Fall 2018 Graduate Course Descriptions

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501 AIM Student Seminar — TBD — F 1:00-2pm & 3:00-4pm

Prerequisites: *You must be a graduate student in the AIM program to register for this course.*

Course Description: The Applied and Interdisciplinary Mathematics (AIM) student seminar is an introductory and survey course in the methods and applications of modern mathematics in the natural, social, and engineering sciences. Students will attend the weekly AIM Research Seminar where topics of current interest are presented by active researchers (both from U-M and from elsewhere). The other central aspect of the course will be a seminar to prepare students with appropriate introductory background material. The seminar will also focus on effective communication methods for interdisciplinary research. MATH 501 is primarily intended for graduate students in the Applied & Interdisciplinary Mathematics M.S. and Ph.D. programs. It is also intended for mathematically curious graduate students from other areas. Qualified undergraduates are welcome to elect the course with the instructor's permission. Student attendance and participation at all seminar sessions is required. Students will develop and make a short presentation on some aspect of applied and interdisciplinary mathematics.

Text: None

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520 Life Contingencies I — Moore — TTh 11:30-1pm & 1pm-2:30pm

Prerequisites: *Math 424 and Math 425, or permission from the instructor*

Course Description: Background and Goals: Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of the Math 520-521 sequence is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. The main topics are (1) developing probability distributions for the future lifetime random variable, and (2) using those distributions to price life insurance and annuities.

Text: (Required) Actuarial Mathematics, Second Edition, Bowers et al. Publisher: Society of Actuaries; 2nd edition (May 1997) ISBN-10: 0938959468, ISBN-13: 978-0938959465

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523 Loss Models I — Marker — TTh 8:30am-10am & 10-11:30am

Prerequisites: *Math 425*

Course Description: The goals of this course are to understand parametric distributions for the purpose of (1) using parametric distributions to model the frequency, severity, and aggregate insurance losses, (2) analyzing the effects of insurance coverage modifications, and (3) simulating losses from parametric distributions.

Text: (Required) "Loss Models: From Data to Decisions" by Stuart A. Klugman, Harry H. Panjer, and Gordon E. Willmot (4^{th} Edition) ISBN: ISBN: 978-1-118-31532-3.

Fall 2018 Graduate Course Descriptions

525 Probability Theory — Barvinok — TTh 8:30-10am

Prerequisites: *MATH 451 (Required)* — TTh 10-11:30am

TBD — MWF 8:00-9am

Course Description: This is a fairly rigorous introduction to probability theory with some emphasis given to both theory and applications, although a knowledge of measure theory is not assumed. Topics covered are: probability spaces, conditional probability, discrete and continuous random variables, generating functions, characteristic functions, random walks, branching processes, limit theorems.

Text: (Optional) Probability and Random Processes by G.R. Grimmett and D.R. Stirzaker, 3rd Edition, 2001. ISBN-10: 0198572220, ISBN-13: 978-0198572220

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526 Discrete Stochastic Processes — Saplaouras — TTh 8:30-10am

Prerequisites: *MATH 525 or STATS 525 or EECS 501* — TTh 10-11:30am

Course Description: The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made for some special classes of processes and their applications. Some specific topics include: Markov chains (Markov property, recurrence and transience, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, Brownian martingales). Significant applications and examples will be an important feature of the course.

Text: (Required) Essentials of Stochastic Processes by Rick Durrett, 2nd Edition. ISBN: 9781461436140

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555 Intro to Complex Variable — Viswanath — MW 8:30-10am

Prerequisites:

Course Description: The following topics will be introduced: complex numbers, elementary functions (fractional linear transformations, exponential and trigonometric functions) in the complex plane, Cauchy-Riemann equations, complex integration, power series, Laurent expansion, residue integration, argument principle, Schwarz reflection principle, and conformal maps.

