Winter 2019 Graduate Course Descriptions

501 **AIM STUDENT SEMINAR** **Alben** **F 1:00-2PM & 3:00-4PM**

At least two 300 or above level math courses, and Graduate standing; Qualified undergraduates with permission of instructor only. (1). May be repeated for a maximum of 6 credits. Offered mandatory credit/no credit.

MATH 501 is an introductory and overview seminar course in the methods and applications of modern mathematics. The seminar has two key components: (1) participation in the Applied and Interdisciplinary Math Research Seminar; and (2) preparatory and post-seminar discussions based on these presentations. Topics vary by term.

Text: None

506 **ANALYSIS FOR FINANCE** **Saplaouras** **T/TH 10:00 AM -11:30 AM**

MATH 526, MATH 573. Graduate students or permission of instructor. (3). (BS). May not be repeated for credit.

Stochastic Analysis for Finance --- The aim of this course is to teach the probabilistic techniques and concepts from the theory of stochastic processes required to understand the widely used financial models. In particular concepts such as martingales, stochastic integration/calculus, which are essential in computing the prices of derivative contracts, will be discussed. Pricing in complete/incomplete markets (in discrete/ continuous time) will be the focus of this course as well as some exposition of the mathematical tools that will be used such as Brownian motion, Levy processes and Markov processes.

521 **LIFE CONTINGENCIES II** **Young** **T/TH 8:30 AM -10:00 AM**

MATH 520 with a grade of C- or higher. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

This course extends the single decrement and single life ideas of MATH 520 to multi-decrement and multiple-life applications directly related to life insurance. The sequence 520-521 covers the Part 4A examination of the Casualty Actuarial Society and covers the syllabus of the Course 150 examination of the Society of Actuaries. Concepts and Calculation are emphasized over proof.

524 **LOSS MODELS II** **Young** **T/TH 10:00 AM -11:30 AM**

STATS 426 and MATH 523. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Risk management and modeling of financial losses. Frequentist and Bayesian estimation of probability distributions, model selection, credibility, and other topics in casualty insurance.

525 **PROBABILITY THEORY** **Baik** **T/TH 10:00 AM to 11:30 AM**
T/TH 1:00 PM to 2:30 PM

MATH 451 (strongly recommended). MATH 425/STATS 425 would be helpful. (3). (BS). May not be repeated for credit.

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

526 **DISCRETE STATE STOCHASTIC PROCESSES** **Chakraborty** **T/TH 8:30 AM -10AM & T/TH 10:00 AM – 11:30 AM**
Conlon **T/TH 11:30 AM – 1:00 PM**

MATH 525 or STATS 525 or EECS 501. (3). (BS). May not be repeated for credit.

The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include: Markov chains (Markov property, recurrence and transience, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times, Markov queues); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales). Significant applications will be an important feature of the course.

547 PROB MOD BIOINF Rajapakse T/TH 4:00 PM to 5:30 PM

Open to upper-level undergraduates and graduate students. Basic probability (level of MATH/STATS 425), or molecular biology (level of BIOLOGY 427), or biochemistry (level of CHEM/BIOLCHEM 451), or basic programming skills desirable or permission

Description: This course is open to graduate students and senior undergraduates in applied mathematics, bioinformatics, statistics, and engineering, who are interested in learning from data. Students with other backgrounds such as life sciences are also welcome, provided they have maturity in mathematics. It will cover geometric methods (manifold learning, diffusion maps, etc.) and topological data reduction (clustering and computational homology group, etc.). Also I will bring dynamical systems theory to data. I will give real data examples wherever possible.

This class is motivated by my own experience with data and mathematics. Over the past few years I have worked with Dr. Steve Smale and my funding institute DARPA on many problems that have required knowledge at the interface of mathematics and data. The new DARPA initiative on Artificial Intelligence and within it the Lifelong Learning Machines (L2M) program exemplify types of work at this interface. My own lab has a dual approach of generating time series data in-house and using mathematics to identify patterns in data and determine major unknowns. The methods discussed in this class reflect our approaches and those that are useful in handling data across many fields. Guest lecturers from industry and academia will also participate.

