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501   AIM Student Seminar   Esedoglu   F 12:00-1:00pm & 3-4:00pm

Prerequisites: *You must be a graduate student in the AIM program to register for this course.*

Course Description: Math 501 is a required course for all students enrolled in the Applied and Interdisciplinary Mathematics (AIM) MS and PhD graduate programs. In the Winter term, all first-year AIM students from both programs must sign up for this course. Due to the highly specialized content of the course, enrollment is available only for students in an AIM degree program.

The purpose of Math 501 is to address specific issues related to the process of studying applied mathematics in the AIM program and becoming an active member of the research community. The weekly meetings of the class will be divided among three types of sessions:

1. "Focus on. . . " presentations. These are presentations on various topics, some of immediate practical significance for students and others of a further-reaching nature. These discussions will include aspects of scholarly writing, research, and career development.

2. AIM Faculty Portraits. These are short presentations by faculty members in the Mathematics Department and other partner disciplines who are potential advisors or committee members for AIM students. The AIM faculty portraits provide a direct channel for students to discover what research is being done in various areas by current faculty, and to see what kind of preparation is required for participating in such research.

3. AIM Research Seminar Warm-up talks. One of the course requirements for Math 501 is weekly attendance of the AIM Research Seminar that takes place from 3-4 PM each Friday. The warm-up talks are presentations during the regular course meeting time by particularly dynamic speakers slated to speak in the AIM Research Seminar later the same day as a way to provide background material with the goal of making the AIM Research Seminar lecture more valuable for students.

Weekly attendance both of the course meeting and also of the AIM Research Seminar is required for Math 501. If you are registering for Math 501 you must be available both during the regular class time of 12-1pm on Fridays as well as during the AIM Research Seminar which runs 3-4pm on Fridays. If you are teaching, you should keep both of these obligations in mind when you submit your class/seminar schedule prior to obtaining a teaching assignment. Other requirements, including possible assignments related to topics discussed in the lectures, will be announced by the instructor in class.

Winter 2015 Graduate Course Descriptions

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521   Life Contingencies II   Marker   TTh 8:30-10am & 10-11:30am
Prerequisites: *Math 520 or permission of the instructor.*

Course Description: Background and Goals: This course extends the single decrement and single life ideas of Math 520 to multi-decrement and multiple-life applications directly related to life insurance. The sequence 520--521 covers covers the material for Examination 3L of the Casualty Actuarial Society and for Examination MLC of the Society of Actuaries.

Content: Topics include multiple life models--joint life, last survivor, contingent insurance; multiple decrement models---disability, withdrawal, retirement, etc.; and reserving models for life insurance. This corresponds to chapters 7--11 and 15 of Bowers et al.

Text: Actuarial Mathematics, Second Edition, Bowers et al. Publisher: Society of Actuaries; 2nd edition (May 1997) ISBN-10: 0938959468, ISBN-13: 978-0938959465

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523   Loss Models I   Moore   TTh 11:30am - 1:00pm & 1-2:30pm
Prerequisites: *Math 425 or equivalent*

Course Description: Risk management is of major concern to all financial institutions, especially casualty insurance companies. This course is relevant for students in insurance and provides background for the professional examinations in Actuarial Modeling offered by the Society of Actuaries (Exam C) and the Casualty Actuary Society (Exam 3). Students should have a basic knowledge of common probability distributions (Poisson, exponential, gamma, binomial, etc.) and have at least Junior standing.

Review of random variables (emphasizing parametric distributions), review of basic distributional quantities, continuous models for insurance claim severity, discrete models for insurance claim frequency, the effect of coverage modification on severity and frequency distributions, aggregate loss models, and credibility.

Text: Loss Models: From Data to Decisions (4th edition) by Klugman, Panjer, and Willmot

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524   Loss Models II   Young   TTh 11:30am-1:00pm

Prerequisites: Math 523 and Stats 426, each with a grade of C- or better.

Course Description: Risk management and modeling of financial losses. Frequentist and Bayesian estimation of probability distributions, model selection, credibility, and other topics in casualty insurance.

Text: "Loss Models: From Data to Decisions" by Stuart A. Klugman, Harry H. Panjer, and Gordon E. Willmot, fourth edition, published by John Wiley in 2012.

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525   Probability Theory   Cohen   TTh 10:00-11:30am
                                  Nishry   TTh 1:00-2:30pm

Course Description: The following topics will be covered: sample space and events, random variables, concept and definition of probability and expectation, conditional probability and expectation, independence, moment generating functions, Law of large numbers, Central limit theorem, Markov chains, Poisson process and exponential distribution.

