Fall 2019 Graduate Course Descriptions

501 AIM Student Seminar — **Alben** — **Fri 1:00 PM – 2:00 PM**; **Fri 2:00 PM – 3:00 PM**

At least two 300 or above level math courses, and Graduate standing; Qualified undergraduates with permission of instructor only. (1). May be repeated for a maximum of 6 credits. Offered mandatory credit/no credit.

MATH 501 is an introductory and overview seminar course in the methods and applications of modern mathematics. The seminar has two key components: (1) participation in the Applied and Interdisciplinary Math Research Seminar; and (2) preparatory and post-seminar discussions based on these presentations. Topics vary by term.

520 Life Contingencies I — **Moore** — **T/Th 11:30 AM – 1:00 PM**

MATH 424 and 425 with minimum grade of C-, plus declared Actuarial/Financial Mathematics Concentration. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit

Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of this course is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. The main topics are the development of (1) probability distributions for the future lifetime random variable; (2) probabilistic methods for financial payments on death or survival; and (3) mathematical models of actuarial reserving.

523 Loss Models I — **TBA** — **T/Th 8:30 AM – 10:00 AM**

MATH/STATS 425. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Risk management and modeling of financial losses. Review of random variables (emphasizing parametric distributions), review of basic distributional quantities, continuous models for insurance claim severity, discrete models for insurance claim frequency, the effect of coverage modification on severity and frequency distributions, aggregate loss models, and simulation.

MATH 525 / STATS 525. Probability Theory

Le,Pengyu	**T/Th 8:30 AM – 10:00 AM**
Le,Pengyu	**M/W/F 8:00 AM – 9:00AM**
Chakraborty,S	**T/Th 10:00 AM – 11:30AM**

MATH 451 (strongly recommended). MATH 425/STATS 425 would be helpful. (3). (BS). May not be repeated for credit.

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

MATH 526 / STATS 526
Discrete State Stochastic Processes

Hernandez	**T/Th**	**8:30 AM – 10:00 AM**
Hernandez	**T/Th**	**10:00 AM – 11:30 AM**

MATH 525 or STATS 525 or EECS 501. (3). (BS). May not be repeated for credit.

This is a course on the theory and applications of stochastic processes, mostly on discrete state spaces. It is the second course in probability which should be of interest to students of mathematics and statistics as well as students from other disciplines in which stochastic processes have found significant applications.

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MATH 555 Complex Variable — Schotland,John — T/Th — 1:00 PM – 2:30 PM
Introduction to Functions of a Complex Variable with Applications
MATH 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

This course is an introduction to analysis of complex-valued functions of a complex variable with applications. Topics include the Cauchy-Riemann equations, Taylor series, Cauchy's theorem and integral formula, Laurent expansions, residues, the argument principle, harmonic functions, maximum modulus theorem, and conformal mapping. Concepts and calculations are emphasized over proofs. The prerequisite of a course in undergraduate real analysis is essential. This is a core course for the Applied and Interdisciplinary Mathematics (AIM) program.

MATH 556 — Borcea — T/Th — 10:00 AM – 11:30 AM
Applied Functional Analysis
MATH 217, 419, or 420; MATH 451; and MATH 555. (3). (BS). May not be repeated for credit.

This is an introduction to methods of applied functional analysis. Students are expected to master both the proofs and applications of major results. The prerequisites include linear algebra, undergraduate analysis, advanced calculus and complex variables. This course is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program.

MATH 558. Applied Nonlinear Dynamics — Conlon — T/Th — 2:30 PM – 4:00 PM
MATH 451. (3). (BS). May not be repeated for credit.

This course surveys a broad range of differential equations topics, with a focus on techniques and results that are useful in applications. It is intended for students in mathematics, engineering, and the natural sciences, and is a core course for the Applied and Interdisciplinary Mathematics graduate program. Topics selected from: dynamics in dimension 1 and 1.5, bifurcations, Poincar\'{e} map, existence, uniqueness, and perturbation theory, linear systems, spectral theorems, linearization at equilibria for nonlinear systems, phase plane analysis of linear and nonlinear systems, stable and unstable manifolds, Lyapunov functions, gradient flows and Hamiltonian systems, periodic solutions, stability, omega-limit set, Poincar\'{e}-Bendixson and Bendixson-duLac Theorems, bifurcation theory, and chaotic dynamics.

