

Winter 2018 Graduate Course Descriptions

501 AIM Student Seminar Alben F 1:00pm-2:00pm & 3:00pm-4:00pm

Prerequisites: You must be a graduate student in the AIM program to register for this course.

Course Description: The Applied and Interdisciplinary Mathematics (AIM) student seminar is an introductory and survey course in the methods and applications of modern mathematics in the natural, social, and engineering sciences. Students will attend the weekly AIM Research Seminar where topics of current interest are presented by active researchers (both from U-M and from elsewhere). The other central aspect of the course will be a seminar to prepare students with appropriate introductory background material. The seminar will also focus on effective communication methods for interdisciplinary research. MATH 501 is primarily intended for graduate students in the Applied & Interdisciplinary Mathematics M.S. and Ph.D. programs. It is also intended for mathematically curious graduate students from other areas. Qualified undergraduates are welcome to elect the course with the instructor's permission. Student attendance and participation at all seminar sessions is required. Students will develop and make a short presentation on some aspect of applied and interdisciplinary mathematics.

Text: None

506 Stochastic Analysis for Finance Bayraktar TTh 10:00am-11:30am

Course Description: The aim of this course is to teach the probabilistic techniques and concepts from the theory of continuous time stochastic processes required to understand the widely used financial models. In particular, such concepts as Brownian motion, martingales, stochastic integration/calculus, stochastic differential equations, as well as the associated mathematical tools (e.g the Feynman-Kac formula), are discussed in this course. The theory is illustrated with examples from asset pricing, optimal investment and risk management.

Text: (Required) Stochastic Calculus for Finance II, by Steven Shreve, First Edition, 9781441923110

521 Life Contingencies II Young TTh 8:30am-10:00am

Prerequisites: Math 520

TTh 10:00am-11:30am

Course Description: Background and Goals: Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of the Math 520-521 sequence is to teach the basic actuarial theory of mathematical models for financial

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uncertainties, mainly the time of death. Math 521 extends the single-decrement and single-life applications of Math 520 to multi-decrement and multiple-life applications, as related to life insurance.

We build on the pricing material from Life Contingencies I by (1) determining on-going reserves (money that the insurance company sets aside to pay future claims), (2) modeling insurance products with payment depending on the type decrement (how the individual died), (3) pricing products with payment depending on the life status of more than one person, and (4) including expenses and other business considerations in the price and reserves. This corresponds to chapters 7-11 and 15 of Bowers et al.

Text: (Required) Actuarial Mathematics, Second Edition, Bowers et al. Publisher: Society of Actuaries; 2nd edition (May 1997) ISBN-10: 0938959468, ISBN-13: 978-0938959465

524	Loss Models II	Marker	TTh 11:30am-1:00pm TTh 1:00pm-2:30pm
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Prerequisites: Math 523 and Stats 426, each with a grade of C- or better.

Course Description: Risk management and modeling of financial losses. Frequentist and Bayesian estimation of probability distributions, model selection, credibility, and other topics in casualty insurance.

Text: "Loss Models: From Data to Decisions" by Stuart A. Klugman, Harry H. Panjer, and Gordon E. Willmot, fourth edition, ISBN: 1118315324.

525	Probability Theory	Azimzadeh	TTh 10:00-11:30am TTh 1:00pm-2:30pm
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Prerequisites: MATH 451 (Required)

Course Description: This is a fairly rigorous introduction to probability theory with some emphasis given to both theory and applications, although a knowledge of measure theory is not assumed. Topics covered are: probability spaces, conditional probability, discrete and continuous random variables, generating functions, characteristic functions, random walks, limit theorems, and some more advanced topics (this may include Poisson processes, branching processes, etc.)

Text: (Required) Knowing the odds: an introduction to probability, by John B. Walsh, ISBN: 9780821885321

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526	Discrete Stochastic Processes	Keller	TTh 8:30am-10:00am
		Saplaouras	TTh 10:00am-11:30am
		Saplaouras	TTh 11:30am-1:00pm

Prerequisites: Required: MATH 525 or STATS 525 or EECS 501

Course Description: The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include: Markov chains (Markov property, recurrence and transience, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times, Markov queues); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales). Significant applications will be an important feature of the course.

Text: (Required): Essentials of Stochastic Processes, 2nd ed. (Durrett) ISBN : 9781461436140.

(Optional): Stochastic Processes (Ross) ISBN: 9780471120629.

(Optional): Probability and Measure (Billingsley) ISBN: 9781118122372.

547	Probabilistic Models in Bioinformatics: The 4D Nucleome	Burns Rajapakse	TTh 8:30am-10:00am
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Prerequisites: Basic math: linear algebra, ODE's, calculus. Some programming (Matlab, e.g.) would be useful, as would some familiarity with biology; or permission of instructor.