Text: G. Grimmett and D. Stirzaker, Probability and Random Processes, Third edition, Oxford,2001 (required)

S.M. Ross, Introduction to Probability Models, Tenth Edition, Academic Press
ISBN:9780123756862 (optional)

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526   Discrete Stochastic Processes   Nadtochiy   TTh 10-11:30am
                                              Angoshtari   TTh 11:30am-1:00pm

Prerequisites: *Required: Math 525 or basic probability theory including: probability measures, random variables, expectations, cumulative distribution and probability density functions, conditional probabilities and independence, law of large numbers. Good understanding of advanced calculus covering limits, series, the notions of continuity, differentiation and integration. Recommended: Various types of convergence of random variables (almost sure, in probability, in distribution); limit theorems for sums of random variables (e.g. central limit theorem); interchanging the limit and integration/expectation (monotone and dominated convergence theorems, Fubini's theorem); linear algebra, including matrix calculus, eigenvalues and eigenfunctions.*

Course Description: The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their

applications. Some specific topics include: Markov chains (Markov property, recurrence and transience, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times, Markov queues); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales). Significant applications will be an important feature of the course.

Text: Required: Essentials of Stochastic Processes, 2nd ed. (Durrett). Optional: Introduction to Stochastic Processes (Cinlar), Stochastic Processes (Ross), Probability and Measure (Billingsley).

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542   Financial Enginering I   Saigal   TTh 5:00-6:30pm
Prerequisites: IOE 452 or MATH 423 and a good course in probability and Statistics, like Math 425 or IOE 316.

Course Description: This course covers the concepts and methods for the design and pricing of financial instruments like options, futures and other derivative products. It also covers the use of these products in the management of investment and other financial risks. The mathematical basis for the analysis of these products will be covered. This includes the study and solution of stochastic differential equations using Ito calculus, the use of measure change to price the derivatives in a risk neutral environment and the concept of arbitrage free pricing. Applications to non-financial derivatives and their applications in engineering and other areas will also be covered.

This course is a must for individuals planning careers in the Financial and Actuarial Industries, and will be of interest to those involved in risk assessment in engineering applications.

Text: 1. Stochastic Calculus for Finance, Vols. I and II, Steven Shreve, Springer.

2. Arbitrage Theory in Continuous Time, Tomas Bjork, Oxford.

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555   Applied Complex Analysis   Miller   MW 8:30-10:00am

Prerequisites: *Courses in elementary real analysis (e.g. Math 451) and multivariable calculus (e.g. Math 215 or Math 255) are essential background.*

Course Description: This course is an introduction to the analysis of complex-valued functions of a complex variable with substantial attention to applications in science and engineering. Concepts, calculations, and the ability to apply principles to problems are emphasized alongside rigorous proofs of the basic results in the subject.

Topics covered include differentiation and integration of complex-valued functions of a complex variable, series, mappings, residues, and applications including evaluation of improper real integrals and fluid mechanics.

Text: R. A. Silverman, Complex Analysis with Applications, Dover Publications, 1984. ISBN 0-486-64762-5

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557   Applied Asymptotic Analysis   Miller   TTh 1:00-2:30pm

Prerequisites: *This course will assume a strong background in differential equations, linear algebra, and advanced calculus or real analysis. Even more important is technical skill in complex variables and analysis at the level of Math 555 or Math 596.*

Course Description: Asymptotic analysis is the quantitative study of approximations. The fundamental idea is that one tries to solve a problem in applied mathematics (say, a boundary-value problem for a partial or ordinary differential equation) by embedding it into a family of problems with a parameter. If the problem can be solved exactly for one special value of the parameter, or in a limiting case, then asymptotic analysis can be used to analyze how the solution changes as the parameter is tuned from the special value to a more physically reasonable one. The course will develop the general theory of so-called asymptotic expansions, which are a kind of series in the perturbation parameter that are extremely useful in practice, in a way that is mathematically completely rigorous, despite the strange fact that they frequently fail to converge at all! We will then study how to use asymptotic expansions to evaluate integrals that cannot be computed in closed form and that are also difficult to approximate numerically. Next, we will turn to differential equations and use asymptotic expansions to evaluate solutions near certain singular points and also to study the way that solutions depend on parameters. At the end of the course we will study how the differential equations of diverse physical phenomena can be reduced, with the help of asymptotic expansions, to certain universal model equations that show up again and again in applied mathematics.

