Fall 2021 Graduate Course Descriptions

MATH 501. AIM Student Seminar	**Alben, Silas** **Alben, Silas**	**Fri 1:00 PM-2:00 PM** **Fri 3:00 PM-4:00 PM**

At least two 300 or above level math courses, and Graduate standing; Qualified undergraduates with permission of instructor only. (1). May be repeated for a maximum of 6 credits. Offered mandatory credit/no credit.

MATH 501 is an introductory and overview seminar course in the methods and applications of modern mathematics. The seminar has two key components: (1) participation in the Applied and Interdisciplinary Math Research Seminar; and (2) preparatory and post-seminar discussions based on these presentations. Topics vary by term.

MATH 520. Life Contingencies I	**Natarajan,B Roger** **TBD**	**T/Th 11:30 AM-1:00 PM** **T/Th 10:00 AM-11:30 AM**

MATH 424 and 425 with minimum grade of C-, plus declared Actuarial/Financial Mathematics Concentration. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of the Math 520-521 sequence is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. The main topics are (1) developing probability distributions for the future lifetime random variable, and (2) using those distributions to price life insurance and annuities.

Required Textbook: Actuarial Mathematics for Life Contingent Risks, by Dickson, David C.M./Hardy, Mary R./Waters, Howard R. ISBN-13: 978-1108478083

MATH 523. Loss Models I	**Young,Virginia R** **TBD**	**T/Th 8:30 AM-10:00 AM** **T/Th 1:00 PM-2:30 PM**

MATH/STATS 425. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Risk management and modeling of financial losses. Review of random variables (emphasizing parametric distributions), review of basic distributional quantities, continuous models for insurance claim severity, discrete models for insurance claim frequency, the effect of coverage modification on severity and frequency distributions, aggregate loss models, and simulation.

Textbook: Loss Models: From Data to Decisions, by Stuart A. Klugman, 9781118315323

MATH 525/STATS 525. Probability Theory	**TBD** **Deng, Shuoqing** **Burns, Dan**	**T/Th 8:30 AM-10:00 AM** **T/Th 10:00 AM-11:30 AM** **M/W/F 8:00 AM-9:00 AM**

MATH 451 (strongly recommended). MATH 425/STATS 425 would be helpful. (3). (BS). May not be repeated for credit.

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

The following topics will be covered: sample space and events, random variables, concept and definition of probability and expectation, conditional probability and expectation, independence, moment generating functions, Law of large numbers, Central limit theorem, Markov chains, Poisson process and exponential distribution.

Required Textbook: Probability and Random Processes, by Grimmett, Geoffrey R. / Stirzaker, David R. (9780198572220) - 3RD 01

MATH 526/STATS 526. Discrete State Stochastic Processes Cohen, Asaf T/Th 10:00 AM-11:30 AM
Cohen, Asaf T/Th 8:30 AM-10:00 AM

MATH 525 or STATS 525 or EECS 501. (3). (BS). May not be repeated for credit.

This is a course on the theory and applications of stochastic processes, mostly on discrete state spaces. It is a second course in probability which should be of interest to students of mathematics and statistics as well as students from other disciplines in which stochastic processes have found significant applications.

The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include generating functions; recurrent events and the renewal theorem; random walks; Markov chains; branching processes; limit theorems; Markov chains in continuous time with emphasis on birth and death processes and queueing theory; an introduction to Brownian motion; stationary processes and martingales.

**Testbook: Essentials of Stochastic Processes, by Richard Durrett, 3rd Edition, 9783319456133

MATH 555 Introduction to Functions of a Complex Variable with Applications Lagarias, Jeff T/Th 1:00 PM-2:30 PM

MATH 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

Intended primarily for students of engineering and of other cognate subjects. Doctoral students in mathematics elect Mathematics 596. Complex numbers, continuity, derivative, conformal representation, integration, Cauchy theorems, power series, singularities, and applications to engineering and mathematical physics.