Course Description: The 4D nucleome refers to the 3D structure of the cell nucleus and its dynamics through time. This is a rapidly emerging area enabled by new technologies (such as Hi-C) which allow observation of the geometric configuration of chromosomes and tracing of the relation of these structure or form features with the expression or function features of the genome. New mathematical methods (spectral graph theory, bifurcation theory of dynamical systems, control theory, Peano curves) must be employed to exploit the new insights and data

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provided by these developments. The theoretical foundations go back as far as Alan Turing (1952) and Steve Smale (1976). Some of the visionary biology goes back to Hal Weintraub (1984). We will consider competing theories for the organization of the nucleus, and try to show how they can be analyzed mathematically. The class will be self-contained and students will work together in groups to facilitate working across specializations.

Text: None required. Notes and articles will be posted in a Canvas site. A survey issue of some of these topics appears in *Method*, v.**123** (2017), 1-138, eds., T. Ried and I. Rajapakse (available online through UM Library). This survey may not be accessible to some students until we get into the background topics in class.

550 Intro to Adaptive Systems: Doering MW 8:30am-10:00am
 Evolutionary Game Theory

Prerequisites: Equivalent of four semesters calculus (especially familiarity with differential equations), working knowledge of elementary linear algebra and probability, and computer literacy appropriate for mathematically minded science and engineering students.

Course Description: An introduction to applications and integration of nonlinear dynamical systems and game theory to model population and ecological dynamics and evolutionary processes. Topics include general analyses of Lotka-Volterra systems, replicator dynamics, mutations, and the evolution of cooperation.

Text: (Required) *Evolutionary Games and Population Dynamics*, 1st Edition, Josef Hofbauer and Karl Sigmund. ISBN: 978-0521625708

555 Intro to Complex Variable Silva MW 8:30am-10:00am

Prerequisites: Advanced calculus, including multivariable concepts (e.g. Math 215 or Math 452) and Introductory Analysis at the level of Math 451

Course Description: This course is a first introduction to the theory and application of complex-valued functions of a complex variable, with special emphasis on the class of analytic functions and their uses in solving practical problems. Applications include the elegant evaluation of definite and improper integrals, the

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solution of polynomial equations, steady planar fluid flow, and electrostatics. Math 555 and Math 596 are courses that cover similar material, but Math 555 is more focused on breadth and applications, while Math 596 goes into certain topics with greater depth.

The audience for this course are graduate students in mathematics, engineering, and the physical sciences, and advanced undergraduate students with sufficient background. Math 555 is a core course for the graduate program in Applied and Interdisciplinary Mathematics.

Text: (Required) Complex Analysis with Applications, R.A. Silverman, Reprint of 1974 Edition, ISBN: 0486647625

(Optional) Basic Complex Analysis, Barry Simon. (A comprehensive course in analysis; part 2A), ISBN 978-1-4704-1100-8

557 Applied Asymptotic Analysis Krasny TTh 1:00pm-2:30pm
Prerequisites: Math 555 or Math 596

Course Description: In calculus we learn techniques of integration and derive explicit closed form solutions of differential equations. Later on we learn that such closed form solutions are rare, but in practice one can still find an approximate solution using a numerical method or asymptotic analysis. Math 557 is an introduction to the techniques of asymptotic analysis commonly used for problems in science and engineering. The topics include: asymptotic expansions, method of steepest descent, method of stationary phase, asymptotic evaluation of Fourier and Laplace transforms, WKB method, turning points, singular perturbations, method of multiple scales, matched asymptotic expansions, boundary layers, plus other topics as time permits. The course grade will be determined by homework assignments.

Text: (Required) Asymptotic Analysis, J.D. Murray. ISBN: 0387909370

(Optional) Applied Asymptotic Analysis, P.D. Miller. ISBN 0821840789

566 Combinatorial Theory: Algebraic Combinatorics Karp MWF 11:00am-12:00pm
Prerequisites: Math 465

Course Description: Introduction to algebraic and enumerative combinatorics at the graduate level. Topics include: algebraic graph theory; enumeration of matchings; electrical networks; generating functions; partially ordered sets; integer partitions and Young tableaux.

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Text: (Optional) Algebraic Combinatorics: Walks, Trees, Tableaux, and More, Richard P. Stanley Springer, 2013 ISBN: 978-1-4614-6997-1. The e-text of this book (without exercises) is available online at <http://www-math.mit.edu/~rstan/algcomb/>.

567 Introduction to Coding Theory Zerbib TTh 2:30pm-4:00pm

Prerequisites: Math 217, 417, 419 or 420. Some knowledge about probability and abstract algebra useful, but will be reviewed as needed

Course Description: Coding Theory is an area of mathematics that has important applications to digital communication and data storage. The goal of coding theory is to encode a digital message such that most of the lost information can be retrieved after the message has been sent over a noisy channel. Coding Theory was developed after Shannon proved some important results about digital communication.

