

# Contents

<b>Introduction</b> .....	<b>5</b>
<b>Pre-Concentration: The First Two Years</b> .....	<b>7</b>
<b>Mechanics of The Concentration</b> .....	<b>8</b>
<b>Declaring the Concentration</b> .....	<b>9</b>
<b>Subconcentration Programs</b> .....	<b>9</b>
Pure Mathematics.....	11
Honors Mathematics.....	12
Mathematical Sciences.....	15
Actuarial Mathematics and .....	21
Mathematics of Finance and Risk Management .....	21
Secondary Mathematics Teaching Certificate .....	23
<b>Graduation Requirements</b> .....	<b>25</b>
<b>Minor in Mathematics</b> .....	<b>25</b>
<b>Department Faculty</b> .....	<b>28</b>
<b>Academic Advising</b> .....	<b>31</b>
<b>Academic Help</b> .....	<b>31</b>
<b>Around the Mathematics Department</b> .....	<b>32</b>
<b>Mathematics Library</b> .....	<b>32</b>
<b>Computing Facilities</b> .....	<b>33</b>
<b>Career Planning</b> .....	<b>33</b>
<b>Personal Counseling</b> .....	<b>34</b>
<b>Complaints and Problems</b> .....	<b>34</b>
<b>Extracurricular Activities</b> .....	<b>34</b>
<b>From Out Graduates</b> .....	<b>38</b>
<b>Courses</b> .....	<b>40</b>
Math 105 - Data, Functions, and Graphs .....	41
Math 107 - Mathematics for the Information Age .....	42
Math 110 - Pre Calculus (Self-Paced).....	43
Math 115 - Calculus I .....	44
Math 116 - Calculus II .....	45
Math 127 - Geometry and the Imagination .....	46
Math 128 - Explorations in Number Theory .....	47
Math 145 - Houghton Scholars Calculus Workshop I .....	48
Math 146 - Houghton Scholars Calculus Workshop II .....	49
Math 147 - Introduction to Interest Theory.....	50
Math 156 - Applied Honors Calculus II .....	51
Math 174 - Proofs in Geometry.....	52

Math 175 - Introduction to Cryptology.....	53	Math 452 - Advanced Calculus II.....	96
Math 176 - Explorations in Topology and Analysis .....	54	Math 454 - Boundary Value Problems for Partial Differential Equations .....	97
Math 185 - Honors Calculus I .....	55	Math 462 - Mathematical Models.....	98
Math 186 - Honors Calculus II .....	56	Math 463 - Math Modeling in Biology .....	99
Math 214 - Linear Algebra .....	57	Math 464 - Inverse Problems.....	100
Math 215 - Calculus III.....	58	Math 466 (EEB 466) - Mathematical Ecology .....	102
Math 216 - Introduction to Differential Equations .....	59	Math 471 - Introduction to Numerical Methods .....	103
Math 217 - Linear Algebra.....	60	Math 472 - Numerical Methods with Financial Applications .....	104
Math 255 - Applied Honors Calculus III .....	61	Math 475 - Elementary Number Theory.....	105
Math 256 - Applied Honors Calculus IV .....	62	Math 476 - Computational Laboratory in Number Theory .....	106
Math 285 - Honors Calculus III .....	63	Math 481 - Introduction to Mathematical Logic .....	107
Math 286 - Honors Differential Equations .....	64	Math 486 - Concepts Basic to Secondary School Math.....	108
Math 289 - Problem Solving.....	65	Math 489 - Math for Elem. and Middle School Teachers.....	109
Math 295 - Honors Mathematics I.....	66	Math 490 - Introduction to Topology.....	110
Math 296 - Honors Mathematics II.....	67	Math 493 - Honors Algebra I.....	111
Math 310 - Elementary Topics: Choice and Chance.....	68	Math 494 - Honors Algebra II.....	112
Math 312 - Applied Modern Algebra.....	69	Math 497 - Topics in Elementary Mathematics .....	113
Math 316 - Differential Equations.....	70	Math 498 - Topics in Modern Mathematics.....	114
Math 327 - Evolution of Mathematical Concepts .....	71	Math 520 - Life Contingencies I.....	115
Math 351 - Principles of Analysis.....	72	Math 522 - Actuarial Theory of Pensions and Social Security .....	117
Math 354 - Fourier Analysis and its Applications.....	73	Math 523 - Risk Theory .....	118
Math 371 (Engin 371) - Numerical Methods .....	74	Math 525 (Stats 525) - Probability Theory .....	119
Math 385 - Math for Elementary School Teachers.....	75	Math 526 (Stats 526) - Discrete State Stochastic Processes.....	120
Math 389 - Explorations in Mathematics .....	76	Math 528 - Topics in Casualty Insurance .....	121
Math 395 - Honors Analysis I.....	77	Math 547 - Biological Sequence Analysis .....	122
Math 396 - Honors Analysis II.....	78	Math 550 (CMPLXSYS 510) - Introduction to Adaptive Systems ...	123
Math 404 - Intermediate Differential Equations .....	79	Math 555 - Introduction to Complex Variables.....	124
Math 412 - Introduction to Modern Algebra .....	80	Math 556 - Methods of Applied Mathematics I.....	125
Math 416 - Theory of Algorithms.....	81	Math 557 - Methods of Applied Mathematics II.....	126
Math 417 - Matrix Algebra I.....	82	Math 558 - Ordinary Differential Equations.....	127
Math 419 - Linear Spaces and Matrix Theory.....	83	Math 559 - Topics in Applied Mathematics.....	128
Math 420 - Advanced Linear Algebra.....	84	Math 561 (OMS 518, IOE 510) - Linear Programming I.....	129
Math 422 (BE 440) - Risk Management and Insurance.....	85	Math 562 (IOE 511) - Continuous Optimization Methods.....	130
Math 423 - Mathematics of Finance .....	86	Math 563 - Advanced Mathematical Methods for the Biological Sciences.....	131
Math 424 - Compound Interest and Life Insurance .....	87	Math 564 - Topics in Mathematical Biology .....	132
Math 425 (Stats 425) - Introduction to Probability.....	88	Math 565 - Combinatorics and Graph Theory.....	133
Math 427 - Retirement Plans and Other Employee Benefits .....	89	Math 566 - Combinatorial Theory.....	134
Math 429 - Internship.....	90	Math 567 - Introduction to Coding Theory .....	135
Math 431 - Topics in Geometry for Teachers .....	91	Math 571 - Numerical Methods for Scientific Computing I.....	136
Math 433 - Introduction to Differential Geometry .....	92	Math 572 - Numerical Methods for Scientific Computing II.....	137
Math 437 - Introduction to Differentiable Manifolds.....	93	Math 575 - Introduction to Theory of Numbers.....	138
Math 450 - Advanced Mathematics for Engineers I.....	94		
Math 451 - Advanced Calculus I.....	95		

Math 582 - Introduction to Set Theory .....	139
Math 590 - An Introduction to Topology .....	140
Math 591 - General and Differential Topology .....	141
Math 592 - An Introduction to Algebraic Topology.....	142
Math 593 - Algebra I.....	143
Math 594 - Algebra II.....	144
Math 596 - Analysis I (Complex).....	145
Math 597 - Analysis II (Real) .....	146

## Introduction

Welcome to the Department of Mathematics! This booklet describes the wide variety of options open to potential mathematics concentrators. There are thirteen subprograms outlined in this booklet ranging from Honors Mathematics to Mathematical Biology to Actuarial Mathematics. Whether you are already committed to being a mathematics concentrator or are still deciding, we hope that this booklet will provide you with useful information. If you have any questions that are not answered here, the first place to turn is the Undergraduate Mathematics Office in 2084 East Hall, (734) 763-4223 or via e-mail: [math-undergrad-office@umich.edu](mailto:math-undergrad-office@umich.edu). This office is the focal point for all matters relating to the undergraduate mathematics concentration and provides a wealth of services including academic advising and career planning.

All of the information in this brochure and more is available online at <http://www.math.lsa.umich.edu/undergrad/>

The department has approximately 100 teaching faculty, more than 140 graduate students, approximately 500 undergraduate concentrators, and about 250 students pursuing a math minor. All of our faculty teach both graduate and undergraduate courses and maintain research programs. Courses offered range from pre-calculus to topics courses and seminars at the frontiers of current research in more than 20 specialty areas.

Our faculty take a strong interest in the undergraduate program and are available to students at all levels. Most of the junior/senior classes are small, often with fewer than 30 students. We encourage our concentrators to meet regularly with Mathematics Department advisors, all of whom are regular faculty members. Appointments may be scheduled online at <http://www.math.lsa.umich.edu/undergrad/>.

Within the mathematics concentration there are options to accommodate a multitude of different interests. The **Pure Mathematics** and **Honors Mathematics Programs** focus on the more theoretical side of mathematics, although there are also advanced courses devoted to a wide variety of applications, such as numerical analysis, modeling, cryptography, biomathematics, and financial mathematics. The **Mathematical Sciences Program** is more oriented towards applications, although students are also exposed to theory. In the

Mathematical Sciences Program you will choose one of eight different fields of study: Discrete and Algorithmic Methods, Numerical and Applied Analysis, Operations Research and Modeling, Probabilistic Methods, Mathematical Economics, Control Systems, Mathematical Physics, or Mathematical Biology. The **Actuarial Mathematics Program** and the **Mathematics of Finance and Risk Management Program** provide preparation for employment in areas such as finance, pensions, and insurance. The **Secondary Mathematics Teaching Certificate Program** is coordinated with the School of Education to prepare students to teach mathematics in K-12 schools. A **Minor** in Mathematics is also an option for students enrolled in many schools and colleges at the University of Michigan. Information about academic minors can be found at <http://www.lsa.umich.edu/bulletin/chapter3/minors>.

There are a variety of extracurricular activities available to our concentrators. The Nesbitt Room (2851 East Hall) is an undergraduate commons room and is the meeting place for our Undergraduate Math Club. The Undergraduate Math Club meets regularly to hear informal mathematical lectures and socialize (for current information see <http://www.math.lsa.umich.edu/undergrad/mathclub.shtml>). The Student Actuaries at Michigan club (<http://www.math.lsa.umich.edu/SAM/index.html>) helps prepare students for careers in the actuarial profession as well as fielding a number of intramural sports teams. The Women in Mathematics club (<http://www.math.lsa.umich.edu/undergrad/WIM.html>) provides social and educational support for women through study nights, social events, and mathematical lectures. The STEM society (<https://sites.google.com/a/umich.edu/stem-society/>) works to improve math and science education in the local community and provide teaching opportunities to undergraduate students. Undergraduates may also participate in several mathematical competitions such as the Putnam Exam, the Virginia Tech Regional Mathematics Contest, and the University of Michigan Undergraduate Mathematics Competition [(UM)<sup>2</sup>C<sup>x</sup>].

Mathematics demands careful, rigorous, analytical reasoning. The intellectual development afforded by an undergraduate concentration in mathematics provides an excellent background for a wide variety of careers. Graduates of the University of Michigan Department of Mathematics have gone on to successful careers in law, medicine, politics, and business, as well as every aspect of science, computer science, technology, and, of course, mathematics itself. Some have chosen to pursue further education, but many others have taken

interesting and challenging positions in industry, business, and government directly after graduation. For more information on careers in mathematics please see <http://www.math.lsa.umich.edu/career>.

## **Pre-Concentration: The First Two Years**

The ideal high school background for a student who intends to concentrate in mathematics (or science or engineering) includes one and one-half years of algebra, one year of geometry, one-half year of trigonometry, and one year of pre-calculus and/or calculus. Inclusion of a high-school calculus course is not essential; courses 115 and 185 do not assume previous calculus experience. A student whose background is less thorough may need to begin college mathematics with Math 105 (Data, Functions and Graphs) and should expect that completion of a mathematics concentration will require at least one additional semester.