Textbook: Introductory Complex Analysis by Silverman, Richard A., 9780486646862

MATH 556. Applied Functional Analysis Borcea, Liliana T/Th 10:00 AM-11:30 AM

MATH 217, 419, or 420; MATH 451; and MATH 555. (3). (BS). May not be repeated for credit.

This is an introduction to methods of applied functional analysis. Students are expected to master both the proofs and applications of major results. The prerequisites include linear algebra, undergraduate analysis, advanced calculus and complex variables. This course is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program.

Required Textbook: Applied Analysis - by Hunter, John K. (9789812705433) – 01
Optional: Lecture Notes on Functional Analysis by Bressan, Alberto (9780821887714) – 13

MATH 558. Applied Nonlinear Dynamics Miller, Peter T/Th 2:30 PM-4:00 PM

MATH 451. (3). (BS). May not be repeated for credit.

Differential equations model systems throughout science and engineering and display rich dynamical behavior. This course emphasizes the qualitative and geometric ideas which characterize the post-Poincare era. The course surveys a broad range of topics with an emphasis on techniques, and results that are useful in applications. It is intended for students in mathematics, engineering, and the natural sciences and is a core course for the Applied and Interdisciplinary Mathematics graduate program. Proofs are given. Homework and exams concentrate on using rather than proving.

**Required Textbook: Nonlinear Ordinary Differential Equations by Jordan, Dominic / Smith, Peter (9780199208258) - 4TH 07

MATH 561/IOE 510/TO 518. Linear Programming I Lee,Jon M/W 9:00 AM-10:30 AM

MATH 217, 417, or 419. (3). (BS). May not be repeated for credit. F, W, Sp.

Formulation of problems from the private and public sectors using the mathematical model of linear programming. Development of the simplex algorithm; duality theory and economic interpretations. Postoptimality (sensitivity) analysis application and interpretations. Introduction to transportation and assignment problems; special purpose algorithms and advanced computational techniques. Students have opportunities to formulate and solve models developed from more complex case studies and to use various computer programs.

Textbook: https://github.com/jon77lee/JLee_LinearOptimizationBook/blob/master/JLee.3.12.pdf

MATH 565. Combinatorics and Graph Theory **Lam, Thomas** **T/Th 10:00 AM-11:30 AM**

MATH 465. (3). (BS). May not be repeated for credit.

This course has two somewhat distinct halves devoted to (1) graph theory and (2) topics in the theory of finite partially ordered sets. Students should have taken at least one proof-oriented course.

Textbook: Course in Combinatorics, by Jacobus H. Vanlint & Richard M. Wilson, 9780521006019

MATH 568/BIOINF 568 Mathematical and Computational Neuroscience **Booth,Victoria** **M/W 10:00 AM-11:30 AM**

MATH 463 or 462 (for undergraduate students) or Graduate standing. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Computational neuroscience investigates the brain at many different levels, from single cell activity to small, local network computation to the dynamics of large neuronal populations. This course introduces modeling and quantitative techniques used to investigate neural activity at all these different levels. Topics covered include passive membrane properties, the Nernst potential, derivation of the Hodgkin-Huxley model, action potential generation, action potential propagation in cable and multi-compartmental models, reductions of the Hodgkin-Huxley model, phase plane analysis, linear stability and bifurcation analysis, synaptic currents, excitatory and inhibitory network dynamics.

MATH 571. Numerical Linear Algebra **Viswanath,Divakar** **T/Th 8:30 AM-10:00 AM**

MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit.

Math 571 is an introduction to numerical linear algebra, a core subject in scientific computing. Three types of problems are considered: (1) linear systems, (2) eigenvalues, (3) least-squares problems. These problems arise in many scientific applications and we'll study the accuracy, efficiency, and stability of the methods that have been developed for their solution. As an application, we'll consider finite-difference schemes for boundary value problems in 1D and 2D.

Textbook: Numerical Linear Algebra, by Lloyd N. Trefethen and David Bau. 9780898713619

MATH 573. Financial Mathematics I **Norgilas,Dominykas** **T/Th 1:00 PM-2:30 PM**

(3). (BS). May not be repeated for credit.

