Winter 2020 Graduate Course Descriptions

MATH 501	**AIM Student Seminar**	**Alben**	**Fri 1:00 PM – 2:00 PM** **Fri 2:00 PM – 3:00 PM**

At least two 300 or above level math courses, and Graduate standing; Qualified undergraduates with permission of instructor only. (1). May be repeated for a maximum of 6 credits. Offered mandatory credit/no credit.

MATH 501 is an introductory and overview seminar course in the methods and applications of modern mathematics. The seminar has two key components: (1) participation in the Applied and Interdisciplinary Math Research Seminar; and (2) preparatory and post-seminar discussions based on these presentations. Topics vary by term.

No book for this course.

MATH 506/IOE	**Stochastic Analysis for Finance**	**Bayraktar, E.**	**TR 10:00 AM - 11:30 AM**

Math 526. Graduate students or permission of instructor. (3). (BS). May not be repeated for credit.

The aim of this course is to teach the probabilistic techniques and concepts from the theory of stochastic processes required to understand the widely used financial models. In particular concepts such as martingales, stochastic integration/calculus, which are essential in computing the prices of derivative contracts, will be discussed. The specific topics include: Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales), martingales in continuous time, stochastic integration (including It\^{o}'s formula), stochastic differential equations (including Feynman-Kac formula), change of measure (including Girsanov's theorem and change of numeraire), and, time permitting, stochastic control (including Merton problem). Applications from various areas of Finance (including, pricing of derivatives, risk management, etc) are used to illustrate the theory.

MATH 521	**Life Contingencies II**	**Natarajan, R.**	**TR 10:00 AM - 11:30 AM**

MATH 520 with a grade of C- or higher. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

This course extends the single decrement and single life ideas of MATH 520 to multi-decrement and multiple-life applications directly related to life insurance. The sequence 520-521 covers the Part 4A examination of the Casualty Actuarial Society and covers the syllabus of the Course 150 examination of the Society of Actuaries. Concepts and Calculation are emphasized over proof.

MATH 524	**Loss Models II**	**Young, J.** **TBD**	**TR 8:30 AM - 10:30 AM** **TR 1:00 PM - 2:30 PM**

STATS 426 and MATH 523. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Risk management is of major concern to all financial institutions, especially casualty insurance companies. This course is relevant for students in insurance and provides background for the professional examination in Short-Term Actuarial Modeling offered by the Society of Actuaries (Exam STAM). Students should have a basic knowledge of common probability distributions (Poisson, exponential, gamma, binomial, etc.) and have at least Junior standing.

Content: Frequentist and Bayesian estimation of probability distributions, model selection, credibility, simulation, and other topics in casualty insurance.

MATH 525/STATS	**Probability Theory**	**Deng, S.** **TBA**	**TR 10:00 AM - 11:30 AM** **TR 1:00 PM - 2:30 PM**

MATH 451 (strongly recommended). MATH 425/STATS 425 would be helpful. (3). (BS). May not be repeated for credit.

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

MATH 526/STATS	**Stochastic Processes with Discrete State Spaces**	**Chakraborty, S.** **Chakraborty, S.** **Conlon, J.**	**TR 8:30 AM - 10:00 AM** **TR 10:00 AM - 11:30 AM** **TR 1:00 PM - 2:30 PM**

MATH 525 or STATS 525 or EECS 501. (3). (BS). May not be repeated for credit.

The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include: Markov chains (Markov property, recurrence and transience, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times, Markov queues); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales). Significant applications will be an important feature of the course.

Winter 2020 Graduate Course Descriptions

MATH 547/BIOINF/STATS Probabilistic Modeling in Bioinformatics TR 4:00 PM - 5:30 PM
MATH, Flexible, due to diverse backgrounds of intended audience. Basic probability (level of MATH/STATS 425), or molecular biology (level of BIOLOGY 427), or biochemistry (level of CHEM/BIOLCHEM 451), or basic programming skills desirable or permission. (3). (BS). May not be repeated for credit.

Probabilistic models of proteins and nucleic acids. Analysis of DNA/RNA and protein sequence data. Algorithms for sequence alignment, statistical analysis of similarity scores, hidden Markov models. Neural networks, training, gene finding, protein family profiles, multiple sequence alignment, sequence comparison and structure prediction. Analysis of expression array data.

MATH 550/CMPLXSYS 510 Introduction to Adaptive Systems TBD MW 8:30 AM - 10:00 AM
Prerequisites: Equivalent of four semesters of calculus (especially familiarity with differential equations) and working knowledge of elementary linear algebra and probability.