Background: Basic Linear Algebra, Ordinary Differential Equations (Math 216), Multivariable Calculus (215). Some exposure to more advanced mathematics e.g. Advanced Calculus (Math 450/451) or Advanced Mathematical Methods (Math 454).

MATH 565. Combinatorics and Graph Theory — Nguyen — T/Th — 10:00 AM – 11:30 AM
MATH 465. (3). (BS). May not be repeated for credit.

Topics in the graph theory part of the course include (if time permits) trees, k-connectivity, Eulerian and Hamiltonian graphs, tournaments, graph coloring, planar graphs, Euler's formula, the 5-Color theorem, Kuratowski's theorem, and the matrix-tree theorem. The second part of the course will deal with topics in the theory of finite partially ordered sets. This will include material about Mobius functions, lattices, simplicial complexes, and matroids.

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MATH 568 / BIOINF 568 **Booth,Victoria** **M/W 10:00 AM – 11:30 AM**
Mathematical and Computational Neuroscience
MATH 463 or 462 (for undergraduate students) or Graduate standing. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Computational neuroscience investigates the brain at many different levels, from single cell activity to small, local network computation to the dynamics of large neuronal populations. This course introduces modeling and quantitative techniques used to investigate neural activity at all these different levels.

Topics to be covered include: Passive membrane properties, the Nernst potential, derivation of the Hodgkin-Huxley model, action potential generation, action potential propagation in cable and multi-compartmental models, reductions of the Hodgkin-Huxley model, synaptic currents, excitatory and inhibitory network dynamics, synaptic plasticity, neural coding.

MATH 571 Numerical Linear Algebra **Viswanath** **T/Th 8:30 AM – 10:00 AM**
Viswanath **T/Th 10:00 AM– 11:30 AM**
MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit.

The main topics of this class are the numerical solution of linear systems, finding eigenvalues and singular values, and solving linear least squares problems. We will discuss condition numbers, numerical stability, QR factorization, SVD, the QR algorithm as well as iterative methods (GMRES, Arnoldi, Conjugate Gradients, Lanczos).

The following applications are included: KKT conditions, convergence of the perceptron, reproducing kernel Hilbert spaces, and kernel-based regression. The homework assignments will use either Python or Matlab, with the choice left to the student.

Required Textbook: Numerical Linear Algebra by Trefethen and Bau.

MATH 573 Financial Mathematics I **Norgilas** **T/Th 1:00 PM – 2:30 PM**
(3). (BS). May not be repeated for credit.

This is a core course for the quantitative finance and risk management masters program and introduces students to the main concepts of Financial Mathematics. This course emphasizes the application of mathematical methods to the relevant problems of financial industry and focuses mainly on developing skills of model building.

MATH 591. Differentiable Manifolds **Wright** **M/W/F 10:00 AM – 11:00 AM**
MATH 451, 452 and 590. (3). (BS). May not be repeated for credit.

This is one of the basic courses for students beginning the PhD program in mathematics. The approach is rigorous and emphasizes abstract concepts and proofs. The first 2-3 weeks of the course will be devoted to general topology, and the remainder of the course will be devoted to differential topology.

MATH 593 Algebra I **Speyer** **M/W/F 2:00 PM – 3:00 PM**
MATH 412, 420, and 451 or MATH 494. (3). (BS). May not be repeated for credit.

Topics include basics about rings and modules, including Euclidean rings, PIDs, UFDs, and basic constructions such as quotients, localizations and tensor products. We will cover the structure theory of modules over a PID, and standard matrix forms such as Smith normal form, Jordan canonical form and rational normal form. The course will also cover tensor, symmetric, and exterior algebras, and the classification of bilinear forms.

Large portions of the class will involve solving and presenting solutions to problems.