(a) Resolvent equations and linear algebra, (b) Chebyshev polynomials and series, and (c) classical aerofoil theory are among the many applications of complex analysis. We will cover (a) and (b), and if time permits, (c) will also be covered in class.

Text: There is no textbook for the class. However, students will receive a list of half a dozen books, any of which they may use, during the first lecture.

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556 Applied Functional Analysis Gilbert TTh 10-11:30am

Prerequisites: *Undergraduate analysis, linear algebra and complex variables. Some exposure to partial differential equations is desirable but not essential.*

Course Description: Introduction to topics in functional analysis that are used in applications ranging from differential equations, signal processing, and machine learning. Metric and normed linear spaces, Banach spaces and the contraction mapping theorem, Hilbert spaces and spectral theory of compact operators, distributions and Fourier transforms, Sobolev spaces and applications.

Text: (Required) Applied Analysis, Hunter and Nachtergaele

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558 Advanced Ordinary Differential Equations and Dynamical Systems TBD TTh 2:30-4pm

Prerequisites*: Basic Linear Algebra, Ordinary Differential Equations (Math 216 or 316), Multivariable Calculus (Math 215). Some exposure to more advanced mathematics, e.g. Advanced Calculus (Math 451)*

Course Description: Course Description: This course surveys a broad range of differential equations topics, with a focus on techniques and results that are useful in applications. Topics selected from: dynamics in dimension 1 and 1.5, bifurcations, Poincaré map, existence, uniqueness, and perturbation theory, linear systems, spectral theorems, linearization at equilibria for nonlinear systems, phase plane analysis of linear and nonlinear systems, stable and unstable manifolds, Lyapunov functions, gradient flows and Hamiltonian systems, periodic solutions, stability, omega-limit set, Poincaré-Bendixson and Bendixson-duLac Theorems, bifurcation theory, and chaotic dynamics.
Text: (Required) Differential Equations, Dynamical Systems, and an Introduction to Chaos, M. Hirsh, S. Smale, and R. Devaney, 3rd edition, ISBN: 978-0123820105

Fall 2018 Graduate Course Descriptions

565 Combinatorics and Graph Theory TBD TTh 10:00-11:30am

Prerequisites: *Linear Algebra, experience with proofs and abstract mathematics*

Course Description: The goal of the course is to provide an introduction to some basic notions, techniques, and results in combinatorics. The first part of the course will be devoted to graph theory. Topics we may discuss include graph colorings, Ramsey theory, extremal graph theory, and planar graphs. The second part of the course will cover a number of topics related to partially ordered sets, matroids, projective geometries and hyperplane arrangements.

Text: : (Required) A Course in Combinatorics, J. H. Van Lint and R. M. Wilson, Second Edition, ISBN: 9780521567244

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568 Mathematical & Computational Neuroscience Booth MW 10-11:30am

Prerequisites: *Differential equations (215 or 316), linear algebra (217 or 417), for undergraduates MATH 463 required*

Course Description: Computational neuroscience investigates the brain at many different levels, from single cell activity to small, local network computation to the dynamics of large neuronal populations. This course introduces modeling and quantitative techniques used to investigate neural activity at all these different levels.

Topics to be covered include: Passive membrane properties, the Nernst potential, derivation of the Hodgkin-Huxley model, action potential generation, action potential propagation in cable and multi-compartmental models, reductions of the Hodgkin-Huxley model, phase plane analysis, bifurcation analysis, synaptic currents, excitatory and inhibitory network dynamics.

Text: None

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571 Numerical linear algebra Borcea TTH 8:30-10am

Prerequisites: *Linear algebra on the level of Math 417, 419 or Math 513, working knowledge of MATLAB*

Course Description: This course is an introduction to numerical methods for solving linear systems of equations and for computing eigenvalues of a matrix. Topics include singular value decomposition, QR factorization, Gram-Schmidt orthogonalization, least squares problems, condition number, Gaussian elimination, iterative methods (Arnoldi, GMRES, conjugate gradient), preconditioning, methods for computing eigenvalues (e.g. power method, QR algorithm, shifts). The course should be useful for students in applied and computational mathematics, and in any area of scientific computing and engineering.

