Winter 2017 Graduate Course Descriptions

---

501 AIM Student Seminar — Alben — F 1:00-2pm & 3:00-4pm

Prerequisites: *You must be a graduate student in the AIM program to register for this course.*

Course Description: The Applied and Interdisciplinary Mathematics (AIM) student seminar is an introductory and survey course in the methods and applications of modern mathematics in the natural, social, and engineering sciences. Students will attend the weekly AIM Research Seminar where topics of current interest are presented by active researchers (both from U-M and from elsewhere). The other central aspect of the course will be a seminar to prepare students with appropriate introductory background material. The seminar will also focus on effective communication methods for interdisciplinary research. MATH 501 is primarily intended for graduate students in the Applied & Interdisciplinary Mathematics M.S. and Ph.D. programs. It is also intended for mathematically curious graduate students from other areas. Qualified undergraduates are welcome to elect the course with the instructor's permission. Student attendance and participation at all seminar sessions is required. Students will develop and make a short presentation on some aspect of applied and interdisciplinary mathematics.

Text: None

---

506 Stochastic Analysis for Finance — Stoev — TTh 8:30-10am

Course Description: The aim of this course is to teach the probabilistic techniques and concepts from the theory of stochastic processes required to understand the widely used financial models. In particular concepts such as martingales, stochastic integration/calculus, which are essential in computing the prices of derivative contracts, will be discussed. The specific topics include: Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales), martingales in continuous time, stochastic integration (including Ito's formula), stochastic differential equations (including Feynman-Kac formula), change of measure (including Girsanov's theorem and change of numeraire), and, time permitting, stochastic control (including Merton problem). Applications from various areas of Finance (including, pricing of derivatives, risk management, etc) are used to illustrate the theory.

Text: (Required) Stochastic Calculus for Finance II, by Steven Shreve, First Edition, 9781441923110

---

521 Life Contingencies II — Marker — TTh 8:30-10am & 11:30-1PM

Prerequisites: *Math 520 or permission of the instructor.*

Course Description: Background and Goals: Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of the Math 520-521 sequence is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. This course extends the single decrement and single life ideas of Math 520 to multi-decrement and multiple-life applications related to life insurance. We build on the pricing material from Life Contingencies I by (1) determining on-going reserves (money that the insurance company sets aside to pay future claims), (2) modeling insurance products with payment depending on the mode of decrement (how the individual died), (3) pricing products with payment depending on the life status of more than one person, and (4) including expenses and other business considerations in the price and reserves. This corresponds to chapters 7-11 and 15 of Bowers et al.

Text: (Required) Actuarial Mathematics, Second Edition, Bowers et al. Publisher: Society of Actuaries; 2nd edition (May 1997) ISBN-10: 0938959468, ISBN-13: 978-0938959465

---

524 Loss Models II — Elie — TTh 11:30am-1pm & 1-2:30pm

Prerequisites: *Math 523 and Stats 426, each with a grade of C- or better.*

Course Description: Risk management and modeling of financial losses. Frequentist and Bayesian estimation of probability distributions, model selection, credibility, and other topics in casualty insurance.

Text: "Loss Models: From Data to Decisions" by Stuart A. Klugman, Harry H. Panjer, and Gordon E. Willmot, fourth edition, published by John Wiley in 2012.

Winter 2017 Graduate Course Descriptions

525 Probability Theory Vershynin TTh 10:00-11:30a TTh 1:00-2:30pm

Prerequisites: *MATH 451 (Required)*

Course Description: This is a fairly rigorous introduction to probability theory with some emphasis given to both theory and applications, although a knowledge of measure theory is not assumed. Topics covered are: probability spaces, conditional probability, discrete and continuous random variables, generating functions, characteristic functions, random walks, limit theorems, and some more advanced topics (this may include Poisson processes, branching processes, etc.)

Text: (Required) Knowing the odds: an introduction to probability, by John B. Walsh, ISBN: 9780821885321

---

526 Discrete Stochastic Processes Nadtochiy TTh 8:30-10am TTH 10-11:30am
Nishry TTH 11:30-1pm

Prerequisites: *Required: MATH 525 or STATS 525 or EECS 501*

Course Description: The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include: Markov chains (Markov property, recurrence and transience, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times, Markov queues); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales). Significant applications will be an important feature of the course.

Text: (Required): Essentials of Stochastic Processes, 2nd ed. (Durrett)ISBN : 9781461436140. Optional: Stochastic Processes (Ross), Probability and Measure (Billingsley) ISBN: 9781118122372 .

