Fall 2024 Graduate Course Descriptions

MATH 501. AIM Student Seminar — **Alben, Silas** — **Fri 1:00 PM-2:00 PM**
Alben, Silas — **Fri 3:00 PM-4:00 PM**

At least two 300 or above level math courses, and Graduate standing; Qualified undergraduates with permission of instructor only. (1). May be repeated for a maximum of 6 credits. Offered mandatory credit/no credit.

MATH 501 is an introductory and overview seminar course in the methods and applications of modern mathematics. The seminar has two key components: (1) participation in the Applied and Interdisciplinary Math Research Seminar; and (2) preparatory and post-seminar discussions based on these presentations. Topics vary by term.

MATH 507. Mathematical Methods for Algorithmic Trading — **Ekren, Ibrahim** — **T/Th 11:30 AM-1:00 PM**

This is a graduate level course focused on using tools of stochastic optimal control for designing trading strategies. The aim is to teach the relevant techniques from Probability, Statistics, PDEs, and Optimization, as well as to introduce students to the wide range of specific problems and existing models related to price impact and algorithmic trading.

MATH 520. Life Contingencies I — **Kausch, David** — **T/Th 10:00 AM-11:30 AM**

MATH 424 and 425 with minimum grade of C-, plus declared Actuarial/Financial Mathematics Concentration. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of the Math 520-521 sequence is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. The main topics are (1) developing probability distributions for the future lifetime random variable, and (2) using those distributions to price life insurance and annuities.

Required Textbook: Actuarial Mathematics for Life Contingent Risks, by Dickson, David C.M./Hardy, Mary R./Waters, Howard R. ISBN-13: 978-1108478083

MATH 523. Loss Models I — **Natarajan, Roger** — **T/Th 8:30 AM-10:00 AM**

The goal of this course is to understand parametric distributions for the purposes of (1) modeling frequency, severity, and aggregate insurance losses, (2) analyzing the effects of insurance coverage modifications, and (3) simulating losses from those parametric distributions.

The immediate pre-req is Math 425, with a grade of C- or better. To take Math 524 next semester, you'll need this course and Stats 426, each with a grade of C- or better.

The required text for the course is Loss Models: From Data to Decisions by Stuart A. Klugman, Harry H. Panjer, and Gordon E. Willmot, fourth edition, published by John Wiley in 2012.

I also rely on An Introductory Guide in the Construction of Actuarial Models: A Preparation for the Actuarial Exam C/4, which I label CGuide, by Marcel B. Finan, published in 2017. The latter is available on our Canvas web page under Files.

MATH 525/STATS 525. Probability Theory — **TBD** — **T/Th 11:30 AM-1:00 PM**
TBD — **T/Th 1:30 PM-2:30 PM**

MATH 451 (strongly recommended). MATH 425/STATS 425 would be helpful. (3). (BS). May not be repeated for credit.

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

The following topics will be covered: sample space and events, random variables, concept and definition of probability and expectation, conditional probability and expectation, independence, moment generating functions, Law of large numbers, Central limit theorem, Markov chains, Poisson process and exponential distribution.

Required Textbook: Probability and Random Processes, by Grimmett, Geoffrey R. / Stirzaker, David R. (9780198572220) - 3RD 01

MATH 526/STATS 526. Discrete State Stochastic Processes **Cohen, Asaf** **T/Th 10:00 AM-11:30 AM**
Cohen, Asaf **T/Th 11:30 AM-1:00 PM**
TBD **T/Th 8:30 AM-10:00 AM**

MATH 525 or STATS 525 or EECS 501. (3). (BS). May not be repeated for credit.

This is a course on the theory and applications of stochastic processes on discrete state spaces. Some specific topics include:
(1) Markov chains – Markov property,– recurrence and transience, – stationarity, – ergodicity, – coupling, – exit probabilities and expected exit times;
(2) Markov decision processes – optimal control, – Banach fixed point theorem;
(3) Exponential distribution and Poisson processes – memoryless property,– thinning and superposition, – compound Poisson processes;
(4) Markov chains in continuous time – generators and Kolmogorov equations, – embedded Markov chains,– stationary distributions and limit theorems, – exit probabilities and expected exit times,– Markov queues;
(5) Martingales – conditional expectations, – gambling (trading) with martingales, – optional sampling,
– applications to the computation of exit probabilities and expected exit times, – martingale convergence.