Text: (Required) Numerical Linear Algebra, L.N. Trefethen and D. Bau. ISBN: ISBN-13: 978-0898713619

Fall 2018 Graduate Course Descriptions

573 Financial Math I — Bayraktar — TTh 1-2:30pm

Prerequisites: *A good knowledge of probability theory, real analysis, and linear algebra is essential for this course. Familiarity with basic notions of measure theory and discrete-time stochastic processes is helpful.*

Course Description: This is an introductory course in Financial Mathematics. This course starts with the basic version of Mathematical Theory of Asset Pricing and Hedging (Fundamental Theorem of Asset Pricing in discrete time and discrete space). This theory is applied to problems of Pricing and Hedging of simple Financial Derivatives. Finally, the continuous time version of the proposed methods is presented, culminating with the Black--Scholes model. A part of the course is devoted to the problems of Optimal Investment in discrete time (including Markowitz Theory and CAPM) and Risk Management (VaR and its extensions). This course shows how one can formulate and solve relevant problems of financial industry via mathematical (in particular, probabilistic) methods. Although Math 526 is not a prerequisite for Math 573, it is strongly recommended that either these courses are taken in parallel, or Math 526 precedes Math 573.

Text: (Required) Stochastic Finance: An Introduction in Discrete Time, Hans Föllmer, Alexander Schied, 4th edition. ISBN: 978-3110463446.

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591 Differentiable Manifolds — Uribe — MWF 10-11:00am

Prerequisites: *Math 451, 452, 590*

Course Description: This is one of the basic courses for students beginning the PhD program in mathematics. The approach is rigorous and emphasizes abstract concepts and proofs. The first 2-3 weeks of the course will be devoted to general topology, and the remainder of the course will be devoted to differentiable manifolds.
Note: this course has been fairly recently restructured and is more advanced than it was in years past.

Topics may include: Product and quotient topology, topological manifolds, smooth manifolds, manifolds with boundary, smooth maps, partitions of unity, tangent vectors and differentials, the tangent bundle, vector fields, Lie brackets, submersions, immersions and embeddings, smooth submanifolds, Sard's theorem, the Whitney embedding theorem, transversality, Lie groups, multilinear algebra, vector bundles, differential forms, exterior derivatives, orientation, De Rham cohomology groups, homotopy invariance, degree theory.

Text: (Required) An Introduction to Manifolds, by Loring W. Tu. Second Edition.
ISBN: 978-1-4419-7399-3

Fall 2018 Graduate Course Descriptions

593 Algebra 1 TBD MWF 2:00-3:00pm

Course Description: Math 593 and Math 594 provide an introduction to algebra for incoming PhD students, and these courses provide excellent preparation for the PhD Qualifying Exam in algebra. It is assumed that students have had a year long sequence in algebra at the advanced undergraduate level (e.g., Math 493-494).

Math 593 will cover: (1) basic ring theory, including localization, normalization, PIDs, UFDs, DVRs and valuations, the prime and maximal spectra of a ring; (2) the theory of modules over a ring, including tensor and exterior algebra, direct and inverse limits, the rudiments of homological algebra; (3) the structure theory of finitely generated modules over a PID with applications to classification theorems in linear algebra; and (4) bilinear algebra, including symmetric, hermitian, and alternating maps.

Text: (Optional) Lang's "Algebra". Material will also be drawn from the more advanced parts of Dummit and Foote's "Abstract Algebra".