This course is an introduction to applications and integration of dynamical systems and game theory to model population and ecological dynamics and evolutionary processes. Topics include Lotka-Volterra systems, non-cooperative games, replicator dynamics and genetic mechanisms of selection and mutation, and other adaptive systems.

Attend lectures and complete the substantial homework assignments. There will also be a midterm test and the final exam.

Applied math and other theoretically and mathematically minded natural science and engineering students. Students of biology, ecology, economics and other social sciences, natural resources, bioengineering, and bioinformatics—with suitable math backgrounds—are especially welcome.

MATH 555 Introduction to Complex Variables Viswanath, D. MW 8:30 AM - 10:00 AM
MATH 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

The following topics will be introduced: complex numbers, elementary functions (fractional linear transformations, exponential and trigonometric functions) in the complex plane, Cauchy-Riemann equations, complex integration, power series, Laurent expansion, residue integration, argument principle, Schwarz reflection principle, and conformal maps.

Applications: (a) Resolvent equations and linear algebra, (b) classical aerofoil theory, and (c) Chebyshev polynomials are among the many applications of complex analysis. We will cover (a) and (b), and if time permits, (c) will also be covered in class.

There is no textbook for the class. However, students will receive a list of half a dozen books, any of which they may use, during the first lecture.

MATH 557 Applied Asymptotic Analysis Krasny, R. TR 1:00 PM - 2:30 PM
MATH 217, 419, or 420; MATH 451; and MATH 555 or 596. (3). (BS). May not be repeated for credit.

Math 557 is an introduction to the techniques of asymptotic analysis commonly used in science and engineering. The topics include: asymptotic expansions, method of steepest descent, method of stationary phase, asymptotic evaluation of Fourier and Laplace transforms, WKB method, turning points, singular perturbations, method of multiple scales, matched asymptotic expansions, boundary layers, plus other topics as time permits.

Required Textbook: **Asymptotic Analysis**, J.D. Murray, latest edition, ISBN-10 0387909370
Optional: **Applied Asymptotic Analysis** by P.D. Miller, AMS, ISBN 0-8218-4078-9

MATH 561 / IOE / TO Linear Programming I MW 9:00 AM - 10:30 AM
MATH 217, 417, or 419. (3). (BS). May not be repeated for credit. F, W, Sp.

Formulation of problems from the private and public sectors using the mathematical model of linear programming. Development of the simplex algorithm; duality theory and economic interpretations. Postoptimality (sensitivity) analysis application and interpretations. Introduction to transportation and assignment problems; special purpose algorithms and advanced computational techniques. Students have opportunities to formulate and solve models developed from more complex case studies and to use various computer programs.

MATH 562 / IOE 511 Continuous Optimization Methods Epelman,Marina A TR 9:00 AM - 10:30 AM
MATH 217, 417, or 419. (3). (BS). May not be repeated for credit.

Survey of continuous optimization problems. Unconstrained optimization problems: unidirectional search techniques; gradient, conjugate direction, quasi-Newton methods. Introduction to constrained optimization using techniques of unconstrained optimization through penalty transformations, augmented Langrangians, and others. Discussion of computer programs for various algorithms.

MATH 566 Combinatorial Theory Stembridge, J. MWF 11:00 AM - 12:00 PM
MATH 465 group theory and abstract linear algebra. (3). (BS). May not be repeated for credit.

Algebraic Combinatorics

This course will be a graduate-level introduction to algebraic combinatorics. Previous exposure to combinatorics will not be necessary, but experience with algebra including basic group theory and abstract linear algebra will be needed.

Most of the topics we cover will be centered around enumeration and generating functions. But this is not to say that the course is only about enumeration--counting formulas are often manifestations of of deeper structure.

Highlights along the way should include cancellation methods, combinatorial factorization, Lagrange inversion, the permanent-determinant method, the matrix-tree theorem, the transfer matrix method, and exponential generating functions from a categorical perspective.

Note that the text is available as a free download from the author's website. **Enumerative Combinatorics, Vol. 1**, Richard P. Stanley, 2nd edition.

MATH 567 Introduction to Coding Theory Zheng, H. TR 2:30 PM - 4:00 PM
One of MATH 217, 419, 420. (3). (BS). May not be repeated for credit.

We will cover Chapters 1-5 and a few topics from Chapter 6 of the course textbook. This includes introduction to entropy, Shannon's theorem and channel capacity, noiseless coding theorem and data compression, basic examples of codes including Golay, Hamming, BCH, Reed-Muller, Reed-Solomon codes, linear codes and cyclic codes, introduction to decoding and fundamental asymptotic bounds on coding efficiency.