This is an introductory course in Financial Mathematics. This course starts with the basic version of the Mathematical Theory of Asset Pricing and Hedging (Fundamental Theorem of Asset Pricing in discrete time and discrete space). This theory is applied to problems of Pricing and Hedging of simple Financial Derivatives. Finally, the continuous-time version of the proposed methods is presented, culminating with the Black–Scholes model. A part of the course is devoted to the problems of Optimal Investment in discrete time (including Markowitz Theory and CAPM) and Risk Management (VaR and its extensions). This course shows how one can formulate and solve relevant problems of the financial industry via mathematical (in particular, probabilistic) methods. Although Math 526 is not a prerequisite for Math 573, it is strongly recommended that either these courses are taken in parallel, or Math 526 precedes Math 573.

Textbook: Stochastic Finance, by Hans Follmer & Alexander Schied, 9783110463446

MATH 591. Differentiable Manifolds **Uribe,Alejandro** **M/W/F 10:00 AM-11:00 AM**

MATH 451, 452 and 590. (3). (BS). May not be repeated for credit.

This is one of the basic courses for students beginning the PhD program in mathematics. The approach is rigorous and emphasizes abstract concepts and proofs.

Topics: Product and quotient topology, group actions, topological groups, topological manifolds, smooth manifolds, manifolds with boundary, smooth maps, partitions of unity, tangent vectors and differentials, the tangent bundle, submersions, immersions and embeddings, smooth submanifolds, Sard's Theorem, the Whitney Embedding Theorem, transversality, Lie groups, vector fields, Lie brackets, Lie algebras, multilinear algebra, vector bundles, differential forms, exterior derivatives, orientation, Stokes' Theorem, introduction to De Rham cohomology groups, homotopy invariance.

Textbook: An Introduction to Manifolds, Loring W. Tu, Second Edition, ISBN: 978-1-4419-7399-3

MATH 593. Algebra I **Speyer, David** **M/W/F 2:00 PM-3:00 PM**

MATH 412, 420, and 451 or MATH 494. (3). (BS). May not be repeated for credit.

Topics include basics about rings and modules, including Euclidean rings, PIDs, UFDs, and basic constructions such as quotients, localizations and tensor products. We will cover the structure theory of modules over a PID, and standard matrix forms such as Smith normal form, Jordan canonical form and rational normal form. The course will also cover tensor, symmetric, and exterior algebras, and the classification of bilinear forms. Large portions of the class will involve solving and presenting solutions to problems.

MATH 596. Analysis I **Conlon, Joseph** **T/Th 2:30 PM-4:00 PM**
Topic Title: Complex Analysis

MATH 451. (3). (BS). May not be repeated for credit. Students with credit for MATH 555 may elect MATH 596 for two credits only.

This course covers the Complex Analysis portion of the syllabus for the Qualifying Review Exam in Analysis. Topics to be covered include:

- Complex elementary functions, conformal mapping, the Riemann sphere, linear fractional transformations, rational functions
- Complex derivatives, Cauchy-Riemann equations
- Contour integration, Cauchy's theorem, Cauchy-Green formula, Cauchy's integral formula and consequences, power series expansion and consequences
- Harmonic functions, maximum principle, Dirichlet's problem
- Isolated singularities, residues, application to computation of definite integrals, meromorphic functions, argument principle, Rouche's theorem
- Equicontinuity, Montel's theorem, Schwartz's lemma, Riemann mapping theorem
- Homework will be assigned weekly, and midterm and final exams will be given.

Text: Complex Analysis, by Gamelin, Springer 2001, 978-0-387-95069-3

MATH 602. Real Analysis II **Bieri, Lydia** **M/W 2:30 PM-4:00 PM**

MATH 590 and 597. (3). (BS). May not be repeated for credit.