Math 538. Lie Algebras **TBD** **W/F 11:30 AM-1:00 PM**

TBD

MATH 555. Introduction to Functions of a Complex Variable with Applications **Esedoglu, Selim** **T/Th 1:00 PM-2:30 PM**

MATH 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

This course is an introduction to the theory of functions of a complex variable. Applications to physical problems will be stressed throughout the course, but the presentation will be rigorous, and students will be expected not only to perform calculations but also to write proofs. Topics will include differentiation of functions of a complex variable, Cauchy-Riemann equations, contour integrals, Cauchy's theorem, Taylor and Laurent series, residue theory and its applications, conformal maps.

MATH 556. Applied Functional Analysis **Bieri, Lydia** **T/Th 10:00 AM-11:30 AM**

MATH 217, 419, or 420; MATH 451; and MATH 555. (3). (BS). May not be repeated for credit.

This is an introduction to methods of applied functional analysis. Students are expected to master both the proofs and applications of major results. The prerequisites include linear algebra, undergraduate analysis, advanced calculus and complex variables. This course is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program. This course focusses on topics in functional analysis that are used in the analysis of ordinary and partial differential equations. Metric and normed linear spaces, Banach spaces and the contraction mapping theorem, Hilbert spaces and spectral theory of compact operators, distributions and Fourier transforms, Sobolev spaces and applications to (elliptic) PDEs.

MATH 558. Applied Nonlinear Dynamics **Viswanath, Divakar** **T/Th 2:30 PM-4:00 PM**
Topic: Advanced Ordinary Differential Equations

MATH 216, 217, and 451/452. (3). (BS). May not be repeated for credit.

Differential equations model systems throughout science and engineering and display rich dynamical behavior. This course emphasizes the qualitative and geometric ideas which characterize the post Poincare era. The course surveys a broad range of topics with emphasis on techniques, and results that are useful in applications. It is intended for students in mathematics, engineering, and the natural sciences and is a core course for the Applied and Interdisciplinary Mathematics graduate program.

Course material will be taken from Chapters 1-10, and Chapter 15 of the text.

There will be weekly homeworks, midterm and final exams.

Required Textbook: M. Hirsh, S. Smale, and R. *Devaney, Differential Equations, Dynamical Systems, and an Introduction to Chaos*, 3rd ed., Elsevier.

MATH 559. Special Topics: Computational Symmetry in AI & Robotics | **Ghaffari, Maani** | **T 10:30 AM-1:30 PM**

MATH 217, 419, or 420; MATH 451; and MATH 555. (3). (BS). May not be repeated for credit.

Symmetry describes the intrinsic structure and properties of a subject. In this course, we will explore the symmetries in geometry and rigorously express them in mathematics, covering relevant topics in group theory, differential geometry, representation theory, and Lie groups. Then, we will use them as tools to achieve computationally efficient and generalizable algorithms for learning, perception, estimation, and control with applications in many domains, such as AI, computer vision, and robotics. The course will cover novel topics in geometric learning and explores state of the art in symmetry-preserving and geometric learning methods.

MATH 565. Combinatorics and Graph Theory | **TBD** | **T/Th 8:30 AM-10:00 AM**

TBD

MATH 568/BIOINF 568. Mathematical and Computational Neuroscience | **Booth, Victoria** | **M/W 10:00 AM-11:30 AM**

MATH 463 or 462 (for undergraduate students) or Graduate standing. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Computational neuroscience investigates the brain at many different levels, from single cell activity to small, local network computation to the dynamics of large neuronal populations. This course introduces modeling and quantitative techniques used to investigate neural activity at all these different levels. Topics to be covered include: Passive membrane properties, derivation of the Hodgkin-Huxley model for action potential generation, action potential propagation in cable and multi-compartmental models, reductions of the Hodgkin-Huxley model, phase plane analysis, linear stability and bifurcation analysis, synaptic currents, excitatory and inhibitory network dynamics, synaptic plasticity, firing rate models, neural coding.