MATH 596 Analysis I **Barrett** **M/W/F** **11:00 AM– 12:00 PM**

Topic Title: Complex Analysis

MATH 451. (3). (BS). May not be repeated for credit. Students with credit for MATH 555 may elect MATH 596 for two credits only.

This course covers the Complex Analysis portion of the syllabus for the Qualifying Review Exam in Analysis. Topics to be covered include:

- Complex elementary functions, conformal mapping, the Riemann sphere, linear fractional transformations, rational functions
- Complex derivatives, Cauchy-Riemann equations
- Contour integration, Cauchy's theorem, Cauchy-Green formula, Cauchy's integral formula and consequences, power series expansion and consequences
- Harmonic functions, maximum principle, Dirichlet's problem
- Isolated singularities, residues, application to computation of definite integrals, meromorphic functions, argument principle, Rouche's theorem
- Equicontinuity, Montel's theorem, Schwartz's lemma, Riemann mapping theorem

Homework will be assigned weekly, and midterm and final exams will be given.
Text: Gamelin, "Complex Analysis"

MATH 602 Real Analysis II **Rudelson** **M/W/F** **12:00 PM – 1:00 PM**

Topic Title: Functional Analysis

MATH 590 and 597. (3). (BS). May not be repeated for credit.

MATH 606 **Tan, Xiaolu** **T/Th** **11:30 AM – 1:00 PM**

Advanced Stochastic Analysis for Finance

Consent of instructor required. Strong background in Probability Theory (Math 625), Measure Theory, Real Analysis & Functional Analysis. (3). May not be repeated for credit.

This is a PhD level course in Stochastic Analysis and applications to Quantitative Finance. The aim of this course is to teach the advanced probabilistic techniques and concepts from the theory of stochastic processes, in order to provide a sufficient knowledge base for the students to conduct research in Financial Mathematics. It is a research-level extension of Math 506.

MATH 612 Algebra II **Stembridge** **M/W/F** **1:00 PM – 2:00 PM**

Title: Lie Algebras and their Representations

MATH 593 and 594; and Graduate standing. (3). (BS). May not be repeated for credit.

Semisimple Lie algebras and their representations are at the crossroads of many important branches of mathematics--witness the ubiquity of Dynkin diagrams, for example. This course should be valuable for those interested in representation theory and Lie theory, as well as for those with interests in allied areas, such as algebraic combinatorics or non-commutative algebra.

We will cover the basic theory of Lie algebras, with emphasis on the complex semisimple case. We plan to cover most of the topics in Humphreys' book, with priority given to the finite-dimensional representation theory.

MATH 614 Commutative Algebra **Smith** **T/Th** **10:00 AM – 11:30 AM**

MATH 593 and Graduate standing. (3). (BS). May not be repeated for credit.

Review of commutative rings and modules. The prime spectrum of a ring and the Zariski Topology. Hilbert's Nullstellensatz. Local rings and localization. Noetherian and Artinian rings. Integral independence. Valuation rings, Dedekind domains, completions, graded rings. Dimension theory.

Textbook: We will use notes of Mel Hochster. No other books required.

MATH 623 / IOE 623. Computational Finance **Guo** **T/Th** **10:00 AM – 11:30 AM**

MATH 316 and MATH 425 or 525. (3). (BS). May not be repeated for credit.