Specific applications to be addressed in the course as time permits include the small-viscosity theory of shock waves, the theory of quantum mechanics in the semiclassical limit, aspects of the theory of special functions, vibrations in nonlinear lattices, and surface water waves.

Text: P. D. Miller, Applied Asymptotic Analysis, Graduate Studies in Mathematics, volume 75, American Mathematical Society, Providence, RI, 2006. ISBN 0-8218-4078-9.

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| 564 | Topics in Mathematical Biology: Clocks Rhythms and Oscillations | Forger | TTh 11:30am-1:00pm |

Prerequisites: Differential equations and linear algebra. Math 463 is an excellent preparation.

Course Description: From sleeping patterns, heartbeats, locomotion and firefly flashing to the treatment of cancer, diabetes and neurological disorders, oscillations are of great importance in biology and medicine. Mathematical modeling and analysis are needed to understand what causes these oscillations to emerge, properties of their period and amplitude and how they synchronize to signals from other oscillators or from the external world. The goal of this course will be to teach students how to take real biological data, convert it to a system of equations and simulate and/or analyze these equations. Models will typically use ordinary differential equations. Mathematical techniques introduced in this course include 1) the method of averaging 2) methods for fitting data 3) Fourier techniques 4) entrainment and coupling of oscillators 4) phase plane analysis and 5) various techniques from the theory of dynamical systems. Emphasis will be placed on primary sources (papers from the literature) particularly those in the biological sciences. Consideration will be given in the problem sets and course project to interdisciplinary student backgrounds. Teamwork will be encouraged.

Text: No textbook is required.

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| 566 | Combinatorial Theory: Algebraic Combinatorics | Fomin | TTh 1:00-2:30pm |

Prerequisites: Math 465 or 565, or some other course in combinatorics.

Course Description: This course is an overview of applications of algebra (mostly linear algebra) to combinatorics (mostly enumerative combinatorics). Topics include: introduction to algebraic graph theory; applications of linear algebra to enumeration of matchings, tilings, and spanning trees; combinatorics of electric networks; partially ordered sets, integer partitions, and Young tableaux. The course will emphasize problem solving (as opposed to theory-building).

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Text: Optional textbook: R. P. Stanley, Algebraic Combinatorics: Walks, Trees, Tableaux, and More, Springer, 2013. The text of this book (without exercises) is available online at <http://www-math.mit.edu/~rstan/algcomb/>.

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567   Introduction to Coding Theory   Ho   TTh 1:00-2:30pm
Prerequisites: *Linear algebra (217, 417, or 419) required. Abstract algebra helpful.*

Course Description: Introduction to coding theory focusing on the mathematical background for error-correcting codes. Shannon's Theorem and channel capacity. Bounds on the sizes of codes. Basic examples of codes such as Hamming, BCH, cyclic, and Reed-Solomon. Weight distributions. More advanced codes over rings and from algebraic geometry.

Text: Fundamentals of Error Correcting Codes, by W. Cary Huffman and Vera Pless, Cambridge University Press

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571   Numerical Linear Algebra   Alben   TTh 10:00-11:30am
Prerequisites: *Math 217, 417, 419, or equivalent.*

Course Description: Numerical Linear Algebra --- Direct and iterative methods for solving systems of linear equations (Gaussian elimination, Cholesky decomposition, conjugate gradients), QR factorization, Least Squares problems, Stability and Conditioning, Singular Value Decomposition, methods for computing eigenvalues and eigenvectors, applications to elliptic partial differential equations.

Text: Numerical Linear Algebra by Trefethen and Bau.

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572   Numerical Methods for Scientific Computing II   Karni   TTh 11:30-1:00pm
Prerequisites: *Solid background in advanced calculus, linear algebra and working knowledge of a computing programming language (such as C, C++ or Fortran) or a computing environment (such as Matlab or Python).*

Course Description: This course is an introduction to numerical methods for boundary-value and initial-value problems. The course will cover numerical methods for ordinary differential equations and for elliptic, parabolic and hyperbolic partial differential equations. Nonlinear hyperbolic partial differential equations may also be discussed, if time permits.

The course will focus on the derivation of methods, on their accuracy, stability and convergence properties with brief comments on practical aspects of efficient implementation of the methods. The course should be useful to students in mathematics, physics and engineering.