Some of the topics that we will discuss are: entropy (a measure for information), Huffman codes (for data compression), channels and channel capacity, Shannon's theorem, error correcting block codes, finite fields and constructions of various codes (Hamming codes, Golay codes, Reed-Muller codes, cyclic codes etc.), linear codes, bounds for codes, and more.

Text: (Optional): Introduction to Coding and Information Theory, Steven Roman.

ISBN:0387947043

(Optional) Coding and Information Theory, Steven Roman. ISBN:3540978127

(Optional) Introduction to Coding Theory, J.H. Van Lint, 3rd Edition. ISBN: 3540641335

571 Numerical Linear Algebra Alben TTH 10:00am-11:30am

Prerequisites: One semester of linear algebra (e.g. Math 217, 417, 419, 513, or equivalent) and some knowledge of a high level computer language (Fortran, C, Matlab, etc.) Matlab is the recommended language; help will be provided for those new to it.

Course Description: This course is an introduction to numerical methods for solving linear systems of equations and for computing eigenvalues of a matrix. Topics include singular value decomposition, QR factorization, Gram-Schmidt orthogonalization, least squares problems, condition number, Gaussian elimination, iterative methods (Arnoldi, GMRES, conjugate gradient), preconditioning, methods for computing eigenvalues (e.g. power method, QR algorithm, shifts). The course should be very useful for students in applied and computational mathematics, and in any area of scientific computing and engineering.

Text: None

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572 Numerical Methods

Esedoglu

TTh 11:30am-1:00pm

for Scientific Computing II

Prerequisites: Advanced calculus and linear algebra

Course Description: Math 572 is an introduction to numerical methods for differential equations, focusing on finite differences. This is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program, and should also appeal to graduate students from engineering and science departments, or anyone interested in scientific computing. It covers methods for ordinary and partial differential equations, including derivation of numerical schemes and systematic study of their accuracy, stability, and convergence. A solid background in advanced calculus and linear algebra, and proficiency in a computer language such as C, Fortran, or Matlab is a must.

Topics:

Finite differences, their derivation and truncation error. Two point boundary value problems, elliptic equations. Consistency, stability, and convergence. Efficient solution of resulting sparse linear systems (Jacobi, Gauss-Seidel, SOR, conjugate gradients, preconditioning). Multistep, Runge-Kutta methods for initial value problems. Absolute stability, stiff problems, and A-stability. Barrier theorems. Explicit and implicit finite difference schemes for parabolic equations. Stability and convergence analysis via the maximum principle, energy methods, and the Fourier transform. Operator splitting techniques, the alternating direction implicit method. Advection equation. Lax-Wendroff, upwind methods, the CFL condition. Hyperbolic systems, initial boundary value problems.

Text: (Required) Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems by R.J. LeVeque, ISBN: 978-0-898716-29-0.

574 Financial Math II

Herrmann

TTh 1:00pm-2:30pm

Prerequisites: MATH 526, 573

Course Description: This is a continuation of Math 573. This course discusses Mathematical Theory of Continuous-time Finance. The course starts with the general Theory of Asset Pricing

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and Hedging in continuous time and then proceeds to specific problems of Mathematical Modeling in Continuous-time Finance. These problems include pricing and hedging of (basic and exotic) Derivatives in Equity, Foreign Exchange, Fixed Income and Credit Risk markets. In addition, this course discusses Optimal Investment in Continuous time (Merton's problem), High-frequency Trading (Optimal Execution), and Risk Management (e.g. Credit Value Adjustment). Although Math 506 is not a prerequisite for Math 574, it is strongly recommended that either these courses are taken in parallel, or Math 506 precedes Math 574.

Text: (Required) Arbitrage Theory in Continuous Time, Bjork, Thomas, 3rd edition.
ISBN: 9780199574742

575 Introduction to the Theory of Numbers Kim MWF 10:00am-11:00am

Prerequisites: None

Course Description: This course will be an introduction to number theory. Basic topics to be covered include factorization, congruences and classical reciprocity laws such as quadratic and cubic reciprocity. Time permitting a selection of more advanced topics may be covered.

Text: (Required): A Classical Introduction to Modern Number Theory, Ireland, Kenneth and Rosen, Michael, 2nd Edition. ISBN: 9780387973296

582 Introduction to Set Theory Fernandez Breton MWF 11:00am-12:00pm

Prerequisites: The official prerequisite, "Math 412 or 451 or equivalent experience with abstract mathematics," means that students should be comfortable with writing mathematical proofs.