Prospective mathematics concentrators may choose from a variety of entry-level sequence:

The sequence Math 115-116-215 is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof. All sections are given two uniform midterms and a uniform final exam.

Math 174 (Proofs in Geometry) and Math 175 (Intro. to Cryptology) require permission from the honors mathematics placement advisor, but are also open to students not enrolled in the LS&A Honors Program. The natural sequel to either Math 174 or Math 175 is Math 176 (Explorations in Topology and Analysis); a course that is taught in an inquiry based learning method and is intended to prepare students for math 285 (Honors Calculus III).

Math 156, 255, and 256 form the applied honors calculus sequence for science concentrators and engineering students. It is assumed that students are well-acquainted with the material of high school Calculus (as evidenced by a score of 4 or 5 on the AB or BC Calculus exam, for example). To provide students with the solid mathematical background they will need for subsequent coursework in

mathematics, engineering, and science, the course gives equal time to applications and theory. In particular, while definitions are stated precisely and theorems are derived, technical details are often omitted.

The sequence Math 185-186-285-286 is an introduction to calculus at the honors level. Math 185 (Honors Calculus I) presupposes a higher level of accomplishment in high school math courses and covers somewhat more of the theory behind the calculus. Math 186 is usually followed by Math 285; after completion of Math 285, a prospective honors concentrator should take Math 286 followed by Math 451 and Math 217. Admission to Math 185 requires permission of the honors mathematics placement advisor, but is also open to students not enrolled in the LS&A Honors Program.

Math 295 (Honors Mathematics I), together with the succeeding courses Math 296-395-396, provides an intensive introduction to theoretical mathematics. From the beginning, students are exposed to abstract concepts. These courses require a higher level of interest and commitment than either of the other honors introductory sequences. The student who completes Math 396 is prepared to explore the world of mathematics at the advanced undergraduate and graduate level. Admission to Math 295 requires permission from the honors mathematics placement advisor, but is also open to students not enrolled in the LS&A Honors Program.

Students who enter the University with a score of 4 or 5 on either the AB or BC Advanced Placement exam in Mathematics have several additional options. A student should check with their School's general advisor or a Mathematics faculty advisor as to which mathematics course would be the most appropriate to complete their first year. Advanced Placement credit is granted in varying amounts as explained by a memo available from the Undergraduate Office and online at <http://www.math.lsa.umich.edu/undergrad/apcredit.shtml>.

## **Mechanics of The Concentration**

If you are interested in a mathematics concentration, we urge you to learn about the requirements as early as possible, but no later than the end of your sophomore year. Inappropriate course selections may result in difficulties in fulfilling requirements in a timely manner. We recommend that you make an appointment with a concentration

advisor to discuss the programs and to answer questions you may have. Appointments may be scheduled online at <http://www.math.lsa.umich.edu/undergrad/> or by visiting us at 2084 East Hall.

## **Declaring the Concentration**

The decision to concentrate in mathematics should be made by the end of the sophomore year and officially registered by filling out a [Concentration Declaration Form](#) in the Undergraduate Mathematics Office. During your first counseling session as a declared mathematics concentrator, you should make a tentative decision about which concentration program you want to pursue and should plan a possible sequence of courses to fulfill its requirements. Of course, as you progress through the program you may and will make many changes in this initial plan. Before you register for courses for each subsequent semester, you should make a counseling appointment to review your progress with an advisor and revise your plan for the remaining semesters. Regular counseling is your best guarantee for completing the program in a timely manner.

## **Subconcentration Programs**

There are five distinct concentration programs in mathematics: Pure Mathematics, Honors Mathematics, Mathematical Sciences, Actuarial Mathematics and Mathematics of Finance and Risk Management, and the Secondary Mathematics Teaching Certificate. The Mathematical Sciences Concentration is designed for the student interested in mathematics and its applications and splits up into eight subprograms depending on the particular applications. Although each of these programs has its own requirements and conditions, the prerequisites and basic courses are the same throughout.

A student may pursue 2 (or more) concentrations. However, unless precluded by requirements, for any pair of concentrations the set of courses used to satisfy conditions III and IV for either concentration must be distinct from the set of courses used to fulfill requirements I-IV of the other concentration (requirements listed later in this section). Exception: this rule does not apply if one of the pair is the Secondary Mathematics Teaching Certificate Program.

**Students are urged to discuss their ultimate career goals with an advisor at an early stage to ensure that an appropriate program is planned.**

All mathematics students are strongly encouraged to use Physics 140-240 as their Natural Science distribution requirement and to acquire a working knowledge of computers and their languages. Many of the careers open to mathematics concentrators involve heavy use of computing, and students preparing for such a career should take several computing courses.

Upper-level courses taken at another college or university can be used to satisfy concentration requirements only with written permission of the Director of Undergraduate Programs. Documentation (syllabus and text in English) is required to verify the equivalence of the external course. Students are required to complete at least 24 credit hours for the Mathematics Concentration (parts I-IV) in residence. At least 6 of these credit hours should be from the basic courses (part II), and at least 9 of these credit hours should be from the elective requirement (part III) and the cognate requirement (part IV).

**Students who intend to concentrate in mathematics and receive a grade of C- or lower in Math 217 should repeat this course before proceeding further.** By LS&A rules, to be awarded a degree in Mathematics a student must maintain an average grade point average of at least 2.0 (a C average) in all mathematics courses and other courses used to fulfill concentration requirements. If a course was taken in residence and a grade of A+ through C-, P, CR, or S was earned, then repetition of this course results in no additional credit or honor points. The course and grade appear on the transcript with the notation "Not for Credit." A student repeating a course in which a D+ through D- was previously earned will receive honor points but no additional credit towards a degree. The course appears on the transcript with the notation "Repetition." Repetition of a course in which an E, F, or U grade was originally earned produces both credit towards a degree and honor points for courses elected on the graded patten; there is no special transcript notation. In all such cases, the first election and grade earned remain on the transcript. The grades earned by repetition of courses are not averaged and posted as a single entry; but are posted as separate elections.

## Pure Mathematics

The Pure Mathematics Program is designed to provide broad training in basic modern mathematics including an introduction to the methods of rigorous mathematical proof and exposure to the major branches of mathematics: Algebra, Analysis, and Geometry/Topology.

**I.** Prerequisite to concentration in Pure Mathematics is one of the pair of courses 215&217, 255&217, 285&217, or 295&296. Note that Math 216 is not intended for Pure Mathematics concentrators.

All Pure Mathematics concentrators are also strongly encouraged to take Physics 140-141 and 240-241 and to acquire a working knowledge of a high-level computer language (e.g. *Fortran*, *C*, or *C++*) at a level equivalent to completion of EECS 183.

The concentration program must include at least nine courses: four basic courses (**II.**), four elective courses (**III.**), and one cognate course (**IV.**) as described below.

**II.** The basic courses consist of one from each of the following groups completed with a grade of at least C-:

1. **Modern Algebra:** Math 412 or 512
2. **Differential Equations:** Math 256, 286, or 316
3. **Analysis:** Math 351 or 451
4. **Geometry/Topology:** Math 433, 490, or 590

More advanced students, such as those who have completed Math 396, may substitute higher level courses with the approval of a concentration advisor.

**Following Math 215 all students intending to concentrate in Pure Mathematics should elect Math 217 (Linear Algebra) rather than Math 216 (Introduction to Differential Equations).** Math 216 is not intended for Pure Mathematics concentrators, who generally take Math 316 (Differential Equations) after completing Math 217.

**III.** The four elective courses must be chosen in consultation with an advisor to provide a cohesive program that explores an area of mathematics in some depth. There is a good deal of freedom here, but a random selection of courses will not satisfy this requirement. The courses should be chosen from the following list or have a course

number of 600 or above. Math 289 is a repeatable 1-credit course and can be used to satisfy the elective requirement only if taken for a total of 3 credits.

289 Problem Solving	310 Elementary Topics
354 Fourier Anal. and its Applic.	362 Applic. of Calc. and Lin. Alg.
389 Explorations in Mathematics	404 Inter. Diff. Equations
416 Theory of Algorithms	423 Mathematics of Finance
425 Introduction to Probability	433 Intro. to Differential Geom.
437 Intro. to Diff. Manifolds	450 Adv. Math for Engineers I
452 Advanced Calculus II	454 Bound. Val. Prob. for PDE
462 Mathematical Models	463 Math Modeling in Biology
464 Inverse Problems	465 Introduction to Combinatorics
471 Intro. to Numerical Methods	475 Elementary Number Theory
481 Intro. to Mathematical Logic	490 Introduction to Topology
498 Topics in Modern Math	525 Probability Theory
526 Discrete State Stoch Process	550 Intro. to Adaptive Systems
555 Intro. to Complex Variables	556 Methods of Applied Math I
557 Methods of Applied Math. II	558 Ordinary Diff. Equations
559 Topics in Applied Math	561 Linear Programming I
562 Cont. Optimization Meth.	563 Advanced Mathematical Bio
565 Combin. and Graph Theory	567 Intro. to Coding Theory
571 Num. Meth. for Sci. Comp. I	572 Num. Meth. for Sci. Comp II
575 Intro. to Theory of Numbers	582 Introduction to Set Theory
590 An Intro. to Topology	591 General & Diff. Topology
592 Intro. to Algebraic Topology	593 Algebra I
594 Algebra II	596 Analysis I (Complex)
597 Analysis II (Real)	

**IV. One cognate course** should be chosen from some field other than mathematics. Almost any field is acceptable, but the course must be at the 300+ level and should have significant mathematical content, at least at the level of Math 215. A list of suggested courses is available online at <http://www.math.lsa.umich.edu/undergrad/cognates.shtml>, but in all cases **approval of a concentration advisor is required.**

## Honors Mathematics

A student who is either in the LS&A Honors Program or is approved by the Departmental Honors Committee may declare an honors concentration in mathematics. The Honors concentrator will acquire a greater command of abstractions and of the subtleties of mathemati-

cal rigor. Members of the Honors Committee of the Department of Mathematics counsel honors concentrators. Honors students who complete an honors concentration with distinction may receive on their diplomas the designations “with honors,” “with high honors,” or “with highest honors.” However, these designations are not restricted to students officially enrolled in the Honors Program; any student whose course selection has followed the pattern of an honors concentrator may ask the Chair of the Honors Committee to be considered for an honors designation. An honors citation will be awarded to any student who completes the honors concentration requirements with a concentration GPA of at least 3.25 and an LS&A cumulative GPA of at least 3.4 at the time of graduation. Honors will automatically remove students without a 3.4 GPA. Citations of high and highest honors are awarded at the discretion of the Honors Committee on the basis of superior performance in advanced courses as attested by grades and individual faculty evaluations.

**I.** Students intending an honors concentration in mathematics are advised to take one of the Honors introductory sequences 156-256, 175-286, 185-286, 295-396, or some combination of these four. Please note that the sequence 295-396 is very theoretical. It is recommended that students in the 156-256, 175-286, and 185-286 tracks also complete Math 217.

All Honors Mathematics concentrators are also strongly encouraged to take Physics 140-141 and 240-241 and to acquire a working knowledge of a high-level computer language (e.g. *Fortran*, *C*, or *C++*) at a level equivalent to completion of EECS 183.

The Honors concentration program must include at least nine courses: four basic courses (**II.**), four elective courses (**III.**), and one cognate course (**IV.**) as described below.