טופיצס: Finite difference approximations; boundary-value and initial-value ODEs, consistency, stability and convergence, Lax equivalence theorem, Gaussian elimination, Gauss-Seidel, Jacobi, and SOR, Runge-Kutta and multistep methods, methods for stiff ODEs, elliptic equations, diffusion equation, Crank-Nicolson, stability analysis by Fourier and energy methods, maximum principle, ADI, linear advection equation, CFL condition, upwind, Lax-Wendroff, Lax Friedrichs, scheme. Numerical boundaries for the wave equation. Conservation laws and shocks. Finite volume methods (time permitting)

Text: Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems by R.J. LeVeque, SIAM, 2007.

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582   Introduction to Set Theory   Blass   MWF 12:00-1:00pm
Prerequisites: Math 412 or 451 or equivalent experience with abstract mathematics

Course Description: This is an introductory course in axiomatic set theory. Topics include: The intuitive concept of set; paradoxes. Type theory and the cumulative hierarchy of sets. The Zermelo-Fraenkel axioms for set theory. Set-theoretic representation of the fundamental concepts of mathematics (e.g., function, number) and proofs of basic properties of these concepts (e.g., mathematical induction). Infinite cardinal and ordinal numbers and their arithmetic. The axiom of choice and equivalent axioms (e.g., Zorn's Lemma). Additional topics may be discussed if time permits. The official prerequisite, "Math 412 or 451 or equivalent experience with abstract mathematics," means that students should be comfortable with writing mathematical proofs. No specific knowledge of set theory will be presupposed.

Text: "Elements of Set Theory" by H. B. Enderton

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583   Probabilistic and Interactive Proofs   Strauss   MWF 1:00-2:00pm
Prerequisites: Graduate students and advanced undergraduates in Math, Computer Science, and Philosophy. Students should have an interest in theoretical computer science, logic, randomized algorithms and graphs, and/or proofs and knowledge.

Course Description: Can we be convinced that a proof is correct, even if we only check it in three places? Can a proof convince us that a statement is true, while giving us no aid in convincing anyone else that the statement is true? The answer to both is affirmative. How? Using randomness and interaction, two elements missing from traditional deductive proofs. Why? Checking a proof in just a few places is useful for checking computer-generated proofs

that are too long to read (an example bigdata algorithm); there are also surprising connections to showing that certain functions cannot be computed or even approximated efficiently. A "zero-knowledge proof" might be used, for example, for a customer to prove to a merchant that the customer is the rightful owner of a credit card, without giving the merchant any ability to prove (fraudulently) that the merchant is the owner of that credit card.

Other topics: We will also look at IP=PSPACE: converting a game, G, to another, G', such that if Peggy beats any expert in G, then she beats any expert in G'; if Peggy loses to an expert in G, she loses to a random (non-expert) player in G'. Thus the prover Peggy can convince a non-expert verifier Victor that she wins G, even though the game tree is too big for Victor to read. We will also look at Multi-prover Interactive Proof systems and show that they are more powerful than single-prover systems, answering the question posed so eloquently by Professors Click and Clack: Do two people who don't know what they are talking about know more or less than one person who doesn't know what he's talking about?

Content: Probabilistically-checkable proofs, zero-knowledge proofs, and interactive proofs are studied and their computational, cryptographic, and other advantages discussed. The course will include, as modules, a presentation of the necessary background material, which (it turns out) are important tools for many other topics in discrete math and theoretical computer science. These modules include the Chernoff tail bound and other topics from probability theory, error-correcting codes, expander graphs, and randomized computation. Motivations and applications in other fields, such as secure computation and the philosophical nature of proof and knowledge, are briefly discussed.

Expected work: Students will transcribe lecture notes and present papers.

Contact Martin Strauss, martinjs, with questions.

Text: None. Readings and handouts provided.

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590   Intro to Topology   Foster   MWF 12:00-1:00pm
Prerequisites: *Math 451*

Course Description: This course is an introduction to topology with an emphasis on the set-theoretic aspects of the subject. It is quite theoretical and requires extensive construction of proofs. Content will include topological and metric spaces, continuous functions, homeomorphism, compactness and connectedness, fundamental theorem of algebra, and other topics as time allows.

Text: Topology (2nd Edition), by James Munkres

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592   An introduction to algebraic topology   Bhatt   MWF 10:00-11:00am

Prerequisites: *Previous exposure to point-set topology and familiarity with abstract algebra will be assumed.*

Course Description: The goal of this course is to introduce the basic objects in algebraic topology: fundamental groups and covering spaces, singular homology and cohomology, and higher homotopy groups. Time permitting, we will discuss more advanced topics, such as the Lefschetz fixed point theorem or Serre's techniques for computing homotopy groups.