This is a course on computational methods in finance and financial modeling. Using financial mathematics (like many branches of applied mathematics) in practice involves three tasks. First, one needs to develop mathematical models that accurately describe the real-life phenomena that one wishes to study – in the present case, probabilistic models for the evolution of prices, interest rates, and other relevant quantities. Once a model is chosen, the second task is to derive theoretical equations, or formulas, which establish relations between various objects in the financial markets: for example, the prices of derivative securities (options, bonds, etc), and the risk profiles of investment portfolios, as functions of risk factors. Finally, one needs to design and implement numerical methods to perform computations based on these formulas and equations. This course is concerned with the latter task, and it has three components. In the first part, we will study the lattice (or, tree) methods, which correspond to the models based on discrete time Markov chains (e.g. the binomial model). We will discuss the pricing and hedging of financial derivatives in such models, using the arbitrage theory, or, more specifically, the risk-neutral pricing. We will, then, proceed to analyze the diffusion-based models of financial mathematics (including, e.g., the Black-Scholes model) and the associated Partial Differential Equations (PDEs). We will discuss the finite difference methods, which provide numerical approximations for solutions to these PDEs. Both explicit and implicit schemes will be studied, the concepts of stability and convergence will be introduced, and a connection between the finite difference schemes and lattice methods will be established. After that, we will turn to the Monte Carlo simulations – the most general computational method for probabilistic equations. This method is based on generating a large number of paths of the underlying stochastic processes, in order to approximate the expectations of certain functions of these paths (which, e.g., may determine prices, portfolio weights, default probabilities, etc.). In addition to the standard Monte Carlo algorithms, we will study the variance reduction techniques, which are often necessary to obtain accurate results. The computational methods presented in this course will be illustrated using the popular models of equity markets (e.g. Black-Scholes, Heston), fixed income (e.g. Vasicek, CIR, Hull-White, Heath-Jarrow-Morton) and credit risk (e.g. Merton, Black-Cox, reduced-form models).

Prerequisites: Good understanding of the concepts from Probability Theory (probability measures, random variables, expectations, cumulative distribution and probability density functions, conditional probabilities and independence, law of large numbers, central limit theorem), Stochastic Analysis (stochastic processes, martingales, Brownian motion, stochastic integration, Itˆo’s formula, SDEs), Differential Equations (ODEs, as well as elliptic and parabolic PDEs), Mathematical Finance (arbitrage theory, binomial models, Black-Scholes and other diffusion-based models), basic

Numerical Methods (numerical methods for systems of linear equations, interpolation methods, numerical integration, bisection method, Newton's method), and Computer Programming (MatLab).

MATH 625 / STATS 625. Cohen T/Th 10:00 AM – 11:30 AM
Probability and Random Processes I
MATH 597 and Graduate standing. (3). (BS). May not be repeated for credit.

Axiomatics; measures and integration in abstract spaces. Fourier analysis, characteristic functions. Conditional expectation, Kolmogoroff extension theorem. Stochastic processes; Wiener-Levy, infinitely divisible, stable. Limit theorems, law of the iterated logarithm.

Pre-reqs: Commutative algebra (at the level of Atiyah-Macdonald) and basic algebraic topology

MATH 631 Introduction to Algebraic Geometry Bhatt T/Th 11:30 AM – 1:00 PM
MATH 594 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

This the first half of a year long sequence in algebraic geometry. In the first semester, we will cover schemes, sheaves and some aspects of the cohomology of sheaves. Homework will be an essential component of the class.

Background: Commutative algebra (at the level of Atiyah-Macdonald) and basic algebraic topology
Optional Textbook: Algebraic Geometry, Hartshorne

MATH 636 Topics in Differential Geometry Burns T/Th 11:30 AM – 1:00 PM
MATH 635 and Graduate standing. (3). (BS). May not be repeated for credit.

Symplectic Geometry: Modern Invariants and Relations to Quantum Mechanics

The course will start with basics of symplectic geometry, but then study the modern invariants introduced into the subject by Gromov, Floer, Hofer and others. The tools needed will include basic manifolds and "alpha-level"complex analysis. We will discuss the existence and moduli of J-holomorphic curves (holomorphic curves in an almost-complex manifold) and Morse-Floer type homologies built from them, as well as measures of the size of symplectic diffeomorphisms coming from variational properties of Hamiltonians generating them. Open problems related to the interpretation of these invariants in quantum mechanics will be discussed at the end, time permitting.

There will be a couple of problem sets for those wishing to take the course for credit.

Prerequisites: Basic maniflolds, "alpha-level" complex analysis.
References: McDuff-Salamon, "Intro. to Symplectic Topology".
M. Audin, "Theorie de Morse et Homologie de Floer".
M. Audin and J. Lafontaine, "Holomorphic curves in symplectic geometry".
Recent papers of L. Polterovich and collaborators.