Winter 2017 Graduate Course Descriptions

555 Intro to Complex Variable Borcea MW 8:30-10am

Prerequisites: *Courses in elementary real analysis (e.g. Math 451) and multivariable calculus (eg., Math 215 or Math 255) are essential background.*

Course Description: This course is an introduction to the analysis of complex valued functions of a complex variable with substantial attention to applications in science and engineering. Concepts, calculations, and the ability to apply principles to problems are emphasized alongside rigorous proofs of the basic results in the subject.
Topics covered include the Cauchy-Riemann equations, Taylor series, Laurent expansions, Cauchy integral formula, residues, the argument principle, harmonic functions, maximum modulus theorem, conformal mappings and applications including evaluation of improper real integrals and fluid mechanics.
There is no textbook for the class. However, students will receive a list of half a dozen books, any of which they may use, during the first lecture.

Text: (Required) Complex Analysis with Applications, R.A. Silverman, Dover Publications, ISBN: 0486647625

---

557 Applied Asymptotic Analysis Miller TTh 1-2:30pm

Prerequisites: *Differential Equations (e.g. 216, 256, 316, or 404), Linear Algebra (e.g. 214, 419, 420), Real Analysis (e.g. 451), and Complex Analysis (e.g. 555 or 596).*

Course Description: Asymptotic analysis is the quantitative study of approximations. The fundamental idea is that one tries to solve a problem in applied mathematics (say, a boundary-value problem for a partial or ordinary differential equation) by embedding it into a family of problems with a parameter. If the problem can be solved exactly for one special value of the parameter, then asymptotic analysis can be used to analyze how the solution changes as the parameter is tuned from the special value to a more physically reasonable one. The course will develop the general theory of so-called asymptotic expansions, which are a kind of series in the perturbation parameter that are extremely useful in practice, in a way that is mathematically completely rigorous, despite the strange fact that they frequently fail to converge at all! We will then study how to use asymptotic expansions to evaluate integrals that cannot be computed in closed form and that are also difficult to approximate numerically. Next, we will turn to differential equations and use asymptotic expansions to evaluate solutions near certain singular points and also to study the way that solutions depend on parameters. At the end of the course we will study how the differential equations of diverse physical phenomena can be reduced, with the help of asymptotic expansions, to certain universal model equations that show up again and again in applied mathematics.

Specific applications to be addressed in the course as time permits include the small-viscosity theory of shock waves, the theory of quantum mechanics in the semi classical limit, aspects of the theory of special functions, vibrations in nonlinear lattices, and surface water waves. Grades will be determined based upon homework sets and a final project.

Text: (Required) P. D. Miller, Applied Asymptotic Analysis, Graduate Studies in Mathematics, volume 75, American Mathematical Society, Providence, RI, 2006. ISBN 0-8218-4078-9.

---

564 Topics in Mathematical Biology: Rhythms and Oscillations in Biology — Booth — TTh 2:30-4pm

Prerequisites*: 463 or permission of the instructor*

Course Description: Rhythmic and oscillatory processes occur in a multitude of biological processes such as heartbeats, locomotion, sleep-wake patterns, intracellular transcription-translation dynamics as well as predator-prey interactions and firefly flashing. Mathematical modeling and analysis can help to understand what causes oscillations to emerge, the properties of period and amplitude and how the coupling of oscillatory processes leads to synchronized behaviors. This course will cover mathematical models of different rhythmic processes in biology, usually consisting of ordinary differential equations, and mathematical techniques used to analyze and understand their solutions. Course requirements include homework assignments, involving numerical solution of differential equation models, and a course project.

Text: None

---

566 Combinatorial Theory: Algebraic Combinatorics — Fomin — TTh 1-2:30pm

Prerequisites: *Math 420, 465*

Course Description: This course is an introduction to algebraic and enumerative combinatorics at the beginning graduate level. Topics include: fundamentals of algebraic graph theory; applications of linear algebra to enumeration of matchings, tilings, and spanning trees; combinatorics of electric networks; partially ordered sets; integer partitions and Young tableaux.

Text: (Optional) R. P. Stanley, Algebraic Combinatorics: Walks, Trees, Tableaux, and More, Springer, 2013 ISBN:978-1-4614-6998-8. The text of this book (without exercises) is available online at <http://www-math.mit.edu/~rstan/algcomb/>.

Winter 2017 Graduate Course Descriptions

567 Introduction to Coding Theory Gelaki TTh 2:30-4pm

Prerequisites: *Math 217, 417, 419 or 420.*

Course Description: Background and Goals:
This course is designed to introduce mathematics concentrators to an important area of applications in the communications industry. Using linear algebra it will cover the foundations of the theory of error- correcting codes and prepare a student to take further EECS courses or gain employment in this area. For EECS students it will provide a mathematical setting for their study of communications technology.
Content:
Introduction to coding theory focusing on the mathematical background for error-correcting codes. Topics include: Shannon's Theorem and channel capacity; review of tools from linear algebra and an introduction to abstract algebra and finite fields; basic examples of codes such and Hamming, BCH, cyclic, Melas, Reed-Muller, and Reed-Solomon; introduction to decoding starting with syndrome decoding. Further topics range from asymptotic parameters and bounds to a discussion of algebraic geometric codes in their simplest form.