Course Description: This is an introductory course in axiomatic set theory, whose main objective is to convince the student that most everyday mathematical objects can be conceived as sets of some sort.

Topics include:

- * The intuitive concept of set; paradoxes.
- * Type theory and the cumulative hierarchy of sets.
- * The Zermelo-Fraenkel axioms for set theory.
- * Set-theoretic representation of the fundamental concepts of mathematics (e.g., function, number) and proofs of basic properties of these concepts (e.g., mathematical induction).
- * Infinite cardinal and ordinal numbers and their arithmetic.
- * The axiom of choice and equivalent axioms (e.g., Zorn's Lemma).

Additional topics may be discussed if time permits.

No specific previous knowledge of set theory will be presupposed.

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Text: (Optional) Elements of Set Theory by H. B. Enderton, ISBN: 9780122384400
(Optional) Introduction to Set Theory by Karel Hrbacek; ISBN: 9780585243412

590 Intro to Topology Zheng MWF 12:00pm-1:00pm

Prerequisites: none

Course Description: This is a course on point-set topology, which emphasizes the set-theoretic aspects of topology. The course will focus on the notions of continuity, connectedness, compactness, and other topics. The class is quite theoretical and requires extensive construction of proofs.

Text: (Required) Topology (2nd Edition), by James Munkres. ISBN:131816292

592 Introduction to Algebraic Topology Bhatt MWF 10:00am-11:00am

Prerequisites: Previous exposure to point-set topology and familiarity with abstract algebra will be assumed.

Course Description: The goal of this course is to introduce the basic objects in algebraic topology: fundamental groups and covering spaces, singular homology and cohomology, and higher homotopy groups. Time permitting, we will discuss more advanced topics, such as the Lefschetz fixed point theorem or Serre's techniques for computing homotopy groups.

Text: (Optional): Algebraic Topology, Allen Hatcher. ISBN: 978-0521795401
(Optional) Algebraic Topology, Edwin H. Spanier. ISBN: 978-0387944265
(Optional) A Concise Course in Algebraic Topology, J. P. May. ISBN: 978-0226511832

594 Algebra II: Groups and Galois Theory Derksen TTh 2:30pm-4:00pm

Prerequisites: A solid knowledge of linear algebra over arbitrary fields is needed (math 420, math 593 or equivalent)

Course Description: The first part of the course is group theory. We will discuss topics such as the permutation group and alternating group, group actions on sets, simple, solvable and nilpotent groups, p -groups and the Sylow theorems.

In the second part of the course we discuss field extensions and Galois theory. We will study the Galois correspondence, solving equations by radicals, finite fields and transcendental field extensions among other things.

Text: (Required): Abstract Algebra, Dummit & Foote, 3rd Edition. ISBN: 0471433349

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597 Analysis II (Real Analysis) Baik MWF 12:00pm-1:00pm
Prerequisites: None

Course Description: MATH 597 (Real Analysis) is one of two core analysis courses for the beginning students in the PhD program for mathematics. Topics will include measure theory on the real line, abstract measure space, integration theory, Fubini theorem, differentiation theory, L_p spaces, introduction to Hilbert space, and introduction to Fourier analysis.

Text: (Required) Real Analysis: Modern Techniques and Their Applications, Gerald B. Folland.

ISBN: 978-0-471-31716-6

(Optional) An Introduction to Measure Theory, Terence Tao. ISBN: 978-0-8218-6919-2

604 Complex Analysis II Barrett MWF 9:00am-10:00am
Prerequisites: first-year graduate analysis

Course Description: This course will cover a number of fundamental topics in one-dimensional complex analysis, such as

(1) the Cauchy transform of a measure; use of the inhomogeneous Cauchy-Riemann equations to construct holomorphic and meromorphic functions;

(2) zeros and growth of holomorphic functions on the plane and on the unit disk; factorization for function spaces on the unit disk;

(3) construction of mappings with prescribed angular distortion (Beltrami's equation);

(4) geometric function theory: univalent functions, Schwarzian derivatives, conformal metrics;

(5) potential theory: harmonic and subharmonic functions, Perron's solution of the Dirichlet problem;

(6) Riemann surfaces: the Riemann surface of a holomorphic function, fractional power series, Newton polygons, algebraic functions; Riemann surfaces in general; uniformization theorem (every Riemann surface is covered by the disk, the plane, or the sphere); all Riemann surfaces admit non-constant meromorphic functions;

(7) complex structures on the torus, elliptic functions.

If time permits, additional topics may include logarithmic and analytic capacity and/or a brief

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introduction to complex dynamics. (Choices here may reflect the background and interests of the enrolled students.)

