Department of Mathematics
Fall 2013 Graduate Course Descriptions

501 AIM Student Seminar Vershynin F 12-1:00pm & 3-4pm

Prerequisites: *You must be a graduate student in the AIM program to register for this course.*

Math 501 is a required course for all students enrolled in the Applied and Interdisciplinary Mathematics (AIM) MS and PhD graduate programs. In the Winter term, all first-year AIM students from both programs must sign up for this course. Due to the highly specialized content of the course, enrollment is available only for students in an AIM degree program.

The purpose of Math 501 is to address specific issues related to the process of studying applied mathematics in the AIM program and becoming an active member of the research community. The weekly meetings of the class will be divided among three types of sessions:

1. "Focus on. . . " presentations. These are presentations on various topics, some of immediate practical significance for students and others of a further-reaching nature. These discussions will include aspects of scholarly writing, research, and career development.

2. AIM Faculty Portraits. These are short presentations by faculty members in the Mathematics Department and other partner disciplines who are potential advisors or committee members for AIM students. The AIM faculty portraits provide a direct channel for students to discover what research is being done in various areas by current faculty, and to see what kind of preparation is required for participating in such research.

3. AIM Research Seminar Warm-up talks. One of the course requirements for Math 501 is weekly attendance of the AIM Research Seminar that takes place from 3-4 PM each Friday. The warm-up talks are presentations during the regular course meeting time by particularly dynamic speakers slated to speak in the AIM Research Seminar later the same day as a way to provide background material with the goal of making the AIM Research Seminar lecture more valuable for students.

Weekly attendance both of the course meeting and also of the AIM Research Seminar is required for Math 501. If you are registering for Math 501 you must be available both during the regular class time of 12-1pm on Fridays as well as during the AIM Research Seminar which runs 3-4pm on Fridays. If you are teaching, you should keep both of these obligations in mind when you submit your class/seminar schedule prior to obtaining a teaching assignment. Other requirements, including possible assignments related to topics discussed in the lectures, will be announced by the instructor in class.

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520 Life Contingencies 1 Marker TTh 8:30-10:00 & 10:00 - 11:30

Prerequisites: *Math 424 and 425 or permission from the instructor.*

Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of this course is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. In addition to actuarial students, this course is appropriate for anyone interested in mathematical modeling outside of the physical sciences.

The main topics are the development of (1) probability distributions for the future lifetime random variable; (2) probabilistic methods for financial payments on death or survival; 6and (3) mathematical models of actuarial reserving.

TEXT: Actuarial Mathematics (Second Edition) 2nd Bowers, Gerber, Hickman, Jones and Nesbitt (Society of Actuaries), Required

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523 Risk Theory Young TTh 8:30-10:00am

Prerequisites: *Math 425*

This course focuses on topics covered by professional examinations C (Society of Actuaries and 4 (Casualty Actuarial Society. Topics include: models of loss frequency and severity; mixture models; compound frequency models; aggregate loss models; incorporating deductibles, limits and co-insurance; parameter estimation; model selection criteria; continuous time ruin models; Brownian motion-based ruin models in insurance and finance; credibility theory. There will be an emphasis on applications and computer-based implementation and assignments.

TEXT: Loss Models: From Data to Decisions, Klugman, Panjer, and Willmot, 4th ed, 9781118315323, Required

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525 Probability Theory Yu MWF 8:30am - 9:30am

Prerequisites: *MATH 451, MATH 425* Vershynin TTh10-11:30am & 11:30am-1:00pm

The aim of this course is to provide mathematically rigorous basics of probability and some useful results on the subject. Some applications will be given through examples and exercises. There is substantial overlap with MATH 425 (Intro. to Probability), but here more sophisticated mathematical tools are used and there is greater emphasis on proofs of major results. MATH 451 is preferable to MATH 450 as preparation, but either is acceptable. This course is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program.