Textbooks:

1. Cichocki, A., Zdunek, R., Phan, A. H., & Amari, S. I. (2009). Nonnegative Matrix and Tensor Factorizations: Applications to Exploratory Multi-Way Data Analysis and Blind Source Separation. John Wiley & Sons.
2. Smale, S. (1980). The Mathematics Of Time (pp. 133-4). New York etc.: Springer.

555 INTRODUCTION TO FUNCTIONS OF A COMPLEX VARIABLE WITH APPLICATIONS Patel M/W 8:30 AM – 10:00AM

MATH 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

Intended primarily for students of engineering and of other cognate subjects. Doctoral students in mathematics elect Mathematics 596. Complex numbers, continuity, derivative, conformal representation, integration, Cauchy theorems, power series, singularities, and applications to engineering and mathematical physics.

557 APPLIED ASYMPTOTIC ANALYSIS Miller T/TH 1:00 PM -2:30 PM

Differential Equations (e.g. 216, 256, 316, or 404), Linear Algebra (e.g. 214, 217, 419, 420), Real Analysis (e.g. 451), and Complex Analysis (e.g. 555 or 596).

Topic Title: *Asymptotic Analysis*

Asymptotic analysis is the quantitative study of approximations. The fundamental idea is that one tries to solve a problem in applied mathematics (say, a boundary-value problem for a partial or ordinary differential equation) by embedding it into a family of problems with a parameter. If the problem can be solved exactly for one special value of the parameter, then asymptotic analysis can be used to analyze how the solution changes as the parameter is tuned from the special value to a more physically reasonable one. The course will develop the general theory of so-called asymptotic expansions, which are a kind of series in the perturbation parameter that are extremely useful in practice, in a way that is mathematically completely rigorous, despite the strange fact that they frequently fail to converge at all! We will then study how to use asymptotic expansions to evaluate integrals that cannot be computed in closed form and that are also difficult to approximate numerically. Next, we will turn to differential equations and use asymptotic expansions to evaluate solutions near certain singular points and also to study the way that solutions depend on parameters. At the end of the course we will study how the differential equations of diverse physical phenomena can be reduced, with the help of asymptotic expansions, to certain universal model equations that show up again and again in applied mathematics.

Specific applications to be addressed in the course as time permits include the small-viscosity theory of shock waves, the theory of quantum mechanics in the semiclassical limit, aspects of the theory of special functions, vibrations in nonlinear lattices, and surface water waves.

Grades will be determined based upon homework sets and a final project.

563 ADVANCED MATHEMATICAL METHODS FOR THE BIOLOGICAL SCIENCES Booth T/TH 2:30 PM - 4:00 PM

216 or 316, 217 or 417 and 463 (or permission by instructor). Graduate standing. (3). (BS). May not be repeated for credit.

This course will focus on mathematical modeling of how biological objects, such as molecules, cells or whole organisms, move and interact in time and in space. Depending on the biological process, time scales for interaction can vary widely, from milliseconds to days, and spatial scales can vary from microns to miles, however we can use similar mathematical techniques to capture behavior of these diverse systems. The techniques stem from the theory of random walks and involve partial differential equations (PDEs). The course will cover analytical and numerical techniques for solving random walk systems and associated PDE equations in the context of modeling diverse biological processes. Agent-based modeling will also be introduced to model highly complex and variable interactions among biological objects. Grades will be based on biweekly homework sets, consisting of analytical and numerical problems, and a modeling project. Programming experience, particularly with Matlab, is required.

MATH 566 Combinatorial Theory Stembridge M/W/F 11:00 AM – 12:00 PM

MATH 465, basic group theory and abstract linear algebra (3). (BS). May not be repeated for credit.

Topic Title: *Algebraic Combinatorics*

This course will be a graduate-level introduction to algebraic combinatorics. Previous exposure to combinatorics will not be necessary, but experience with algebra including basic group theory and abstract linear algebra will be needed.