Required Textbook: **Introduction to Coding and Information Theory**, by S. Roman

MATH 571 Numerical Linear Algebra TBD TR 8:30 AM - 10:00 AM
Borcea TR 10:00 AM - 11:30 AM
MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454 or permission from the instructor.. (3). (BS). May not be repeated for credit.

This course is an introduction to numerical linear algebra, a core subject in scientific computing. Three types of problems are considered: (1) linear systems, (2) eigenvalues and eigenvectors, and (3) least squares problems. Topics include: Gram-Schmidt orthogonalization, QR factorization, singular value decomposition (SVD), normal equations, vector and matrix norms, condition number, backward error analysis, LU factorization, Cholesky factorization, reduction to Hessenberg and tridiagional form, power method, inverse iteration, rayleigh quotient iteration, QR algorithm, Krylov subspace methods, Arnoldi iteration, GMRES, steepest descent, conjugate gradient method, preconditioning.

Required: **Numerical Linear Algebra**, by Lloyd N. Trefethen and David Bau; ISBN-13: 978-0898713619

MATH 572 Numerical Methods for Differential Equations Esedoglu, S. TR 11:30 AM - 1:00 PM
MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit.

Course Description: Math 572 is an introduction to numerical methods for differential equations, focusing on finite differences. This is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program, and should also appeal to graduate students from engineering and science departments, or anyone interested in scientific computing. It covers methods for ordinary and partial differential equations, including derivation of numerical schemes and systematic study of their accuracy, stability, and convergence. A solid background in advanced calculus and linear algebra, and proficiency in a computer language such as C, Fortran, or Matlab is a must.

Topics:
Finite differences, their derivation and truncation error. Two point boundary value problems, elliptic equations. Consistency, stability, and convergence. Efficient solution of resulting sparse linear systems (Jacobi, Gauss-Seidel, SOR, conjugate gradients, preconditioning). Multistep, Runge-Kutta methods for initial value problems. Absolute stability, stiff problems, and A-stability. Barrier theorems. Explicit and implicit finite difference schemes for parabolic equations. Stability and convergence analysis via the maximum principle, energy methods, and the Fourier transform. Operator splitting techniques, the alternating direction implicit method. Advection equation. Lax-Wendroff, upwind methods, the CFL condition. Hyperbolic systems, initial boundary value problems.

Textbook**:** (Required) **Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems** by R.J. LeVeque, ISBN: 978-0-898716-29-0

MATH 574 Financial Mathematics II Norgilas, D. TR 11:30 AM - 1:00 PM
MATH 526 and MATH 573. (Prerequisites enforced at registration.) Although MATH 506 is not a prerequisite for MATH 574, it is strongly recommended that either these courses are taken in parallel, or MATH 506 precedes MATH 574. (3). (BS). May not be repeated for credit.

This course discusses Mathematical Theory of Continuous-time Finance. The course starts with the general Theory of Asset Pricing and Hedging in continuous time and then proceeds to specific problems of Mathematical Modeling in Continuous-time Finance. These problems include pricing and hedging of (basic and exotic) Derivatives in Equity, Foreign Exchange, Fixed Income and Credit Risk markets. In addition, this course discusses Optimal Investment in Continuous time (Merton's problem), High-frequency Trading (Optimal Execution), and Risk Management (e.g. Credit Value Adjustment).

Required Text: Arbitrage Theory in Continuous Time, by Tomas Björk, 3rd 978- 0199574742

MATH 575 Introduction to Theory of Numbers I Prasanna, K. TR 10:00 AM - 11:30 AM

MATH 451 and 420 or permission of instructor. (1 - 3). (BS). May not be repeated for credit.

This course will be an introduction to number theory. Basic topics to be covered include factorization, congruences, Gauss and Jacobi sums, classical reciprocity laws such as quadratic and cubic reciprocity and some basic algebraic number theory.

No formal prerequisites, but some familiarity with abstract algebra, including the theory of groups, rings and fields will be expected.

Required Textbook: **A classical introduction to modern number theory**,(Springer GTM 84), by Ireland and Rosen, 2nd edition, ISBN: 978-0387973296, (Note: this textbook is available free to UM users on SpringerLink).