TEXT:
Probability and random processes 3rd Geoffrey R. Grimmett and David R. Stirzaker 9780123756862 Required
Introduction to probability models 10th Sheldon M. Ross 9780123756862 Optional

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526 Stochastic Processes Nadtochiy TT 8:30am-10am

Prerequisites: *Required: Math 525 or basic probability theory including: probability measures, Random variables, expectations, conditional probabilities and independence.*
Recommended: Good understanding of advanced calculus covering limits, series, the notions of continuity, differentiation and integration; interchanging the limit and integration/expectation (monotone and dominated convergence theorems); linear algebra, including matrices, eigenvalues and eigenfunctions.

The covers both discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include: Markov chains (Markov property, recurrency and transiency, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times, Markov queues); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales); time-permitting, introduction to stochastic integration and Ito's formula. Significant applications will be an important feature of the course.

TEXT: Essentials of Stochastic Processes, 2nd ed. (R. Durrett) 2 R. Durrett Required
Introduction to Stochastic Processes (E. Cinlar) E. Cinlar Optional

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550 Adaptive Dynamics: Dynamic Population Models — Simon — MW 2:30pm-4:00pm

Prerequisites: *Four semesters calculus or permission of instructor*

Overview: In Fall 2013, Math 550/CSC 510 will present a systems-based and in-depth examination of the mathematical foundations of dynamic population growth. Since change, resilience, evolution, optimality, and trade-offs are central issues in population growth, focal TOPICS of this course will be the theory and applications of dynamical systems and optimal control – single and multiple dimensional, deterministic and stochastic, simple, complex and chaotic.

Single Species Models
Difference Equations (one variable)
...Linear, nonlinear, logistic
...Stability of steady states
...Harvesting
...Delays
Differential Equations (one variable)
...Linear, nonlinear, logistic, solvable
...Geometric solutions, phase diagrams
...Stability of steady states
...Harvesting
...Stochastic birth and death processes
Special topics
...Bifurcation,chaos, Branching processes, Optimal Control I
Multiple Species Models
...Review of matrices and eigenvalues
...Linear systems, Nonlinear systems
...Stability of steady states
...Competing species
...Predator-Prey models
...Epidemiology models
...Liapunov functions and first integrals
...Optimal control II
Spatially structured models
...Partial differential equations (PDEs), Diffusion
Age or Sex structured models
...Discrete time: Leslie matrices

...Continuous time: McKendrick-von Foerster PDE

TEXT: Elements of Mathematical Ecology 2001, paper Mark Kot 0 521 00150 Required

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555 Intro to Complex Variables Rauch TTh 1-2:30pm

Prerequisites: *Math 450 or 451*

This course is an introduction to the theory of complex valued functions of a complex variable with substantial attention to applications in science and engineering. Concepts, calculations, and the ability to apply princip les to physical problems are emphasized over proofs, but arguments are rigorous. The prerequisite of a course in advanced calculus is essential. This course is a core course for the Applied and Intersciplinary Mathematics (AIM) graduate program

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556 Applied Functional Analysis Gilbert TTh 2:30 - 4:00pm

Prerequisites: *undergraduate analysis, linear algebra and complex variables. Some exposure to partial differential equations is desirable but not essential.*

This course provides an introduction to topics in functional analysis that are used in the analysis of differential equations. We will cover the following topics: metric and normed linear spaces, Banach spaces and the contraction mapping theorem, Hilbert spaces and spectral theory of compact operators, distributions and Fourier transforms, Sobolev spaces and applications.

TEXT: Applied Analysis, Hunter and Nachtergaele, Optional

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558 Applied Nonlinear Dynamics Rauch TTh 10-11:30

Prerequisites: *Math 451*

This course is an introduction to dynamical systems (differential equations and iterated maps). The aim is to survey a broad range of topics in the theory of dynamical systems with emphasis on techniques and results that are useful in applications. Chaotic dynamics will be discussed. This course is a core course for the Applied and Intersciplinary Mathematics (AIM) graduate program.

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565 Combinatorics and Graph Theory Speyer TuTh 11:30 AM - 1 PM

Prerequisites: *Linear algebra and previous exposure to rigorous mathematics with proofs.*

We will study various aspects of graph theory, including the enumerations of various graph theoretic objects, algorithms used for optimization on graphs, the structure and geometry of planar graphs, and the uses of matroid theory within graph theory. We will try to emphasize examples which are useful in computer science, or which have connections to other areas of mathematics.