Text: (Optional): Introduction to Coding and Information Theory, Steven Roman. ISBN:0387947043
(Optional) Coding and Information Theory, Steven Roman. ISBN:3540978127

---

571 Numerical Methods for Scientific Computing I Esedoglu TTH 10-11:30am

Prerequisites: *"Prerequisites: One semester of linear algebra (e.g. Math 217, 417, 419, 513, or equivalent) and some knowledge of a high level computer language (Fortran, C, Matlab, etc.) Matlab is the recommended language; help will be provided for those new to it.*

Course Description: Background and Goals: Numerical linear algebra is at the core of much of scientific computing, and is a fundamental skill for anyone with any interest in numerical/computational mathematics; the course is a core course for the AIM program. We will cover robust, accurate, and efficient methods for finding (1) solutions of linear systems of equations, (2) eigenvalues and eigenvectors of matrices, and (3) solution of least squares problems. These standard problems arise in all venues of science and engineering.
Topics will include: (1) Orthogonal matrices, vector and matrix norms, singular value decomposition (SVD). (2) QR factorization, Householder triangularization, least squares problems. (3) Stability: Condition numbers, floating point arithmetic, backward error analysis. (4) Direct methods: Gaussian elimination, pivoting, LU and Cholesky factorizations. (5) Eigenvalues and eigenvectors: Reduction to Hessenberg or tridiagonal form, Rayleigh quotient,

inverse iteration, the QR algorithm, computing the SVD. (6) Iterative methods: Classical methods (Jacobi, Gauss-Seidel, SOR), Krylov subspace methods, conjugate gradients, Arnoldi iteration, GMRES, preconditioning.

Text: (Required) Numerical Linear Algebra by Trefethen and Bau. ISBN: 9780898713619

---

572 Numerical Methods for Scientific Computing II — Veerapaneni — TTh 11:30-1pm

Prerequisites: None

Course Description: Computer simulation is pervasive in modern scientific research; it is routinely used in engineering and science, and increasingly in other fields as well such as finance and medicine. However, computer simulations can be challenging - using a faster computer is no guarantee of success and sometimes one must use a smarter method. Math 572 is an introduction to numerical methods used in solving differential equations. The course will focus on finite-difference schemes for initial value problems for ordinary and partial differential equations. Theoretical concepts and practical computing issues will be covered.

Text: (Optional) Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems by R.J. LeVeque, SIAM, 2007.

---

574 Financial Math I — Keller — TTh 1-2:30pm

Prerequisites: *MATH 526, 573*

Course Description: This is a continuation of Math 573. This course discusses the mathematical theory of continuous-time finance. The course starts with the general theory of asset pricing and hedging in continuous time and then proceeds to specific problems of mathematical modeling in continuous-time finance. These problems include pricing and hedging of (basic and exotic) derivatives in equity, foreign exchange, fixed income and credit Risk markets. In addition, this course discusses optimal investment in continuous time (Merton’s problem), high-frequency trading (optimal execution), and risk management (e.g. credit value adjustment).

Students develop modeling skills so that they are able to formulate a well-posed mathematical problem from a description in financial language, carry out relevant mathematical analysis using tools of stochastic analysis and probability theory, implement the results using advanced numerical methods, and interpret and make decisions based on these results.

Text: (Required) Arbitrage Theory in Continuous Time, Bjork, Thomas, 3rd edition.
ISBN: 9780199574742

---

575 Introduction to the Theory of Numbers Prasanna TTh 10-11:30pm

Prerequisites: None

Course Description: This course will be an introduction to number theory. Basic topics to be covered include factorization, congruences and classical reciprocity laws such as quadratic and cubic reciprocity. Time permitting a selection of more advanced topics may be covered.

Text: (Required)**:** A Classical Introduction to Modern Number Theory, Ireland, Kenneth and Rosen, Michael, 2nd Edition.

---

582 Introduction to Set Theory Fernandez Breton MWF 11:00-12pm

Prerequisites: *The official prerequisite, "Math 412 or 451 or equivalent experience with abstract mathematics," means that students should be comfortable with writing mathematical proofs.*

Course Description: This is an introductory course in axiomatic set theory, whose main objective is to convince the student that most everyday mathematical objects can be conceived as sets of some sort.
Topics include:
* The intuitive concept of set; paradoxes.
* Type theory and the cumulative hierarchy of sets.
* The Zermelo-Fraenkel axioms for set theory.
* Set-theoretic representation of the fundamental concepts of mathematics (e.g., function, number) and proofs of basic properties of these concepts (e.g., mathematical induction).
* Infinite cardinal and ordinal numbers and their arithmetic.
* The axiom of choice and equivalent axioms (e.g., Zorn's Lemma).