**II.** The basic courses consist of one from each of groups 1, 2, 3 and 4 or groups 1, 3, 5 and 6 below **completed with a grade of at least C-**:

1. **Analysis:** Math 451
2. **Modern Algebra:** Math 493
3. **Linear Algebra:** Math 420 or 494
4. **Geometry/Topology:** Math 433, 490, or 590
5. **Probability:** Math 525
6. **Differential Equations:** Math 404, 454, 556, 557, or 558

**A student who has completed Math 295-296, with a grade of at least a C- is exempt from Math 451. A student who completed Math 295-395, with a grade of at least a C- is exempt from Math 420.**

**III.** The four elective courses must be chosen in consultation with an honors advisor to provide a cohesive program that explores an area of mathematics in some depth. There is a good deal of freedom allowed here, but a random selection of courses will not satisfy this requirement. The courses should be chosen from the following list or have a course number of 600 or above. Math 289 is a repeatable 1-credit course and can be used to satisfy the elective requirement only if taken three times. An honors counselor may approve another mathematics course or a course from another department with advanced mathematical content as one of these elective courses. The honors counselor may ask that the student arrange supplemental work in a given class not listed below to conform to expectations for an honors elective. **A student who completes the requirements for part II by choosing courses from groups 1, 3, 5 and 6 must complete a course in Complex Analysis.**

289 Problem Solving	310 Elementary Topics
389 Explorations in Mathematics	416 Theory of Algorithms
433 Intro. to Differential Geom.	452 Advanced Calculus II
462 Mathematical Models	463 Math Modeling in Biology
464 Inverse Problems	465 Introduction to Combinatorics
471 Intro. to Numerical Methods	481 Intro. to Mathematical Logic
490 Introduction to Topology	525 Probability Theory
526 Disc. Stochastic Processes	537 Differentiable Manifolds
555 Intro. to Complex Variables	556 Methods of Applied Math I
557 Methods of Applied Math II	558 Ordinary Diff. Equations
559 Topics in Applied Math	561 Linear Programming I
563 Adv. Mathematical Biology	565 Combin. and Graph Theory
566 Combinatorial Theory	567 Intro. to Coding Theory
571 Num. Meth. for Sci. Comp. I	572 Num. Meth. for Sci. Comp. II
575 Intro. to Theory of Numbers	582 Introduction to Set Theory
590 An Intro. to Topology	591 General and Diff. Topology
592 An Intro. to Algebraic Topology	593 Algebra I
594 Algebra II	596 Analysis I (Complex)
597 Analysis II (Real)	

**IV.** One cognate course should be chosen from some field other than mathematics. Almost any field is acceptable, but the course must be at the 300+ level and should have significant mathematical content, at least at the level of Math 215. A list of suggested courses is available online at <http://www.math.lsa.umich.edu/undergrad/cognates>.

[shtml](#), but in all cases **approval of a concentration advisor is required.**

## Mathematical Sciences

The Mathematical Sciences Program is designed to provide broad training in basic mathematics together with some specialization in an area of application of mathematics. Each student must select one of the eight Program Options as a special area.

Because of the somewhat more specific requirements of the Program Options, careful planning and frequent consultation with your advisor are essential to ensure timely completion of the program.

**I.** Prerequisite to concentration in Mathematical Sciences is one of the pair of courses 215&217, 255&217, 285&217, or 295&296. In addition, students must acquire a working knowledge of a high-level computer language (e.g. *Fortran*, *C*, or *C++*) at a level equivalent to completion of EECS 183. For those not pursuing either a Physics concentration or a Physics minor, Physics 140-141 and 240-241 are required for the Numerical and Applied Analysis and Mathematical Physics options and strongly recommended for the other options. Some of the options have additional requirements as noted below.

The concentration program must include at least nine courses: four basic courses (**II.**), three courses from one of the Program Options (**III.**), and two additional courses (**IV.**) as described below. At least two of the five optional courses must be mathematics courses.

**II.** The basic courses consist of one from each of the following groups **completed with a grade of at least C-**:

1. **Differential Equations:** Math 256, 286, or 316
2. **Discrete Math/Modern Algebra:** Math 312, 412, 465, or 493
3. **Analysis:** Math 351, 354, 450, 451, or 454
4. **Probability:** Math 425 or 525

More advanced students, such as those who have completed Math 396, may substitute higher level courses with the approval of a concentration advisor. All students are strongly encouraged to include in their program one of the more theoretical courses: Math 412, 451, 493, 494, or 525.



**III.** A concentrator in the Mathematical Sciences Program must choose one of the eight Options (a-h) below and complete at least three of the courses listed under that option. This requirement is designed to provide focus and depth to the program and can only be waived by a concentration advisor in favor of a program which provides this depth in some equivalent way. An acceptable program must include some of the more difficult courses. Advice should be sought from a concentration advisor before selecting an option. As an initial guide, we give a brief description of the options.

**(a) Discrete and Algorithmic Methods**

Discrete and algorithmic methods are concerned with the analysis of finite structures such as graphs, networks, codes, incidence structures, and combinatorial structures. The rapid growth of this area has been driven largely by its role as the mathematical core of computer science. Typical problems of this field involve optimization, emulation, or probabilistic estimation. Students choosing this option are encouraged to take EECS 280 and 281.

416 Theory of Algorithms	420 Advanced Linear Algebra
475 Elementary Number Theory	481 Intro. to Mathematical Logic
561 Linear Programming I	565 Combin. and Graph Theory
566 Combinatorial Theory	567 Intro. to Coding Theory
575 Intro. to Theory of Numbers	EECS 280 Prog. and Intro Data
EECS 281 Data Structures	EECS 376 Found. of Computer Sci.
EECS 477 Intro. to Algorithms	EECS 550 Information Theory
EECS 574 Comput. Complexity	EECS 586 Design/Anal. Algorithms
EECS 587 Parallel Computing	IOE 614 Integer Programming

**(b) Numerical and Applied Analysis**

As computers become more powerful, they are being used to solve increasingly complex problems in science and technology. Examples of such problems include developing high-temperature superconducting materials, determining the structure of a protein from its amino acid sequence, and methods to model global climate change. Industrial and government research laboratories require personnel who are trained in applying numerical and analytical techniques to solve such problems. Numerical techniques are algorithms for computer simulation, and analytical techniques may rely on series expansions such as the Taylor or Fourier series expansions. There is a close connection between numerical and analytical techniques. A new analytical technique often leads to more effective numerical algorithms; a good example is the development of wavelets and their applications in signal processing. Students wishing to enter

this field must acquire a strong background in mathematics, science, and computing. Students are encouraged to include EECS 283 and MATH 451 in their program, and to also consider doing a minor in another scientific discipline.

Students in this program may choose to pursue a dual concentration in Informatics (especially the Data Mining and Information Analysis track). This combination is a powerful recommendation to a prospective employer that the student can think quantitatively about information; collect, manage, analyze, and visualize massive datasets; and that the student has both the computational tools and the rigorous analytical methods to reason about information.

420 Advanced Linear Algebra	354 Fourier Anal. and its Applic.
404 Intermediate Diff. Equations	423 Mathematics of Finance
451 Advanced Calculus I	452 Advanced Calculus II
454 Bound. Val. Prob. for PDE	462 Mathematical Models
463 Math Modeling in Biology	464 Inverse Problems
471 Intro. to Numerical Methods	550 Intro. to Adaptive Systems
555 Intro. to Complex Variables	AERO 225 Intro. to Gas Dynamics
EECS 283 Prog. for Sci. and Eng.	ME 240 Dynam. and Vibrations
PHYS 340/341 Waves, Heat, Light	PHYS 401 Inter. Mechanics
STATS 406 Statistical Computing	STATS 426 Intro. to Theor. Stat.

**(c) Operations Research and Modeling**

Mathematical modeling refers generally to the representation of real-world problems in mathematical terms. In some sense this is necessary for any application of mathematics, but the term is used more often to refer to the more recent applications of mathematics to biological, mechanical, and human systems. Analysis of such systems involves complex mathematical descriptions and leads to large problems which can be solved only by use of a computer. Operations Research studies integrated systems including health care, education, manufacturing processes, finance, and transportation. Because the emphasis is on the analysis and operation of systems, practitioners are also qualified to deal with managerial problems. Career opportunities are available in many parts of industry and government. Most students should include Math 561 and Stats 426.

420 Advanced Linear Algebra	433 Intro. to Differential Geom.
462 Mathematical Models	463 Math Modeling in Biology
561 Linear Programming I	562 Contin. Optimization Meth.
CHE 510 Math. Meths in ChemE	IOE 515 Stochastic Processes
IOE 543 Scheduling	IOE 610 Linear Programming II
IOE 611 Nonlinear Programming	IOE 612 Network Flows

**(d) Probabilistic Methods**

Probability theory deals with the mathematics of randomness and its applications. It is the basis of mathematical statistics, where the goal is to draw inferences from samples. Non-statistical applications are found in many branches of the social, biological, and physical sciences, as well as in engineering. Because of the intimate connection between probability and statistics, students electing this option will usually include statistics courses in their program and **sometimes have a dual concentration in statistics.** Students electing this option must complete Math 525.

Students in this program may choose to pursue a dual concentration in Informatics (especially the Data Mining and Information Analysis track). This combination is a powerful recommendation to a prospective employer that the student can think quantitatively about information; collect, manage, analyze, and visualize massive datasets; and that the student has both the computational tools and the rigorous analytical methods to reason about information.

- |                                  |                                  |
|----------------------------------|----------------------------------|
| 423 Mathematics of Finance       | 523 Risk Theory                  |
| 525 Probability Theory           | 526 Discrete State Stoch. Proc.  |
| 547 Biological Sequence Analysis | EECS 502 Stochastic Processes    |
| STATS 406 Intro. to Stat. Comp.  | STATS 426 Intro. to Theor. Stat. |
| STATS. 430 Applied Probability   | STATS 466 Stat. Quality Control  |
| STATS 500 Applied Statistics I   | STATS 501 Applied Statistics II  |
| STATS 550 Bayesian Decision      |                                  |

**(e) Mathematical Economics**

One definition of economics is the study of the optimal allocation of scarce resources. Several mathematical techniques are fundamental to this study: constrained optimization using Lagrange multipliers, n-dimensional calculus, especially the Implicit Function Theorem (dependence of a solution on parameters), dynamics, probability and statistics to deal with inherent uncertainty, game theory to deal with decisions in which the actions of one agent affect the options of others, and proofs for understanding the derivation of economic principles. To ensure coverage of these topics, students choosing the Mathematical Economics option will usually choose Math 351 or 451 as their basic analysis course; Math 423, Stat 426, or Econ 406 as courses from the options list; and Econ 401 and a mathematics course at the 400-level or above as their related courses. A student who completes this option should find opportunities available in busi-

ness, government, and research organizations which collect, analyze, and model data having economic, social, and political parameters. **Many students in this program pursue a dual concentration in economics;** this combination is a powerful recommendation to a prospective employer that the student can think quantitatively, understand complex reasoning, and work with economic models.

- |                                   |                                 |
|-----------------------------------|---------------------------------|
| 420 Advanced Linear Algebra       | 423 Mathematics of Finance      |
| 424 Compound Interest & Life Ins. | 452 Advanced Calculus II        |
| 462 Mathematical Models           | 471 Intro. to Numerical Methods |
| 472 Num. Meth. with Fin. App.     | 523 Risk Theory                 |
| 561 Linear Programming I          | 562 Cont. Optimization Meth.    |
| 623 Computational Finance         | ECON 406 Intro. to Econometrics |
| STATS 426 Intro. to Theor. Stat.  | ECON 409 Game Theory            |

**(f) Control Systems**

Control Systems is a fascinating field which draws upon knowledge in many areas of mathematics. It pervades industry, and its practitioners can be found in such diverse fields as automotive pollution control, avionics, and process control in manufacturing. A control designer will need to interface with the modeling group to develop a mathematical description of the system to be controlled, and perhaps with the testing group to characterize disturbances or other uncertainties affecting the system. The required performance of the system will then be ascertained from the intended use and translated into a set of mathematical specifications for a closed-loop system. At this stage the designer will select from an arsenal of tools for the controller analysis and synthesis—this generally requires a solid foundation in linear algebra, differential equations, real analysis, and probability. Students planning to pursue graduate study in this area are recommended to include Math 451 and EECS 476 in their programs.