TEXT: Introduction to Graph Theory 2, West, 0-13-014400-2, Required

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571 Numerical Methods for Scientific Computing I — Veerapaneni — TTh 8:30-10:00am

Prerequisites: *a course in linear algebra (e.g. Math 217, 417, 419, 513, or equivalent); computing will be required on some homework - Matlab is recommended.*

Math 571 is an introduction to numerical linear algebra, a core subject in scientific computing. The following topics will be covered: (1) Direct solvers for linear system of equations - Gaussian elimination, LU factorization, pivoting, Cholesky factorization; (2) Iterative solvers - Krylov subspace methods, Arnoldi iteration, GMRES, conjugate gradient method, preconditioning; (3) Computing eigenvalues and eigenvectors of a matrix – power method, inverse iteration, shifts, Rayleigh quotient iteration, QR algorithm, SVD; (4) Least squares problems, QR factorization, normal equations, Gram-Schmidt orthogonalization and Householder triangularization. Time permitting, multi-resolution techniques and modern preconditioning strategies will be reviewed. Throughout the course, applications in science and engineering will be discussed. homework = 30%, midterm exam = 30%, final exam = 40%

TEXT: Numerical Linear Algebra, L.N. Trefethen and D. Bau, optional

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575 Introduction to Number Theory — Lagarias — MWF 10:00am-11:00am

Prerequisites: *Students should be familiar with groups, rings and fields to the level of Math 412. They should be comfortable with writing proofs, at the level of Math 451. Proofs are emphasized but are often pleasantly short.*

Number theory has long been admired for its beauty and elegance, and for its rich legacy of fundamental unsolved problems in mathematics. It has recently turned out to have many applications in coding theory and cryptography. This is a first course in number theory-also

called the higher arithmetic. It is faster paced than Math 475.
Topics covered will include:
divisibility and prime numbers, factorization and primality testing, congruences, public key cryptography (RSA), p-adic numbers, arithmetic functions and distribution of prime, quadratic reciprocity and binary quadratic forms. It will include special topics as time permits. These could include: continued fractions Diophantine equations, solutions of equations over finite fields, partitions.

TEXT:
An Introduction to the theory of numbers, Fifth Edition, 1991. I. Niven, H. S. Zuckerman, H. L. Montgomery, Required

A Classical Introduction to Modern Number Theory Second Edition, 1990 K. Ireland and M. Rosen

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591 General and Differential Topology Scott MWF10:00 - 11:00am

Prerequisites: *Math 451*

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

The course will cover the following topics:

General topology: topological and metric spaces, continuity, subspaces, products and quotient topology, compactness and connectedness, extension theorems, topological groups, topological manifolds.

Differential topology: smooth manifolds, tangent spaces, vector fields, submanifolds, inverse function theorem, immersions, submersions, partitions of unity, Sard's theorem, embedding theorems, transversality, classification of surfaces.

TEXT:
Topology Second edition Munkres, James R. 0-13-181629-2, Required
Differential Topology, Victor Guillemin and Alan Pollack , Required

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593 Algebra I — Derksen — TTh 8:30-10:00 am

This course covers part of the required topics for the QR algebra exam for graduate students. The following areas will be covered: rings and their properties, ideals and quotient rings, the Chinese Remainder Theorem, Euclidean domains, principal ideal domains, unique factorization domains, polynomial rings, irreducibility criteria for polynomials, modules over rings, tensor products and principal ideal domains, tensor products of modules and vector spaces, exterior powers of vector spaces, the rational canonical form and the Jordan form of a matrix, real symmetric bilinear forms and their signature.

TEXT: Abstract Algebra, 3rd Ed, Dummit, Foote, 978-0-471-43334-7, Required

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596 Analysis I (Complex) — Burns — MWF 9-10:00 am

This is a theoretical and rigorous introductory course for complex analysis for beginning graduate students. Advanced undergraduate students may also take this course. We will discuss homomorphic functions, Cauchy's theorem, power series, meromorphic functions, analytic continuation, conformal mappings, Gamma and zeta functions, and elliptic functions. Time permitting, an introduction to some aspect of current research in the area.