Text: No text is required, but the following are recommended reading:

(1) "Algebraic topology" by Allen Hatcher.
(2) "A concise course in algebraic topology" by Peter May.
(3) "Algebraic topology" by Edwin Spanier.

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594   Algebra II: Groups and their Actions   Smith   MWF 2:00-3:00pm
Prerequisites: *Admission to the Math PhD program OR Math 493-494.*

Course Description: This is the second course in the year-long required sequence in algebra for math PhD students. The main topic is group actions---understanding the different ways a group can act on a vector space, or a set, or some object in some other category. When that vector space is some field extension L of a fixed field k, we arrive at Galois theory of L/k, and we will spend some time looking into this subject and its amazing applications (including the fact that the there is no analog of the quadratic formula for polynomials of degree five or more). We also cover in depth the beautiful story of the representations a finite group on a complex vector space (the character table, etc). We also derive the Sylow theorems, which are helpful in classifying finite (non-abelian) groups (and for QR exam problems ;).).

Text: Dummit and Foote Abstract Algebra (Same as Math 593)

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597   Analysis II (Real Analysis)   Barrett   MWF 11:00am-12:00pm
Prerequisites: *Math 451; Math 490 or 590 strongly recommended*

Course Description: This is one of the core courses for the mathematics doctoral program.

Topics will include: Lebesgue measure in R^n; general measures; measurable functions; integration; monotone convergence theorem; Fatou's lemma; dominated convergence theorem; product measures; Fubini's theorem; function spaces; Holder and Minkowski inequalities; functions of bounded variation; differentiation theory; modes of convergence; Fourier analysis. Additional topics such as Hausdorff dimension and Sobolev spaces to be covered as time permits.

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Text: T. Tao, An Introduction to Measure Theory

T. Tao, An Epsilon of Room, 1: Real Analysis

(Additional remarks about the textbooks will be sent by email in late December.)

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605   Several Complex Variables   Jonsson   TTh 10:00-11:30am
Prerequisites: *First-year graduate analysis.*

Course Description: Analysis in several complex variables is formally just the extension of complex analysis in one variables to the higher-dimensional case. However, it has a much more geometric flavor, and the analytic techniques it provides can be quite powerful in fields such as complex algebraic geometry. The course will start out with basic properties of holomorphic functions in several variables and some surprising phenomena, such as the Hartogs extension theorem. We will also study local properties of analytic sets, that is, zero loci of holomorphic functions.

After that we will focus on L^2 methods, one of the main techniques for constructing holomorphic functions. Along the way, we will study pseudoconvex sets and plurisubharmonic functions, the complex cousins of convex sets and functions in real Euclidean space.

Finally we will turn to geometric applications. We will study basic properties of complex manifolds and extend some of the results obtained in C^n. Towards the end of the course, we will prove Kodaira's Embedding Theorem, which gives a criterion for a compact complex manifold to embed into some projective space, and hence be algebraic.

Text: The main reference will be the book "An Introduction to Complex Analysis in Several Variables", 3rd ed. by Lars Hörmander. It will be complemented by various notes.

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613   Homological Algebra   Bhatt   MW 11:30am-1:00pm
Prerequisites: *Previous exposure to homological ideas (such as, for example, in the setting of algebraic geometry, algebraic topology, or commutative algebra) will be very useful.*

Course Description: Homological algebra began life more than a century ago in the study of combinatorial topology. In the time since, it has developed into a mature mathematical subject, with applications to a variety of mathematicial disciplines including topology, algebraic geometry, number theory, commutative algebra, representation theory, and group theory.

The first quarter of this course will introduce the basic objects: abelian categories, derived functors, spectral sequences. The rest of the semester will be spent on applications, with possible topics being a subset of:

(1) Commutative algebra: Tor and Ext, Koszul homology, local cohomology, Serre's regularity criterion.

(2) Topology: the Leray spectral sequence and some homotopy groups of spheres.

(3) Non-commutative algebra: Hochschild homology and connections to deformation theory, cyclic homology

(as an example of Tate cohomology) and connections to de Rham cohomology.

(4) Derived categories, local cohomology, derived completions, Greenlees-May duality.

Text: No text is required, but the following are recommended reading:

(1) "An introduction to homological algebra" by Charles Weibel.
(2) "Homological algebra" and "Methods of homological algebra" by Gelfand and Manin.