Additional topics may be discussed if time permits.
No specific previous knowledge of set theory will be presupposed.

Text: (Optional) Elements of Set Theory by H. B. Enderton, ISBN: 9780122384400
(Optional) Discovering modern set theory. I, The basics by W. Just and M. Weese, ISBN 0821802666.
(Optional) Introduction to Set Theory by Karel Hrbacek; ISBN: 9780585243412

Winter 2017 Graduate Course Descriptions

590 Intro to Topology Janda MWF 12-1:00pm

Prerequisites: *Math 451*

Course Description: This is a course on point-set topology, which emphasizes the set-theoretic aspects of topology. The course will focus on the notions of continuity, connectedness, compactness, and other topics. The class is quite theoretical and requires extensive construction of proofs.

Text: (Required) Topology (2nd Edition), by James Munkres. ISBN:131816292

---

592 An Introduction to Algebraic Topology Kriz MWF 10:00-11am

Prerequisites: *Previous exposure to point-set topology and familiarity with abstract algebra will be assumed.*

Course Description: Description: We will cover the basic concepts of algebraic topology, starting with the fundamental group, Seifert-Van Kampen theorem, and covering spaces, CW complexes and their fundamental group. We will then move to homology, covering singular homology, the Eilenberg-Steenrod axioms, the homology of a CW complex and the use of the degree of a map, with some examples. We will also give applications in geometry, including Jordan's separation theorem and invariance of domain. We will also talk about trace, Euler characteristic, and Lefschetz fixed point theorem.

Text: No text is required, but the following are recommended reading:
(1) "A concise course in algebraic topology" by Peter May. ISBN:9780226511832
(2) "Elements of Algebraic Topology" by James R. Munkres. ISBN:9780201162782

---

594 Algebra II: Groups and Galois Theory Speyer TTh 2:30-4pm

Prerequisites: *Prior exposure to the definitions of groups, rings, modules and fields. Abstract linear algebra over arbitrary fields. PID's and unique factorization in Z and k[x]. 513 and 593 are certainly enough; please talk to me if you have questions about your background.*

Course Description: Modern abstract algebra began with Galois' study of the roots of polynomial equations, and their groups of symmetries. This course will cover finite groups, and will then focus on the beautiful subject of Galois theory -- connecting group theory with the behavior of roots of equations. As texts, we will use James Milne's freely available course notes, downloadable at http://jmilne.org/math/CourseNotes/gt.html and http://jmilne.org/math/CourseNotes/ft.html .

Text: None

Winter 2017 Graduate Course Descriptions

597 Analysis II (Real Analysis) Rudelson MWF 12:00-1pm

Prerequisites: *Math 451, 452*

Course Description: This is one of the basic courses for students beginning the study towards a Ph. D. degree in mathematics. The topics include general construction of a measure, Lebesgue measure on R and R^n, measurable functions, integration, Fubini theorem, complex and signed measures, Lebesgue-Radon-Nikodim theorem, maximal function, differentiation of measures, L_p spaces, introduction to Hilbert space and Fourier analysis.
Grades will be based on homeworks a midterm, and a final exam.

Text: (Optional) Real Analysis: Modern Techniques and Their Applications, Gerald B. Folland.
ISBN: 978-047-1317-16

---

605 Several Complex Variables Barrett MWF 11:00-12 pm

Prerequisites: *Background in single-variable complex analysis and Lebesgue integration (at the first-year graduate level).*

Course Description: Domains in the complex plane always have a rich enough collection of holomorphic functions to solve a number of natural problems; for example, any complex-valued function on a discrete subset of the domain extends to a holomorphic function on the whole domain.

In higher dimension the situation is different: the extension problem stated above is automatically solvable on some domains, such as the unit ball, but not on others, such as the punctured ball. This last statement follows from the astounding fact that all isolated singularities are removable in higher dimension. There are also more subtle examples where all holomorphic functions on one domain automatically extend to a fixed larger domain, a phenomenon completely absent in one variable.

Notions developed to sort out these and many related phenomena include the concepts of pseudoconvexity for sets and plurisubharmonicity for functions. Both of these notions are generalizations of the real-variable notion of convexity. These and related concepts from the theory of several complex variables have important applications in many other parts of mathematics, including algebraic geometry and symplectic geometry.

An important tool for building holomorphic functions of several variables is the theory of the inhomogeneous Cauchy-Riemann equations. The study of these equations has also had important consequences elsewhere in mathematics, such as Hans Lewy's surprising discovery of a linear partial differential equation with no local solutions.

Math 605 will provide an introduction to complex analysis in several variables, including the topics mentioned above. Some details of the course may be adjusted according to the interests and backgrounds of the students enrolled.