Complex Analysis (Princeton Lectures in Analysis), Elias Stein and Rami Shakarchi, 978-0691113852, Optional

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602 Real Analysis II — Wu — MW 2:30-4:00 pm

Prerequisites: *Math 451, 419 or 513, Math 590 and Math 597*

Functional analysis is a core subject in mathematics. It has connections to probability and geometry, and is of fundamental importance to the development of analysis, differential equations, quantum mechanics and many other branches in mathematics, physics, engineering and theoretical computer science. The goal of this course is to introduce students to the basic concepts, methods and results in functional analysis. Topics to be covered include linear spaces, normed linear spaces, Banach spaces, Hilbert spaces, linear operators, dual operators, the Riesz representation theorem, the Hahn-Banach theorem, uniform boundedness theorem, open mapping theorem, closed graph theorem, compact operators, Fredholm Theory, reflexive Banach spaces, weak and weak* topologies, spectral theory, and applications to classical analysis and partial differential equations.

TEXT:
Functional Analysis (Pure and Applied Mathematics: A Wiley Series of Texts, Monographs and Tracts), 1st ed., Peter Lax, 0-471-55604-1, Required
Principles of functional analysis 2nd ed. Martin Schechter, Optional

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612 Lie Algebras Prasad Tu-Th 11:30am-1 pm

Prerequisites: *A course on Linear Algebra and matrix theory*

Math 612 is a detailed introduction to the theory of finite dimensional Lie algebras. The contents of the book on Lie Algebras by J.E. Humphreys will be covered. This course should be useful for students interested in learning Lie group theory, algebraic groups/algebraic geometry and combinatorics of root systems and Weyl groups.

TEXT: Lie Algebras Graduate Texts in Mathematics, Springer-Verlag J.E.Humphreys Required

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623 Computational Finance Conlon TuTh 10-11.30am, 1.00pm-2.30pm

Prerequisites: *Differential equations (e.g. Math 316); basic probability theory (e.g. Math 425, Stat 515); numerical analysis (Math 471); mathematical finance (Math 423 and Math 542/IOE 552 or permission from instructor); programming (e.g. C, Matlab, Mathematica, Java).*

This class is a course on mathematical finance with an emphasis on numerical and statistical methods. It is assumed that the student is familiar with basic theory of arbitrage pricing of equity and fixed income (interest rate) derivatives in discrete and continuous time. The course will focus on numerical implementations of these models as well as statistical methods for calibration, i.e. obtaining the parameters of the models. Specific topics include finite-difference methods, trees and lattices and Monte Carlo simulations with extensions.

TEXT:
The Mathematics of Financial Derivatives Cambridge, 1995 Wilmott, Howison, Dewynne 0-521-49789 Optional
Monte Carlo Methods in Finance Wiley, 2002 Jaeckel 0 Optional

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625 Probability with Martingales — Bayraktar — TTh 10-11:30

Prerequisites: *A strong background in Analysis.*

A graduate level course on probability theory and the theory of martingales in discrete time.

Topics include measure theory and integration; characteristic functions; convergence concepts; limit theorems; conditional expectation; martingales (uniform integrability, martingale convergence theorems, optional sampling theorem). Time permitting we will also cover some important stochastic analysis results

TEXT: Probability theory with martingales, David Williams, 521406056, Required

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631 Algebraic Geometry I — Smith — MWF 1-2:00pm

Prerequisites: *593, 594, 596, 591, 592*

This course will be a basic and broad introduction to the classical algebraic geometry of affine and projective varieties over an algebraically closed field, suitable for students who have completed the alpha courses.

Algebraic geometry is one of the oldest and most central branches of mathematics, so just about any mathematician will benefit from knowing its basics. Math 631 is an essential course for students planning to specialize in number theory, commutative or non-commutative algebra, representation theory, complex geometry/analysis, combinatorics (of the sort done at Michigan), or of course, algebraic geometry.