Functional analysis is a core subject in mathematics. It has connections to probability and geometry, and is of fundamental importance to the development of analysis, differential equations, quantum mechanics and many other branches in mathematics, physics, engineering and theoretical computer science. The goal of this course is to introduce students to the basic concepts, methods and results in functional analysis. Topics to be covered include linear spaces, normed linear spaces, Banach spaces, Hilbert spaces, linear operators, dual operators, the Riesz representation theorem, the Hahn-Banach theorem, uniform boundedness theorem, open mapping theorem, closed graph theorem, compact operators, Fredholm Theory, reflexive Banach spaces, weak and weak* topologies, spectral theory, and applications to classical analysis and partial differential equations.

***Material will be taken from the first 33 chapters of the text. Grades will be based on a few sets of homework and attendance and participation.

***Required Textbook: Functional Analysis, Peter D. Lax, ISBN-13: 978-0471556046, ISBN-10: 0471556041

MATH 614. Commutative Algebra **Bhatt, Bhargav** **T/Th 10:00 AM-11:30 AM**

MATH 593 and Graduate standing. (3). (BS). May not be repeated for credit.

Commutative algebra is a field that interacts strongly with many other areas of mathematics, including algebraic geometry, algebraic combinatorics, algebraic number theory, and several complex variables. This course is an introduction that will include material on the uses of the prime spectrum, behavior of primes under integral extensions of rings, Noetherian rings and modules, Noether normalization, the Hilbert basis theorem, an introduction to affine algebraic geometry, primary decomposition, normal rings, discrete valuation rings, Dedekind domains, Artinian rings, flatness, completion, and dimension theory, including the Krull height theorem. Some basic material from category theory will also be introduced. There is no text: lecture notes will be provided.

MATH 623/IOE 623. Computational Finance Chen, Tao T/Th 10:00 AM-11:30 AM

MATH 316 and MATH 425 or 525. (3). (BS). May not be repeated for credit.

This is a course in computational methods in finance and financial modeling. Particular emphasis will be put on interest rate models and interest rate derivatives. Specific topics include Black-Scholes theory, no-arbitrage and complete markets theory, term structure models, Hull and White models, Heath-Jarrow-Morton models, the stochastic differential equations and martingale approach, multinomial tree and Monte Carlo methods, the partial differential equations approach, finite difference methods.

Textsbooks: Monte Carlo methods in financial engineering / Paul Glasserman/ 0387004513
The Mathematics Of Financial Derivatives A Student Introduction, Paul Wilmott, Sam Howison, Jeff Dewynne, 9780511812545

MATH 625/STATS 625. Probability and Random Processes I Rudelson, Mark T/Th 10:00 AM-11:30 AM

MATH 597, Measure theory at the level of Math 597 and Graduate standing. (3). (BS). May not be repeated for credit.

The goal of this course is to develop some of the major ideas of probability theory. Two central themes throughout the course are:
(a) The strong law of large numbers (SLLN).
(b) The central limit theorem (CLT).

The strong law is the law of averages, that for example the fraction of heads in N tosses of a fair coin tends to 1/2 as N becomes large, with probability 1. The central limit theorem proves that the fluctuation in the number of heads from N/2 is approximately the square root of N times a Gaussian random variable with mean 0.

The course begins with the proof of theorems (a) and (b) for independent random variables. Specializing to Bernoulli variables, we then interpret these theorems in terms of random walk on the integers Z. With the introduction of some further ideas, most notably zero-one laws and the reflection principle, we prove for Bernoulli variables the law of the iterated logarithm, which is an optimal strengthening of SLLN.

The remainder of the course develops deep generalizations of the ideas discussed in the previous paragraph. The first of these is the idea of a Martingale, which is ubiquitous in current research, particularly in mathematical finance. Next is the notion of measure preserving transformation and ergodicity, from which one obtains an optimal SLLN for general independent random variables. Finally Brownian motion is constructed as a generalized CLT. Time permitting, there will be also some discussion of Markov chains and information theory.

Pre-req Knowledge: complex analysis

Textbooks: Probability Theory and Examples, R. Durrett, 5th edition, 2019, 9781108473682
Probability by L. Breiman, SIAM reprint in classics of applied mathematics series 1992. ISBN: 9780898712964

MATH 627/ BIOSTAT 680 Applications of Stochastic Processes I Wen,Xiaoquan M/W 8:30 AM-10:00 AM

Graduate standing; BIOSTAT 601, 650, 602 and MATH 450. (3). (BS). May not be repeated for credit.