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596 Analysis I Burns MWF 11:00am-12:00pm

Course Description: This is a theoretical and rigorous introductory course for complex analysis at the beginning graduate level. Topics to be discussed are directed towards the departmental qualifying review examination in analysis and include complex elementary functions, conformal mapping, the Riemann sphere, linear fractional transformations, rational functions ; complex derivatives, Cauchy-Riemann equations; contour integration, Cauchy's theorem, Cauchy-Green formula, Cauchy's integral formula and consequences; power series expansion and consequences; harmonic functions, maximum principle, Dirichlet's problem; isolated singularities, residues, application to computation of definite integrals; meromorphic functions, argument principle, Rouche's theorem; equicontinuity, Montel's theorem, Schwartz's lemma, Riemann mapping theorem.

Homework will be assigned weekly, and midterm and final exams will be given. Highly advanced undergraduate students may also take this course but they should also consider the option of taking the alternative course 555.

Text: (Required) Complex Analysis, T. Gamelin
(Optional, Recommended) Complex Analysis, L. Ahlfors.

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602 Real Analysis II Jonsson MWF 12:00-1:00pm

Prerequisites: *Linear algebra, real and complex analysis, point set toplogy*

Course Description: This is a course on functional analysis, the study of vector spaces endowed with additional structure such as a topology, norm, or inner product.
It is a core subject in analysis, and of crucial importance to the study of PDEs, harmonic analysis, probability theory, quantum mechanics, computer science...

Topics to be covered include a review of linear algebra, convexity, normed and topological vector spaces, Banach spaces, Hilbert spaces, the Hahn-Banach theorem, the Baire category theorem and consequences, compact operators, Fredholm Theory and spectral theory. Time permitting, we will also cover some applications to classical analysis and partial differential equations.

Text: (Required) Functional Analysis, Sobolev Spaces and Partial Differential Equations by H. Brezis

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614 Commutative Algebra Jeffries TTH 11:30am-1:00pm

Prerequisites: *Math 593 and Math 594 or permission of instructor*

Course Description: The course is an introduction to commutative algebra, a subject that has interactions with algebraic geometry, number theory, combinatorics, and several complex variables. The emphasis will be on Noetherian rings.

Topics will include the Noetherian property, integral extensions, Hilbert's Nullstellensatz, Noether normalization, localization, chains of prime ideals, Krull dimension, Artinian rings, normal Noetherian rings, flatness, primary decomposition, symbolic powers, Rees rings and the Artin-Rees Lemma, the Krull height theorem, and completion of local rings.

There is no required text for the course. Recommended sources include: "Introduction to Commutative Algebra" by Atiyah and Macdonald, Mel Hochster's 614 lecture notes, "Commutative Ring Theory" by Hideyuki Matsumura, "Introduction to Commutative Algebra and Algebraic Geometry" by Ernst Kunz, and Mel Hochster's 614 lecture notes.

Text: None

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623 Computational Finance Guo TTh 10-11:30am

Prerequisites*: Differential equations (e.g. Math 316); probability theory (e.g. Math 525/526, Stat 515); numerical analysis (Math 471 or Math 472); mathematical finance (Math 423 or Math 574, Math 506 or permission from instructor); programming (e.g. C, Matlab, Mathematica, Java).*

Course Description: This is a course in computational methods in finance and financial modeling. The course starts with the introduction of numerical methods for solving differential equations of evolution, including partial differential equations (PDEs) of parabolic type.

Convergence and stability of explicit and implicit numerical schemes are analyzed. Examples include the generalized Black-Scholes PDE for pricing European, American and Asian options. Another part of the course is concerned with Monte Carlo methods. This includes pseudo random number generators (with applications to option pricing) and numerical methods for solving stochastic differential equations (with applications to stochastic volatility models). A part of the coursework requires programming in a high-level language.