Most of the topics we cover will be centered around enumeration and generating functions. But this is not to say that the course is only about enumeration--counting formulas are often manifestations of of deeper structure.

Highlights along the way will include cancellation methods, combinatorial factorization, Lagrange inversion, the permanent-determinant method, the matrix-tree theorem, the transfer matrix method, and exponential generating functions from a categorical perspective.

Note that Volume 1 of the text is available as a free download from the author's website, and electronic versions of Volume 2 should be free from the publisher's website for all U-M students.

MATH 567 INTRODUCTION TO CODING THEORY Nguyen T/TH 2:30 PM – 4:00 PM

One of MATH 217, 419, 420. (3). (BS). May not be repeated for credit.

This is course on the mathematical foundations of modern coding and information theory, it is NOT about cryptography, which deals with the security aspect of data. Here, we assume data is being transmitted over a noisy channel, and find efficient ways encode it to deal with loss/errors, and at the same time keeping the encoding/decoding process manageable. The subject became important after Shannon's work in the forties, and is now widely applied in industry.

Some of the topics that we will discuss are: entropy (a measure for information), Huffman codes (for data compression), channels and channel capacity, Shannon's theorem, error correcting block codes, finite fields and constructions of various codes (Hamming codes, Golay codes, Reed-Muller codes, cyclic codes etc.), linear codes, bounds for codes, and more.

MATH 571 Numerical Linear Algebra Esedoglu T/TH 10:00 AM – 11:30 AM

MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit

Direct and iterative methods for solving systems of linear equations (Gaussian elimination, Cholesky decomposition, Jacobi and Gauss-Seidel iteration, SOR, introduction to multi-grid methods, steepest descent, conjugate gradients), introduction to discretization methods for elliptic partial differential equations, methods for computing eigenvalues and eigenvectors.

MATH 572 NUMERICAL METHODS for DIFFERENTIAL EQUATIONS Krasny 11:30 AM – 1:00 PM

MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit.

Computer simulation is routinely used in many branches of science and engineering, and increasingly in other fields such as finance and medicine. However, computer simulations can be challenging; using a faster computer is no guarantee of success and sometimes one must use a better algorithm. Math 572 is an introduction to numerical methods used in solving differential equations. The course will focus on finite-difference schemes for initial value problems involving ordinary and partial differential equations. Theory and practical computing issues will be covered.

574 FINANCIAL MATHEMATICS II Herrmann T/TH 1:00 PM – 2:30 PM

MATH 526 and MATH 573. (Prerequisites enforced at registration.) Although MATH 506 is not a prerequisite for MATH 574, it is strongly recommended that either these courses are taken in parallel, or MATH 506 precedes MATH 574. (3). (BS). May not be repeated for credit

This is a continuation of Math 573. This course discusses Mathematical Theory of Continuous-time Finance. The course starts with the general Theory of Asset Pricing and Hedging in continuous time and then proceeds to specific problems of Mathematical Modeling in Continuous-time Finance. These problems include pricing and hedging of (basic and exotic) Derivatives in Equity, Foreign Exchange, Fixed Income and Credit Risk markets. Although Math 506 is not a prerequisite for Math 574, it is strongly recommended that either these courses are taken in parallel, or Math 506 precedes Math 574.

575 INTRODUCTION TO THE THEORY OF NUMBERS Gazaki T/TH 10:00 AM - 11:30 AM

Math 451 and either Math 412 or preferably Math 493, 494

This course is an introductory course in modern number theory. We will start by defining the notions of congruence modulo m and of a unique factorization domain. Among the main topics that will be covered in the course will be the quadratic reciprocity law, basic theory of finite fields, and unique factorization of ideals in an algebraic number field. If time permits, we will discuss more advanced topics, like the definition and properties of the Riemann zeta function.
Textbook: A Classical Introduction To Modern Number Theory by Ireland & Rosen, 2nd Edition.