BIOSTAT 680 is a cross listed courses. Contact BIOSTAT for course details.

MATH 631. Introduction to Algebraic Geometry Perry, Alex T/Th 11:30 AM-1:00 PM

MATH 594 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

This is the first half of a year-long sequence in algebraic geometry. In the first semester, we will introduce the basic notions and objects of modern algebraic geometry - sheaves and schemes. We will be loosely following Ravi Vakil's notes "Foundations of Algebraic Geometry".

Prerequisites: General topology. Familiarity with the language of category theory. Commutative algebra is recommended but not essential; you should have a solid grasp of localizations (of rings/modules) and tensor products though."

Textbook: Algebraic Geometry by R. Hartshorne, 9781475738490

MATH 636. Topics in Differential Geometry **Ji, Lizhen** **M/W/F 3:00 PM-4:00 PM**
Topic: Introduction to Riemann Surfaces

MATH 635 and Graduate standing. (3). (BS). May not be repeated for credit.

The notion of Riemann surfaces was introduced by Riemann in his thesis in 1851 in order to provide the natural domains of holomorphic functions under analytic continuation, and was used by him crucially in his classical paper ``Theory of Abelian functions" in 1857 to study integrals of algebraic functions and hence to solve the Jacobi inversion problem.

Riemann surfaces are basic and essential notions in mathematics. In the preface of his book on Riemann surfaces, Donaldson wrote: “The theory of Riemann surfaces occupies a very special place in mathematics. It is a culmination of much of traditional calculus, making surprising connections with geometry and arithmetic. It is an extremely ‘useful’ part of mathematics, knowledge of which is needed by specialists in many other fields. It provides a model for a large number of more recent developments in areas including manifold topology, global analysis, algebraic geometry, Riemannian geometry and diverse topics in mathematical physics.”

The study of Riemann surfaces is indeed the starting point of many subjects, for example, the algebraic topology, the complex geometry, the Hodge theory, moduli spaces of complex manifolds, discrete subgroups of Lie groups and locally symmetric spaces etc.

In an autobiographic article, Kodaira said that his work in complex geometry and algebraic geometry was motivated by an attempt to generalize the theories and results for Riemann surfaces contained in the classical book by Weyl on Riemann surfaces.

In this course, we will give an introduction to Riemann surfaces and moduli spaces of Riemann surfaces with some emphasis on the historical development, applications of the uniformization theorem and the Riemann-Roch Theorem, and connection with other subjects.

Optional Textbooks: Reimann Surfaces, Simon Donaldson, 978-0-19-960674-0

MATH 650. Fourier Analysis **Rudelson, Mark** **T/Th 10:00 AM-11:30 AM**

Fourier analysis is one of the most powerful tools in PDEs, probability, analytic number theory, as well as signal processing and computer science. This course will cover the basics of Fourier analysis starting from Fourier series, Dirichlet and Fejer kernel and Fejer-Lebesgue theorem. We will continue with Fourier integral in one and multiple dimensions, Schwartz functions, theory of distributions and its applications. At a more advanced stage of the course we will discuss Fourier transform in complex domain leading to boundary behavior of harmonic and analytic functions and Paley-Wiener theory.

No textbook for this course.

MATH 651. Topics in Applied Mathematics **Alben, Silas** **M/W 10:00 AM-11:30AM**
Topic: Modeling and Mechanics

Math 450 or 454. (3). (BS).

The course will develop mathematical modeling methods that are useful for solving problems in continuum mechanics. We will discuss some applications including fluid-structure interactions and the mechanics of organisms. Many of the modeling methods are useful in other areas of applied mathematics, physics, engineering, and biology. The course is aimed at graduate students in applied mathematics, engineering, and the sciences.

No textbook for this course

MATH 656. Introduction to Partial and Differential Equations **Wu, Sijue** **T/Th 2:30 PM-4:00 PM**

MATH 558, 596 and 597 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

Partial differential equations are at the core of models in science, technology, economics and related fields. These equations and their solutions have interesting structures that are studied by methods of analysis, geometry, probability and other mathematical fields.