MATH 582 Introduction to Set Theory Cho, S. TR 1:00 PM - 2:30 PM

MATH 412 or 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

The main topics covered are set algebra (union, intersection), relations and functions, orderings (partial, linear, well), the natural numbers, finite and denumerable sets, the Axiom of Choice, and ordinal and cardinal numbers.

MATH 590 Introduction to Topology Heffers, J. MWF 12:00 PM - 1:00 PM
MATH 451. (3). (BS). May not be repeated for credit. Rackham credit requires additional work.

Topics include metric spaces, topological spaces, continuous functions and homeomorphisms, separation axioms, quotient and product topology, compactness, and connectedness. We will also cover a bit of algebraic topology (e.g., fundamental groups) as time permits.

Required Textbook: **Topology**, by James Munkres, ISBN: 978-0134689517

MATH 592 Introduction to Algebraic Topology Kriz, I. MWF 10:00 AM - 11:00 AM
MATH 591. (3). (BS). May not be repeated for credit.

Algebraic topology studies topological invariants, i.e. algebraic structures constructed from topology which can help distinguish when two topological spaces are homeomorphic (i.e. "the same") or not. In the first part of the course, we study the fundamental group, its computation, and the theory of covering spaces. Some group theory is included, and some basic examples, such as compact surfaces. In the second part of the course, we introduce singular homology, as well as CW complexes and their homology, and examples of computation of homology. We also include geometric applications, such as Jordan's separation theorem in any dimension, and Invariance of domain.

No book for this course.

MATH 594 Algebra II Speyer, R. MWF 2:00 PM - 3:00 PM
MATH 593. (3). (BS). May not be repeated for credit.

Topics include group theory, permutation representations, simplicity of alternating groups for n>4, Sylow theorems, series in groups, solvable and nilpotent groups, Jordan-Holder Theorem for groups with operators, free groups and presentations, fields and field extensions, norm and trace, algebraic closure, Galois theory, and transcendence degree.

MATH 597 Analysis II Rudelson, M. MWF 9:00 AM - 10:00 AM
Rudelson, M. MWF 11:00 AM - 12:00 PM
MATH 451 and 420; or MATH 395. (3). (BS). May not be repeated for credit.

This is one of the basic courses for students beginning the study towards a Ph. D. degree in mathematics. The topics include general construction of a measure, Lebesgue measure on R and R^n, measurable functions, integration, Fubini theorem, complex and signed measures, Lebesgue-Radon-Nikodim theorem, maximal function, differentiation of measures, L_p spaces, introduction to Hilbert space and Fourier analysis.

Grades will be based on homeworks a midterm, and a final exam.
Textbook: (Optional) Real Analysis: Modern Techniques and Their Applications, Gerald B. Folland. ISBN: 978-047-1317-16

MATH 604 Complex Analysis II Barrett MWF 11:00 AM -12:00 PM
Prerequisites: first-year graduate analysis

This is a second course in one-dimensional complex analysis, serving as a follow-up to a course such as Math 596. A particular theme of this version of this course will be potential theory, including such topics as harmonic and subharmonic functions, equilibrium measures, solvability of the Dirichlet problem, harmonic measure, capacity, transfinite diameter and rate of polynomial approximation.

Other topics will be covered as time permits: these include inhomogeneous Cauchy-Riemann equations, the Cauchy transform, zeros and growth of holomorphic functions on the plane and on the unit disk, geometric function theory, Riemann surfaces, the uniformization theorem, angular distortion and Beltrami's equation.

Textbook: None

MATH 615 Commutative Algebra II Hochster,Mel MWF 2:00 PM - 3:00 PM
MATH 614 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

This is a topics course in commutative algebra. One major focus will be on Cohen-Macaulay rings and modules (including big Cohen-Macauly modules and algebras) and their applications. Homological methods will be developed as needed. The technique of reduction to positive characteristic will be used in several different ways to prove significant theorems. Methods for proving that large classes of rings are Cohen-Macaulay will be discussed. There will be discussion of a substantial number of open questions.

Lecture notes and other needed materials will be made available: there is no textbook.

MATH 623 Computational Finance Guo, G. TR 8:30 AM - 10:00 AM

This is a course on computational methods in finance and financial modeling. Using financial mathematics (like many branches of applied mathematics) in practice involves three tasks. First, one needs to develop mathematical models that accurately describe the real-life phenomena that one wishes to study – in the present case, probabilistic models for the evolution of prices, interest rates, and other relevant quantities. Once a model is chosen, the second task is to derive theoretical equations, or formulas, which establish relations between various objects in the financial markets: for example, the prices of derivative securities (options, bonds, etc), and the risk profiles of investment portfolios, as functions of risk factors. Finally, one needs to design and implement numerical methods to perform computations based on these formulas and equations.