Text: None

615 Commutative Algebra II: Jeffries MWF 10:00am-11:00am
Local Cohomology

Prerequisites: Math 614

Course Description: This is a course centered on a cohomology theory for commutative rings called local cohomology. Local cohomology modules measure many interesting features of rings and modules, but present challenges to study, since they are often not finitely generated. Topics will include the structure theory of injective modules over noetherian rings, Matlis duality, definitions and basic properties of local cohomology, vanishing theorems, Cohen-Macaulay and Gorenstein rings, local duality, arithmetic rank, and connections with differential operators.

Text: No textbook is required. Recommended texts include Twenty-Four Hours of Local Cohomology by Iyengar et al., Cohen-Macaulay Rings by Bruns and Herzog, and Local Cohomology by Brodmann and Sharp.

623 Computational Finance Nadtochiy TuTh 8:30am-10:00am

Prerequisites: None

Course Description: This is a course on computational methods in finance and financial modeling. Using financial mathematics (like many branches of applied mathematics) in practice involves two tasks. First, one has to develop mathematical models that accurately describe the real-life phenomena that one wishes to study (i.e. a model has to be consistent with the observations). In our case, this typically means finding models, based on probability theory, for the evolution of prices of financial instruments, interest rates, and other relevant quantities. Once a model is chosen, one can derive theoretical equations, or formulas, which establish relations between various objects in the financial markets: for example, the prices of derivative securities (options, bonds, etc) and the risk profiles of investment portfolios can be determined as functions of the underlying stochastic factors. In this course, we study the problem of implementing these equations, in order to obtain actual numbers. The course has three components. In the first part, we will study the lattice (or, tree) methods, which correspond to the models based on discrete time Markov chains (e.g. the binomial model). We will discuss the pricing and hedging of financial derivatives in such models, using the arbitrage theory, or, more specifically, the risk-neutral pricing. We will, then, proceed to analyze the diffusion-based models of financial mathematics (e.g. the Black-Scholes model). In such models, the prices of derivative contracts can be characterized by means of partial differential equations (PDEs). We will discuss the finite difference methods, which provide numerical approximations for solutions to these PDEs. Both explicit and implicit schemes will be studied, the concepts of

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stability and convergence will be introduced, and a connection between the finite difference schemes and lattice methods will be established. After that, we will turn to the Monte Carlo simulations, which is a very general method for computing the derivatives' prices and trading strategies, as well as for analyzing the risk profiles, numerically. This method is based on generating a large number of paths of the underlying stochastic processes, in order to approximate the expectations of certain functions of these paths (which determined the prices, portfolio weights, or default probabilities). In addition to the standard Monte Carlo methods, we will study the variance reduction techniques, which are often necessary to obtain accurate results. The mathematical methods presented in this course will be illustrated using the popular models of equity markets (e.g. Black-Scholes, Heston), fixed income (e.g. Vasicek, CIR, Hull-White, Heath-Jarrow-Morton) and credit risk (e.g. Merton, Black-Cox, reduced-form models). We will also address the issue of calibration -- i.e. the problem of finding the values of the model's parameters that are consistent with the available financial data. In the homeworks, which form an integral part of the class, you will implement many of these models and the associated computational methods in MatLab.

Text: (Optional): Monte Carlo Methods in Financial Engineering, Glasserman.
ISBN: 978-0-387-21617-1.

632	Algebraic Geometry II: Cohomology on Algebraic Varieties	Mustata	TTh 11:30am-1:00pm
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Prerequisites: Math 631

Course Description: The course will be a continuation of Math 631. In the first part of the course, we will discuss cohomology of sheaves on algebraic varieties, with applications to curves and surfaces. The second part will be devoted to Hodge Theory on complex algebraic varieties.

Text: None

635	Differential Geometry	Spatzier	MWF 1:00pm-2:00pm
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Prerequisites: Basic topology and manifold theory as in MATH 591 and 592

Course Description: Differential geometry has been at the core of geometry and topology for over a century, and remains at the forefront of research. This course is an introduction. We will discuss the basic ideas of connections, Riemannian metrics, curvature and the basic tools in the subject, especially variational methods, Jacobi fields, and comparison theorems. After those more local ideas, we will turn to global differential geometry which relates geometric ideas to the underlying topology. Examples are sphere theorems, structural results both in positive and negative curvature, and rigidity theorems. If time permits we will pursue more advanced topics.

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Text: (Optional): Riemannian Geometry, Do Carmo, Birkhauser. ISBN: 0817634908.