582 INTRODUCTION TO SET THEORY T/TH 1:00 PM – 2:30 PM

MATH 412 or 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

The main topics covered are set algebra (union, intersection), relations and functions, orderings (partial, linear, well), the natural numbers, finite and denumerable sets, the Axiom of Choice, and ordinal and cardinal numbers.

583 PROBABILISTIC AND INTERACTIVE PROOFS Strauss M/W 8:30 AM – 10:00 AM

MATH 412, 451 or permission of instructor. May not be repeated for credit.

“Can we be convinced that a proof is correct, even if we only check it in three places? Can a proof convince us that a statement is true, while giving us no aid in convincing anyone else that the statement is true? The answer to both is affirmative. How? Using randomness and interaction, two elements missing from traditional deductive proofs. Why? Checking a proof in just a few places is useful for checking computer-generated proofs that are too long to read (an example bigdata algorithm); there are also surprising connections to showing that certain functions cannot be computed or even approximated efficiently. A ""zero-knowledge proof"" might be used, for example, for a purchaser Peggy to prove to a vender Victor that Peggy is the rightful owner of a credit card, without giving Victor any ability to prove (fraudulently) that Victor is the owner of that credit card.

Other topics: We will also look at IP=PSPACE: converting a game, G, to another, G', such that if Peggy beats any expert in G, then she beats any expert in G'; if Peggy loses to an expert in G, she loses to a random (non-expert) player in G'. Thus the prover Peggy can convince a non-expert verifier Victor that she wins G, even though the game tree is too big for Victor to read. We will also look at Multi-prover Interactive Proof systems and show that they are more powerful than single-prover systems, answering the question posed so eloquently by Professor Click and the late Professor Clack: Do two people who don't know what they are talking about know more or less than one person who doesn't know what he's talking about?"

590 INTRODUCTION TO TOPOLOGY Wilson M/W/F 12:00 – 1:00 PM

MATH 451. (3). (BS). May not be repeated for credit. Rackham credit requires additional work.

The purpose of this course is to introduce basic concepts of topology. Most of the course will be devoted to the fundamentals of general (point set) topology.

592 INTRODUCTION TO ALGEBRAIC TOPOLOGY Kriz M/W/F 10:00 AM – 11:00 AM

MATH 591. (3). (BS). May not be repeated for credit.

This course treats the introductory topics of algebraic topology: Fundamental Group, Covering Spaces, Homology. We will develop these concepts, and discuss, with proof, their basic properties, aimed at calculation. We will also discuss plenty of examples and applications throughout the course.

594 ALGEBRA II Snowden T/TH 2:30 PM – 4:00 PM

MATH 593. (3). (BS). May not be repeated for credit.

Topic Title: *Groups and Galois Theory*

Topics include group theory, permutation representations, simplicity of alternating groups for n>4, Sylow theorems, series in groups, solvable and nilpotent groups, Jordan-Holder Theorem for groups with operators, free groups and presentations, fields and field extensions, norm and trace, algebraic closure, Galois theory, and transcendence degree.

597 ANALYSIS II (REAL ANALYSIS) Barrett M/W/F 11:00 AM – 12 Noon

MATH 451 and 420; or MATH 395. (3). (BS). Math 451; Math 490 or 590 strongly recommended May not be repeated for credit.

This is one of the core courses for the mathematics doctoral program. Topics will include: Lebesgue measure in R^n; general measures; measurable functions; integration; monotone convergence theorem; Fatou's lemma; dominated convergence theorem; product measures; Fubini's theorem; function spaces; Holder and Minkowski inequalities; functions of bounded variation; differentiation theory; modes of convergence; Fourier analysis. Additional topics such as Hausdorff dimension and Sobolev spaces to be covered as time permits.

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605 SEVERAL COMPLEX VARIABLES Jonsson M/W/F 11:00 AM – 12 Noon

MATH 596 and 597. Graduate standing. (3). (BS). May not be repeated for credit.

Analysis in several complex variables is formally just the extension of complex analysis in one variables to the higher-dimensional case. However, it has a much more geometric flavor, and the analytic techniques it provides can be quite powerful in fields such as complex algebraic geometry.