This course is concerned with the latter task, and it has three components. In the first part, we will study the lattice (or, tree) methods, which correspond to the models based on discrete time Markov chains (e.g. the binomial model). We will discuss the pricing and hedging of financial derivatives in such models, using the arbitrage theory, or, more specifically, the risk-neutral pricing. We will, then, proceed to analyze the diffusion-based models of financial mathematics (including, e.g., the Black-Scholes model) and the associated Partial Differential Equations (PDEs).

We will discuss the finite difference methods, which provide numerical approximations for solutions to these PDEs. Both explicit and implicit schemes will be studied, the concepts of stability and convergence will be introduced, and a connection between the finite difference schemes and lattice methods will be established. After that, we will turn to the Monte Carlo simulations – the most general computational method for probabilistic equations. This method is based on generating a large number of paths of the underlying stochastic processes, in order to approximate the expectations of certain functions of these paths (which, e.g., may determine prices, portfolio weights, default probabilities, etc.). In addition to the standard Monte Carlo algorithms, we will study the variance reduction techniques, which are often necessary to obtain accurate results. The computational methods presented in this course will be illustrated using the popular models of equity markets (e.g. Black-Scholes, Heston), fixed income (e.g. Vasicek, CIR, Hull-White, Heath-Jarrow-Morton) and credit risk (e.g. Merton, Black-Cox, reduced-form models).

Required: **Credit Risk Modeling**, by Lando (9780691089294)

MATH 626/ STATS Probability and Random Processes II Cohen, A. TR 11:30 AM – 1:00 PM
MATH 625/STATS 625 and Graduate standing. (3). (BS). May not be repeated for credit.

The course is dedicated to basic concepts of stochastic processes in continuous-time, such as stochastic integrals, stochastic differential equations, PDEs and dynamic programming. In the first half of the semester, we will do a close reading of Pham's book. The second half will take a seminar form, where each student will present a paper of his choice from a list I will provide.

Required Textbook: **Continuous-time Stochastic Control and Optimization with Financial Applications**, by Huyen Pham, ISBN: 978-3-540-89500-8
https://link.springer.com/content/pdf/10.1007%2F978-3-540-89500-8.pdf

MATH 632 Algebraic Geometry II Witaszek, J. TR 11:30 AM - 1:00 PM
MATH 631 and Graduate standing. (3). (BS). May not be repeated for credit.

In this course we will cover cohomology of sheaves, algebraic curves, and some aspects of the theory of algebraic surfaces. Background: basic scheme theory

Optional Textbook: Algebraic Geometry by Robin Hartshorne

MATH 635 Differential Geometry Uribe, A. MWF 1:00 PM - 2:00 PM
591 or equivalent. Consent of instructor required. (3). (BS). May not be repeated for credit.

This is an introduction to Riemannian geometry. We will study the notions of connections, Riemannian metrics, geodesics, curvature, and Jacobi fields. We will cover the Hopf-Rinow and the Bonnet-Myers theorems. Then we will turn to complex manifolds and we will discuss some basic ideas in Kähler geometry. The book by do Carmo is not required but is highly recommended.

Optional Textbook: **Riemannian Geometry**, by Manfredo Perdigão do Carmo, 2nd Edition, ISBN: 0817634908

MATH 636 Topics in Differential Geometry Pixton, A. TR 11:30 AM - 1:00 PM
MATH 635 and Graduate standing. (3). (BS). May not be repeated for credit.

Topic Title: Moduli of Curves

The moduli space of curves is a fundamental object in geometry; depending on your perspective, points in it either correspond to algebraic curves or to compact Riemann surfaces. In this course, we will survey various features of the moduli space of curves, with particular emphasis on its cohomology. We will switch between algebraic and topological perspectives at different parts in the course, but in either case we will omit some of the more technical details and focus more on describing/using results and building intuition.

Rough list of topics:

1. basic Teichmuller theory
2. cohomological stability and related results
3. algebraic construction of the moduli space of curves
4. the Deligne-Mumford compactification
5. definition/examples of cohomological field theories and Frobenius manifolds
6. classification of semisimple cohomological field theories

Prerequisites: Cohomology (singular and/or de Rham), along with some idea of either what an algebraic curve of genus g looks like or what a Riemann surface of genus g looks like.

MATH 650 Fourier Analysis Hani, Z. MW 2:30 PM - 4:00 PM
MATH 596, 602, and Graduate standing. (3). (BS). May not be repeated for credit.