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615   Topics in Commutative Algebra: Hochster   MWF 2:00-3:00pm
Multiplicities

Prerequisites: *Math 614 and Math 631, at least concurrently. Other prerequisites needed briefly for some segments will be presented seminar style. In class I will assume familiarity with Tor and Ext, but I will give some supplementary lectures in a time slot to be determined introducing these functors for those unfamiliar with them.*

Course Description: This course will discuss various notions of multiplicity, including intersection multiplicity, Hilbert-Samuel multiplicity and Hilbert-Kunz multiplicity and the relations among them. Many open questions will be discussed, including positivity of intersection multiplicities and Lech's conjecture. The course will include a brief treatment of spectral sequences.

Text: There is no textbook. I will provide written materials on the course Web site.

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626   Stochastic Analysis/Control: Bayraktar   TTh 10:00-11:30am
Applications of Stochastic Analysis in Finance

Prerequisites: *Math 625 and or instructor approval.*

Course Description: This is an advanced graduate course which will cover selected topic in stochastic analysis, stochastic control and mathematical finance.

Text: Tentative titles:

1) I.KARATZAS & S.E.SHREVE (1991) Brownian Motion and Stochastic Calculus. Springer.

2) Huyen Pham (2009) Continuous-time Stochastic Control and Optimization with Financial Applications, Springer.

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632   Algebraic Geometry II   Speyer   MWF 3:00-4:00pm

Prerequisites: *631 (Algebraic Geometry) or other prior exposure to algebraic geometry. Point set topology. Prior exposure to algebraic topology (592) and differential geometry (537) will be very helpful but not logically needed.*

Course Description: An introduction to scheme theory and sheaf cohomology. Sheaves, construction of Spec X, definition of schemes, line bundles, sheaf cohomology, vanishing theorems. Ideally, we will have time for some classical applications to the theory of surfaces at the end. This is not a first course in algebraic geometry; you should have already worked with algebraic varieties (as in Shafarevich's book) somewhere else.

Text: Algebraic Geometry, Hartshorne, ISBN 9780387902449

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635   Differential Geometry   Bieri   MWF 11:00am-12:00pm

Prerequisites: *Basics from topology, algebraic topology and manifolds.*

Course Description: In this course we will discuss important local and global aspects of differential geometry as well as the relation with the underlying topology. We will start with manifolds, connections, Riemannian metrics, curvature and the basic tools such as variational methods, Jacobi fields and comparison theorems. Then we will continue to study sphere theorems, rigidity theorems and related topics. If time permits we will consider more advanced topics.

Text: Do Carmo, Riemannian Geometry, Birkhauser. The book is optional but highly recommended.

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636   Topics in Differential Geometry: Burns   TTh 11:30am-1:00pm
           Hodge Theory and Geometry

Prerequisites: *Basic differential geometry, complex analysis. Some algebraic geometry would be very useful.*

Course Description: We will focus on two standard problems relating differential geometric techniques and algebraic geometry: the Hodge problem and the Torelli problem. Both conjecture the existence of algebraic sub-varieties of certain complex submanifolds of projective space under topological and analytical conditions. Both are known to be false in complete generality, but there appears to be a very interesting kernel of truth in what they suggest. This term we will study: (0.) review of basics; (1.) the parameter spaces of Hodge structures and their differential and geometric properties; (2.) the invariants associated to cycles in projective manifolds; (3.) the topology of smooth projective manifolds, and auxiliary

varieties. We will discuss many of the known examples and counter-examples to these two main problems, and specific partial open problems.

Every effort will be made to keep the course accessible to anybody who is interested in learning about the subject.

Text: Suggested books: there are several very good textbooks on these topics. The basic reference for the course will be Claire Voisin, "Hodge Theory and Complex Algebraic Geometry, vols. I & II". There is also the new several-author review/textbook, "Hodge Theory", edited by Cattani, El Zein, Griffiths and Le.

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637   Lie Groups   Prasad   TTh 11:30 -1:00pm
Prerequisites: linear algebra and differential topology.

Course Description: This is a basic introduction to Lie groups. We will begin with the definition of Lie groups, and give many examples. We will define the Lie algebra of a Lie group, and connect them via left-invariant vector fields and the exponential map (prior knowledge of Lie algebras is not assumed). We will establish some structure theorems in Lie theory.

Towards the end of the course we will discuss representation theory, with an emphasis towards representations of compact groups.