The course will start out with basic properties of holomorphic functions in several variables and some surprising phenomena, such as the Hartogs extension theorem. We will also study local properties of analytic sets, that is, zero loci of holomorphic functions.

After that we will focus on L^2 methods, one of the main techniques for constructing holomorphic functions. Along the way, we will study pseudoconvex sets and plurisubharmonic functions, the complex cousins of convex sets and functions in real Euclidean space. .

Finally we will turn to geometric applications. We will study basic properties of complex manifolds and extend some of the results obtained in C^n. Towards the end of the course, we will prove Kodaira's Embedding Theorem, which gives a criterion for a compact complex manifold to embed into some projective space, and hence be algebraic.

Problem sets will be distributed about every other week.

615 COMMUTATIVE ALGEBRA II Hochster M/W/F 10:00 AM – 11:00 AM

MATH 614 or permission of instructor. Graduate standing. (3). (BS). May not be repeated for credit.

Topic Title: *Multiplicities and Integral Closure*
This course will deal with various notions of multiplicity (Hilbert-Samuel, Hilbert-Kunz-Monsky, and Serre intersection multiplicities), including both asymptotic and other methods of finding multiplicities, and the related topic of integral closure of ideals, including the Lipman-Sathaye proof of the Briancon-Skoda theorem. We will also introduce tight closure and do the tight closure proof of the Lipman-Sathaye theorem. Many open questions will be discussed. There is no text. Lecture Notes will be provided.

626 RANDOM PROCESSES Rudelson T/TH 10:00 AM – 11:30 AM

MATH 625 or Instructor's Approval

The course will focus on discrete time Markov chains and ergodic theory. After covering the basics of Markov chain theory, we will concentrate on mixing in finite chains. Mixing time characterizes how fast a Markov chain approaches the stationary distribution. This theory has seen a rapid progress in the last 20 years. Mixing in Markov chains plays a key role in many sampling and approximate counting algorithms in computer science.

632 ALGEBRAIC GEOMETRY II Fulton T/TH 11:30 AM – 1:00 PM

MATH 631 and Graduate standing. (3). (BS). May not be repeated for credit.

This course will develop some of the basic tools of algebraic geometry, with emphasis on examples and applications. Here are some of the topics we hope to discuss:

> Ringed spaces, abstract algebraic varieties.Sheaves and cohomology. Coherent sheaves on projective varieties. Grassmann and flag bundles. Representable functors. Toric varieties. Blowups, tangent and normal cones. Schemes. Intersection multiplicities. Picard groups, class groups. Grothendieck groups of vector bundles and sheaves. Chern classes. Grothendieck Riemann-Roch Theorem.

Mastering these tools requires a good deal of heavy lifting, much of which needs to be done individually. Signing up for this course includes an agreement that any paper passed in contains only the work of those whose names are on it. Prerequisites are Math 631 and a good background in commutative algebra. No required text, beyond Shafarevich for 631.

635 DIFFERENTIAL GEOMETRY Pengyu M/W/F 1:00 PM – 2:00 PM

Consent of instructor required. (3). (BS). May not be repeated for credit

Second fundamental form, Hadamard manifolds, spaces of constant curvature, first and second variational formulas, Rauch comparision theorem, and other topics chosen by the instructor

636 TOPICS IN DIFFERENTIAL GEOMETRY Burns T/TH 11:30 AM – 1:00 PM

MATH 635 and Graduate standing. (3). (BS).

Topic Title: Geometric Quantization

Quantum mechanics is usually formulated axiomatically in terms of self-adjoint operators on a Hilbert space, the eigenvalues corresponding to physical quantities of interest. Geometric quantization is in general a program to associate in as canonical a way as possible, the Hilbert space of quantum mechanics in terms of geometric spaces of functions or sections on spaces associated with the classical mechanical system, e.g., phase space of a system of moving particles.

