Barvinok — TTh 1:00 - 2:30 pm

Prerequisites: *Good knowledge of linear algebra*

Course Description: Integer points (points with integer coordinates) and polytopes (convex hulls of finitely many points) play an important role in algebra, number theory, combinatorics, and optimization. I plan to cover some classical results regarding integer points in polytopes and convex bodies in general, such as Pick's formula, Minkowski's Convex Body Theorem(s), Ehrhart polynomial, reciprocity, as well as more recent developments related to valuations on rational polyhedra, the Lawrence-Khovanski-Pukhlikov Theorem, Brion's Theorem, integer semigroups, ''local'' formulas for the coefficients of the Ehrhart polynomial due to Berline and Vergne and some asymptotic results. Connections with other areas will be discussed as well, such as continued fractions and their extensions, relations to commutative algebra and Hilbert series, algorithmic applications and integer programming, connections to coding and sphere packing.

Grading: We will have a number of homework problem sets.

Text: There is no required text. Some of the material can be found in A. Barvinok "Integer Points in Polyhedra", Zurich Lectures in Advanced Mathematics, EMS 2008.

---

678 Modular Forms Prasanna TTh 2:30-4:00pm

Prerequisites: *Will assume some familiarity with the basic theory of modular forms and elliptic curves. Class field theory will be reviewed but some familiarity with it will be helpful as well.*

Course Description: This course will focus on rational points on elliptic curves and the conjecture of Birch and Swinnerton-Dyer. We will discuss the work of Gross-Zagier and Kolyvagin, as well as more recent developments.

Text: No textbook is required. Lecture notes will be provided.

---

697 Topics in Topology: Ruan MWF 11:00 - 12:00pm
B-model Topological String

Prerequisites: *Basic topology and geometry. Some familiarity with Hodge theory is helpful, but not necessary*

Course Description: String theory has inspired a great deal of mathematics. A famous example is mirror symmetry which predicts an equivalence between A-model of a Calabi-Yau manifold and B-model of its mirror. Among several different models, B-model topological string is the best understood model. One of best examples is Klemm's striking predication of Gromov-Witten invariant of quintic 3-fold up to genus 51 while mathematician can only prove it for genus zero and one. Facing such a huge gap, there is a great deal of recent activities in mathematical community to understand B-model rigorously. This course is designed to help young people from both mathematics and physics to get to understand the recent actions. This is the part of special semester on B-model Gromov-Witten theory supported by FRG grant. The course will be co-lectured by our residence physicist Albrecht Klemm. Other major contributors includes Kentaro Hori, Hiroshi Iritani and Si Li.

Text: Mirror symmetry, Clay Mathematics Monograph
www.claymath.org/publications/Mirror_Symmetry/

The first part of course will cover topics in the book. The second part of course plans to cover more recent topics such as the construction of higher genus B-model invariants, holomorphic anomaly equation and modularity.

---

732 Topics in Algebraic Geometry II: Berkovich Spaces | Jonsson | MWF 12:00-1:00pm

Prerequisites: *Some analysis, algebraic geometry and commutative algebra.*

Course Description: Berkovich spaces are analogues of complex manifolds that appear when replacing complex numbers by the elements of a general normed field, e.g. p-adic numbers or formal Laurent series. They were introduced in the late 1980's by Vladimir Berkovich as a more geometric alternative to the rigid spaces earlier used by Tate. In recent years, Berkovich spaces have seen a large and growing range of applications to complex analysis, tropical geometry, complex and arithmetic dynamics, the local Langlands program, Arakelov geometry, etc.

The first part of the course will be devoted to the basic theory of Berkovich spaces (affinoids, gluing, analytifications). In the second part, we will discuss various applications or specialized topics, partly depending on the interest of the audience.

Text: We will use the original text "Spectral Theory and Analytic Geometry over non-Archimedean Fields" by V. G. Berkovich. It will be complemented by lecture notes. The book "Non-Archimedean Analysis" by S. Bosch, U. Guentzer and R. Remmert will also be used as a reference.

---

756 Topics in Analysis: General Relativity | Smoller | TTh 8:30-10:00am

Prerequisites: *Some mathematical maturity obtained from a graduate course in analysis or applied mathematics.*

Course Description: General Relativity is Einstein's theory of gravity. But it is more than this; it is a theory of space and time and therefore of the dynamics of the entire universe. It is based on pure geometry, is indisputably elegant and is important in math and physics. The course will start off with a discussion of tensors, connections and curvature. We will then consider Einstein's equations, and some special solutions, including black holes. Stellar dynamics and cosmology will also be discussed. If time permits we will consider Einstein/Yang Mills equations and the anomalous acceleration of the universe.

---

775 Analytic Number Theory II Vaughan MWF 1:00-2:00pm

Prerequisites: *Math 575 or equivalent*

Course Description: In the last few years there have been a number of sensational developments in prime number theory.

1. Goldston, Pintz and Yildirim have shown that there are relatively small gaps between consecutive primes.
2. Yitang Zhang has adapted their method to show that there are infinitely many pairs of primes p, p' with 0 < p-p'< 70 x 10^6, and as a further development Maynard has given a relatively simple proof that there are infinitely many pairs of primes p, p' with 0 < p-p'< 700.
3. Green and Tao have shown that there are arbitrarily long arithmetic progressions in the primes.
4. Helfgott has shown that every odd natural number bigger than 5 is the sum of three prime numbers (the Goldbach ternary problem).

In Math 775 we will build on Math 675 to cover background further and use this to study several of these developments in detail.

Topics covered include

The Selberg sieve.
The large sieve.
Bombieri's theorem on primes in arithmetic progression, which tells us that the generalized Riemann hypothesis is true on average.
If not covered in Math 675 the following
The approximate functional equation for L-functions.
Mean value theorems.
Distribution of zeros.
The Vinogradov three primes theorem and a proof that almost all even natural numbers are the sum of two primes.
The Goldston, Pintz and Yildirim theorem that liminf((p-p')/log p) =0.
Zhang's theorem.
Some aspects of the Green, Tao work.
If time allows, Waring's problem.

There is no recommended text.