636 Topics in Differential Geometry: Koch TTh 11:30am-1:00pm
The Topology and Dynamics of Rational Maps

Prerequisites: MATH 596, 592

Course Description: One aspect of complex dynamical systems concerns the study of iterating rational maps on the Riemann sphere. A wealth of complicated and deep behavior can emerge when a rational map is iterated; this behavior is governed by the orbits of the critical points of the map under iteration. The rational map is said to be "postcritically finite" if every critical point is eventually periodic. Postcritically finite maps are particularly nice to study. Loosely speaking, in the space of all rational maps, those that are postcritically finite play a role akin to the rational numbers in the reals (we will make this notion precise in the course!).

In the 1980s, William Thurston established a topological characterization of postcritically finite rational maps: he proved that postcritically finite rational maps are determined by topological invariants, and he completely characterized which invariants arise. Recently, Dylan Thurston established a complementary result. Taken together, these theorems have extensive applications to understanding not only the dynamics of rational maps, but also to understanding the geometry of parameter spaces.

The central goal of this course will be to understand these theorems, providing proofs of each. The proofs are quite different: one involves iteration on Teichmueller space, and the other uses elastic networks and the corresponding Dirichlet energy. We will begin by establishing necessary background in geometry and complex dynamics. Then, as we prove these theorems, we will illustrate some of their applications by navigating our way through the Mandelbrot set.

The math department is very glad to have Dylan Thurston visiting in W2018; he will help design this course and give numerous guest lectures.

Text: None

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650 Fourier Analysis: Wu TTH 1:00pm-2:30pm
Introduction to Harmonic Analysis

Prerequisites: Math 596, 597

Course Description: Harmonic analysis is a beautiful and core subject in mathematics, and an important tool in the modern study of partial differential equations, probability, analytic number theory and many others. The goal of this course is to introduce students to some basic methods and tools in treating nonlinear operators. Topics covered include the Fourier transform, the Calderon-Zygmund operators, A_p weights, Hardy spaces, BMO, Carleson measure, para-products, the T1 Theorem and the Cauchy integral of Calderon. Time permits, we will discuss applications in Partial Differential Equations.

Course materials will be taken from the references below.

Text: (Optional) Introduction to Fourier Analysis on Euclidean Spaces by Stein & Weiss
(Optional) Calderon-Zygmund operators, pseudo-differential operators, and the Cauchy integral of Calderon by Jean-Lin Journé
(Optional) Wavelets: Calderon-Zygmund and multilinear operators by Coifman & Meyer
(Optional) Harmonic Analysis: real-variable methods, orthogonality, and oscillatory integrals by Elias M. Stein

651 Topics in Applied Math: Miller TTH 11:30am-1:00pm
Integrable Systems and Riemann-Hilbert Problems

Prerequisites: 555 or 596; 454 or 556; 557 would be helpful

Course Description: A Hamiltonian system of ordinary differential equations on the phase space \mathbb{R}^{2n} is said to be (Liouville-Arnol'd) integrable if there exist n functionally independent constants of motion that are in involution with respect to the underlying Poisson bracket. In practical terms such dynamical systems can be regarded as explicitly solvable, because there exists a change of coordinates that linearizes the dynamics. This is a classical notion.

In the late 1960's it was discovered that some initial-value problems for nonlinear partial differential equations (PDE) describing wave propagation (e.g., the Korteweg-de Vries equation and the cubic nonlinear Schrödinger equation) can be seen as infinite-dimensional analogues of classical integrable Hamiltonian systems, and a new technique was advanced for constructing the change of coordinates that trivializes the dynamics for these problems. This technique immediately yields a huge variety of exact solutions (solitons and their relatives) and is based on a nonlinear analogue of the Fourier transform (the latter of course trivializes the

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dynamics of linear constant-coefficient equations on \mathbb{R}^n) now called the scattering transform. The direct scattering transform considers the solution of the PDE at hand at a fixed time t to be a coefficient in some linear differential equation with a complex spectral parameter λ and maps it to a set of λ -dependent scattering data. The inverse scattering transform has to reconstruct the coefficient in the scattering equation from its scattering data. The latter is an inverse problem of some interest in many other areas of applied mathematics (e.g., tomography, remote sensing) as well.

Because appropriately selected solutions of the direct-scattering equation frequently depend analytically on λ in some domains of \mathbb{C} , the inverse scattering transform can often be formulated as a Riemann-Hilbert problem, in which a function analytic in various domains of the complex plane has to be reconstructed from “jump conditions” that relate the traces on curves that bound adjacent domains. Such a problem can in turn be recast as a system of singular integral equations with Cauchy-type kernels.

Riemann-Hilbert problems have other applications as well. They can be used to characterize polynomials orthogonal with respect to arbitrary weights, and this leads to a useful representation of various statistics of random matrices. There is also a class of integral operators that can be inverted with the help of an associated Riemann-Hilbert problem.