- |                                   |                                  |
|-----------------------------------|----------------------------------|
| 354 Fourier Anal. and its Applic. | 420 Advanced Linear Algebra      |
| 451 Advanced Calculus I           | 454 Bound. Val. Prob. for PDE    |
| 462 Mathematical Models           | 471 Intro. to Numerical Methods  |
| 555 Intro. to Complex Variables   | 561 Linear Programming I         |
| 562 Cont. Optimization Meth.      | EECS 376 Found. of Comp. Sci.    |
| EECS 460 Control Sys. Anal./Des.  | EECS 476 Theor. of Internet App. |
| EECS 560 Linear Systems Theory    | EECS 561 Digital Control Sys.    |
| EECS 562 Nonlinear Systems        | EECS 565 Linear Feedback         |
| EECS 567 Intro. to Robotics       | STATS 426 Intro. to Theor. Stat. |

**(g) Mathematical Physics**

Among all of the disciplines which make significant use of math-

ematics, physics has the longest history. Indeed, several areas of mathematics were developed for the purpose of solving problems in physics. This option allows a student to pursue interests in physics which use undergraduate mathematics. **It is designed to facilitate a concurrent concentration in Physics.** Every program must include at least two of the Physics courses from the list below. Note that although Physics 401 is a prerequisite to several of these, it does not count as one of the option courses.

404 Intermediate Diff. Equations	433 Intro. to Differential Geom.
454 Bound. Val. Prob. for PDE	471 Intro. to Numerical Methods
555 Intro. to Complex Variables	PHYS 351 Meth. of Theor. Phys. I
PHYS 405 Intermed. Elec. & Mag.	PHYS 406 Stat. & Thermal Phys.
PHYS 413 Intro. Nonlinear Dyn.	PHYS 435 Gravitational Phys.
PHYS 452 Meth. of Theor. Phys. II	PHYS 453 Quantum Mechanics

#### (h) **Mathematical Biology**

Mathematical Biology is a relatively new area of applied mathematics and is growing with phenomenal speed. Ever since the advent of high-powered computing, it has become obvious that mathematics can contribute a great deal to biological and medical research. Indeed, in many cases mathematical approaches can answer questions that cannot be addressed by other means, and thus mathematics is often an indispensable tool for biological research. Guided by campus-wide interest, the Department of Mathematics organized a Mathematical Biology Research Group consisting of undergraduates, graduate students, post-docs, and faculty that provides an informal setting for Mathematical Biology concentrators to learn about current research in the field. Typical areas of application include such diverse areas as the topology of DNA, genetic algorithms, cell physiology, cancer biology and control strategies, micro-circulation and blood flow, the study of infectious diseases such as AIDS, the biology of populations, neuroscience and the study of the brain, developmental biology and embryology, the study of hormone secretion and endocrine control, and bioinformatics. The Mathematical Biology option will thus be appropriate for any student with an interest in biology or medicine and a desire to apply the mathematics they learn to current and important biological problems.

An **additional prerequisite** to this concentration is completion of the Introductory Biology sequence (Bio 171 and 172). Math 463 (Mathematical Biology), and at least one course from Math 404, 452, 454, 462, 471, 558, 563, or 571 must be included in the concentration program. The last of the three elective courses for this sub-option

must be at an advanced level (numbered over 300) and chosen from one of the following areas: Biology, Physiology, Microbiology/Immunology, Neuroscience, Bioinformatics, or Natural Resources and Environment, with the approval of your mathematics concentration advisor. It is recommended that STATS 426 or 510 be a cognate course.

**IV.** To complete the concentration program each student should elect two additional advanced courses in mathematics or a related area. Every student must include, either here or elsewhere in his/her program, a course numbered 300 or above taught outside the department which emphasizes the application of significant mathematical tools (at least at the level of Math 215) in another discipline. A list of suggested courses is available from the Undergraduate Office, but in all cases **approval of a concentration advisor is required.** This is a very flexible requirement designed to accommodate special interests and may be satisfied by a broad range of courses in other departments (generally numbered 300 or above) or by mathematics courses numbered 400 or above.

**V.** At least two of the courses in **III.** and **IV.** must be MATH courses.

## **Actuarial Mathematics and Mathematics of Finance and Risk Management**

The **Actuarial Mathematics** Program is designed to provide broad training in the basic mathematics underlying the operations of private and social insurance and employee benefit plans. The courses are organized to assist the student to prepare for several of the examinations of the Casualty Actuarial Society and the Society of Actuaries.

Non-credit review classes for some professional actuarial examinations are organized each term; ask your actuarial advisor about the time and place of these classes. It is strongly recommended that some of these exams be passed before graduation. Summer internships are an important component of the educational program, and students are strongly encouraged to seek an internship no later than the conclusion of their junior year. Students are encouraged to take either Math 422 or 427 to satisfy their upper-level writing requirement.

The program in **Mathematics of Finance and Risk Management** (or **Financial Mathematics** for short) is designed to provide a broad education in the quantitative aspects of risk management and finance. Today's financial instruments require sophisticated mathematical techniques for their valuation. These techniques come from the fields of probability, statistics, and differential equations.

**I.** Prerequisite to concentration in **Actuarial Mathematics** or **Financial Mathematics** is one of the pair of courses 215&217, 255&217, 285&217, or 295&296. In addition, each student must complete two introductory courses in Economics (Econ 101 and 102), and acquire a working knowledge of a high-level computer language (e.g. *Fortran*, *C* or *C++*) at a level equivalent to completion of EECS 183.

The concentration program must include at least ten courses: four basic courses (**II.**), four special Actuarial or Financial Mathematics courses (**III.**), and two courses in related areas as described below (**IV.**).

**II.** The basic courses consist of one from each of the following groups **completed with a grade of at least C-**:

1. **Differential Equations:** Math 216, 256, 286, or 316
2. **Probability:** Math 425
3. **Finance:** Math 423
4. **Statistics:** Stats 426

More advanced students, such as those who have completed Math 396, may substitute higher level courses with the approval of a concentration advisor.

**III.** The special **Actuarial Mathematics** courses must include Math 424 and Math 520, and at least one of Math 521 or 522, as well as Math 523. Note that Math 424 and Math 425 or 525 are prerequisite to Math 520, which in turn must precede Math 521 or 522. Since 520 is not always offered every semester, careful planning is essential.

The special **Financial Mathematics** courses must include Math 451, 471 or 472 (472 is preferred), 525, and 526. As noted above, careful planning is essential.

**IV.** To complete the concentration program each student should elect two additional intermediate or advanced courses related to **Actu-**

**arial** or **Financial Mathematics**. Some, but not all, of the courses numbered 300 and above offered by Accounting, Computer Science, Economics, Finance, Industrial and Operations Engineering, and Statistics are appropriate here. Some specific approved courses are:

- Math 422, 427, 462 and 623;
- all Accounting courses over 300, except 471;
- Econ 401, 402, 406 and 409;
- Finance 302 and 310;
- IOE 310 and 452;
- LHC (Bus. Ad.) 305;
- Statistics courses numbered above 400;
- Sociology 330; and
- Philosophy 361, 414 and 429.

Actuarial Mathematics students interested in receiving VEE credits from the Society of Actuaries are encouraged to elect Economics 406 and IOE 452 for their cognates.

Other courses may sometimes be appropriate and in all cases **approval of the advisor in Actuarial Mathematics or Financial Mathematics is required.**

## Secondary Mathematics Teaching Certificate

The Secondary Mathematics Teaching Certificate program is designed to provide the broad training in mathematics necessary to be a successful teacher of mathematics at the secondary level, grades 7-12. The requirements for a secondary teaching certificate with a concentration in mathematics may be met while earning a degree from the College of Literature, Science, and the Arts (A.B., B.S., or B.G.S.) or from the School Education (A.B. Ed. or B.S. Ed.). Students must apply for admission to the certification program by January 15 of their sophomore year. Concentration requirements are identical for all of these degrees, although LS&A and Education have somewhat differing language and distribution requirements. LS&A degree candidates must earn at least 104 LS&A credits and at least 30 Education credits. Please note that the LS&A B.S. degree requires 60 credits in physical and natural science and mathematics; students with less than 60 credits are eligible for an A.B. degree. Appointments with a Mathematics Department teaching certificate advisor may be scheduled online at <http://www.math.lsa.umich.edu/>

undergrad/. For information specific to the School of Education, students should contact the SoE Office of Student Services, 1033 SEB, (734) 764-7563, <http://www.soe.umich.edu/osa/index.html>.

It is essential that students planning to obtain a teaching certificate consult a teaching certificate advisor, either in Mathematics or Education, prior to beginning their concentration program.

I. The math prerequisites for the teaching certificate program are Math 217 and a working knowledge of a high-level computer language (e.g. *Fortran*, *C* or *C++*) at a level equivalent to completion of EECS 183.

II. The basic courses for a candidate for a teaching certificate consist of one course from each of the following five groups (chosen with the approval of a teaching certificate advisor), **completed with a grade of at least a C-**:

1. **Modern Algebra:** Math 312, 412, or 512
2. **Geometry:** Math 431 or 531
3. **Probability:** Math 425 or 525
4. **Analysis:** Math 351 or 451
5. **Secondary Mathematics:** Math 486

III. The program requires ten specific Education courses, listed below, totaling 30 credits. These are elected in the junior and senior years in a specified order. Consult with the School of Education Office of Student Services for the order and timing of these courses.

Methods of Teaching Mathematics (or minor field): Ed 413 (3 credits)  
Practicum in Teaching Methods: Ed 307I and 307II (4 credits)  
Educational Psychology: Ed 391 (3 credits)  
Reading and Writing: Ed 402 (3 credits)  
Education in a Multi-cultural Society: Ed 392 (3 credits)  
Directed Teaching: Ed 302 (10 credits)  
Problems and Principles of Secondary Education: Ed 304 (2 credits)  
Teaching with Technology: EDU 490-003 (1 credit)  
Teaching Students with Exceptionalities: EDU 490-004 (1 credit)

The last four of these are to be elected concurrently. We repeat: students must apply for admission to the certification program by January 15 of their sophomore year. Forms are available at the School of Education, Office of Student Services (1033 SEB).

IV. Additionally, every student must successfully complete: an introductory course in psychology (should be taken before Ed 391) and Michigan's licensure requirements.

V. Every Teaching Certificate student must also obtain a concentration or minor (SOE, not LS&A) in another academic field. This normally requires 20-24 credits in a structured program in an area other than mathematics. Consult the *Bulletin* of the School of Education for acceptable programs.

## **Graduation Requirements**

The Departmental requirements for each of the concentration programs are described in detail above. The final certification that you have satisfied the requirements for your program is provided to the Senior Auditors on the Concentration Release Form (available in the Undergraduate Office, 2084 East Hall). This form is to be signed by your concentration advisor **before** the beginning of your final semester in the program. It will list your current and future courses and declare that if these courses are completed satisfactorily, the concentration requirements will be satisfied. Early submission of this form is very important to allow time for any required adjustments. At the same time you should use Wolverine Access to Apply for Graduation electronically. The College of LS&A has a number of further requirements which must be satisfied before you can graduate. These are described in the LS&A Bulletin. General Counselors in the LS&A Academic Advising Office (1255 Angell Hall), (734) 764-0332, are trained in the administration of these regulations and should be consulted regularly to ensure that all requirements will be satisfied by the time you expect to graduate.