MATH 571. Numerical Linear Algebra | **Esedoglu, Selim** | **T/Th 8:30 AM-10:00 AM**

MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit.

Numerical linear algebra is at the core of much of scientific computing, and is a fundamental skill for anyone with any interest in numerical/computational mathematics; the course is a core course for the AIM program. We will cover robust, accurate, and efficient methods for finding (1) solutions of linear systems of equations, (2) eigenvalues and eigenvectors of matrices, and (3) solution of least squares problems. These standard problems arise in all venues of science and engineering.

Topics will include: (1) Orthogonal matrices, vector and matrix norms, singular value decomposition (SVD). (2) QR factorization, Householder triangularization, least squares problems. (3) Stability: Condition numbers, floating point arithmetic, backward error analysis. (4) Direct methods: Gaussian elimination, pivoting, LU and Cholesky factorizations. (5) Eigenvalues and eigenvectors: Reduction to Hessenberg or tridiagonal form, Rayleigh quotient, inverse iteration, the QR algorithm, computing the SVD. (6) Iterative methods: Classical methods (Jacobi, Gauss-Seidel, SOR), Krylov subspace methods, conjugate gradients, Arnoldi iteration, GMRES, preconditioning.

MATH 573. Financial Mathematics I | **Kolliopoulos, Nikolaos** | **T/Th 1:00 PM-2:30 PM**

(3). (BS). May not be repeated for credit.

This is an introductory course in Financial Mathematics. This course starts with the basic version of the Mathematical Theory of Asset Pricing and Hedging (Fundamental Theorem of Asset Pricing in discrete time and discrete space). This theory is applied to problems of Pricing and Hedging of simple Financial Derivatives. Finally, the continuous-time version of the proposed methods is presented, culminating with the Black–Scholes model. A part of the course is devoted to the problems of Optimal Investment in discrete time (including Markowitz Theory and CAPM) and Risk Management (VaR and its extensions). This course shows how one can formulate and solve relevant problems of the financial industry via mathematical (in particular, probabilistic) methods. Although Math 526 is not a prerequisite for Math 573, it is strongly recommended that either these courses are taken in parallel, or Math 526 precedes Math 573.

Required Textbook: Stochastic Finance: An Introduction in Discrete Time; Hans Föllmer and Alexander Schied, 4th edition; 978-3110463446

Optional Textbook: Economics and Mathematics of Financial Markets by Jaksa Cvitanic and Fernando Zapatero

MATH 591. Differentiable Manifolds **Ji, Lizhen** **M/W/F 10:00 AM-11:00 AM**

MATH 451, 452 and 590. (3). (BS). May not be repeated for credit.

This is one of the basic courses for students beginning the PhD program in mathematics. The approach is rigorous and emphasizes abstract concepts and proofs.

Topics: Product and quotient topology, group actions, topological groups, topological manifolds, smooth manifolds, manifolds with boundary, smooth maps, partitions of unity, tangent vectors and differentials, the tangent bundle, submersions, immersions and embeddings, smooth submanifolds, Sard's Theorem, the Whitney Embedding Theorem, transversality, Lie groups, vector fields, Lie brackets, Lie algebras, multilinear algebra, vector bundles, differential forms, exterior derivatives, orientation, Stokes' Theorem, introduction to De Rham cohomology groups, homotopy invariance.

Optional Textbooks: Introduction to Smooth Manifolds(2nd edition), by John Lee; 978-1-4419-9981-8
An Introduction to Manifolds(2nd edition), by Loring W. Tu; 978-1-4419-7399-3

MATH 593. Algebra I **Speyer, David** **M/W/F 2:00 PM-3:00 PM**

MATH 412, 420, and 451 or MATH 494. (3). (BS). May not be repeated for credit.

Topics include basics about rings and modules, including Euclidean rings, PIDs, UFDs. The structure theory of modules over a PID will be an important topic, with applications to the classification of finite abelian groups and to Jordan and rational canonical forms of matrices. The course will also cover tensor, symmetric, and exterior algebras and bilinear forms.