Text: (Optional) Tools for Computational Finance by Rüdiger U. Seydel, 5th Edition.
ISBN: 978-1447129929

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625 Probability and Random Processes I: Rudelson TTh 10-11:30am

Prerequisites: *Math 597 or an equivalent course in measure theory is required. Some knowledge of undegraduate probability is desirable, but not required.*

Course Description: This is a graduate level measure theory based probability course. The topics include:
- Independence of random variables and zero-one laws.
-Main limit theorems: the law of large numbers and central limit theorem. Both laws deal with the average of N independent identically distributed random variables as N tends to infinity. The law of large numbers asserts that this average converges to the expectation, and the central limit theorem proves that the deviation of the average from the expectation converges to a normal random variable.
-Elements of measure concentration and large deviation theory. These are newer directions in probability, inspired by applications. In many questions of combinatorics, statistics and computer science the number of random variables is large, but fixed. In this case limit theorems provide valuable intuition. However, definite results require explicit probability estimates valid for a fixed dimension. We will prove non-asymptotic counterparts of the law of large numbers and the central limit theorem.
-Conditional expectation and its properties.
-Martingales. Martingales are special sequences of random variables arising in many applications, ranging from harmonic analysis to mathematical finance and computer science. We will consider convergence, stopping times, as well as measure concentration for martingales.

Text: (Optional) Probability: theory and examples, by R. Durrett, Fourth edition. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, Cambridge, 2010. ISBN: 978-0-521-76539-8

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631 Algebraic Geometry I: Speyer MWF 11:00am-12pm

Course Description: This course is an introduction to geometry of algebraic varieties over an algebraically closed field. The intended audience is students who either already know or are concurrently taking commutative algebra, and have seen at least a little bit of topology and of manifolds. Here is an approximate list of the topics I plan to cover:

1) Affine varieties, the Nullstellansatz and decomposition into irreducible components
2) Projective varieties and important classical examples
3) Finite maps and the principle of conservation of number
4) Dimension theory
5) Tangent spaces and differential forms
6) Curves, building up to the Riemann-Roch theorem

Text: Required, "Basic Algebraic Geometry I", by Shafarevich ISBN 978-3642427268

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636 Topics in Differential Geometry: Entropy and Rigidity in Dynamics — Spatzier — TTh 11:30-1pm

Prerequisites:

Course Description: This course will discuss the notions of entropy and exponential growth in dynamical systems and geometry.

Entropy is a key concept in dynamics and ergodic theory, closely related to exponential growth phenomena. I will first discuss the basic ideas, examples and constructions, of both measure theoretic and topological entropy. Classically, entropy has served as an invariant of dynamical systems, distinguishing Bernoulli systems for example, in fact classifying them up to conjugacy by Ornstein's theorem. Later, smooth dynamicists found important relations between Lyapunov exponents and entropy, investigated measures of maximal entropy, and more generally pressure an equilibrium states.

Entropy has also played a central role in several major rigidity results in geometry and dynamics, in particular measure rigidity of unipotent flows (Ratner's theorem) and higher rank actions by abelian groups. Lately, the absence of entropy or sub-exponential behavior has proved crucial in the proof of Zimmer's conjecture on non-existence of actions of higher rank lattices on low dimensional manifolds or in the solution of Burnside's problem for volume preserving diffeomorphisms of the 2-sphere.
I am planning to discuss a good many of these results, and the relevance of entropy in the arguments.

A background in basic real analysis and manifold theory (591 and 597) will be assumed.

Text: None

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654 Fluid Dynamics — Alben — TTH 1-2:30pm

Prerequisites: *Vector calculus. Complex analysis and differential equations at a senior undergraduate level (Math 450). Elementary physics (mechanics).*

Course Description: The course is a broad introduction to fluid dynamics. We'll use classical applied mathematical tools to gain physical insight into fluid phenomena. No knowledge of fluid mechanics is assumed, but some upper-level undergraduate mathematics is (see pre-reqs below). Topics: Continua and conservation laws. Inviscid flow. Irrotational flow. Vorticity. Complex variable methods. Airfoil theory and conformal transformations. Viscosity and the Reynolds number. The limit of zero Reynolds number. Boundary layers. Flow instabilities.