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651   Topics in Applied Mathematics: Schotland   MW 4:00-5:30pm
        Waves and Imaging in Random Media
Prerequisites: Basic partial differential equations; some knowledge of probability theory would be useful, but not essential.

Course Description: This is a special topics course. The focus is on the theory of wave propagation in inhomogeneous media in various asymptotic regimes including: (i) geometrical optics of high-frequency waves (ii) homogenization of low-frequency waves in periodic and random media (iii) radiative transport and diffusion theory for high-frequency waves in low-frequency random media. Applications to inverse problems in imaging will be considered. The necessary tools from asymptotic analysis, scattering theory and probability will be developed as needed. The course is meant to be accessible to graduate students in mathematics, physics and engineering.

Course Outline:

Week 1 – wave propagation

Weeks 2,3 – scattering

Weeks 4,5 – homogenization

Weeks 6,7 – radiative transport

Weeks 8,9 – speckle

Weeks 10–12 – applications to imaging

Homework:

Problem sets will be assigned every 2–3 weeks. Students may work together on the problems. However, the solutions must be written independently.

Final Grade:

Will be determined from an average of the scores on the problem sets.

Text: Lecture notes will be distributed.

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657   Nonlinear partial differential equations   Wu   TTh 2:30-4pm
Prerequisites: *Math 656, Math 596, 597, Math 556 or 602, or permission of the instructor*

Course Description: References: L. C. Evans: Partial Differential Equations. J. Smoller: Shock waves and reaction-diffusion equations, Whitham: Linear and nonlinear waves, C. Sogge: Lectures on non-linear wave equations Partial Differential Equations are mathematical structures for models in science and technology. It is of fundamental importance in physics, biology and engineering design with connections to analysis, geometry, probability and many other subjects. The goal of this course is to introduce students (both pure and applied) to the basic concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations. Topics covered include hyperbolic waves, dispersive waves, second order elliptic equations, and parabolic equations.

Course material will be taken from the references.

Course work: Attend lectures and complete several problem sets.

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669   Topics in Combinatorial Theory: Stembridge   MWF 11:00am-12:00pm
        Coxeter Groups and Root Systems
Prerequisites: Alpha-level algebra.

Course Description: This course will be a survey of Coxeter groups and root systems, with particular emphasis on the combinatorial aspects of the subject. We will not follow any text, although useful references include:

"Groupes et Algebres de Lie", Chp. IV-VI, by N. Bourbaki,

"Reflection Groups and Coxeter Groups", by J. E. Humphreys,

"Combinatorics of Coxeter groups", by A. Bjorner and F. Brenti.

We plan to place special emphasis on two subtopics:

(1) the theory of crystal graphs and Weyl characters, and

(2) the theory and applications of affine reflection groups.

Additional topics (depending on time available and level of interest) may include the Bruhat and Cayley orders; the Coxeter complex and Steinberg torus; and polynomial invariants of reflection groups.

Propaganda: There is an extraordinarily broad range of mathematics that comes into contact with the geometry and combinatorics of root systems and Coxeter groups. Examples include (semisimple) Lie groups and their representation theory, cluster algebras, the geometry of flag manifolds and their generalizations, and the theory of hyperplane arrangements. If you have ever been intrigued by classification theorems in which the objects are labeled by Dynkin diagrams, this is the course for you.

Text: None.

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677   Topics in Galois Theory   Zieve   TTh 10:00-11:30am

Prerequisites: Solid foundation in algebra and some exposure to Riemann surfaces and fundamental groups.

Course Description: This course addresses advanced Galois theory. Topics covered include Hilbert's irreducibility theorem and its modern refinements, Riemann's existence theorem, the rigidity method, and Galois extensions produced via geometric methods. We will also present the needed group-theoretic background, so that students will learn how to apply the powerful results of modern group theory to solve problems in other subjects.

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696   Algebra Topol II: Kriz   MWF 1:00-2:00pm
Homotopical Methods in Algebraic Geometry

Prerequisites: Some familiarity with algebraic varieties (such as Math 631) will be assumed. Schemes are not required. Some familiarity with algebraic topology (such as 695) will also be assumed.

Course Description: In this course, we will study results in algebraic and arithmetic geometry which use, in a substantial way, methods of homotopy theory. In reference to an original idea of Grothendieck, people sometimes speak of 'motivic homotopy theory'. A classical result in this direction is Deligne's development of etale cohomology and his solution to the Weil conjectures. A more recent development is Voevodsky's proof of the Milnor and Bloch-Kato conjectures. We will study these results, and compare the methods of Deligne and Voevodsky from the point of view of homotopy theory. There will be no exams, there will be some HW exercises.