Topics covered will include the basic properties of affine and projective varieties over an algebraically closed field: the Zariski topology, Hilbert's Nullstellensatz, dimension, the Zariski tangent space, smoothness and singularities, the sheaf of regular functions, the local ring of a point, the notion of degree (of a projective variety or map), the Hilbert polynomial, regular and rational maps, maps to projective space, divisors, differential forms, the canonical class, and the Reimann-Roch theorem for curves. Considerable emphasis will be on the many rich examples of classical algebraic geometry: Veronese and Segre maps, Grassmannians, quadrics, determinantal varieties. We'll also talk a bit about varieties as "functors of points" and see degenerations of varieties into "schemes", hoping to develop some intuition for these ideas without treating them in full technical glory (which would require more commutative algebra prerequisites and will be the subject of Math 632.)

This will be seriously graded course, with weekly homework, occasional quizzes, and a final project. Ideal prerequisites include the alpha course Math 593, 594, 596, 591, and Math 614, the latter of which may be taken simultaneously.

The closest book to a text will be Shafarevich's "Basic Algebraic Geometry I," though we'll draw lots of examples from Harris's "Algebraic Geometry: A first course." Of course, most of the topics we discuss can also be found in Hartshorne's classic text on Algebraic Geometry. (All three are published by Springer.)

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636 Topics in Differential Geometry Spatzier MWF 12-1:00pm

Prerequisites: *Mostly standard material from alpha courses*

This class will introduce you to exciting developments in recent years that tie group theory, geometry and dynamics together. Let me give you a few examples.

Groups and geometry have a long and fruitful relationships, especially via the group of symmetries of a geometry. In recent years some new aspects have emerged. For example,

given group G with a finite generation set, one defines the length of an element g is the length of a shortest word in the generators expressing g. This defines the word metric. Remarkably the metric geometry of G determines many group theoretic properties of G. For example, if the number of elements in a ball of radius r is bounded by a polynomial then G contains a nilpotent subgroup of finite index. This is Gromov's famous polynomial growth theorem. Sometimes even the group is determined, for example for SL(n,Z) for n at least 3, a result by Eskin.

There are also close relationships to dynamics. Ergodicity and mixing lead to counting results, e.g. for the number of closed geodesics of manifolds in terms of entropy of the geodesic flow. Using mixing of group actions one can get estimates on the number of integer points on a suitable variety. In turn this is related to representation theoretic properties such as spectral gap. This leads to the Kazhdan property which has had many applications, e.g. to the construction of expander graphs. There are also relations to random walks on groups, Lyapunov exponents in dynamics.

Thin groups are most recent , and present fascinating properties algebraically, dynamically and in applications to number theory such as Apollonian circle packing. I expect to discuss some aspects of this theory.

I will develop the needed background material in class, sometimes as black boxes. No specialized knowledge from geometry or dynamics will be needed. For the most part, this class should be accessible to anyone with a solid understanding of algebra and analysis.

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651 Topics in applied math Borcea TTh 1-2:30pm

This is a special topics course. The focus is on the mathematical analysis of wave propagation and sensor array imaging in random media. In many applications such as nondestructive testing of materials, underwater acoustics, ground penetrating radar, seismic exploration, etc., the medium through which the waves propagate contains numerous small inhomogeneities that cannot possibly be known in detail, and are not even interesting in imaging. It is the

uncertainty about them that we model with random processes, and thus speak of wave propagation and imaging in random media. The course uses tools from linear partial differential equations, asymptotic methods
and stochastic analysis to give a self-contained introduction to the field. We begin with a general description of imaging with sensor arrays. Then, we focus attention on the analysis of the wave equation with random wave speed in waveguides. The application to waveguides allows a rigorous and explicit description of the wave field, which gives physical insight on the cumulative long range wave scattering effects of the random inhomogeneities. We show how to use it for designing novel and robust imaging methods.

Textbook: Lecture notes will be distributed.
Course outline:
Weeks 1-2: Basic wave propagation and introduction to sensor array imaging.
Weeks 3-4: Random media and impediments to coherent imaging.
Weeks 5-7: Theory of wave propagation in random waveguides with bounded cross-section.
Weeks 8-9: Imaging in random waveguides with bounded cross-section.
Weeks 10-11: Theory of wave propagation in random waveguides with unbounded cross-section.
Weeks 12-13: Coherent interferometric imaging in random waveguides with unbounded cross-section.