Harmonic analysis grew out of the study of Fourier analysis into a broader set of ideas and problems in hard analysis aimed at the quantitative understanding of operators (linear and nonlinear), singularities, oscillations that often arise in mathematics and physics.

In addition to being important in its own right, harmonic analysis proved to be a powerful and useful tool in the modern study of partial differential equations, analytic number theory, probability, and many other fields. In this course, we will survey several topics in classical and modern Harmonic analysis such as singular integral operators, Littlewood-Paley theory, pseudo-differential and para-differential operators, maximal functions, etc. Time permitting we will discuss some applications to other fields.

No book for this course.

MATH 651 Mathematical and algorithmic aspects of machine learning Gilbert, A. TR 8:30 AM - 10:00 AM
MATH 451, 555 and one other 500-level course in analysis or differential equations. Graduate standing. (3). (BS). May be elected twice for credit.

Machine learning algorithms are ubiquitous in our modern lives from voice recognition in our phones to recommendation systems when we shop and watch moves online. The astonishing fact is that these algorithms work in practice despite a lack of provable guarantees on their behavior. When they entail solving an optimization problem, do they actually find the best solution or even a pretty good one? When the algorithm assumes a probabilistic model, can it incorporate new evidence and sample from the true posterior distribution? Machine learning works amazingly well in practice, but that doesn't mean we understand why it works so well. In fact, many machine learning problems (such as sparse coding or topic modeling) are provably hard in the worst-case.

In this course we will see many instances where we can design algorithms for machine learning problems that have rigorous guarantees, under the appropriate, reasonable models for our data and using different ways of measuring the complexity of the algorithm. The course will cover topics such as: nonnegative matrix factorization, matrix completion, tensor decomposition, and neural nets and deep learning.

Topics:
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1. Computational and statistical learning theory (PAC learning, VC dimension, basic stuff)
2. NMF and topic models
3. Matrix and spectral methods
4. tensor decompositions
5. Non-convex optimization, including sparse signal recovery and matrix completion
6. Neural nets and deep learning"

MATH 654 Intro to Math Fluid Dynamics Doering, C. MW 8:30 AM - 10:00 AM
MATH 451, 454, 555. Math 556 is recommended. Graduate standing. (3). (BS). May not be repeated for credit.

This is a course on applied analysis of Euler and Navier-Stokes equations. Topics covered include conservation laws for mass, momentum and energy, vorticity & potential flow, viscous flow & hydrodynamic instabilities, turbulence theory & modeling, and challenges to proving existence & regularity of solutions ti the incompressible 3D Navier-Stokes equations.

Required: **Applied Analysis of the Navier-Stokes Equations**, by C.R. Doering & J.D. Gibbon 1st edition, revised printing 2004, ISBN: 052144568X
Optional: **A Mathematical Introduction to Fluid Mechanics,** by A.J. Chorin & J.E. Marsden, Springer-Verlag (3rd edition, corrected fourth printing, 2000), ISBN: 0387979182

MATH 657 Nonlinear Partial Differential Equations Wu, Sijue TR 2:30 PM - 4:00 PM
MATH 656. (3). (BS). May not be repeated for credit.

Partial Differential Equations are mathematical structures for models in science and technology. It is of fundamental importance in physics, biology and engineering design with connections to analysis, geometry, probability and many other subjects. The goal of this course is to introduce students (both pure and applied) to the basic concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations.

Topics to be covered will include Sobolev spaces, second order elliptic equations, parabolic and hyperbolic equations, shock waves, and nonlinear wave equations. Course material will be taken from Chapters 5, 6, 7 and 12 of the text. Grading: Grades will be based on a few sets of homework and attendance and participation.

Required Textbook: **Partial Differential Equations**, by Lawrence C. Evans, 2nd. ISBN-13: 978-0821849743

MATH 669 Topics in Combinatorial Theory Barvinok, A. TR 1:00 PM - 2:30 PM
Good knowledge of linear algebra (3). (BS). May not be repeated for credit.

Combinatorics, Geometry and Complexity of Integer Points

Integer points (points with integer coordinates) play an important role in algebra, number theory, combinatorics and optimization. In this course, we will discuss some of the classical and recent results regarding integer points and lattices, such as:

- Minkowski Theorems and their applications in number theory and analysis,
- sphere packings and error-correcting codes, including recent advances on bounding sphere packing densities via Fourier analysis,
- transference and flatness theorems and their applications to Diophantine approximation and integer programming,
- Lenstra-Lenstra-Lovasz basis reduction algorithm and its applications to coding, polynomial factorization, etc.