We will discuss the analysis of one method of quantization -- the Berezin-Toeplitz quantization. This requires a minimum of background, and is an area of much current interest, in part because of the study of relations of quantum mechanics to hard symplectic topology, and because of cases arising of non-self-adjoint quantum operators. The mathematical applications are to algebraic and symplectic geometry, representation theory and the theory of Bergman and Szego kernels in several complex variables.

Course materials will be distributed through the Canvas site. Books used will be available in online editions through the UM Library. We will likely require a presentation from students taking the course for credit.

References:

Spectral Theory of Toeplitz Operators, V. Guillemin and L. Boutet de Monvel
Quantum Mechanics for Mathematicians, B. Hall
Geometric Quantization, N. Woodhouse
Current literature for the recent parts.

NB: There will be a symplectic reading group in parallel for those interested in the relations to contemporary symplectic geometry.

650 FOURIER ANALYSIS Gilbert T/TH 8:30 AM – 10:00 AM

MATH 596, 602, and Graduate standing. (3). (BS). May not be repeated for credit. Real Analysis, linear algebra, basic probability.

Topic Title: *Computational Harmonic Analysis*

This course will cover techniques and algorithms in computational harmonic analysis as well as some mathematical foundations of the field.

Optional Text: A Wavelet Tour of Signal Processing: The Sparse Way by Stephane Mallat, 3rd edition.

651 TOPICS IN APPLIED MATHEMATICS I Uribe T/TH 4:00 PM – 5:30 PM

MATH 451, 555 and one other 500-level course in analysis or differential equations. Graduate standing. (3). (BS).

Topic Title: *Semi-Classical Methods*

This course is about the asymptotic behavior of linear partial differential operators that include a small parameter, as the parameter tends to zero in a suitable regime. The small parameter is usually referred to as

Planck's constant, h, as the primary example where this theory applies is to the Schrödinger operator of quantum mechanics. The limit h –> 0 is known as the semi-classical limit (or WKB method in the physics literature), and it was investigated from the very beginning of the quantum theory. In this limit quantum mechanics is supposed to "converge" to classical mechanics. The asymptotics can be about the spectrum and eigen functions of the operator, or about the time-dependent evolution of suitable initial conditions under a general Schrödinger equation. In general, the limit involves objects of symplectic geometry, which is the arena where Hamiltonian mechanics takes place.

The course will start with the fundamentals of Hamiltonian mechanics and symplectic geometry. We will then develop the theory of h-pseudo-differential operators in Euclidean space and their symbol calculus. A fundamental question we will address is how to associate to functions and operators on configuration space semi-classical analogues in phase space, which has twice the number of dimensions. We will cover as much of the theory as time allows, and we will skip many proofs but will strive to state precise results. Most of the time we will work in Euclidean space, but towards the end of the course I am planning to cover topics where symplectic geometry enters more heavily and where there are many open questions. These topics will depend to a considerable extent on the interest of the course participants.

The course grade will be based on homework assigned approximately every other week.

657 NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS Hani T/TH 2:30 PM – 4:00 PM

MATH 656. (3). (BS). May not be repeated for credit.

Partial differential equations are at the core of models in science, engineering, economics, and related fields. These equations and their solutions have interesting structures that are studied by a beautiful combinations of methods from analysis, geometry, probability and other mathematical fields.

This course is an introduction to nonlinear partial differential equations for a diverse audience in pure and applied mathematics. It will start by reviewing and introducing some analytical techniques that are central to the modern study of nonlinear PDEs like Fourier analysis, Sobolev spaces, and Littlewood-Paley theory. Afterwards, the course will present various techniques to construct and analyze solutions of nonlinear PDE. The focus will be on models coming from fluids (Euler and Navier-Stokes equations) and nonlinear dispersive/hyperbolic equations (the nonlinear Schr\"odinger and wave equations). Other topics and equations will be covered time permitting.

669 TOPICS IN COMBINATORIAL THEORY Fomin T/TH 1:00 PM – 2:30 PM

MATH 565, 566, or 664; and Graduate standing. (3). (BS).

Topic Title: *Combinatorial Matrix Theory*

This is an introductory course in combinatorial matrix theory, emphasizing its algebraic aspects. It can also be described as a second (or third) course in linear algebra, emphasizing its combinatorial aspects.