## **Minor in Mathematics**

The Minor in Mathematics is designed to enable a student with a significant interest in Mathematics to deepen his/her knowledge while pursuing a concentration in another field. Note that LS&A regulations preclude having both a concentration and a minor in the same field. While the concentration will often be in a field which makes significant use of mathematics, such as a science or a quantitative social science, it may be in any area of study. Students from outside LS&A,

for example those from the College of Engineering, may also pursue a Minor in Mathematics. LS&A regulations allow Advanced Placement credits and prerequisites for the concentration to count also as prerequisites for the minor. For students enrolled in LS&A, only one course may be shared between the requirements of a minor and the requirements of a concentration. This rule does not apply to students enrolled outside of LS&A. Courses used to meet the requirements of a minor may not be taken pass/fail. **All courses for the minor program must be completed with a grade of at least a C-.**

The prerequisite to a Minor in Mathematics is one of the sequences Math 115-116, 175-176, 185-186, or 295-296; or Math 156. These all provide a thorough grounding in the calculus of functions of one variable. Advanced Placement credits in Math 120 and 121 also fulfill the prerequisite requirement.

The Minor consists of courses from the following two lists:

Second-year courses:

Multivariable Calculus:	Math 215, 255, or 285
Linear Algebra:	Math 214, 217, 417, or 419
Differential Equations:	Math 216, 256, or 286

Upper-level courses:

Analysis/Diff. Equations:	Math 316, 351, 354, 404, 450, 451, 452, 454, 555
Algebra/Number Theory:	Math 312, 389, 412, 420, 475, 493, 494, 561, 575
Geometry/Topology:	Math 433, 437, 490
Applied Mathematics:	Math 354, 371, 404, 423, 425, 450, 454, 462, 463, 471, 472, 550, 563
Discrete Mathematics:	Math 310, 312, 389, 412, 416, 420, 425, 465, 475, 481, 512, 561, 566, 567, 582
Financial/Actuarial :	Math 423, 424, 520, 523, 524

A student must select **either**:

- 2 second-year courses and 3 upper-level courses
- or**
- 1 second-year course and 4 upper-level courses.

Existing credit restrictions ensure that at most one second-year

course can be elected from each of the three areas. The upper-level courses must be selected from **exactly two** of the listed areas. Note that a few courses are listed in more than one area.

A student planning to take linear algebra and differential equations should note that not all of Math 215, 216, and 217 will count towards the minor, whereas all of Math 215, 217, and 316 will.

All courses carry 3 or 4 credit hours and the total number of required credit hours is between 15 and 18. Per LSA rule at least 10 credits must be taken in-residence. All in-residence credits must be taken from the Math Department. In all cases, more advanced courses may be substituted with the approval of a math advisor. In particular, students who have satisfied the prerequisite with the honors sequence Math 295-296 will need to consult an advisor for the proper selection of courses. Other modifications can also be made with the approval of a math advisor. Finally, classes offered outside of Mathematics **cannot** be used to satisfy the requirements of the minor.

## Department Faculty

names in bold are research area leaders

### Professors

Jinho Baik, *Probability Theory*

David Barrett, *Complex Analysis*

**Alexander Barvinok, Combinatorics, Applied Mathematics**

Hyman Bass, *Algebraic Geometry*

Erhan Bayraktar, *Actuarial and Financial Mathematics*

**Andreas Blass, Combinatorics, Logic and Foundations**

Anthony Bloch, *Differential Equations*

Daniel Burns, Jr., *Complex Analysis, Algebraic and Differential Geometry*

Richard Canary, *Topology*

Joseph Conlon, *Mathematical Physics*

Stephen DeBacker, *Representation Theory*

**Hendrikus Derksen, Algebra**

Charles Doering, *Applied Mathematics, Mathematical Physics*

Selim Esedoglu, *Applied Mathematics*

Sergey Fomin, *Algebraic Geometry, Theoretical Computer Science*

William Fulton, *Commutative Algebra/Algebraic Geometry*

Anna Gilbert, *Analysis*

Robert Griess, *Group/Lie Theory, Algebra*

Philip Hanlon, *Combinatorics*

Mel Hochster, *Commutative Algebra/Algebraic Geometry*

**Curtis E. Huntington, Actuarial and Financial Mathematics**

Trachette Jackson, *Mathematical Biology*

Lizhen Ji, *Analysis, Functional Analysis*

Mattias Jonsson, *Complex Analysis, Financial Mathematics*

Smadar Karni, *Numerical Analysis*

Robert Krasny, *Applied Mathematics, Numerical Analysis*

Igor Kriz, *Topology*

Jeffrey Lagarias, *Computer Science, Number Theory*

Robert Lazarsfeld, *Algebraic Geometry*

Robert Megginson, *Analysis, Functional Analysis*

Peter Miller, *Applied Mathematics*

Hugh Montgomery, *Number Theory*

Mircea Mustata, *Algebraic Geometry*

Gopal Prasad, *Algebra/Algebraic Geometry, Lie/Representation Theory*

Jeffrey Rauch, *Differential Equations*

**Yongbin Ruan, Symplectic Geometry**

Mark Rudelson, *Analysis*

**John Schotland, Applied Mathematics**

G. Peter Scott, *Topology*

Carl Simon, *Differential Equations*

Peter Smereka, *Differential Equations*

Karen Smith, *Algebra/Algebraic Geometry*

Joel Smoller, *Differential Equations*

Ralf Spatzier, *Differential Geometry*

John Stembridge, *Combinatorics*

Martin Strauss, *Fundamental Algorithms, Comp Security*

Alejandro Uribe, *Geometric Analysis, Mathematical Physics*

**Roman Vershynin, Geometric Functional Analysis**

Divakar Viswanath, *Numerical Analysis*

David Winter, *Algebra/Group Theory, Lie Theory*

Sijue Wu, *Differential Equations*

Virginia Young, *Actuarial and Financial Mathematics*

Michael Zieve, *Algebra and Number Theory*

### Associate Professors

Silas Alben, *Applied Mathematics*

Volker Elling, *Differential Equations*

Daniel Forger, *Mathematical Biology*

Thomas Lam, *Combinatorics*

Kristen Moore, *Actuarial and Financial Mathematics*

**Kartik Prasanna, Number Theory**

David Speyer, *Algebra, Algebraic Geometry, Combinatorics*

### Assistant Professors

Hala Al Hajj Shahedeh, *Applied Mathematics, Differential Equations*

Jose Alcala Burgos, *Financial and Actuarial*

Ashwini Aroskar, *Logic and Foundations*

Burcu Baran, *Number Theory*

Lydia Bieri, *Analysis, Applied Mathematics, Differential Geometry, Differential Equations*

Jonah Blasiak, *Combinatorics*

Henry Boateng, *Applied Mathematics*

Victoria Booth, *Mathematical Biology*

Dmitriy Boyarchenko, *Representation Theory*

Khalid Bou-Rabee, *Geometry and Topology*

Morgan Brown, *Algebra/Algebraic Geometry*

Trevor Clark, *Geometry and Topology*

Michael Dabkowski, *Analysis*

Alexander Duncan, *Algebra/Algebraic Geometry*

Justin Dunmyre, *Mathematical Biology, Differential Equations*

Daniel Erman, *Algebra, Algebraic Geometry*  
 Arash Fahim, *Applied, Financial, and Actuarial Mathematics*  
 Daniel Florilli, *Number Theory*  
 Dusty Grundmeier, *Analysis*  
 Mark Hagen, *Geometry*  
 Brett Hemenway, *Number Theory*  
 Lunmei Huang, *Applied Mathematics*  
 Karl Liechty, *Analysis*  
 Benjamin Linowitz, *Geometry, Number Theory*  
 Ricky Ini Liu, *Combinatorics*  
 Ruochan Liu, *Number Theory*  
 Evelyn Lunasin, *Applied Mathematics, Differential Equations*  
 Christopher Lyons, *Algebra, Algebraic Geometry, Number Theory*  
 Manabu Machida, *Applied Mathematics*  
 Seth Marvel, *Applied Mathematics, Differential Equations*  
 Benson Muite, *Applied Mathematics*  
 Sergey Nadtochiy, *Financial and Actuarial Mathematics*  
 Danny Neftin, *Algebra, Number Theory*  
 Suho Oh, *Combinatorics*  
 Yaniv Plan, *Applied Mathematics*  
 Jenna Rajchgot, *Algebraic Geometry*  
 Ryan Reich, *Algebraic Geometry*  
 Shifra Reif, *Algebra, Combinatorics*  
 Matthew Satriano, *Algebra, Algebraic Geometry*  
 Scott Schneider, *Algebra, Algebraic Geometry*  
 Ian Shipman, *Algebraic Geometry*  
 Matthew Stover, *Geometry and Topology*  
 Shравan Veerapaneni, *Applied Mathematics*  
 Cynthia Vinzant, *Algebraic Geometry, Combinatorics, Applied Math*  
 Bogdan Vioreanu, *Differential Equations*  
 Daniel Visscher, *Geometry and Topology*  
 Li Wang, *Numerical Analysis*  
 Zuoqin Wang, *Analysis, Differential Geometry*  
 Xiang Yu, *Finance and Actuarial*  
 Weiyi Zhang, *Geometry and Topology*

#### Lecturers

Irina Arakelian, *Mathematics Education*  
 Fernando Carreon, *Differential Equations*  
 Harry D'Souza, *Algebra/Algebraic Geometry, Number Theory, Complex Geometry*  
 Brad Hinesman, *Financial and Actuarial*

Paul Kessenich, *Analysis, Differential Equations*  
 Angela Kubena, *Mathematics Education*  
 Gavin LaRose, *Mathematics Education*  
 Joseph Marker, *Financial and Actuarial*  
 Harry Panjer, *Financial and Actuarial*

### **Academic Advising**

You should consult with a concentration advisor before registering for courses each semester and before you make any important changes in your program. Appointments and counseling sessions are held in the Undergraduate Office (2084 East Hall). To make an appointment online please see <http://www.math.lsa.umich.edu/undergrad/>. For advice on general LS&A regulations (distribution, languages, etc.) and reassurance that you understand the rules, you should see a General Counselor in the LS&A Academic Advising Office (1255 Angell Hall), (734) 764-0332. Course information is available from the course guide online at the LS&A website: <http://www.lsa.umich.edu>.

Transfer credit for courses taken elsewhere is administered by the Credit Evaluation division of the Undergraduate Admissions Office (1220 SAB), (734) 763-1074, <http://www.lsa.umich.edu/students/transfer/equivalencies>. If this form lists credit as "departmental credit" an official syllabus for the course can be submitted to [math-transfer-credit@umich.edu](mailto:math-transfer-credit@umich.edu) for review.

For women seeking careers in science or mathematics, the Women in Science and Engineering (WISE) Program offers academic and career counseling, workshops on combining careers with various lifestyles, contact with female role models, lists of scholarships and awards, and a resource center listing opportunities for women in science and engineering; visit <http://www.wise.umich.edu/> for more information, including locations and contact numbers.

### **Academic Help**

Your first source for help with course work is the instructor of the course. For Introductory courses, including Math 105, 115, and 116, the Math Lab (B860 East Hall), (734) 936-0160, <http://www.math.lsa.umich.edu/undergrad/mathlab/>, offers drop-in help during the day and many evening and weekend hours. The



Department also maintains a private tutor list of advanced students willing (for pay) to assist with many undergraduate courses: <http://www.math.lsa.umich.edu/courses/tutor.html>. Additional information can be found at <http://www.lsa.umich.edu/advising/academicsupport/tutoring>

The national engineering honor society Tau Beta Pi (1228 EECS), (734) 615-4187 also offers tutoring in a range of science, engineering, and mathematics courses.