Towards the end, depending on the interests of the audience, we can discuss recent lattice-based “post-quantum cryptography” (that is, how to save e-commerce, should quantum computers be built) or integer points in polytopes and the Ehrhart polynomial.

There is no textbook. Occasionally, lecture notes may be posted.

MATH 679 Arithmetic of Elliptic Curves Hong, S. MWF 1:00 PM - 2:00 PM
MATH 594 and Graduate standing. (3). (BS). May be repeated for credit.

Introduction to p-adic Hodge Theory:

This course aims to provide an introduction to p-adic Hodge theory with minimal prerequisites. It will discuss some fundamental notions and classical results such as p-divisible groups, isocrystals, Dieudonne-Manin classification, Serre-Tate theory, and Hodge-Tate decomposition. Basic knowledge on algebraic geometry (as covered in Math 631) will be required, and some familiarity with cohomology of sheaves on algebraic varieties will be recommended.

No book for this course.

MATH 697 Topics in Topology Canary, R. MWF 2:00 PM - 3:00 PM
Graduate standing. (2 - 3). (BS). May not be repeated for credit.

Anosov Representations:

Fuchsian representations, i.e. discrete faithful representations of surface groups into PSL(2,R), arise from holonomy maps of hyperbolic structures on surfaces. Similarly, holonomy maps of closed hyperbolic manifolds give rise to cocompact representations into SO(n,1). More generally, Yves Benoist studied subgroups of PSL(n+1,R) which act cocompactly on strictly convex domains in the real projective space RP^n. Francois Labourie came up with the notion of an Anosov representation which encompasses all these examples and many other examples of ``geometric'' representations into semi-simple Lie groups.

We will begin by discussing a few of these examples, depending on the interests and background of the audience, and then proceed to develop the general theory of Anosov representations. We will try not to rely too heavily on background material, but some familiarity with geometric group theory and Riemannian geometry would be helpful.

No Textbook required.

MATH 710 Topics in Modern Analysis II Baik, J. TR 1:00 PM - 2:30 PM
MATH 597 and Graduate standing. (3). (BS). May not be repeated for credit.

In random matrix theory we are interested in the behavior of the eigenvalues of random matrices when the size of the matrix is large.
Since the eigenvalues are complicated functions of the entries of the matrix, the eigenvalues are not necessarily independent even if the entries of a matrix are independent random variables.
Instead the eigenvalues of random matrices are strongly correlated and indeed they repel each other.
This correlation between the eigenvalues also turn out to describe the interactions among the particles from a wide class of complicated models.

In this course we will discuss some of the fundamental methods and ideas of random matrix theory.
We will focus on most basic random matrix models and study various properties of them.
Along the way we discuss various topics in analysis such as Coulomb gases, potential theory of electric charges, orthogonal polynomials, method of steepest-descent, and Fredholm determinants.
If time permits, we will also discuss other related topics such as random tiling problems which have certain commonalities with random matrix theory.

The students are assumed to be familiar with complex analysis (there will be plenty residue calculations) and basic probability.

No particular textbook is required. There will be lecture notes.

MATH 715 Advanced Topics in Algebra: Berkovich Spaces Jonsson, M. MWF 12:00 PM - 1:00 PM

Some complex analysis and algebraic geometry. (3). (BS). May not be repeated for credit.

Berkovich spaces are analogues of complex manifolds that appear when replacing complex numbers by the elements of a general normed field, e.g. p-adic numbers or formal Laurent series. They were introduced in the late 1980's by Vladimir Berkovich as a topologically more satisfying alternative to the rigid spaces earlier used by Tate. In recent years, Berkovich spaces have seen a large and growing range of applications to degenerations in complex analysis and geometry, tropical geometry, complex and arithmetic dynamics, Arakelov geometry,..

The first part of the course will be devoted to the basic theory of local (affinoid) and global Berkovich spaces. The study of affinoid algebras is also useful for those mainly interested in the other incarnations of non-Archimedean geometry, such as rigid and adic spaces. In the second part, we will discuss various applications or specialized topics, partly depending on the interest of the audience.

Optional: **Spectral Theory and Analytic Geometry over non-Archimedean Fields** by V. G. Berkovich, ISBN: 0-8218-1534-2
Optional: **Non-Archimedean Analysis**, S. Bosch, U. Guentzer and R. Remmert, ISBN: 3-540-12546-9

MATH 732 Topics in Algebraic Geometry II Fulton, W. TR 1:00 PM - 2:30 PM
MATH 631-632 or equivalent. (3). (BS). May not be repeated for credit.