Text: None

Winter 2017 Graduate Course Descriptions

613 Homological Algebra Derksen MWF 11:00-12pm

Prerequisites: *math 593/594 or equivalent.*

Course Description: Homological algebra originated in the study of algebraic topology (homology of simplicial complexes) and abstract algebra (modules and their syzygies). It gives us a general machinery that is used in many areas, such as topology, algebraic number theory, algebraic geometry and representation theory. In this course we will study among other things abelian categories, chain complexes, derived functors, spectral sequences and derived categories. A textbook is not required, but the main references are "An introduction to homological algebra" by Charles Weibel, "Homological Algebra" by Gelfand and Manin, and "Methods of Homological Algebra" by the same authors. There will be regular homework assignments.

Text: None

---

615 Topics in Commutative Algebra Hochster MWF 10:00-11am

Prerequisites: *Math 614*

Course Description: "The prerequisite for this course is Math 614. Some knowledge of algebraic geometry will also be helpful, especially in motivating the notions we consider. We will discuss Zariski's Main Theorem and its application to the classification theorems for smooth, unramified, and \'etale homomorphisms of rings. We also discuss Henselian local rings and Henselization: the Henselization of a local ring is obtained as a directed union of certain local \'etale extensions, and prove the Artin approximation theorem. This theorem asserts that when equations over certain local rings (R,m) have a solution in the completion of R, that solution can be m-adically approximated by solutions in the Henselization of R, and, consequently, by solutions in an \'etale extension of R. As an application of these techniques, we show that many problems about arbitrary Noetherian rings can be reduced to the study of finitely generated algebras over a field or discrete valuation ring. We discuss a number of open questions to which these techniques may apply. There is no textbook. Lecture Notes for the course will be provided."

Text: None

---

623 Computational Finance Muhle-Karbe TuTh 10-11:30am

Prerequisites*: Differential equations (e.g. Math 316); probability theory (e.g. Math 525/526, Stat 515); numerical analysis (Math 471 and Math 472); mathematical finance (Math 423 and Math 542/IOE 552, Math 506 or permission from instructor); programming (e.g. C, Matlab, Mathematica, Java).numerical analysis (Math 471 or Math 472); mathematical finance (Math 423 and Math 542/IOE 552, Math 506 or permission from instructor); programming (e.g. C, Matlab, Mathematica, Java).*

Course Description: Computational Finance --- This is a course in computational methods in finance and financial modeling. Particular emphasis will be put on interest rate models and interest rate derivatives. The specific topics include; Black-Scholes theory, no arbitrage and complete markets theory, term structure models: Hull and White models and Heath Jarrow Morton models, the stochastic differential equations and martingale approach: multinomial tree and Monte Carlo methods, the partial differential equations approach: finite difference methods.

Text: None

---

626 Probability and Random Processes II: Stochastic Analysis and Control in Mathematical Finance — Bayraktar — TTh 10-11:30am

Prerequisites: *Math 625*

Course Description: Selected topics in continuous time stochastic analysis, stochastic control and their applications to mathematical finance. This is an advanced course for Ph.D. students who would like to pursue research in this area.

Text: (Optional) Brownian Motion and Stochastic Calculus, I.Karatzas & S.E.Shreve. (Optional) Continuous-time Stochastic Control and Optimization with Financial Applications, Huyen Pham. (Optional) Optimal Stochastic Control, Stochastic Target Problems, and Backward Sde, Nizar Touzi.

---

632 Algebraic Geometry II: — Fulton — MWF 3:00-4pm

Prerequisites: *Math 631 or equivalent, and some commutative algebra*

Course Description: This course will develop some of the basic tools of algebraic geometry, with emphasis on examples and applications. Here are some of the topics we plan to discuss:

Abstract algebraic varieties
Sheaves and their cohomology
Toric varieties
Coherent sheaves on projective varieties
Riemann-Roch for curves and surfaces
Blowups, tangent and normal cones
Introduction to intersection multiplicities and intersection theory
Divisors, Picard groups, class groups
Projective, Grassmann and flag bundles
Representable functors
Grothendieck groups of bundles and sheaves
Schemes

We will emphasize examples, many on assignments, from curves, surfaces, and toric varieties. Mastering these abstract notions requires a good deal of work done individually.

Text: None

Winter 2017 Graduate Course Descriptions

635 Differential Geometry — Van Limbeek — MWF 1:00-2pm

Prerequisites: *Differential topology (basic theory of manifolds): Math 591 or equivalent*

Course Description: Differential geometry is a core subject in modern geometry. This course will give an introduction to this area with emphasis on Riemannian geometry. We will discuss the basic ideas of connections, Riemannian metrics, curvature and the basic tools in the subject, especially variational methods, Jacobi fields, and comparison theorems. After those more local ideas, we will turn to global differential geometry which relates geometric ideas to the underlying topology. Examples are the study and classification of spaces of constant curvature, the Cartan-Hadamard Theorem, sphere theorems, structural results both in positive and negative curvature, and rigidity theorems. If time permits we will pursue more advanced topics.