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MATH 651 Topics in Applied Mathematics I **Miller** **M/W 8:30 AM – 10:00 AM**

Introduction to Painleve Equations

MATH 451, 555 and one other 500-level course in analysis or differential equations. Graduate standing. (3). (BS). May be elected twice for credit.

The Painlevé equations are a family of nonlinear, non-autonomous, second-order ordinary differential equations that were discovered in the early 1900's as part of a program to identify all differential equations of a certain form having the property that all solutions can be defined as single-valued functions on a common domain, regardless of initial conditions. This makes the solutions, now called Painlevé transcendents, analogous to classical special functions. In many ways, Painlevé transcendents are playing roles in 21st century pure and applied mathematics that classical special functions like Bessel functions played in the past.
For instance:

- Self-similar solutions or radially-symmetric solutions of many nonlinear partial differential equations are solutions of Painlevé equations. (Compare with classical Bessel functions appearing under radial symmetry in linear PDE.)
- Modern probability distribution functions such as the Tracy-Widom law of extreme eigenvalues in random matrix theory are expressible in terms of Painlevé transcendents. (Compare with classical error functions appearing in linear probability theory.)
- This course is an introduction to Painlevé equations and properties of their solutions.

Some topics that may be covered include:

- The Painlevé property and classification of differential equations.
- Compatibility representations of Painlevé equations (Garnier-Fuchs, Flaschka-Newell, Jimbo-Miwa).
- Solution of initial-value problems for the Painlevé equations by the isomonodromy method.
- Behavior of solutions for large argument, formal scaling arguments and solution of connection problems.
- Applications to nonlinear partial differential equations (similarity solutions, universal wave patterns).
- Hierarchies and "higher" Painlevé equations.
- Schlesinger-Bäcklund transformations and rational solutions of Painlevé equations.
- Special polynomials related to rational solutions. Applications to fluid vortices.

Prerequisites for the course are minimal: undergraduate courses in differential equations and complex variables. Additional background material (e.g., material on elliptic functions) will be introduced as part of the course. Some of the course material will be taken from original papers that will be made available, and some will come from one or more books to be announced later.

The course grade will be based on homework assigned approximately every other week.

MATH 656 **Wu,Sijue** **T/Th 2:30 PM – 4:00 PM**

Introduction to Partial and Differential Equations

MATH 558, 596 and 597 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

Partial Differential Equations are mathematical structures for models in science and technology. It is of fundamental importance in physics, biology and engineering design with connections to analysis, geometry, probability and many other subjects. The goal of this course is to introduce students (both pure and applied) to the basic concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations.

Topics to be covered will include nonlinear first order equations, linear elliptic, hyperbolic and parabolic equations. The method of characteristics,
energy methods, maximal principles, Fourier transform and, if time permits, Sobolev spaces will be introduced.
Content: Course contents will be taken from chapters 1, 2, 4, 5, 6, 7 of the book of F. John and/or Chapters 1-3, 5 of Evans.

Grading: Grades will be based on a few sets of homework and attendance and participation.

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Background: Math 451 and a solid background in analysis
Subsequent Courses: Math 657: Nonlinear Partial Differential Equations

MATH 665. Combinatorial Theory II **Speyer** **M/W/F** **12:00 – 1:00 PM**
Title: Bruhat orders
MATH 664 or equivalent. Graduate standing. (3). (BS). May not be repeated for credit.

We will cover the basic structure and examples of Coxeter groups and then study properties of the weak and strong Bruhat orders on them. The course will include connections to lattice theory, the combinatorics of cluster algebras, flag varieties and total positivity.

MATH 671. Analysis of Numerical Methods I **Karni** **T/Th** **2:30 PM – 4:00 PM**
Topic Title: Numerical Methods for Conservation Laws
MATH. 571, 572, or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

This course discusses the theory and numerical solution of nonlinear hyperbolic systems of conservation laws. It covers (i) the basic mathematical theory of the equations: the notion of weak solutions, entropy conditions, and the wave structure of solutions to the Riemann problem; and (ii) it discusses high resolution shock-capturing methods, including the theory of total variation diminishing (TVD) methods and the use of limiter functions.