Degeneracy Loci in Types A-D

Finding formulas for degeneracy loci can be traced to 1849, when Cayley and Salmon found degrees of loci of matrices of polynomials with less than maximal rank. Others considered similar problems for symmetric and skew-symmetric matrices. By the 1990's it became clear that there is a locus for each element of every Weyl group of classical type A, B, C, or D, and much work since then has been devoted to finding formulas for them.

The work involves some Intersection Theory and Chern Classes from Algebraic Geometry, and some determinantal and Pfaffian formulas from Combinatorics, together with some Lie theory. There will be classical and modern applications, to Enumerative Geometry and Equivariant Cohomology.

The necessary background in these areas will be discussed in class, with handouts for details (from a book in progress with Dave Anderson).

The course should be useful for geometers to see how formulas for geometric loci are found and to study the geometry of flag bundles, and for combinatorialists to see the geometry behind some of their algebra.

Textbook: No book for this course

MATH 775 Topics in Analytic Number Theory Ho, Wei MW 10:00 AM - 11:30 AM
MATH 675. (3). (BS). May be repeated for credit.

Distributions of Arithmetic Invariants

This course will sample a wide range of current research topics in arithmetic statistics. We will survey problems, heuristics, and theorems about the asymptotic behavior or distributions of arithmetic objects, such as low degree number fields, ideal class groups of number fields, rational points on varieties over finite fields, Selmer groups of elliptic curves, etc.

Students are expected to be comfortable with a first course in analytic number theory, algebraic number theory, elliptic curves (at the level of Silverman's first book), and algebraic geometry.

No book for this course.

MATH 781 Topics in Mathematical Logic Blass, A. TR 10:00 AM - 11:30 AM
MATH,Varies according to content. (3). (BS). May not be repeated for credit.

Infinitary Combinatorics

This will be a course in infinitary combinatorics, also known as combinatorial set theory. Its central theme is that rather elementary structures on infinite sets can have surprisingly rich properties. An easily stated illustration of the sort of thing I have in mind is (a special case of) Ramsey's Theorem: If X is an infinite set and if every two elements of X are joined by a red or green thread, then there is an infinite subset Y of X such that all threads joining its elements are the same color.

In the first part of the course, I'll develop several combinatorial results that essentially involve only the smallest infinite sets, the countably infinite ones. I shall also discuss the so-called compactness phenomenon, which relates the behavior of infinite sets and large finite sets. For example, the infinite Ramsey theorem quoted above implies various finite versions, including some that cannot be proved without a detour into the infinite.

In the second part of the course, I'll describe some of the new phenomena that occur in uncountable sets. Here is one example: If X is an uncountable, well-ordered set and if a function f assigns to each member x of X except the first some earlier member f(x), then some single member must be f(x) for uncountably many distinct x. I'll also discuss uncountable analogs of some of the countable phenomena from the first part of the course. For example, in Ramsey's theorem quoted above, how large must X be if we want to guarantee an uncountable Y? Answer: The cardinality of the continuum is not large enough, but any larger cardinal is.

Finally, I plan to discuss properties of the "exponential" function that maps the cardinality of a set X to the cardinality of the family of all subsets of X. In addition to the easily verified (weak) monotonicity, this function has some surprisingly subtle additional properties. There will not be time in the course to treat independence results. Such results will be mentioned where appropriate but not proved. The set-theoretic prerequisites for this course are minimal. Math 582 is more than enough. I will briefly review the necessary material, basic cardinal and ordinal arithmetic, in class. So the only real prerequisite is the "mathematical maturity" ordinarily presupposed in graduate courses.

There will be no textbook. I plan to put on reserve in the library several books whose union includes most of the course material. Grading will be based on several homework assignments.

MATH 797 Advanced Topics in Topology: Kriz, I. MWF 9:00 AM - 10:00 AM
Math 695 or equivalent level of study. (3). (BS). May not be repeated for credit.

Structured Homotopy

In this course, we will delve into some recent methods of homotopy theory, including spectral algebra, which studies analogues of certain structures of algebra such as rings, modules, and Lie algebras in stable homotopy theory. The idea of this course is to get an idea about some current thinking and research trends in algebraic topology. The topics are therefore somewhat flexible and may depend on the audience. Students may get the opportunity to give talks on selected topics.

No book for this course