Text: Riemannian Geometry, Do Carmo, Birkhauser. ISBN: 817634908, The book is optional but highly recommended.

---

636 Topics in Differential Geometry: Introduction to Rigidity Theory — Spatzier — TTh 11:30-1pm

Prerequisites: *MATH 591, 592, 597or equivalent*

Course Description: We will discuss rigidity results in geometry and dynamics. These have been highlights of research during the last five decades and are intricately woven together. We will start with Mostow's Global Rigidity Theorem from the 1960's: Let M be a closed manifold of dimensions at least three. Then M carries at most one metric of constant negative curvature -1. Closely related are Margulis' superrigidity and Arithmeticity theorems from the 1970s that classify all representations of fundamental groups of so-called higher rank symmetric spaces. The latter have later been characterized in terms of Riemannian metrics with sufficiently many flats in the 1980's. They have also been characterized quasi-isometrically. Since, much progress have been made on the dynamics side, starting with measure rigidity results for unipotent groups by Ratner in the 1980s, later for higher rank abelian subgroups of semi simple elements and for actions of lattices. Highlights from the last few years include the classifications of Anosov actions of higher rank abelian groups on tori and nilmanifolds, and lately of higher rank lattice actions on low-dimensions manifolds.

Text: None

Winter 2017 Graduate Course Descriptions

650 Fourier Analysis Uribe TTH 1-2:30pm

Prerequisites: *Measure theory and complex analysis (at the level of Math 596, Math 597), basic Hilbert space notions.*

Course Description: The basic idea behind Fourier analysis is to express a function in euclidean space as a weighted superposition of exponentials exp(ikx). The (complex) weights depend on the frequency k and constitute the Fourier transform of the function. The Fourier transform (as an operator) has extraordinary properties and a huge number of applications. The first part of this course will cover the basic theory of the Fourier transform: Basic estimates, the Plancherel formula, the inversion formula. We will discuss the fundamentals of distribution theory, extend the Fourier transform to tempered distributions, and prove the Paley-Wiener theorem. We will do some Fourier series theory as well, mostly as exercises, and prove the Poisson summation formula.

The rest of the course will be an introduction to microlocal analysis. This is a theory developed by Hörmander and others that studies the singularities of distributional solutions of linear PDEs. We will start with a study of Kohn-Nirenberg pseudodifferential operators (PDOs), a class that includes parametrices of elliptic operators. We will use PDOs to define the wave-front set of a distribution, and prove various theorems on the wave-front sets of solutions to PDEs. We will prove Egorov's theorem and, time permitting, we will discuss Fourier integral operators.

There will be homework problem sets, roughly every other week, and students will be asked to give a 20-30 minute presentation towards the end of the semester. There is no required textbook but there are many good references, the main one being: The Analysis of Linear Partial Differential Operators vols. I and III, by Lars Hörmander (available electronically through the U.M. Library and SpringerLink).

Text: (Optional) The Analysis of Linear Partial Differential Operators vols. I and III, by Lars Hörmander.

---

651 Special Topics: Schotland TTH 11:30-1pm
Waves and Imaging in Random Media

Prerequisites: *Prerequisites: basic partial differential equations; some knowledge of probability theory would be useful, but not essential.*

Course Description: This is a special topics course on the theory of wave propagation in various asymptotic regimes including: (i) geometrical optics of high-frequency waves (ii) homogenization of low-frequency waves in periodic and random media (iii) radiative transport and diffusion theory for high-frequency waves in random media. Applications to inverse problems in imaging will be considered. The necessary tools from asymptotic analysis, scattering theory and probability will be developed as needed. The course is meant to be accessible to graduate students in mathematics, physics and engineering.

Text: None

---

657 Nonlinear Partial Differential Equations Bieri TTH 2:30-4pm

Prerequisites: *Math 451, 454, 555 (556 recommended), elementary physics, Newtonian mechanics, elementary dynamical systems theory, differential equations, and computer literacy as expected of mathematically minded science and engineering graduate students.*

Course Description: Partial differential equations are at the core of models in science, technology, economics and related fields. These equations and their solutions have interesting structures that are studied by methods of analysis, geometry, probability and other mathematical fields.

The goal of this course is to introduce students in pure and applied mathematics to concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations.
This course will focus mainly on important nonlinear partial differential equations. The topics will include Sobolev spaces, 2nd order partial differential equations of elliptic, parabolic and hyperbolic type, shock waves and nonlinear waves.
Course material will be taken partially from chapters 5, 6, 7, 8, 11, 12 of the textbook and from the references.