Tentative list of topics: Combinatorial techniques in linear algebra. Basic algebraic graph theory. Determinantal identities. Matroids and projective geometry. Grassmannians and Schubert cells. Canonical forms and factorizations. Polynomials with real roots. Total positivity.

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679 ARITHMETIC OF ELLIPTIC CURVES Kaletha M/W 11:30 AM – 1:00 PM

MATH 594 and Graduate standing. (3). (BS). May be repeated for credit

This class will introduce the basic concepts in the theory of automorphic forms. We will begin with the GL_1-theory, also known as Tate's thesis, which is a fruitful reformulation of class field theory. After this, we will move to GL_2-theory, also known as Jacquet-Langlands theory, and will discuss elements of the representation theory of real and p-adic groups in the example of GL_2, as well as the concept of an automorphic representation, and its associated L-function. Time permitting, we will see a glimpse of the central tool in the analytic theory -- the Arthur-Selberg trace formula.

681 MATHEMATICAL LOGIC Blass T/TH 10:00 AM – 11:30 AM

Mathematical maturity appropriate for a 600-level MATH course. Graduate standing. (3). (BS). May not be repeated for credit.

Historically, mathematical logic is the mathematical study of mathematics itself, especially of the process of deductive reasoning. A central question is the adequacy of deductive reasoning: Can all the consequences of a set of assumptions be obtained from those assumptions by a sequence of very simple inferences? This question is an instance of a general theme that runs through much of mathematical logic, namely the interplay between mathematical statements (which are to be manipulated in the desired simple inferences) and the possibly very complicated mathematical structures that they describe (which underlie the notion of consequence).

Part of Math 681 is devoted to making this interplay precise and establishing a positive answer to the central question in some situations, including the particularly important case of first-order logic. This logic with its simple inferences serves as the explicit or implicit foundation for essentially all mathematical reasoning. In this connection, I'll also discuss at least one alternative way of verifying correctness in first-order reasoning. This way, though farther from intuition than the simple inferences mentioned above, is in general much more efficient and has found applications in automated theorem-proving.

Another part of the course will indicate why first-order logic plays such a central role in mathematics. It is easy to produce logical systems stronger than first-order logic, and one might be tempted to use them as an improved foundation for mathematics. But the notions of "consequence" for these systems are necessarily far more complicated and cannot be captured by any reasonable step-by-step inferences. A mathematics based on them could not have proofs in anything like the usual sense of the term.

A third part of the course uses the interplay between mathematical statements and the structures they describe, to establish some results in other areas of mathematics. These results do not directly involve logical matters, but their proofs are based on the results from logic proved earlier in the course. One example is the existence of the models used in non-standard analysis.

697 TOPICS IN TOPOLOGY Ruan M/W/F 9:00 AM – 10:00 AM

Graduate standing. (2 - 3). (BS).

Topic Title: *Gromov-Witten Theory and Modularity*

It has been known for a while that Gromov-Witten theory of Calabi-Yau manifold should have certain modularity. In this case, we will focus on some explicit examples of toric Calabi-Yau orbifold and explicit computation using Givental theory.

710 TOPICS IN MODERN ANALYSIS II *(Measure Concentration)* **Rudelson T/TH 1:00 PM – 2:30 PM**

MATH 597 and Graduate standing. (3). (BS). Math 625 or instructor's approval.

Measure concentration is an area which strives to formalize a simple idea: a quantity depending on the influence of many independent random parameters is essentially constant. The simplest manifestation of this phenomenon is the Law of Large Numbers for which the relevant random quantity is the average. Yet, the Law of

Large Numbers, being a limit law, may be hard to apply since it does not capture the behavior for a fixed number of samples. Measure concentration takes a different point of view, and tries to show that a relevant quantity is close to a constant with high probability for a large but fixed number of variables.