Text: (Required) Elementary Fluid Dynamics by D. J. Acheson

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656 Introduction to Partial Differential Equations Conlon TTH 2:30-4pm

Prerequisites: *A solid knowledge of advanced calculus: Math 451, 452.*

Course Description: Partial differential equations are at the core of models in science, technology, economics and related fields. These equations and their solutions have interesting structures that are studied by methods of analysis, geometry, probability and other mathematical fields. This course will introduce students in pure and applied mathematics to concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations. Topics to be covered will include nonlinear first order equations, linear elliptic, hyperbolic and parabolic equations. The method of characteristics, energy methods, maximum principle, Fourier transform and Sobolev spaces. Course material will be taken mainly from the first chapters of Evans and John.

Text: (Optional) Partial Differential Equations by L.C. Evans, 2nd Edition. ISBN: 978-0-8218-4974-3
(Optional) Partial Differential Equations by Fritz John, Springer, ISBN: 978-0-387-90609-6

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665 Combinatorial Theory II: Combinatorial Representation Theory Stembridge MWF 1-2:00pm

Course Description: This course will be an introduction to combinatorial representation theory.

Motivations run in two directions: Some of the most interesting results in combinatorics have been derived by means of representation-theoretic tools. Enumeration of plane partitions, unimodality theorems, and the Rogers-Ramanujan identities are all examples of this. In the opposite direction, the symmetry groups that occur most frequently in nature (the symmetric groups, the classical groups) have representations with intrinsic combinatorial structure.

The course will be divided into three unequal parts.

Part 0 will be a self-contained development of the representation theory of finite groups in characteristic 0, and a less self-contained discussion of compact groups.

Part 1 will be a detailed study of the representation theory of the symmetric groups and closely related groups, and the applications thereof. If there is time, we hope to discuss the W-graphs of Kazhdan-Lusztig theory.

Part 2 will be concerned with a combinatorial approach to the representations of GL(n) and related groups (SL(n), U(n), etc).

We will not follow any text, but "Representation Theory: A First Course" by Fulton and Harris (Springer-Verlag, 1991) is a good companion for the representation theory side. Previous exposure to combinatorics or group representations will be advantageous but not necessary.

Text: None

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671 Numerical Methods I: Fast Algorithms — Veerapaneni — TTH 11:30am-1pm

Prerequisites: *Numerical analysis, basic theory of ordinary and partial differential equations*

Course Description: This course will cover selected topics in fast algorithms research. Emphasis will be given on techniques that can be used for discretization and computational solution of partial differential equations. Topics will include:

• Complexity Analysis, review of linear & nonlinear solvers, Krylov subspace methods
• Cartesian grid methods, multigrid and multiresolution methods
• Fourier spectral methods, nonuniform FFTs, butterfly Algorithms, Ewald summation
• Numerical Methods & fast algorithms for integral equations, fast Gauss transform, adaptive algorithms, tree codes, fast multipole methods

Text: None

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676 Algebraic Number Theory — Zieve — TTH 10-11:30am

Prerequisites:

Course Description:
Text: None

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678 Modular Forms: Arithmetic of L-functions and p-adic L-functions — Prasanna — TTH 2:30-4pm

Prerequisites: *Some familiarity with the theory of modular forms and modular curves, and some basic algebraic geometry (curves, abelian varieties, Jacobians)*

Course Description: This class will focus on applications of modular forms to understanding special values of L-functions and p-adic L-functions. Topics to be covered include: modular forms and p-adic modular forms for GL(2), quaternion algebras and the Jacquet-Langlands correspondence, periods of modular forms, Heegner points and Heegner cycles, Gross-Zagier and its generalizations. If time permits, we will discuss the arithmetic of automorphic forms on other reductive groups and applications of the Langlands program to problems on algebraic cycles.