Finally, there is an analogue of the classical method of steepest descent for the asymptotic expansion of integrals that applies to certain Riemann-Hilbert problems. This method allows important information to be gleaned from Riemann-Hilbert problems in which a large or small parameter appears in the jump conditions. This method has been used to analyze solutions of integrable PDE with extreme precision in both the large time and semiclassical (or weakly-dispersive) limits. It has also been used to obtain large-degree asymptotics for general orthogonal polynomials and to prove universality conjectures in random matrix theory.

This course will be an introduction to these and other related topics. Complex analysis (Math 555 or Math 596) is an essential prerequisite. Students who have taken Asymptotic Analysis (Math 557) will more easily grasp and appreciate the steepest descent method for Riemann-Hilbert problems, but this is not as important of a prerequisite. While much of the course will involve solutions of nonlinear PDE, the relevant background required is more along the lines of Math 454 or Math 556 than of Math 656 or Math 657. There is no textbook, and materials will be distributed by the instructor as the course progresses. Students will be evaluated on the basis of some homework sets and possibly a term project culminating in a presentation to the class on a topic related to the course.

Text: None

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657 Nonlinear Partial Differential Equations Bieri TTH 2:30pm-4:00pm

Prerequisites: Math 656 or permission of instructor

Course Description: Partial differential equations are at the core of models in science, technology, economics and related fields. These equations and their solutions have interesting structures that are studied by methods of analysis, geometry, probability and other mathematical fields.

The goal of this course is to introduce students in pure and applied mathematics to concepts and methods, that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations.

This course will focus mainly on important nonlinear partial differential equations. The topics will include Sobolev spaces, 2nd order partial differential equations of elliptic, parabolic and hyperbolic type, shock waves and nonlinear waves.

Text: (Required) Partial Differential Equations, by L.C. Evans, 2nd Edition.
ISBN: 978-0821849743

669 Combinatorial Theory: Fomin TTH 1:00pm-2:30pm
Cluster Algebras

Prerequisites: none

Course Description: Cluster algebras are a class of commutative rings constructed via a recursive combinatorial process of "seed mutations." They arise in a variety of algebraic and geometric contexts including representation theory of Lie groups, Teichmüller theory, discrete integrable systems, classical invariant theory, and quiver representations. This course will provide an elementary introduction to the basic notions and results of the theory of cluster algebras, and present some of its most accessible applications. Combinatorial aspects will be emphasized throughout. No special background in commutative algebra, representation theory, or combinatorics is required.

Text: None

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679 Elliptic Curves: Arithmetic of Surfaces Ho TTH 1:00pm-2:30pm

Prerequisites: Algebraic number theory (676), Algebraic geometry (631 & 632), Algebraic topology (592)

Course Description: We will study the arithmetic of surfaces, especially K3 surfaces, del Pezzo surfaces, and Enriques surfaces. Topics may include: modularity, rational points, Picard numbers, Brauer groups, rational curves.

Text: None

684 Recursion Theory Blass TTH 1:00pm-2:30pm

Prerequisites: none

Course Description: Recursion theory, also called computability theory, is the study of the extent to which functions F and relations R are computable. Is there an algorithm which, given as input any element x of the domain of F , produces after finitely many steps the output $F(x)$? In the case of relations R , is there an algorithm which, given input x , decides in finitely many steps whether or not R holds of x ? In some important cases, we do not have computability in this sense but only semi-computability, meaning that there is an algorithm that will produce the correct result when $R(x)$ holds but will compute forever with no result when $R(x)$ fails. These and other variants of computability will be rigorously defined and studied in this course. A closely related topic is relative computability: Could F (or R) be computed by an algorithm that has black-box access to information about another function G ? The study of such questions has led to deep combinatorial methods, including the so-called priority method, whose simplest incarnation (finite injury priority arguments) will be covered in this course.

In addition to the general study of computability, the course will treat some of the most important applications of computability in mathematical logic. In particular, I will cover the incompleteness theorems of Gödel, which imply that one cannot deduce all mathematical truths, or even all truths of the arithmetic of natural numbers, from any computable set of true axioms. They also imply that, under reasonable assumptions, a consistent theory cannot prove its own consistency.

The course will not presuppose any prior knowledge of mathematical logic, provided you are willing to believe some facts that I'll state without proof. (Alternatively, those facts are covered in each of Math 481 and 681.) The only real prerequisite for Math 684 is mathematical maturity appropriate for a 600-level mathematics course.

Text: (Optional): Fundamentals of Mathematical Logic, Peter G. Hinman. ISBN: 978-1568812625

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697 Topic in Topology: Kriz MWF 9:00am-10:00am
Homological and Cohomological Operations

Prerequisites: Basic Algebraic geometry and topology.