Statements of this type provide a powerful tool in many areas. In geometric functional analysis, one uses concentration to study the properties of random convex bodies, or properties of deterministic bodies by analyzing their random sections and projections. In combinatorics, concentration is the base of the probabilistic method allowing to prove the existence of certain subsets of a large structure by looking at its random pieces. Measure concentration is also a major tool used to justify the convergence of randomized algorithms in computer science.

In this course we will consider different methods of establishing concentration in both discrete and continuous setting along with the applications. Among possible applications we can discuss the Johnson-Lindenstrauss dimension reduction lemma, Dvoretzky's theorem on the existence of almost Euclidean subspaces of a general normed space, counting copies of a small subgraph in a large random graph etc.

732 TOPICS IN ALGEBRAIC GEOMETRY II Smith M/W/F 2:00 PM – 3:00 PM

Graduate standing and permission of instructor. (3). (BS). May not be repeated for credit. Math 632, Math 614, some familiarity with local cohomology useful. (BS).

Topic Title: Characteristic p Methods in Commutative Algebra and Algebraic Geometry.

Course Description: Characteristic p methods in algebraic geometry and commutative algebra form an active branch of research in both areas, and are useful even for proving theorems about algebraic varieties over the complex numbers (by "reduction to characteristic p"). The secret is the Frobenius---or p-th power map. Consider an affine or projective variety X defined over a field k of prime characteristic p. Its coordinate ring R (or homogenous coordinate ring in the projective case) is a finitely generated algebra over k. Because k has characteristic p, the map R--->R sending each regular function f to f^p is a ring homomorphism, which means that it induces a scheme map X---->X, called the Frobenius map. This map is the identity map on the underlying topological space of X, but of course, is doing something highly non-trivial to the scheme X. For example, one of the first theorems we will prove is a theorem of Kunz characterizing when X is smooth as equivalent to the flatness of the Frobenius map. The Frobenius map induces a natural action on cohomology and other objects associated to X, which can be used to prove theorems about the singularities, cohomology, and birational structure of X. In this course, we will develop this theory, beginning with Kunz's theorem and moving through F-split, F-regular and F-rational singularities and their relation to log canonical, log terminal and rational singularities. We will prove vanishing theorems for cohomology and characterize positivity of the canonical bundle in terms of Frobenius. We may discuss invariants such as the F-pure threshold (and analog of the log canonical threshold), test ideals (and the relationship to multiplier ideals in binational geometry), the F-signature or tight closure. Some of these topics will be driven by student interest.

Textbook: Karl Schwede and Karen Smith, *notes will be provided from a text-book in progress.*

776 TOPICS IN ALGEBRAIC NUMBER THEORY Snowden T/TH 1:00 PM - 2:30 PM

MATH 676 and Graduate standing. (3). (BS).

This class will treat class field theory for local and global fields. Students should have a solid background in basic algebra and algebraic number theory, but no other prerequisites are needed.

797 ADVANCED TOPICS IN TOPOLOGY Canary M/W/F 2:00 PM – 3:00 PM

Graduate standing and permission of instructor. (3). (BS).

Topic Title: *Introduction to Hyperbolic 3-manifolds*

The Geometrization Theorem asserts that every 3-manifold can be canonically cut up into pieces, each of which admits one of 8 geometries. The manifolds modelled on 7 of these geometries have been completely classified. In this course we will develop the theory of hyperbolic 3-manifolds, which is the remaining, and most common, type of geometric 3-manifold. Basic topics include: the geometry of hyperbolic space, constructions of examples, thick-thin decompositions of hyperbolic manifolds, and convex cores for hyperbolic 3-manifolds. We will then develop the theory of geometrically finite hyperbolic manifolds, which are the best understood class of hyperbolic 3-manifolds.

The remainder of the class will be taken up with more advanced topics to be determined by the interests of the class and instructor at that time. Possibilities include: topological and geometric properties of geometrically infinite hyperbolic 3-manifolds, deformation spaces of hyperbolic 3-manifolds, the characteristic submanifold and its relationship to the geometry of hyperbolic 3-manifolds, Patterson-Sullivan theory and/or higher Teichmuller theory.