MATH 596. Analysis I **Chelkak, Dmitry** **T/Th 2:30 PM-4:00 PM**
Topic: Complex Analysis

MATH 451. (3). (BS). May not be repeated for credit. Students with credit for MATH 555 may elect MATH 596 for two credits only.

This is a theoretical and rigorous introductory course on complex analysis on the level of the first year math graduate students. Highly advanced math undergraduate students and graduate students from other disciplines may also take this course but they should expect that the workload is heavy and the pace is fast. Topics to be discussed include holomorphic functions, Cauchy's theorem, Cauchy's integral formula, power series, isolated singularities, meromorphic functions, Laurent series, conformal mappings, infinite product, and so on.

Textbook: Complex Analysis, by Lars Ahlfors; 9781470467678

MATH 602. Real Analysis II **Rudelson, Mark** **M/W 1:00 PM-2:30 PM**
Topic: Functional Analysis

MATH 590 and 597. (3). (BS). May not be repeated for credit.

Functional analysis presents a unified approach to spaces of functions, such as Lp spaces, spaces of continuous and differentiable functions, etc., as linear spaces equipped with a topology. The results and methods of this theory are widely used in analysis, differential equations, probability, and mathematical physics. The topics include: Banach and Hilbert spaces; bounded linear functionals, Hahn-Banach theorem; the role of completeness: Baire category and Uniform boundedness; principle, closed graph and open mapping theorems; duality. Duals of classical Banach spaces; weak and weak* topologies, Banach-Alaoglu theorem; extremal points of convex sets, Krein-Milman theorem; compact sets and compact operators; Fredholm theory and index of a Fredholm operator; spectral theory of bounded self-adjoint operators in a Hilbert space (if time permits).

Optional Textbooks:
Functional Analysis, Peter D. Lax, ISBN-13: 978-0471556046, ISBN-10: 0471556041
Functional Analysis: An Introduction, by Yuli Eidelman, Vitali Milman, & Antonis Tsolomitis, ISBN-13: 978-0821836460, ISBN-10: 0821836463

Grading will be based on several sets of homework.

MATH 614. Commutative Algebra **Snowden, Andrew** **T/Th 10:00 AM-11:30 AM**

MATH 593 and Graduate standing. (3). (BS). May not be repeated for credit.

This course is a basic introduction to commutative algebra for students who have completed the alpha sequence in algebra. Topics covered will include localization of rings and modules, primary decomposition of ideals, dimension theory for rings, tensor products and flatness, and completion of rings. Topics important in number theory, including Dedekind domains and Hensel's Lemma, will be covered. We will also cover a number of topics essential in algebraic geometry, including the prime spectrum of a ring, Hilbert's nullstellensatz, and Noether normalization.

MATH 623/IOE 623. Computational Finance **He, Xihao** **T/Th 10:00 AM-11:30 AM**

MATH 316 and MATH 425 or 525. (3). (BS). May not be repeated for credit.

This is a course in computational methods in finance and financial modeling. Particular emphasis will be put on interest rate models and interest rate derivatives. The specific topics include; Black-Scholes theory, no arbitrage and complete markets theory, term structure models: Hull and White models and Heath Jarrow Morton models, the stochastic differential equations and martingale approach: multinomial tree and Monte Carlo methods, the partial differential equations approach: finite difference methods. Class Notes: Enrollment for this course is by waitlist only. Preference will be given to students in the Quantitative Finance & Risk Management program and Math PhD students. Prerequisites: Differential equations (e.g. Math 316); probability theory (e.g. Math 525/526, Stat 515); numerical analysis (Math 471 and Math 472); mathematical finance (Math 423 and Math 542/IOE 552, Math 506 or permission from instructor); programming (e.g. C, Matlab, Mathematica, Java).

MATH 625/STATS 625. Probability and Random Processes I **Bayraktar, Erhan** **T/Th 11:30 AM-1:00 PM**
Topic: Probability with Martingales

Math 597 or permission of instructor. (3). (BS). May not be repeated for credit.