This course will introduce students in pure and applied mathematics to concepts and methods, that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations.

Topics to be covered will include nonlinear first order equations, linear elliptic, hyperbolic and parabolic equations. The method of characteristics, energy methods, maximum principle, Fourier transform and Sobolev spaces.

Required: Fritz, John: “Partial Differential Equations”, Springer, 4th Ed. 978-0387906096

MATH 663/ IOE 611. Nonlinear Programming **Fattahi, Salar** **T/Th 12:00 PM-1:30 PM**

MATH,MATH 561. (3). (BS). May not be repeated for credit.

IOE 611 is a cross listed courses. Contact IOE for course details.

MATH 665. Topic in Combinatorics **Fomin, Sergey** **T/Th 11:30 AM-1:00 PM**
Topic: Cluster Algebras

MATH 664 or equivalent. Graduate standing. (3). (BS). May not be repeated for credit.

Cluster algebras are a class of commutative rings endowed with a particular kind of combinatorial structure. They arise in a variety of contexts including representation theory of Lie groups, Teichmüller theory, discrete integrable systems, classical invariant theory, and quiver representations. This course will provide an elementary introduction to the basic notions and results of the theory of cluster algebras, and present some of its most accessible applications. Combinatorial aspects will be emphasized. No special background in commutative algebra, representation theory, or combinatorics is required.

No textbook for this course

MATH 679 Elliptic Curves **Ho, Wei** **T/Th 2:30 PM-4:00 PM**

This course is an introduction to elliptic curves, focusing on the arithmetic side of the theory. After covering basic aspects of the geometry of elliptic curves and the complex theory, we will look at the arithmetic theory over finite, local, and global fields, culminating in the proof of the Mordell-Weil theorem. A variety of additional topics might be included, depending on time and interest.

Pre-req knowledge: field theory, Galois theory, basic algebraic number theory, basic algebraic geometry

Required Text: The Arithmetic of Elliptic Curves, Silverman, 978-0-387-09494-6

MATH 682. **Smythe, Iian** **T/Th 1:00 PM-2:30 PM**

Math 681 or equivalent. (3). (BS). May not be repeated for credit

The primary topic of this course will be consistency and independence proofs in set theory. I plan to begin with a rapid but self-contained presentation of the necessary facts from axiomatic set theory, including the Zermelo-Fraenkel (ZF) axioms and their motivation, the set-theoretical representation of basic concepts and constructions of mathematics, cardinal and ordinal numbers, and the axiom of choice. Then I shall present, in order of increasing power and difficulty, the three standard techniques for consistency and independence results.

The weakest of these techniques, introduced by Fraenkel in the 1920s, gives results only concerning the axiom of choice and only in the presence of atoms (non-sets that can be elements of sets). It uses permutation groups to produce models of set theory where the axiom of choice is false.

A significantly stronger method, introduced by Goedel in the 1930s, works with pure set theory (i.e., no atoms) and establishes the consistency of the axiom of choice, the generalized continuum hypothesis, and certain other principles. It shows how to shrink the universe of sets to a sub-universe of so-called constructible sets, in which these principles and the ZF axioms are true.

Finally, the forcing method, introduced by Cohen in the 1960s and intensively developed since then, establishes the consistency of a great many propositions by enabling one to enlarge the universe of all sets in many different ways while keeping the ZF axioms true. The power of the method lies primarily in the control it gives over detailed properties of the extension.

The only absolute prerequisite is the mathematical maturity generally expected in 600 level math courses. Some prior acquaintance with axiomatic set theory is helpful but not absolutely necessary since I will cover the necessary material quickly. Similarly, some prior knowledge of mathematical logic, particularly the completeness and Loewenheim-Skolem theorems would be helpful, but it is not necessary if you are willing to take these theorems on faith; I'll state them but at most sketch the proofs.