Course Description: In this course, I will talk in detail about a basic technique used to calculate in modern algebraic topology, namely cohomological and homological operations. I will discuss the most classical case of Steenrod operations on the cohomology of spaces, as well as various other types of homological and cohomological operations which arise in algebraic topology and spectral algebraic geometry. Examples include algebras over various operads. All of these operations have essentially a common source, which we will explore. I will also discuss some examples of computations that are made using these operations.

There will be no exams. Students are expected to attend and keep up with the material. Homework will be assigned at the rate of approximately one problem per class.

Text: None

710 Topics in Modern Analysis II Jonsson MWF 11:00am-12:00pm

Prerequisites: Some familiarity with several complex variables (Math 605 is more than enough), basic geometric notions such as vector bundles on complex manifolds, and basic facts about Hilbert spaces.

Course Description: This course is about a set of techniques in complex geometry that go under the name L²-methods. They have their roots in Hörmander's work in function theory in several complex variables, and have in recent years seen a large range of applications to complex algebraic geometry.

Possible topics include.

- The Hörmander-Skoda Theorem.
- The Kodaira vanishing and embedding theorems.
- Nadel vanishing
- The Ohsawa-Takegoshi Theorem
- Subadditivity and Demailly approximation.
- Invariance of plurigenera
- Positivity of direct images

Text: None

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732 Topics in Algebraic Geometry II: Fulton MWF 2:00pm-3:00pm
Degeneracy Loci

Prerequisites: Math 632 or permission from instructor

Course Description: The 19th century origins of this subject were concerned with varieties of m by n matrices of rank at most r . Since these varieties are usually defined by many more equations than their codimensions, computing even their degrees was a challenge. In the 20th century this was generalized to the loci where maps of vector bundles have restricted ranks; the classical formulas involve Chern classes of the bundles. Symmetric and skew-symmetric matrices and bundle maps were also studied. Finding formulas for these loci was one of the motivations for developing intersection theory in algebraic geometry.

These loci are now understood as special cases of those one has for each element of a Weyl group in each type A, B, C, and D. Formulas for these loci are given by what are called Schubert polynomials, which have a rich combinatorial structure.

The course will work out this story in each type, including current research as well as history and applications. The intersection theory needed for (and motivated by) this story will be described; students can either take this axiomatically or work out details in a seminar.

Dave Anderson and I are working on a memoir on this subject, and I hope chapters will be available as the course progresses.

Text: None

775 Adv. Topics in Analytic Number Theory: Montgomery MWF 10am-11am

Prerequisites: Math 675 or equivalent background in algebraic number theory

Course Description: Math 675 covers basic material, while Math 775 is an advanced topics course. Where we start will depend on what topics were covered and not covered in Math 675. If there are basic things that were not covered in Math 675, then they will have to be covered first, before going on to advanced topics. Among the advanced topics, we will cover

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Vinogradov's theorem that every sufficiently large odd integer is a sum of three primes. We will also cover mean and large values of Dirichlet polynomials, and applications of that to zero-density theorems for Dirichlet L-functions. The large sieve is included here. We will cover the pair correlation of the zeros of the Riemann zeta function, and the connection with random matrix theory. We will cover the basics of probabilistic number theory, especially the Turan--Kubilius theorem and the mean value theorem of Halasz. If time permits, we will continue from there with a discussion of recent results of Matomaki--Radziwill--Tao concerning correlations among values at translated arguments of the Mobius mu function.

Text: None

797 Advanced Topics in Topology: Canary MWF 2:00pm-3:00pm
 Hyperbolic Groups and their Representations

Prerequisites: Basic knowledge of spaces and fundamental groups

Course Description: In this course we focus on the geometric viewpoint on the study of groups. The basic idea here is that it is often easier to study a group by studying its geometric action on some space. Hyperbolic groups are those groups which act properly discontinuously and cocompactly (i.e. with compact quotient) on a space which "coarsely" has negative curvature. It is one of Gromov's fundamental discoveries that it suffices to work with a very naive notion of coarse negative curvature, e.g. that all geodesic triangles are "thin", and that one can still obtain powerful consequences, e.g. the solvability of the word problem for hyperbolic groups. Therefore, proofs of seemingly complicated result can be reduced to their simple essence and become quite accessible. Examples of hyperbolic groups include free groups, fundamental groups of surfaces of genus at least 2, and fundamental groups of negatively curved manifolds. Gromov's concise and elegant notion of coarse negative curvature has been tremendously influential in a wide swath of mathematics where geometric techniques or ideas are used.

As time permits we will focus on various aspects of representations of hyperbolic groups into Lie groups in the later part of the semester. Possible topics include representations into $PSL(2, \mathbb{C})$, i.e. the theory of hyperbolic 3-manifolds, and Anosov representations into higher rank Lie groups. The actual topics discussed will be chosen in consultation with the class.

Text: None