Ph.D. level course on probability theory and the theory of martingales in discrete time. (Main examples will be drawn from mathematical finance, optimal stopping/control problems.) Topics covered include measure theory and integration; convex analysis on L0+; characteristic functions; convergence concepts; limit theorems; conditional expectation; martingales(uniform integrability, martingale convergence theorems, optional sampling theorem).

MATH 629. Machine Learning for Finance II **TBD** **F 1:00 PM- 2:30PM**

This is a graduate level course intended for students in the Master's Program in Quantitative Finance and Risk Management (Quant Program).

TBD

MATH 631. Introduction to Algebraic Geometry **Pixton, Aaron** **T/Th 11:30 AM-1:00 PM**

MATH 594 or permission of instructor. Graduate standing. Previous knowledge: General topology. Familiarity with the language of category theory. Commutative algebra is recommended but not essential; you should have a solid grasp of localizations (of rings/modules) and tensor products though. (3). (BS). May not be repeated for credit.

This is the first half of a year-long sequence in algebraic geometry. In the first semester, we will introduce the basic notions and objects of modern algebraic geometry - sheaves and schemes. We will be loosely following Ravi Vakil's notes "Foundations of Algebraic Geometry".

MATH 636. Topics in Differential Geometry **Spatzier, Ralf** **M/W/F 3:00 PM-4:00 PM**
Topic: Rigidity in Geometry and Dynamics

Math 592

We will discuss rigidity results in geometry and dynamics. Here is a typical problem: Can you "measure the shape of the drum", i.e. do the lengths of the closed geodesics in a compact Riemannian manifold determine the manifold up to isometry? For surfaces this turns out to be true (for lengths as a function on the fundamental group). It is conjectured to be true in any dimension in negative sectional curvature, and has been proved if one of the spaces is highly symmetric, e.g. if it has constant sectional curvature -1.

Closely related is the classical problem if one can "hear the shape of the drum" (still largely unresolved).

Other highlights are: Rigidity in representation theory: Mostow, Margulis, Prasad, etc.; Rank rigidity in nonpositive curvature: Ballman, Brin and Burns. Hamenstadt, Spatzier.; Hopf Conjecture - metrics without conjugate points on tori are flat: Burago-Ivanov; Entropy rigidity: Besson,Courtois, Gallot; Zimmer Program on Rigidity in homeomorphism and diffeomorphism groups; Rigidity in homogeneous dynamics: Measure rigidity, Ratner's Theorem and the Littlewood Conjecture.; Rigidity in positive curvature.

These have been highlights of research during the last five decades. Remarkably, both geometric and dynamical ideas play a crucial role, and are intricately woven together.

I will discuss some of these topics, and develop the necessary background materials from geometry and dynamics

Prerequisites: basic point set topology, manifold theory, real analysis and basic theory of groups and Lie groups.

MATH 656. Introduction to Partial and Differential Equations **Hani, Zaher** **M/W 2:30 PM-4:00 PM**

MATH 558, 596 and 597 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

Partial differential equations are at the core of models in science, engineering, economics, and related fields. It is a field of math that has been growing vigorously for over 400 years, while receiving a constant flux of new problems from inside and outside mathematics. This made PDE a field of immense breadth, diversity, and importance.
Attempts at classifying PDE into classes, and proving theorems applicable to all equations in each class have had limited success and utility. This is partly because of the immense diversity of PDE mentioned above, as well as the fact that each PDE often comes with its own character and complexities. As such, the modern study of PDE is more "equation specific". In this course, we will survey some of the powerful tools used to study some of the most important and representative equations. Each chapter below scratches the surface of a large field of mathematical research, and it is up to your interests to go deeper.

MATH 668. Advanced Combinatorics **Fomin, Sergey** **T/Th 1:00 PM-2:30 PM**
Topic: Coxeter groups and root systems

Coxeter groups and root systems are simple algebraic/geometric/combinatorial gadgets that arise in multiple mathematical contexts, including representation theory, invariant theory, algebraic geometry, and mathematical physics.