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656 Introduction to Partial Differential Equations Smoller TTh 8:30-10am

Prerequisites: *One graduate level in analysis, or the equivalent.*

This will be an introductory course in Partial Differential Equations.

Topics covered will include: Characteristics, wave equations, energy integrals, Holmgren's uniqueness theorem, initial-value problem for hyperbolic equations, weak solutions and the theory of distributions, second order elliptic equations, second order parabolic equations, introduction to shock waves.

The grading will be done on the basis of some homework assignments.

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665 Combinatorial Theory Lam MWF 11am-12pm

Prerequisites: *A solid background in linear algebra is necessary. Some experience with representation theory, especially that of complex simple Lie algebras will be helpful.*

A matrix with real entries is totally positive if all its minors are positive. This notion has its roots in analysis, but in the last 20+ years there has been an explosion of interest in this notion from the algebraic and combinatorial point of view.

I plan to develop the theory of total positivity from a combinatorial point of view, and discuss applications/relations to representation theory, such as: representation theory of the infinite symmetric group, the parametrization of crystal and canonical bases, and cluster algebras and coordinate rings of Bruhat cells and their relatives.

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671 Topics in Scientific Computing Viswanath TTh 1:00-2:30 pm

Prerequisites: *The prerequisites include knowledge of numerical methods at the level of Math 471 or higher, and some exposure to Fortran or C or C++. Background in mathematics or computer science beyond these prerequisites is not needed and will not be assumed. The grade for the class will be based on six homework assignments. There will be no exams nor is there a textbook for the class. Notes will be handed out for all the lectures. Feel free to email for more details.*

As computing hardware has improved remarkably over the past two decades, the need to understand scientific algorithms, computer architecture, and scientific software in a unified way has increased. The purpose of this class is to teach each of these elements but most of all to emphasize the interaction between them.

The class will be a mixture of basic mathematical ideas and issues that arise at the interface of science with high performance computing. It will consist of segments devoted first to compilation units, linking and naming conventions, use of makefiles, mixed language programming (Fortran/C/C++), and the use of scientific libraries (BLAS, LAPACK, MKL, FFTW), and then to computer architecture including basic 64 bit x86 assembly programming, x86 register architecture, compiler optimizations, memory hierarchy (with an emphasis on shared memory programming using OpenMP), networks (with an emphasis on MPI), and GPU architecture. The Intel Phi is just becoming available and is almost certain to be a major step forward in scientific computing. The class will most probably include a thorough discussion of the Intel Phi.

Overall the goal of this class is to introduce scientific computing with an orientation towards high performance computing and the creation of substantial scientific codes. Our aim is not to exhaustively detail the syntax of OpenMP and MPI, but to understand what happens beneath the hood and what sort of performance bottlenecks to expect, and to do so in the context of scientific problems. And a lot happens beneath the hood: a finite difference code, which is written carefully but without expert knowledge, may not even approach 2% of the peak performance or an MPI user unaware of the nature of network architecture may use blocking calls and lose 50% of the network bandwidth right away. Knowledge of register architecture is indispensable to GPU programming.

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675 Analytic Theory of Numbers Vaughan MWF 2-3pm

Prerequisites: *575, 596.*

In the last few years there have been a number of sensational developments in prime number theory.

1. Goldston, Pintz and Yildirim have shown that there are relatively small gaps between consecutive primes.
2. Yitang Zhang has adapted their method to show that there are infinitely many pairs of primes p, p' with 0 < p-p'< 70 x 10^6.
3. Green and Tao have shown that there are arbitrarily long arithmetic progressions in the primes.
4. Helfgott has shown that every odd natural number bigger than 5 is the sum of three prime numbers (the Goldbach ternary problem).

The object of Math 675 is to give the necessary background from classical analytic number theory. In particular one of the central objects of study is the Riemann zeta function. Note that the Riemann hypothesis concerning its zeros is probably the most important unsolved question in mathematics.