Background: The course is pretty much self-contained. Numerical analysis background (471, 571, and particularly 572) is an advantage. Speaking a computer language is essential (Fortran, C, C++ or Matlab).

MATH 675. Analytic Theory of Numbers **Lagarias** **M/W** **11:30 AM – 1:00 PM**
Prerequisites: It would be useful to know number theory equivalent to Math 575, one complex variable equivalent to Math 596, ability to write proofs.

This is a first course in analytic number theory. The methods of analytic number theory are useful and applicable in many areas of mathematics. It will consider multiplicative number theory and properties of primes as a focus. It will emphasize methods. It will start with real analysis methods following Tenenbaum, arithmetic functions, prime number theory, hyperbola method, Euler-Maclaurin summation. For complex analysis methods and Dirichlet series it will follow Davenport and Montgomery. It will include basic theory of Riemann zeta function and Dirichlet L-functions to primes in arithmetic progressions. If time permits the course may include other topics, Selberg sieve, large sieve and a topic in probabilistic number theory.

Grading will be based on frequent homework.

MATH 678. Modular Forms **Zydor** **T/Th** **2:30 PM – 4:00 PM**
MATH 575, 596, and Graduate standing

The general focus of the course will be on L-functions and their special values. The L-functions in questions are certain holomorphic functions generalizing the Riemann zeta function. They will be associated with modular forms and their higher dimensional generalizations. The method of analyzing the special values will be through periods (integrals) of the corresponding modular forms. We will also discuss applications to analytic and algebraic number theory.

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MATH 682. Set Theory **Blass** **T/Th 1:00 PM – 2:30 PM**

Topic Title: Consistency and Independence Proofs

MATH 681 or equivalent

The primary topic of this course will be consistency and independence proofs in set theory. I plan to begin with a rapid but self-contained presentation of the necessary facts from axiomatic set theory,including the Zermelo-Fraenkel (ZF) axioms and their motivation, the set-theoretical representation of basic concepts and constructions of mathematics, cardinal and ordinal numbers, and the axiom of choice. Then I shall present, in order of increasing power and difficulty, the three standard techniques for consistency and independence results.

The weakest of these techniques, introduced by Fraenkel in the 1920's, gives results only concerning the axiom of choice and only in the presence of atoms (non-sets that can be elements of sets). It uses permutation groups to produce models of set theory where the axiom of choice is false.

A significantly stronger method, introduced by Goedel in the 1930's, works with pure set theory (i.e., no atoms) and establishes the consistency of the axiom of choice, the generalized continuum hypothesis, and certain other principles. It shows how to shrink the universe of sets to a sub-universe of so-called constructible sets, in which these principles and the ZF axioms are true. Finally, the forcing method, introduced by Cohen in the 1960's and intensively developed since then, establishes the consistency of a great many propositions by enabling one to enlarge the universe of all sets in many different ways while keeping the ZF axioms true. The power of the method lies primarily in the control it gives over detailed properties of the extension.

There will be no official textbook, but references to several relevant books (and a paper or two) will be given.

The only absolute prerequisite is the mathematical maturity generally expected in 600 level math courses. Some prior acquaintance with axiomatic set theory is helpful but not absolutely necessary, since I will cover the necessary material quickly. Similarly, some prior knowledge of mathematical logic, particularly the completeness and Loewenheim-Skolem theorems would be helpful, but it is not necessary if you are willing to take these theorems on faith; I'll state them but at most sketch the proofs.

MATH 695 Algebra Topol I **Kriz** **M/W/F 9:00 AM – 10:00 AM**

Prerequisites: Fundamental Group And Homology

This is a second year course in algebraic topology. Its purpose is to cover techniques of algebraic topology and homotopy theory which, beside topology, are needed in a variety of other mathematical fields, including algebraic number theory, algebraic geometry, and commutative algebra. In addition to basic concepts such as products in cohomology and duality, we may also venture to more advanced topics such as derived categories, simplicial sets, sheaves, spectral sequences, and generalized cohomology theories. There will be no exams, homework will be assigned and collected weekly.