Grading: The course grade will be based on homework and a final project.

Text: (Required) Partial Differential Equations, by L.C. Evans, 2nd Edition.
(Optional) C. Sogge, Lectures on non-linear wave equations.

---

669 Combinatorial Theory: Barvinok MWF 11:00-12pm
Combinatorics and Complexity of Partition Functions

Prerequisites: *An aptitude for analysis and combinatorics*

Course Description: Partition functions originated in statistical physics in an effort to explain and predict magnetization, transition from gas to liquid and other “phase transition” phenomena. In plain terms, a partition function is a multivariate polynomial with positive integer coefficients and great many monomials enumerating combinatorial structures of a particular type, such as matchings or cuts in graphs.

We will cover some classical results, such as the Lee-Yang Circle Theorem for the Ising model (or, in plain terms, for cuts in graphs) and the Heilmann-Lieb Theorem for the monomer-dimer model (on, in plain terms, for matchings in graphs) as well as recent advances, such as the Gurvits approach to the van der Waerden conjecture for permanents. Bayati et al. approach to the correlation decay for the matching polynomial and Weitz results for independent sets in a graph (known as the “hard core model” in statistical physics).
Grading: we will have a number of homework problem sets.

Text: None.

---

671 Topics in Scientific Computing: Particle Methods — Krasny — TTh 10-11:30am

Prerequisites: *None*

Course Description: The course is a survey of topics related to particle methods in scientific computing. Particles interact with each other through fields, and we'll start by considering boundary value problems for fields. Specifically, we'll consider finite-difference methods and contrast them with spectral and Green's function methods. Examples of particle systems will be chosen from fluid dynamics (point vortices) and plasma dynamics (point charges). Much of the course will deal with methods for evaluating the potential energy and forces due to long-range particle interactions, an important component in molecular dynamics and Monte-Carlo simulations. In a system with N particles, $O(N^2)$ operations are needed to evaluate the pairwise interactions by direct summation. Faster methods have been developed and a prime example is the fast Fourier transform (FFT), which reduces the operation count to $O(N\log N)$. The FFT can be applied when the particles are uniformly spaced, but different ideas are needed for nonuniform distributions. We'll consider particle-mesh methods such as particle-in-cell (PIC), and hierarchical algorithms such as tree codes and the fast multipole (FMM) method. We'll discuss the spherical harmonics expansion for the Coulomb potential on which the treecode and FMM are based. We'll also discuss Ewald summation for periodic systems. Depending on student background and interest and time permitting we'll discuss iterative multigrid methods for linear systems arising from finite-difference discretization of boundary value problems. There is no required textbook; lecture notes will be made available on the course website after each class. Several homework sets will be assigned.
Text: None.

Winter 2017 Graduate Course Descriptions

679 Perfectoid spaces Bhatt MW 1-2:30pm

Prerequisites: *I will assume familiarity with the basic language of algebraic geometry (Math 631, Math 632). Prior experience with some form of rigid analytic geometry (such as Mattias Jonssons's class on 'Berkovich spaces' offered in Fall 2016 as Math 731) will be very useful, but not necessary.*

Course Description: Perfectoid spaces are a class of spaces in arithmetic geometry introduced in 2012 by Peter Scholze in his PhD thesis. Despite their youth, these spaces have had stunning applications to many different areas of mathematics, including number theory, algebraic geometry, representation theory, and commutative algebra. The key to this success is that perfectoid spaces provide a functorial procedure to translate certain algebro-geometric problems from characteristic 0 (or mixed characteristic) to characteristic p; the latter can often be more accessible thanks to the magic of Frobenius.
Plan: A major portion of this class will be devoted to setting up the basic theory of perfectoid spaces. En route, we will encounter Huber's approach to nonarchimedean geometry via his language of adic spaces, Faltings' theory of 'almost mathematics' conceived in his proof of Fontaine's conjectures in p-adic Hodge theory, and the basic algebraic geometry of perfect schemes in characteristic p. The highlight of this part of the course will be the 'almost purity theorem', a cruder version of which forms the cornerstone of Faltings' aforementioned work. The rest of the course will focus on a single application of perfectoid spaces. There are several choices here: we could go either in the arithmetic direction (such as Scholze's work on the weight-monodromy conjecture or some recent progress in p-adic Hodge theory) or in a more algebraic direction (which is the lecturer's inclination). A final choice will be made during the semester depending on audience makeup and interest.
References: The main reference is Scholze's paper titled Perfectoid spaces.