Topics covered include:
Dirichlet Series
The prime number estimates of Chebyshev and of Mertens
Principles and First Examples of Sieve Methods
Primes in Arithmetic Progressions
The inverse Mellin transform
The Prime Number Theorem
A zero-free region
Numbers composed of small primes
Primitive Characters and Gauss Sums
Analytic Properties of the Zeta Function and L-Functions
Exceptional zeros
The prime number theorem for arithmetic progressions
Explicit Formulae.
Depending on time some of the following with be covered.
The approximate functional equation for L-functions.
Mean value theorems.
Distribution of zeros.
Zeros on the critical line.

Recommended text: H. L. Montgomery & R. C. Vaughan, Multiplicative Number Theory I. Classical Theory, Cambridge University Press, xii + 516pp, 2006.

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679 Elliptic Curves — Snowden — TTh 2:30 - 4:00pm

Prerequisites: *A good understanding of basic algebraic number theory and algebraic geometry will be very important. I'd recommend having some familiarity with modular forms and elliptic curves as well. The article of Diamond and Im offers a good overview.*

The study of torsion points on elliptic curves has been a central theme in arithmetic geometry in the last 50 years. A major result in this area was obtained by Mazur, who classified the possible orders of torsion points over the rational numbers. The main purpose of this course is to go through the proof of this theorem. Along the way, we will encounter many topics that occur throughout modern arithmetic geometry, such as Galois representations, Hecke algebras, modular forms, Neron models, group schemes, and moduli of elliptic curves. I will give some introduction to each of these, as needed.

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697 Advanced Algebraic Topology — Kriz — MWF 11-12

Prerequisites: *Math 695 or Math 592, or some knowledge of beginning to intermediate*

The purpose of this course is an introduction to topics which are of main-stream interest in algebraic topology today. Mostly, this means generalized homology and cohomology theories and their main geometric examples, K-theory and cobordism. We will touch on both foundations and calculations. Calculational techniques may include cohomological operations and spectral sequences.

This is an advanced graduate course, so there are no exams. Some homework will be assigned.

While there are no textbooks listed for this class, a reading list consisting of various texts will be discussed during the course, and some notes may be distributed in class. Also, notes from the lectures will be posted on the instructor's web page.

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711 Representation Theory of Reductive p-adic groups — DeBacker — MWF 11-noon

This course will serve as an introduction to the representation theory of reductive p-adic groups. The main goal of the course is to present the Bernstein decomposition of the category of smooth representations of a p-adic group. This is a very beautiful result which uses most of the basic ideas in this branch of representation theory. We shall begin the course by working out the representation theory of the (non-reductive) Heisenberg group and of GL_1. We shall then present some structure theory results (sticking to the example of GL_n). After this, we shall really start: we'll discuss the nature of supercuspidal representations, parabolic induction, Jacquet restriction, and the relations among these concepts. After presenting the Bernstein decomposition, we shall turn our attention to the Bernstein center, the Langlands classification, and various other topics, time permitting.

The course should be useful for anyone interested in representation theory or number theory. Moreover, it will be accessible to most everyone that has done well in algebra (though it might help to recall what exactly a p-adic field is -- I won't dwell on that for long).

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731 Spaces of arcs and singularities in birational geometry — Mustata — TTh 11:30-1:00

Prerequisites: *Math 631*

The space of arcs of an algebraic variety parametrizes formal arcs on the variety. It is typically an infinite-dimensional space (scheme), but can be studied through its finite-dimensional approximations, the spaces of jets. This space has attracted a lot of attention due to its role in the theory of motivic integration, due to Kontsevich, Batyrev, and Denef-Loeser. The first part of the course will give an introduction to the space of arcs, and to its connection to various topics, such as:

1. Motivic integration and its applications to Hodge numbers of birational Calabi-Yau varieties.

2. The McKay correspondence

3. Invariants of singularities of higher-dimensional algebraic varieties.

4. The Nash problem, relating arc spaces and resolutions of singularities

The second part of the course will be devoted to applications of singularities to global geometry. Iskovskikh and Manin proved that quartic 3-folds are non-rational, in spite of the fact that they can be unirational. The techniques they have introduced have been recast in the modern language of birational geometry by Corti, Pukhlikov, and others, and have been applied to many other examples. We will discuss both the general framework, and some important examples.