The Sweetland Center for Writing (1310 North Quad), (734) 764-0429 offers free individual assistance on a specific writing project or with general writing skills.

### **Around the Mathematics Department**

Aside from the Undergraduate Office, there are several other departmental offices which are relevant to undergraduate students. Questions, complaints, praise, and comments about graduate students, graders, and Graduate Student Instructors should be addressed to the Graduate Program Office (2082 East Hall), (734) 764-7436. Similar messages concerning faculty may be addressed to the Associate Chair for Education (2084 East Hall), (734) 763-4223. For other administrative resources, undergraduate students may contact the Chair's Office (2074 East Hall), (734) 764-0335.

A great deal of information about the Department is available online—start with the Mathematics website at <http://www.math.lsa.umich.edu> and follow the desired links.

### **Mathematics Library**

The Mathematics Library (3175 Shapiro Library), (734) 936-2327, <http://www.lib.umich.edu/science/math/>, is an important resource for students and faculty alike. The Library houses one of the best mathematics collections in the world including most of the major periodicals, monographs, and textbooks. Course reserve books are kept here and there are terminals for accessing the on-line general library catalog MIRLYN. Free access is also available to MathSciNet, the American Mathematical Society's Mathematical Reviews Database online. It contains bibliographic data and reviews of mathematical research literature from articles published in almost all mathematical

journals in the world.

### **Computing Facilities**

The Department has arranged for several software packages of special interest to students of mathematics to be available at many of the University's public computing stations. These include *Matlab*, *Maple*, and *Mathematica*. The Department has 4 computing lab rooms equipped for use by students of Math courses that require labs. They are available to all faculty and graduate students and to undergraduates in the REU program (<http://www.math.lsa.umich.edu/undergrad/REU/>).

### **Career Planning**

One of the main goals of the Undergraduate Office is to acquaint students with the many career options that the study of mathematics affords. The office collects and makes available information on graduate study, careers in the industrial and service sectors, and mathematics education, as well as opportunities for summer employment in mathematical fields. For those students who may be considering graduate study either in mathematics or a related field, the Undergraduate Office maintains files on graduate programs at many universities. These include descriptions of the programs, application information, and national rankings of the departments. To help graduating students secure entry-level positions in private companies, government agencies, and non-profit organizations, the Undergraduate Office provides notice of employment opportunities and on-campus recruiting. In the Fall semester, the Department invites Mathematics Alumni/ae as well as outside companies to participate in a Career Fair. Students will have the opportunity to speak with companies hiring mathematics majors as well as learn about what types of careers past students have gone on to in industry, government, and mathematics education. We also maintain an extensive web page on career options: <http://www.math.lsa.umich.edu/career/>.

Because summer employment in a mathematical field is one of the best ways to decide whether a career in mathematics is what you want, the Undergraduate Office assists students in finding meaningful summer experiences that will enhance their mathematical skills and give them a taste of using mathematics in the "real world." Information on opportunities for summer employment in business or

industry, and summer research positions are sent to students in the weekly "math missive." In addition, there are many programs which support undergraduate students doing mathematical research during the summer, both at U-M and across the country. Details are available in the Undergraduate Office (2084 East Hall).

### **Personal Counseling**

Counseling and Psychological Services (CAPS, 3100 Michigan Union, (734) 764-8312), provides individual and group counseling on a wide range of concerns. They also offer workshops which focus on relaxation techniques, strategies for managing study time, and methods for coping with test-taking anxiety. Counseling is free to students on a walk-in basis or by appointment: <http://www.umich.edu/~caps>.

### **Complaints and Problems**

We hope you won't have any, but, just in case, here are a few strategies. General problems concerning instructors that cannot be resolved privately should be directed to the Undergraduate Program Office (2084 East Hall), (734) 763-4223. In particular, any case of unprofessional attitudes or actions by an instructor or grader should be reported immediately. In many cases the Undergraduate Office can serve as an intermediary to help resolve the problem quickly. Problems that cannot be resolved in the Department might be directed to the Office of Student Academic Affairs (1255 Angell Hall), (734) 764-7297, the Affirmative Action Office (734) 763-0235, or the Ombudsman (6015 Fleming Administration), (734) 763-3545.

### **Extracurricular Activities**

Your undergraduate experience in mathematics will be greatly enriched by participating in some of the math-related extracurricular activities sponsored by the department. Interaction with your fellow concentrators is an integral part of a well-rounded undergraduate education.

The **C.J. Nesbitt Common Room** (2075 East Hall) is the nerve center for extracurricular activity in mathematics. This undergraduate common room is a place to relax, study, read, or talk to other mathematics concentrators. It was funded by alumni donations to honor Professor Cecil J. Nesbitt (October 10, 1912 to October 22, 2001)

who directed our very well-respected program in Actuarial Mathematics for many years.

The **Undergraduate Math Club** is run for and by undergraduate math concentrators, with the assistance of a faculty moderator. It is an informal organization which sponsors social events and talks by faculty and students. A typical meeting begins with free pizza and drinks, followed by a 45 minute talk on an interesting mathematical problem, application, or idea (or all three!). The selected topic is something which isn't usually seen in the standard curriculum. Some of these topics lead to important concepts in theoretical or applied research, while others explain a clever solution to an interesting problem. Everything is formulated to be accessible to students whose technical background consists only of calculus and some exposure to the methods of proof. A few of the meetings will also feature information regarding graduate schools, internships, and career planning. For more information see the web page: <http://www.math.lsa.umich.edu/undergrad/mathclub.shtml>.

The **Student Actuaries at Michigan** (SAM) club is an organization for undergraduates and graduates interested in the field of actuarial and financial mathematics. There are monthly meetings on topics of interest, sometimes featuring speakers from industry. The group organizes study groups for the professional examinations and coordinates visits to campus by industry recruiters. It also sponsors a variety of athletic and social activities including an end-of-year picnic. For more information see the web page: <http://www.math.lsa.umich.edu/SAM/>.

The **Women in Mathematics** (WIM) club seeks to provide social and educational support for women studying mathematics and statistics. The club provides opportunities to hear from female speakers in mathematics, allows members to meet others with similar academic interests, and provides peer support for classes and career options. WIM holds study nights, brings in speakers, and hosts group social events. For more information see the web page: <http://www.math.lsa.umich.edu/undergrad/WIM.html>.

The **STEM Society** is a student group that aims to expose K-12 student, especially in lower socioeconomic areas, to inquiry-based learning in the fields of science and math. We hope to disprove common stereotypes that students may have about science and math as well as to increase their awareness of the diverse opportunities available in these fields. In addition, they strive to provide undergraduates

with an interest in teaching or sharing their passion for science and math with opportunities to do so. For more information see their webpage: <https://sites.google.com/a/umich.edu/stem-society/>.

The **Research Experience for Undergraduates (REU)** is designed to provide outstanding undergraduates with the opportunity to pursue research under the tutelage of experienced faculty members. Typically students work with a faculty member on a project of mutual interest for 6-8 weeks during the summer. The projects range anywhere from mathematical modeling in the sciences and engineering to solving abstract and conceptual problems, depending on the background and the interests of student and faculty member. Each student will be required to write a report about the project, which is due at the project's completion. Each student who participates in the REU program is paid a stipend of approximately \$4,300 during the summer. Students have the chance to expand their knowledge, learn new material, and combine professional development with close contact between faculty and student peers. If you are interested in participating and would like to learn more, please contact the Undergraduate Office, (734) 763-4223. For summer REU positions, applications will be available in the Undergraduate Office in February and are due in March.

The University of Michigan's **SUBMERGE (Supplying Undergraduate Biology and Mathematics Education and Research Group Experiences) program** is ideal for students with an interest in interdisciplinary science and at least 2 years of study remaining. For more information about the program please visit: <http://www.math.lsa.umich.edu/submerge/index.html>. Applications are usually available on-line in November and due in January.

The **William Lowell Putnam** competition is a nationwide mathematics competition sponsored annually by the Mathematical Association of America (MAA). All full-time undergraduate students may participate and each year about 30 students compete. The competition consists of an examination on topics common to the undergraduate curriculum and is given in early December. The emphasis is on ingenuity rather than knowledge. There are both team and individual prizes, including a graduate fellowship. In 2011, the University of Michigan team placed 8th overall, receiving an Honorable Mention. As preparation for the Putnam exam, the Department offers a Problem-Solving seminar which meets weekly during the Fall and Winter terms. Students may receive one hour of course credit for this seminar by enrolling in Math 289. In this seminar students are

exposed to interesting mathematical material not found in other courses. They also develop their problem-solving abilities. The seminar is open to undergraduate students at all levels. As additional preparation, students are encouraged to participate in the **Virginia Tech Mathematical Competition** administered within the department each November.

The Department also sponsors a local competition. The **University of Michigan Undergraduate Mathematics Competition [(UM)<sup>2</sup>C\*]** consists of an examination given towards the end of the winter semester each year. It is open to all full-time undergraduates on the Ann Arbor campus and offers substantial cash prizes which vary from year to year depending on financing. The Problem-Solving seminar is also recommended as preparation for the [(UM)<sup>2</sup>C\*].

Also listed in the **weekly departmental bulletin** are a variety of other special events. Many are specialized seminars that are not at a level accessible to undergraduates, but some events include more general colloquium talks, special lectures, and mathematical movies. It is a good idea to check the bulletin every Monday morning. It is posted in the hallway near the main Department Office, 2074 East Hall, and on the web <http://www.math.lsa.umich.edu/seminars>.

An excellent way to improve your mastery of mathematics is to get involved in teaching it. There are positions available to undergraduates as **graders** and **tutors**. Graders correct and grade homework assignments in the larger courses. A student is eligible to grade any course he/she has passed with a grade of B or better, but the positions are competitive. Applications should be submitted to the Graduate Program Office (2082 East Hall) during the first few days of classes each term. Undergraduate graders are currently paid a minimum of \$12 per hour. Undergraduates wanting tutor positions should pick up an application from the Undergraduate Program Office (2084 East Hall). Irina Arakelian (3859 East Hall), (734) 936-1775, [arakel@umich.edu](mailto:arakel@umich.edu) is the current Mathematics Lab coordinator. Tutors are available for drop-in help in courses through Math 217 and are paid \$12 per hour.