This course is a gentle introduction to the subject, emphasizing combinatorial aspects. No graduate-level prerequisites will be assumed.

The main structural results are two classifications: of finite Coxeter groups and of finite crystallographic root systems. Other potential topics include rings of invariants and coinvariants, combinatorics of reduced words, weak and Bruhat orders, permutohedra and associahedra, and more.

MATH 695. Algebraic Topology I **Kriz, Igor** **M/W/F 9:00 AM-10:00 AM**

MATH 591 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

The purpose of this course is to give an introduction to the methods of algebraic topology as used in multiple mathematical fields today. Most students in this course have seen introductory homology and the fundamental group theory on the level of, say, Math 592, but it is not absolutely required. In 695, we take a larger perspective. Singular homology and cohomology is treated with all its properties including products and duality. Homological algebra leads to spectral sequences, as well as derived functors such as Tor and Ext, and ultimately to derived categories. Stability leads to generalized homology and cohomology and spectra, with K-theory and cobordism as notable examples. There are no exams in Math 695, grading is based on homework completion (3-4 problems per week).

MATH 709. Topics in Analysis **Rudelson, Mark** **M/W 10:00 AM-11:30 AM**
Topic: Brownian Motion

The course will discuss the Brownian motion, which is one of the most fundamental objects in probability. Brownian motion enjoys the universality property similar to that of a normal random variable. The Central Limit Theorem asserts that a normal random variable is the limit of a scaled of sums of independent random variables. If we consider the sequence of such sums, we would obtain a random walk, and the scaled limit of such random walk will be a Brownian motion. We will start with constructing a one-dimensional Brownian motion and then discuss both its probabilistic and analytic properties. This includes the Markov property and Donsker's theorem, which plays the role of the Central Limit Theorem, as well as the smoothness properties of the sample paths. We will also discuss Ito calculus and its applications.

Optional Textbook: Brownian Motion, by Peter Morters & Yuval Peres, ISBN-13: 978-0521760188, ISBN-10: 0521760186

Grading will be based on several sets of homework.

MATH 731. Topics in Algebraic Geology I **Jonsson, Mattias** **M/W 1:00 PM-2:30 PM**
Topic: Berkovich Spaces

Berkovich spaces are analogues of complex manifolds that appear when replacing complex numbers by the elements of a general normed field, e.g. p-adic numbers or formal Laurent series. They were introduced in the late 1980's by Vladimir Berkovich as a topologically more satisfying alternative to the rigid spaces earlier used by Tate. In recent years, Berkovich spaces have seen a large and growing range of applications to complex analysis, tropical geometry, complex and arithmetic dynamics, the local Langlands program, Arakelov geometry,...

The first part of the course will be devoted to the basic theory of local (affinoid) and global Berkovich spaces. In the second part, we will discuss various applications or specialized topics, for example degeneration problems in complex geometry.

The main prerequisites for the course is some familiarity with commutative algebra and algebraic geometry. Knowledge of other topics, like complex analysis, is useful but not strictly speaking necessary.

I am writing a textbook on Berkovich spaces and hope to make further progress on it during the course. Otherwise, the main texts are:
V. G. Berkovich. Spectral Theory and Analytic Geometry over non-Archimedean Fields.
S. Bosch, U. Guentzer and R. Remmert. Non-Archimedean Analysis.

MATH 738. Topics in Representation Theory **Zelingher, Elad** **W/F 10:00 AM-11:30 AM**
Topic: Complex representations of finite general linear groups: constructions of local constants

A very prolific way to study objects in number theory is to define L-functions attached to them. This method has been used successfully for automorphic representations of classical groups in many cases. The aim of this course is to discuss constructions of such L-functions and their properties in an accessible way, by considering complex representations of finite general linear groups and defining suitable local constants attached to them. These local constants are analogous to L-functions, and important properties of the representations can be read from them. Topics include: Parabolic induction, cuspidal representations, generic representations, Whittaker models and their uniqueness. Gamma factor constructions of Tate, Godement--Jacquet, Jacquet-Piatetski-Shapiro--Shalika, and Langlands--Shahidi.