Text: None

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681 Mathematical Logic Blass TTh 1-2:30pm

Prerequisites: *No specific topics, but mathematical maturity appropriate for a 600-level course*

Course Description: Historically, mathematical logic is the mathematical study of mathematics itself, especially of the process of deductive reasoning. A central question is the adequacy of deductive reasoning: Can all the consequences of a set of assumptions be obtained from those assumptions by a sequence of very simple inferences? This question is an instance of a general theme that runs through much of mathematical logic, namely the interplay between mathematical statements (which are to be manipulated in the desired simple inferences) and the mathematical structures that they describe (which underlie the notion of consequence).

Part of Math 681 is devoted to making this interplay precise and establishing a positive answer to the central question in some situations, including the particularly important case of first-order logic. This logic with its simple inferences serves as the explicit or implicit foundation for essentially all mathematical reasoning. In this connection, I'll also discuss at least one alternative way of verifying correctness in first-order reasoning. This way, though farther from intuition than the simple inferences mentioned above, is in general much more efficient and has found applications in automated theorem-proving.

Another part of the course will indicate why first-order logic plays such a central role in mathematics. It is easy to produce logical systems stronger than first-order logic, and one might be tempted to use them as an improved foundation for mathematics. But the notions of "consequence" for these systems are necessarily far more complicated and cannot be captured by any reasonable step-by-step inferences. A mathematics based on them could not have proofs in anything like the usual sense of the term.

A third part of the course uses the interplay between mathematical statements and the structures they describe, to establish some results in other areas of mathematics. These results do not directly involve logical matters, but their proofs are based on the results from logic proved earlier in the course. One example is the existence of the models used in non-standard analysis.

Text: (Optional) Fundamentals of Mathematical Logic, Peter G. Hinman**.** ISBN: 1568812620

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695 Algebraic Topology I — Kriz — MWF 9:00am-10:00am

Prerequisites: *Math 592, or a first course in algebraic topology*

Course Description: In the 1990's, this course served to treat some topics which do not fit into 592, such as cohomology, Tor, Ext, products and Poincare duality. The present course mentions these topics, and sets them into a framework of modern homotopy theory in the language of derived categories and derived functors. We will learn how to build and use derived categories of chain complexes, spaces, simplicial sets and spectra (i.e. "spaces stabilized under suspension") via the technique of (co)localization. Time permitting, derived categories of sheaves may also be mentioned. We will also talk about abstract duality, and show how it applies to topology. Weekly homework will be assigned and graded. There are no exams.

Text: None

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697 Topics in Topology: Introduction to Riemann surfaces — Ji — MWF 12:00-1pm

Course Description: Riemann surfaces have played a fundamental role in mathematics
since they were introduced by Riemann in his thesis in 1851.
They are 1-dimensional complex manifolds and algebraic curves.
Besides their own interest, they are basic ingredients in Teichmuller theory
and the theory of moduli spaces. They have also motivated
a lot of problems and results on higher dimensional complex manifolds and algebraic varieties.
As Donadlson wrote in the preface to his recent book on Riemann surfaces:
``The theory of Riemann surfaces occupies a very special place in mathematics.
It is a culmination of much of traditional calculus, making surprising
connections with geometry and arithmetic. It is an extremely useful part of
mathematics, knowledge of which is needed by specialists in many other fields.
It provides a model for a large number of more recent developments in areas
including manifold topology, global analysis, algebraic geometry, Riemannian
geometry and diverse topics in mathematical physics."
In this course, we will start from basics and give an introduction to important results
on Riemann surfaces such as the Riemann-Roch Theorem, the uniformization theorem,
and some basic results on deformation and moduli spaces of Riemann surfaces,
and the geometry of algebraic curves.
We will indicate directions of how results on Riemann surfaces can be generalized.
The book by Donaldson will be one of the main references.
We will use other books and papers.