Textbook: Set Theory: An Introduction to Independence Proofs, by Kenneth Kunen, North-Holland 1980 (available free through ScienceDirect)

MATH 695. Algebraic Topology I **Kriz, Igor** **M/W/F 9:00 AM-10:00 AM**

MATH 591 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

Many areas of mathematics today use the idea of "working up to homotopy." We will explore methods of making this precise, which typically involve identifying much more than maps homotopic in the usual sense. For topological spaces, we will study the notion of *weak equivalence*, allowing us to treat spaces essentially as algebraic objects. For chain complexes, we will introduce the analogous notion of a *derived category* which leads to concepts of homological algebra such as Tor and Ext. Basic computational tools will be developed, such as products in cohomology, duality, and spectral sequences. Then we will apply the methods we learned to introducing generalized homology and cohomology, which are central concepts of modern algebraic topology, with many applications in other fields, including analysis, arithmetic and algebraic geometry, and representation theory.

There will be no exams or quizzes, homework will be collected on a weekly basis.

Optional Texts: A concise course in Algebraic Topology, Peter May, ISBN 13: 9780226511825

MATH 697. Geometry Group Theory **Canary, Dick** **M/W/F 12:00 PM-1:00 PM**

Graduate standing. (3). (BS). May be repeated for credit.

In this course we focus on the geometric viewpoint on the study of groups. The basic idea here is that it is often easier to study a group by studying its geometric action on some space. Hyperbolic groups are those groups which act properly discontinuously and cocompactly (i.e. with compact quotient) on a space which "coarsely" has negative curvature. It is one of Gromov's fundamental discoveries that it suffices to work with a very naive notion of coarse negative curvature, e.g. that all geodesic triangles are "thin", and that one can still obtain powerful consequences, e.g. the solvability of the word problem for hyperbolic groups. Therefore, proofs of seemingly complicated results can be reduced to their simple essence and become quite accessible. Examples of hyperbolic groups include free groups, fundamental groups of surfaces of genus at least 2, and fundamental groups of negatively curved manifolds. Gromov's concise and elegant notion of coarse negative curvature has been tremendously influential in a wide swath of mathematics where geometric techniques or ideas are used.

After covering the basic theory of hyperbolic groups, we will spend the remainder of the semester on an advanced topic to be chosen in consultation with the students. Possible advanced topics include CAT(0) groups, symbolic dynamics for hyperbolic groups and Anosov representations of hyperbolic groups.

The only prerequisite for this course is the basic material on covering spaces and fundamental groups, which occurs in Math 592. There will be no textbook for the course.

MATH 731. Topics in Algebraic Geometry **Snowden, Andrew** **M/W 1:00 PM-2:30 PM**
Topic: Algebraic Groups

Graduate standing. Basic Algebraic Geometry (e.g. Math 631). (3). (BS). May be repeated for credit.

This course will introduce the basic theory of affine algebraic groups over algebraically closed fields. We will discuss the structure of reductive groups in detail. If time permits, we may discuss the setting of non-algebraically closed fields.

No book for this course

MATH 776. Topics in Algebraic Number Theory **Prasanna, Kartik** **T/Th 1:00 PM-2:30 PM**

Graduate standing. Math 676. (3). (BS). May be repeated for credit.

The Gross-Zagier theorem is a celebrated result in number theory proved in the 1980s, that relates heights of Heegner points (on elliptic curves) to derivatives of L-functions. This course will be devoted to understanding instead a more recent related result (from reference [1] below) that connects p-adic logarithms of Heegner points to p-adic L-functions.

Prerequisites: familiarity with the basic theory of modular forms, modular curves and elliptic curves, roughly at the level of the books of Shimura and Silverman on these topics.

Topics to be covered include: p-adic modular forms, p-adic differential operators, p-adic integration on curves, p-adic L-functions, and the relations between these.

No textbook is required. The following are some key references, others will be provided during the course.

[1] Bertolini, Darmon and Prasanna, Generalized Heegner cycles and p-adic Rankin L-series, Duke 162, 2013.
[2] Gross and Zagier, Heights of Heegner points and derivatives of L-series, Inventiones Math 84, 1986.