## From Out Graduates

*Math is among the pinnacles of human knowledge, and it is increasingly relevant in our world. All the modern technology you use (computers, smartphones, GPS, etc.) is based on complex mathematical theories. Even if you don't become a "mathematician", the jobs that require some degree of mathematical or logical thinking are growing to dominate the modern workplace. If you can do math, you can probably do those other things as well, because math puts them all into a larger, well-developed context. And lastly, if you like problem solving, math is a ton of fun.*

(Ibrahim T., B.S. 2005)

*I work in the development of a large-scale, distributed storage system for a local business. I use my math degree almost every day. Not only do subjects I learned explicitly, like number theory and abstract and linear algebra, come into constant use when trying to do novel things on very large scales, but the background and rigor that the U of M math department taught me gave me a good foundation for teaching myself whatever new math I need to solve a problem.*

(Adam E., B.S. 2009)

*Majoring in math was one of the best decisions I have ever made. The critical thinking and logical reasoning skills that I developed studying math provided the fundamentals that I needed to succeed in the real world (even though I have never ever had to use stochastic processes, ito's formula, and the other 70% of material I learned), and when I do run into real world math problem, it is very comforting to know that I used to know the stuff. Also, the best aspect of being a math major is that when we tell other people we studied math in college, they automatically stereotype us as smart, which is definitely a good stereotype to have. The free math T-shirt was also pretty sweet. Go blue!*

(Austin B., B.S. 2011)

*No matter what you major in college, you are bound to worry about what to do upon graduation. But if you major in math, you will have versatile skills sets that will set you apart from others. Whether it be business, medical, legal or any other field you can think of, you will have necessary background to do virtually anything.*

(Young A., B.S. 2005)

*I started a job as a Data Manager in a healthcare analytics company right out of college. I had no experience when I started, not even an internship to put on my resume. Five years and four promotions later, I am now an Operations Manager with 27 people reporting up through me. My friends and family ask me all the time whether or not I use my math degree in my job; they assume that because I majored in math that I should be sitting around crunching numbers all day. The truth is that I don't do complex calculations very often, and I have probably forgotten almost all of the theorems I once knew, but a math education is so much more than calculations and theorems. My math curriculum exposed me to concepts of logic and reasoning that are applicable in both programming and conflict resolution. It taught me to relate variables and data sets which helped me understand databases on a deeper level. I can quickly analyze large volumes of data and draw conclusions based on summarized results or determine the best course of action given several alternatives. I see details that others miss and ask the right questions to identify root causes of issues. Et cetera, et cetera. In short, math taught me problem-solving and critical thinking. For a long time, I took this for granted thinking that everyone was equipped with comparable abilities. After five years in the real world, I realize that it was actually these problem-solving and critical thinking skills that set me apart from my peers and allowed me to excel in a field I originally knew nothing about, and it was my math education that taught me those skills. Math was once a discipline reserved for professors and rocket scientists, but I can say from experience that the possibilities for math majors in the modern workforce are endless. You can't go wrong with a math degree.*

(Nicole S., B.S. 2007)

## **Courses**

### **Math 105 - Data, Functions, and Graphs**

**Prerequisites:** 3-4 years HS math including trigonometry

**Frequency:** Fall (I), Winter (II), Summer (IIIb)

**Student Body:** First-year students

**Credit:** 4 Credits. No credit granted for those who have completed any Math course numbered 110 or higher. 2 credits granted to those who have completed 103.

**Recent Texts:** Functions Modeling Change: A Preparation for Calculus (3rd edition) by Connally, Hughes-Hallett, Gleason et. al.

#### ***Background and Goals:***

Math 105 serves both as a preparatory class to the calculus sequences and as a class for students who are interested in strengthening their math skills. Students who successfully complete 105 are fully prepared for Math 115.

#### ***Content:***

This course presents the concepts of precalculus from four points of view: geometric (graphs), numeric (tables), symbolic (formulas), and written (verbal descriptions). The emphasis is on the mathematical modeling of real-life problems using linear, polynomial, exponential, logarithmic, and trigonometric functions. Students develop their reading, writing, and questioning skills in an interactive classroom setting.

#### ***Alternatives:***

None

#### ***Subsequent Courses:***

The course prepares students for Math 115 (Calculus I).

## Math 107 - Mathematics for the Information Age

**Prerequisites:** 3-4 years HS math

**Frequency:** Sporadically

**Student Body:** First-year students (non-mathematics concentrators) who are not necessarily required to take calculus

**Credit:** 3 Credits.

**Recent Texts:**

### **Background and Goals:**

The course will investigate topics relevant to the information age in which we live. An investigation of cryptography and coding methods, including prime numbers, randomness, and data compression will lead to the mathematics of the Web. Use of interactive web sites and web data are an integral part of the course. The course will emphasize the representation of mathematical data in graphical, tabular, and symbolic forms and investigate the inferences that can be drawn from these models. Emphasis will be placed on the development of estimation skills, the ability to determine reasonableness of answers, and the ability to find alternative approaches to a problem.

### **Content:**

Typical topics include cryptography, coding, politics, biological data, populations, chaos, and game theory. Topics will be presented as modules, generally a week or two in length.

### **Alternatives:**

None.

### **Subsequent Courses:**

Math 127 or Math 128 could be taken after Math 107.

## Math 110 - Pre Calculus (Self-Paced)

**Prerequisites:** 3-4 years HS math

**Frequency:** Fall (I), Winter (II)

**Student Body:** First-year students

**Credit:** 2 Credits. No credit granted for those who have completed a pre-calculus course

**Recent Texts:** Functions Modeling Change: A Preparation for Calculus (3rd edition) by Connally, Hughes-Hallett, Gleason et. al.

### **Background and Goals:**

Math 110 is a condensed, half-term version of Math 105 designed specifically to prepare students for Math 115. It is open only to students who have enrolled in Math 115 and whose performance on the first uniform examination indicates that they will have difficulty completing that course successfully. This self-study course begins shortly after the first uniform examination in Math 115, and is completed under the guidance of an instructor without regular classroom meetings. Students must receive permission from the Math 115 Course Director or other designated representative to enroll in the course, and they must visit the Math Lab as soon as possible after enrolling to receive printed course information. Enrollment opens the day after the first Math 115 uniform examination, and enrollment must be completed by the Friday of the following week.

### **Content:**

The course is a condensed, half-term version of Math 105 designed for students who appear to be prepared to handle calculus but are not able to successfully complete Math 115. Students may enroll in Math 110 only on the recommendation of a mathematics instructor after the third week of classes in the Fall and must visit the Math Lab to complete paperwork and receive course materials. The course covers data analysis by means of functions and graphs.

### **Alternatives:**

Math 105 (Data, Functions and Graphs) covers the same material in a traditional classroom setting.

### **Subsequent Courses:**

The course prepares students for Math 115 (Calculus I).

## Math 115 - Calculus I

**Prerequisites:** 3-4 years HS math including trigonometry

**Frequency:** Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

**Student Body:** First-year students

**Credit:** 4 Credits. No credit after Math 116, 185, 215, or 216

**Recent Texts:** Calculus: Single Variable (5th edition) by Hughes-Hallett, et. al.

### **Background and Goals:**

The sequence Math 115-116-215 is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof. All sections are given two uniform midterms and a final exam.

### **Content:**

The course presents the concepts of calculus from four points of view: geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing, and questioning skills, as well as their ability to work cooperatively. Topics include functions and graphs, derivatives and their applications to real-life problems in various fields, and an introduction to integration. The classroom atmosphere is interactive and cooperative. Both individual and team homework is assigned.

### **Alternatives:**

Math 185 (Honors Calculus I ) is a more theoretical course which covers some of the same material. Math 175 (Intro to Cryptology) is a non-calculus alternative for students with a good command of first-semester calculus. Math 295 (Honors Mathematics I) is a much more intensive and rigorous course. A student whose preparation is insufficient for Math 115 should take Math 105 (Data, Functions and Graphs).

### **Subsequent Courses:**

Math 116 (Calculus II) is the natural sequel. A student who has done very well in this course could enter the honors sequence at this point by taking Math 156 (Applied Honors Calculus II).

## Math 116 - Calculus II

**Prerequisites:** Math 115 or AP credit

**Frequency:** Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

**Student Body:** First-year students

**Credit:** 4 Credits. No credit after Math 156, 176, 186, 215, 216, 255, 256, 285, or 286.

**Recent Texts:** Calculus: Single Variable (5th edition) by Hughes-Hallett, et. al.

### **Background and Goals:**

The sequence Math 115-116-215 is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof. All sections are given two uniform midterms and a final exam.

### **Content:**

The course presents the concepts of calculus from four points of view: geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing and questioning skills, as well as their ability to work cooperatively. Topics include techniques of integration, applications of integration, Taylor series, an introduction to differential equations, and infinite series. The classroom atmosphere is interactive and cooperative. Both individual and team homework is assigned.

### **Alternatives:**

Math 186 (Honors Calculus II) is a somewhat more theoretical course which covers much of the same material. Math 156 (Applied Honors Calculus II) also covers much of the same material using MAPLE and emphasizing applications to science and engineering.

### **Subsequent Courses:**

Math 215 (Calculus III) is the natural sequel. A student who has done very well in this course could enter the honors sequence at this point by taking Math 255 (Applied Honors Calculus III) or Math 285 (Honors Calculus III).

## Math 127 - Geometry and the Imagination

**Prerequisites:** 3 years HS math

**Frequency:** Winter (II)

**Student Body:** First-year students and sophomores

**Credit:** 4 Credits. No credit after completing any 200+ level math course, except 385, 489, and 497.

**Recent Texts:** Beyond the Third Dimension: Geometry, Computer Graphics and Higher Dimensions by Banchoff

### **Background and Goals:**

This course introduces students to the ideas and some of the basic results in Euclidean and non-Euclidean geometry. Beginning with geometry in ancient Greece, the course includes the construction of new geometric objects from old ones by projecting and by taking slices. The course is intended for students who want an introduction to mathematical ideas and culture. Emphasis is on conceptual thinking — students will do hands-on experimentation with geometric shapes, patterns, and ideas.

### **Content:**

The course begins with the independence of Euclid's Fifth Postulate and with the construction of spherical and hyperbolic geometries in which the Fifth Postulate fails; we discuss how spherical and hyperbolic geometry differ from Euclidean geometry. We then study the geometry of higher dimensions: coordinization — the mathematician's tool for studying higher dimensions; construction of higher-dimension analogues of some familiar objects like spheres and cubes; discussion of the proper higher-dimensional analogues of some geometric notions (length, angle, orthogonality, etc.).

### **Alternatives:**

None

### **Subsequent Courses:**

This course does not provide preparation for any further study of mathematics.

## Math 128 - Explorations in Number Theory

**Prerequisites:** 3 years HS math

**Frequency:** Fall (I)

**Student Body:** First-year students and sophomores

**Credit:** 4 Credits. No credit after completing any 200+ level math course, except 385, 485, 489, and 497.

**Recent Texts:** Coursepack

**Area:** Number Theory

### **Background and Goals:**

This course is intended for students who want to engage in mathematical reasoning without having to take calculus first. It is particularly well-suited for non-science concentrators or those who are thoroughly undecided. Students will make use of software to conduct numerical experiments and to make empirical discoveries. Students will formulate precise conjectures and in many cases prove them. Thus the students will, as a group, generate a logical development of the subject.

### **Content:**

After studying factorizations and greatest common divisors, emphasis will shift to the patterns that emerge when the integers are classified according to the remainder produced upon division by some fixed number (congruences). Once some basic tools have been established, applications will be made in several directions. For example, students may derive a precise parameterization of Pythagorean triples.

### **Alternatives:**

None

### **Subsequent Courses:**

This course does not provide preparation for any further study of mathematics.



## Math 145 - Houghton Scholars Calculus Workshop I

**Prerequisites:** Concurrent enrollment in Math 115 and permission of instructor

**Frequency:** Fall (I)

**Student Body:** First year students

**Credit:** 2 Credits.

**Recent Texts:** None

### **Background and Goals:**

This course is an intensive supplemental workshop experience for students in the Douglas Houghton Scholars Program. In a small-class setting, students work in small groups on problems more challenging than those in the regular section of Math 115. The goal is to develop the students' problem-solving capabilities and promote their interest in challenging problems.

### **Content:**

The workshop follows the structure of Math 115 (Calculus I). Content includes geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing, and questioning skills, as well as their ability to work cooperatively. Topics include functions and graphs, derivatives and their applications to real-life problems in various fields, and an introduction to integration.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 146 (Houghton Scholars Calculus Workshop II)

## Math 146 - Houghton Scholars Calculus Workshop II

**Prerequisites:** Concurrent enrollment in Math 116 and permission of instructor

**Frequency:** Winter (II)

**Student Body:** First year students

**Credit:** 2 Credits.

**Recent Texts:** None

### **Background and Goals:**

This course is an intensive supplemental workshop experience for students in the Douglas Houghton Scholars Program. In a small-class setting, students work in small groups on problems more challenging than those in the regular section of Math 116. The goal is to develop the students' problem-solving capabilities and promote their interest in challenging problems.