Text: none, some material may be distributed in class.

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710   Topics in Modern Analysis II: Baik   TTh 1:00-2:30pm
Random Matrix Theory

Course Description: In random matrix theory we are interested in the behavior of the eigenvalues of random matrices, especially when the size of the matrix is large. Since the eigenvalues are complicated functions of the entries of a matrix, the eigenvalues are not necessarily independent even if the entries of a matrix are independent random variables. Instead the eigenvalues of random matrices are strongly correlated. In fact they repel each other and one can evaluate the strength of the repulsion explicitly. This correlation between

the eigenvalues also turn out to describe the interactions among the particles from a wide class of complicated models in mathematics and other sciences.

The study of random matrices has a long history and has connections with diverse areas of mathematics and science. Sometimes such connections are not so clear as in the examples of the distribution of the non-trivial zeros of the Riemann-zeta function, the arctic boundary of random tiling, and the distribution of randomly growing interface. In other times, large random matrices arise naturally: for example, a collection of multi-dimensional data in statistics constitutes a random matrix. Large random matrices also model more complicated systems such as the statistical behavior of the energy levels of heavy atoms in physics. Some of the other areas that have something to do with random matrices include enumeration of maps, random permutations, quantum chaos, Laplace growth, and information theory, to name a few.

In this course we will discuss some of the fundamental methods and ideas of random matrix theory. A special attention is given to the random matrix models (and related models) whose statistics can be evaluated ``exactly'' by using various classical analysis method. The idea of the universality is also emphasized. Along the way we discuss various topics in analysis such as potential theory of electric charges, orthogonal polynomials, method of steepest-descent, Fredholm determinants, and Riemann-Hilbert problems.

The course is aimed at the second-year level graduate students. The students are assumed to be familiar with complex analysis (there will be plenty residue calculations), linear algebra and basic probability. Some familiarity with functional analysis is also useful. If you are sure of your background, please contact the instructor. There will be lecture notes. No particular textbook is required.

Note: This course is independent from 709, Topics in Modern Analysis I. The courses, MATH 709 and 710 are topics courses and they run independently from each other. The students do not need to have taken 709 before taking 710.

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715   Adv Topics in Algebra: Lam   TTh 1:00-2:30pm
         Quantum Groups and Crystal Bases

Prerequisites: I will assume that students are familiar with the structure theory and representation theory of complex semisimple Lie algebras.

Course Description: This course will be an introduction to the theory of quantum groups, canonical bases, and crystal graphs.

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Quantum groups, or quantized enveloping algebras, are Hopf algebras that deform the universal enveloping algebra of a complex semisimple Lie algebra. In the first half of the course I will discuss the definition and representation theory of quantum groups.

In the second half of the course, I will discuss the "canonical bases" or "global bases". These are distinguished bases for irreducible representations of quantum groups, or for the upper-half of the quantized enveloping algebra. At q=0, we obtain crystal graphs, which are a combinatorial model for irreducible representations of complex semisimple algebras.

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732   Topics in Algebraic Geometry: Fulton   TTh 11:30am- 1:00pm
Equivariant Cohomology in Algebraic Geometry

Prerequisites: The prerequisites are the alpha courses in topology and algebra, and a first course in algebraic geometry (Math 631). Some basic material about the topology of algebraic varieties, and basic facts from representation theory and symmetric functions, will be reviewed in lectures or presented by interested students.

Course Description: Varieties with group actions have an enriched cohomology that takes the group action into account. In the best situations, one can hope to calculate cohomology and other topological invariants by analyzing the geometry around the fixed points. This course will concentrate on varieties likeGrassmannians, flag varieties, and other homogeneous varieties, aswell as toric varieties, for which this approach has been particularly successful. There is a rich interplay among algebraic geometry, topology, representation theory, and combinatorics, making the course of interest to students in any of these fields.

Text: none

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776   Topics in Algebraic Number Theory: Snowden   TTh 2:30- 4:00pm
Class Field Theory
Prerequisites: Background in algebraic number theory, at the level of Math 676.

Course Description: The absolute Galois group of a number field is an extremely important and equally complicated object. The purpose of class field theory is to describe a piece of it: the maximal abelian quotient. This turns out to be connected to a number of other topics such as quadratic reciprocity and its generalizations, division algebras, and Galois cohomology. This course will prove the main theorems of class field theory (both local and global) and discuss some of these connections.

Text: No textbook required.