MATH 697 Topics-Topology **Ji,Lizhen** **M/W/F 12:00 PM – 1:00 PM**

Topic Title: Introduction to Lie groups and homogeneous spaces

Lie groups are central objects in mathematics and are needed in many different subjects such as differential geometry, geometric and algebraic topology, algebraic geometry and number theory. Besides the rich structures of Lie groups themselves, there is also a large theory of several naturally related classes of objects: discrete subgroups of Lie groups, homogeneous spaces, and locally homogeneous spaces.

In this course, we will start from basics of Lie groups, structures of semisimple Lie groups, and then homogeneous spaces, in particular symmetric spaces and locally symmetric spaces. We will also explain their applications and connections to other subjects as mentioned above.

We will use various books and papers as references.

MATH 709 Topics in Modern Analysis I **Rudelson** **M/W/F** **10:00 AM – 11:00 AM**
Topic Title: Brownian Motion

The course will discuss the Brownian motion, which is one of the most fundamental objects in probability. Brownian motion enjoys the universality property similar to that of a normal random variable. The Central Limit Theorem asserts that a normal random variable is the limit of a scaled of sums of independent random variables. If we consider the sequence of such sums, we would obtain a random walk, and the scaled limit of such random walk will be a Brownian motion.

We will start with constructing a one-dimensional Brownian motion and then discuss both its probabilistic and analytic properties. This includes the Markov property and Donsker's theorem, which plays the role of the Central Limit Theorem, as well as the smoothness properties of the sample paths. We will also consider a multidimensional Brownian motion and discuss fractal structures associated with it.

Optional Text: Brownian Motion by Morters, Peter; Peres, Yuval. ISBN: 978-0-521-76018-8

MATH 731 Topics in Algebraic Geometry **Mustata** **M/W/F** **2:00 PM – 3:00 PM**
Topic Title: Introduction to Hodge Theory
Graduate standing. (3). (BS). May not be repeated for credit.
The goal of the course is to give an introduction to some of the basic constructions and results in Hodge theory. Topics to be covered:

1. The analytic space associated to a complex algebraic variety
2. The de Rham complex and singular cohomology
3. Compact Kahler manifolds and the Hodge decomposition (this chapter will occupy us for a large part of the semester).
4. Applications of the Hodge decomposition (the Hard Lefschetz theorem and Kodaira's vanishing theorem)
5. Variations of Hodge structures
6. Pure and mixed abstract Hodge structures
7. The mixed Hodge structure on the cohomology of a singular complex algebraic variety

Background: Familiarity with the algebraic varieties, as covered in Math 631, and with cohomology of sheaves on algebraic varieties, as covered in Math 632.

No book for this course

MATH 756 - Adv Part Diff Equ **Hani** **T/Th** **1:00 PM – 2:30 PM**
Topic Title: Nonlinear Wave Theory (Deterministic and Probablistic Methods)

This course will survey some topics on one of the most fundamental and rich classes of partial differential equations (PDE), namely nonlinear dispersive and wave equations. Such equations arise in numerous areas of physics and engineering, ranging from quantum mechanics, field theories, nonlinear optics, plasma physics, all the way to ocean science and general relativity. The underlying phenomenon in all these equations/systems is the nonlinear interaction of waves/frequencies, the study of which is commonly referred to as nonlinear wave theory.

The mathematical study of dispersive PDE has witnessed an explosion of activity in the past thirty years, as it featured a beautiful combination of ideas and tools from various fields of mathematics, most notably harmonic analysis, dynamical systems, probability, and math physics.

In this course, we will give an introduction to this rich topic, and try to convey how tools from analysis and Hamiltonian dynamics are used in the deterministic study of such equations, and how tools from probability theory can be useful to understand questions that are too complex to answer deterministically.

No book for this course.