Text: (Optional) Riemann Surfaces by Simon Donaldson. ISBN: 199606749
(Optiional) H. Farkas, I. Kra, Riemann surfaces. Second edition. Graduate Texts in Mathematics, 71. Springer-Verlag, New York, 1992. xvi+363 pp.

(Optional) Phillips Griffiths, Introduction to algebraic curves. Translated from the Chinese by Kuniko Weltin. Translations of Mathematical Monographs, 76. American Mathematical Society, Providence, RI, 1989. x+221 pp.

(Optional) R. Miranda, Algebraic curves and Riemann surfaces. Graduate Studies in Mathematics, 5. American Mathematical Society, Providence, RI, 1995. xxii+390 pp.

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709 Modern Analysis I: Integrable Probability — Baik — TTh 1-2:30pm

Course Description: This course is about probabilistic systems that can be analyzed by exact methods. The simplest such example in elementary probability is the result of De Moivre and Laplace who considered the sum of i.i.d. Bernoulli random variables in the 18th and the 19th century. They evaluated the probability distribution of the sum explicitly in terms of factorials and then used the Striling's formula to deduce that the fluctuations converge to the normal distribution. Of course this is a special case of the central limit theorem which is established in the 20th century using a method that is applicable to general random variables rather than only to the Bernoulli random variable. In modern probability theory, the fluctuations of more complicated systems are of great interest but often times the analogue of the central limit theorem is lacking for general random variables. This is especially the case for the KPZ universality class (named after the influential work of Kardar-Parisi-Zhang in 1986.) However, over the last 20 years, a growing list of models in the KPZ class are found to be analyzable exactly; we are in the ``De Moivre and Laplace'' stage for this class. We will discuss some of these ``integrable" models in this class.

The ``integrable" models come in variety of contexts; random growth processes, interacting particle systems, random tiling problems, and random matrices. Such models are analyzed in two steps; the first part is algebraic and the second part is analytic. The algebraic part is to compute the distribution or the moment-generating function exactly. This is often done with the help of symmetric functions or other combinatorial methods. The analytic part is to compute the formula obtained by algebraic tools asymptotically. This is an analogue of proving the Stirling's formula for factorials. This often involves method of steepest-descent. We will emphasize both aspects in this class; the claim that one found an ``explicit" formula depends on whether one can perform the asymptotic analysis in the end.

The concrete topics we plan to discuss include: Schur measures, asymmetric simple exclusion processes, Macdonald processes, and stochastic six-vertex models. We do not assume any advanced background on probability, combinatorics, and analysis other than the first-year analysis courses (complex and real.) In particular, the combinatorial component will be self-contained. The final grade is based on short presentations.

Text: None

711	Advanced Algebra: Tensors	Derksen	MWF 10:00am-11:00am

Prerequisites: *Knowledge of basic algebraic geometry and commutative algebra*

Course Description: A order d tensor is an element in a tensor product of d vector spaces. Such tensor can be viewed as d-dimensional arrays. Order 2 tensors are matrices. The matrix rank generalizes to higher order tensors, but the behavior of the tensor rank becomes more complicated. We will study various notions of rank such as the tensor rank, border rank and symmetric rank and algebraic methods for obtaining lower and upper bounds. We will explore the connections between tensor rank and the algebraic complexity of matrix multiplication, and Valiants algebraic P vs. NP problem. The study of tensors that are invariant under a group action is related to classical invariant theory and representation theory. We will also discuss various applications, for example in chemometrics, and quantum computing. We will not follow any book closely, but some of the sources used include:

J.M. Landsberg, Tensors: Geometry and Applications, Graduate Studies in Mathematics 128, American Mathematical Society, 2012.

Peter Buergisser, Michael Clausen, Amin Shokrollahi, Algebraic Complexity Theory, Springer, 1997.

Hanspeter Kraft, Claudio Procesi, Classical Invariant Theory - A Primer. Can be downloaded from http://kraftadmin.wixsite.com/hpkraft

Text: None