Text: None

---

681 Mathematical Logic Blass TTh 10-11:30am

Prerequisites: *None*

Course Description: Historically, mathematical logic is the mathematical study of mathematics itself, especially of the process of deductive reasoning. A central question is the adequacy of deductive reasoning: Can all the consequences of a set of assumptions be obtained from those assumptions by a sequence of very simple inferences? This question is an instance of a general theme that runs through much of mathematical logic, namely the interplay between mathematical statements (which are to be manipulated in the desired simple inferences) and the mathematical structures that they describe (which underlie the notion of logical consequence). Part of Math 681 is devoted to making this interplay precise and establishing a positive answer to the central question in some situations, including the particularly important

case of first-order logic. This logic with its simple inferences serves as the explicit or implicit foundation for essentially all mathematical reasoning. In this connection, I'll also discuss at least one alternative way of verifying correctness in first-order reasoning. This way, though farther from intuition than the simple inferences mentioned above, is in general much more efficient and has found applications in automated theorem-proving.
Another part of the course will indicate why first-order logic plays such a central role in mathematics. It is easy to produce logical systems stronger than first-order logic, and one might be tempted to use them as an improved foundation for mathematics. But the notions of "consequence" for these systems are necessarily far more complicated and cannot be captured by any reasonable step-by-step inferences. A mathematics based on them could not have proofs in anything like the usual sense of the term.
A third part of the course uses the interplay between mathematical statements and the structures they describe, to establish some results in other areas of mathematics. These results do not directly involve logical matters, but their proofs are based on the results from logic proved earlier in the course. One example is the existence of the models used in non-standard analysis.
Text: (Optional) Fundamentals of Mathematical Logic, Peter G. Hinman**.**

---

697 Topic in Topology: Quiver Varieties Ruan MWF 11:00-12pm

Prerequisites: *Basic Algebraic geometry and topology.*

Course Description: Quiver varieties appear in many places in geometry and physics. In mathematics, they are some of most beautiful examples in algebraic geometry with a lot of elegant combinatorial structures. Furthermore, it is closely related to geometric representation theory. In physics, it appears as so called vacuum moduli space of gauge theory. The intersection of geometry and physics on the quiver varieties leads to many recent developments.
We will begin the course with a review of GIT quotient technique and then use it to construct quiver varieties. Then, we will focus on its rich topological and combinatorial structure as well connection to geometric representation theory. We will study plenty of examples.

Text: none

---

732 Topics in Algebraic Geometry II: Rationality of Algebraic Varieties Mustata TTh 11:30am- 1pm

Prerequisites: *Basic knowledge of algebraic geometry, as covered in Math 631 and 632.*

Course Description: A fundamental problem in algebraic geometry is to determine which varieties are rational, that is, birational to the projective space. Several important

developments in the field have been motivated by this question. The main goal of the course is to describe two recent directions of study in this area. One approach goes back to Iskovskikh and Manin, who proved that smooth, 3-dimensional quartic hypersurfaces are not rational. This relies on ideas and methods from higher-dimensional binational geometry. The second approach, based on recent work of Claire Voisin and many other people, relies on a systematic use of decomposition of the diagonal and invariants such as Chow groups, Brauer groups, etc, to prove irrationality. Both directions will give us motivation to introduce and discuss some important concepts and results in algebraic geometry.
The following is a rough outline of the course:
Introduction
1. Rationality in dimension 2 (Castelnuovo's criterion).
Part I
2. Introduction to Chow groups and intersection theory.
3. Decomposition of the diagonal and non-stable-rationality.
4. Examples of classes on non-stably-rational varieties.
Part II
5. Introduction to singularities of pairs and vanishing theorems.
6. Binational rigidity of Fano hyper surfaces of index 1. "

Text: none

---

776 Topics in Number Theory: Class Field Theory — Lagarias — TTh 2:30- 4pm

Prerequisites: *Math 676*

Course Description: This course traditionally covers Class Field Theory.
Class Field Theory concerns the description of properties of abelian Galois extensions of a field k in terms of invariants of k. Global class field theory concerns k a number field or function field over finite field, and the invariants of k are (ray) class groups and units. Local class field theory concerns k a one-dimensional local field, such as a p-adic field, and invariants are field elements and norm groups. These two theories embody the GL(1) part of the (local and global) Langlands program.

I am currently interested in the Intra-Universal-Teichmuller Theory (IUT)
of S. Mochizuki, which asserts to prove the ABC Conjecture.
Class Field Theory is one of many inputs into this theory.
The course will cover selected topics that might help in understanding
the IUT theory. It will initially follow the book of Neukirch,

Class Field Theory, Springer 1986, which proceeds via infinite Galois theory. I hope to cover other topics to be chosen, around elliptic curves, Tate curves, p-adic logarithm and theta functions, ABC conjecture proofs in complex case, etc. I hope for class participation in an IBL- related manner, with an aim being to extract useful things from the IUT theory. This will be discussed at the first meeting of the course.

Text: (Optional) Class Field Theory, J. Neukirch, ISBN: 978-3-642-82467-8