### **Content:**

The workshop follows the structure of Math 116 (Calculus II). Content includes geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing and questioning skills, as well as their ability to work cooperatively. Topics include techniques of integration, applications of integration, Taylor series, an introduction to differential equations, and infinite series.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 215 (Calculus III), Math 216 (Intro. to Differential Equations)

## Math 147 - Introduction to Interest Theory

**Prerequisites:** Math 115

**Frequency:** Fall (I), Winter (II)

**Student Body:** First and second year students

**Credit:** 3 Credits. No credit after completing any 200+ level math course, except 385, 489, or 497.

**Recent Texts:** Mathematics of Finance (6th edition) by Zima and Brown

**Area:** Actuarial & Financial

### **Background and Goals:**

This course is designed for students who seek an introduction to the mathematical concepts and techniques employed by financial institutions such as banks, insurance companies, and pension funds. Actuarial students, and other mathematics concentrators, should elect Math 424 which covers the same topics but on a more rigorous basis requiring considerable use of calculus. The course is not part of a sequence. Students should possess financial calculators.

### **Content:**

Topics covered include: various rates of simple and compound interest, present and accumulated values based on these; annuity functions and their application to amortization, sinking funds, and bond values; depreciation methods; introduction to life tables, life annuity, and life insurance values.

### **Alternatives:**

Math 424 (Compound Interest and Life Ins) covers the same material in greater depth and with a higher level of mathematical content.

### **Subsequent Courses:**

None

## Math 156 - Applied Honors Calculus II

**Prerequisites:** Score of 4 or 5 on Advanced Placement AB or BC Calculus exam

**Frequency:** Fall (I)

**Student Body:** First-year students

**Credit:** 4 Credits. Credit is granted for either 116 or 156, but not both.

**Recent Texts:** Single Variable Calculus (UM edition) by Stewart

### **Background and Goals:**

Math 156 is part of the applied honors calculus sequence for engineering and science concentrators. The course is an alternative to Math 116 for students with strong mathematical ability. Applications and concepts receive equal treatment. Theorems are stated precisely and are derived, but technical details are omitted. Examples are given to illustrate the theory. Critical thinking and class participation are encouraged. The goal is to provide students with the solid background needed for subsequent courses in mathematics, engineering, and science.

### **Content:**

Riemann sums, definite integral, fundamental theorem of calculus, applications of integral calculus (e.g. arclength, surface area, work, center of mass, probability density functions), improper integrals, infinite sequences and series, geometric series, alternating series, power series, Taylor series, differential equations, complex numbers. Students are introduced to MAPLE.

### **Alternatives:**

Math 116 (Calculus II).

### **Subsequent Courses:**

Math 255 (Applied Honors Calculus III) is the natural sequel.

## Math 174 - Proofs in Geometry

**Prerequisites:** Permission of honors advisor.

**Frequency:** Fall (I)

**Student Body:** First-year seminar

**Credit:** 4 Credits. No credit granted to those who have completed a 200-level or higher Mathematics course.

**Recent Texts:** Geometry Revisited by Coxeter and Greitzer

### **Background and Goals:**

The course will be very interactive, eliciting suggestions towards proof from the students so that all the problems are eventually solved by a joint effort between the students and the instructor. This format has worked well in the past for honors courses. To enhance the visualization, we plan to develop software for two-dimensional geometric constructions. This software will be able to produce multi-color pictures of geometric configurations. In the long run, such software will save us time in creating problem sets, handouts and perhaps slides. Additional topics may be added depending on the interest and abilities of the students.

### **Content:**

As a precursor to the mathematics, the course will use familiar games such as the old game Mastermind where player A has a code which player B has to use. Students will pair off and play the game, with the important additional feature that the guesser must write down what (s)he knows and can deduce after each guess, and therefore motivate his/her next guess. This should help set the mood and instill the idea of analyzing the facts at hand and making logical deductions. After this, the course will develop some basic theorems of Euclidean geometry. An example of such a theorem is that the angle bisectors (or medians, or altitudes, or perpendicular bisectors) of a triangle are concurrent. These results are fairly straightforward but exemplify the spirit of the course by providing a good introduction to rigorous proofs. Then we move to some more difficult, but beautiful, theorems from geometry such as Ceva's theorem, the Euler line, the nine-point circle theorem, Ptolemy's theorem, and Morley's theorem.

### **Alternatives:**

none

### **Subsequent Courses:**

none

## Math 175 - Introduction to Cryptology

**Prerequisites:** Permission of honors advisor

**Frequency:** Fall (I)

**Student Body:** First-year students

**Credit:** 4 Credits. No credit granted to those who have completed a 200-level or higher Mathematics course.

**Recent Texts:** None

### **Background and Goals:**

This course is an alternative to Math 185 as an entry to the honors calculus sequence. The course stresses discovery as a vehicle for learning. Students will be required to experiment throughout the course on a range of problems and will participate each semester in a group project. Grades will be based on homework and projects, with a strong emphasis on homework. Personal computers will be a valuable experimental tool in this course and students will be asked to learn to program in one of BASIC, PASCAL, or FORTRAN.

### **Content:**

This course gives a historical introduction to Cryptology, from ancient times up to modern public key encryption, particularly RSA, and introduces a number of mathematical ideas involved in the development and analysis of codes. Mathematical topics include some enumeration, probability, and statistics, but the bulk of the course is devoted to elementary number theory. Students also work throughout the course on effectively communicating mathematics, both written and orally. Moreover, students will develop rigorous mathematical proof writing skills, and a primary goal of the course is to not only understand how various cryptosystems work, but why.

### **Alternatives:**

Math 115 (Calculus I), Math 185 (Honors Calculus I), or Math 295 (Honors Mathematics I).

### **Subsequent Courses:**

Math 176 (Dynamical Systems and Calculus), or Math 116 (Calculus II).

## Math 176 - Explorations in Topology and Analysis

**Prerequisites:** Math 175 or permission of instructor

**Frequency:** Winter (II)

**Student Body:** First-year students

**Credit:** 4 Credits. Credit is granted for either Math 176 or 186, but not both

**Recent Texts:** None

### **Background and Goals:**

Students are expected to have some previous experience with the basic concepts and techniques of calculus. The course stresses discovery as a vehicle for learning. Students will be required to experiment throughout the course on a range of problems and will participate each semester in a group project. UNIX workstations will be a valuable experimental tool in this course and students will run preset lab routines on them using Matlab and MAPLE.

### **Content:**

The general theme of the course will be discrete-time and continuous-time dynamical systems. Examples of dynamical systems arising in the sciences are used as motivation. Topics include: iterates of functions, simple ordinary differential equations, fixed points, attracting and repelling fixed points and periodic orbits, ordered and chaotic motion, self-similarity, and fractals. Tools such as limits and continuity, Taylor expansions of functions, exponentials, logarithms, eigenvalues, and eigenvectors are reviewed or introduced as needed. There is a weekly computer work-station lab.

### **Alternatives:**

Math 116 (Calculus I) or Math 186 (Honors Calculus II)

### **Subsequent Courses:**

Math 285 (Honors Calculus III)

## Math 185 - Honors Calculus I

**Prerequisites:** Permission of honors advisor during orientation

**Frequency:** Fall (I)

**Student Body:** First-year students

**Credit:** 4 Credits.

**Recent Texts:** Calculus (4th ed) by Michael Spivak, Calculus and Linear Algebra (UM edition) by Wilfred Kaplan

### **Background and Goals:**

Most students take calculus in high school, and it may seem that there isn't much new to learn. The goal of this course is to develop the familiar concepts of calculus using a more rigorous and theoretical approach. In particular, with its emphasis on how to use appropriate mathematical language, this course lays a solid foundation for future math courses, and is suitable for students intending to pursue a concentration in mathematics, science, or engineering who desire a more complete understanding of the underpinnings of calculus. This sequence is not restricted to students enrolled in the LSA Honors Program.

### **Content:**

Topics covered include functions, graphs, continuity, limits, derivatives, and integrals. Tuesday meetings are usually devoted to introducing linear algebra.

### **Alternatives:**

Math 115 (Calculus I) is a less theoretical course which covers much of the same material. Math 295 (Honors Mathematics I) gives a much more theoretical treatment of much of the same material.

### **Subsequent Courses:**

Math 186 (Honors Calculus II) is the natural sequel.

## Math 186 - Honors Calculus II

**Prerequisites:** Permission of honors advisor or Math 185

**Frequency:** Winter (II)

**Student Body:** First-year students

**Credit:** 4 Credits. Credit is granted for either Math 176 or 186, but not both.

**Recent Texts:** Calculus (4th ed) by Michael Spivak

### **Background and Goals:**

Most students take calculus in high school, and it may seem that there isn't much new to learn. The goal of this course is to develop the familiar concepts of calculus using a more rigorous and theoretical approach. In particular, with its emphasis on how to use appropriate mathematical language, this course lays a solid foundation for future math courses, and is suitable for students intending to pursue a concentration in mathematics, science, or engineering who desire a more complete understanding of the underpinnings of calculus. This sequence is not restricted to students enrolled in the LSA Honors Program. This course is a continuation of Math 185.

### **Content:**

Topics include integral calculus, transcendental functions, infinite sequences and series (including Taylor's series), and – time permitting – some simple applications to elementary differential equations. Tuesdays are mostly devoted to an introduction to linear algebra.

### **Alternatives:**

Math 116 (Calculus II) is a less theoretical course which covers much of the same material. Math 156 (Applied Honors Calculus II) is more application based, but covers much of the same material.

### **Subsequent Courses:**

Math 285 (Honors Calculus III) is the natural sequel.

## Math 214 - Linear Algebra

**Prerequisites:** Math 115 and 116

**Frequency:** Fall (I), Winter (II)

**Student Body:** Engineering students, particularly in Industrial and Operations Engineering

**Credit:** 4 Credits. No credit granted to those who have completed or are enrolled in Math 217, 417, 419, or 420.

**Recent Texts:** Linear Algebra with Applications (4th edition) by Bretscher

### **Background and Goals:**

An introduction to matrices and linear algebra. This course covers the basics needed to understand a wide variety of applications that use the ideas of linear algebra, from linear programming to mathematical economics. The emphasis is on concepts and problem solving. The sequence 214-215 is not for math concentrators. It is designed as an alternate to the sequence 215-216 for engineering students who need more linear algebra and less differential equations background.

### **Content:**

An introduction to the main concepts of linear algebra... matrix operations, echelon form, solution of systems of linear equations, Euclidean vector spaces, linear combinations, independence and spans of sets of vectors in Euclidean space, eigenvectors and eigenvalues, similarity theory. There are applications to discrete Markov processes, linear programming, and solutions of linear differential equations with constant coefficients.

### **Alternatives:**

Math 217 (Linear Algebra) is a more theoretical course which covers much of the material of Math 214 at a deeper level. Mathematics concentrators are required to take Math 217.

### **Subsequent Courses:**

Math 561 (Linear Programming I), Math 462 (Mathematical Models), Math 571 (Numer. Meth. for Sci. Comput. I).

## Math 215 - Calculus III

**Prerequisites:** Math 116, 156, or 186

**Frequency:** Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

**Student Body:** Sophomores

**Credit:** 4 Credits. Credit is granted for only one course among Math 215, 255, and 285.

**Recent Texts:** Multivariable Calculus (6th edition) by Stewart

### **Background and Goals:**

The sequence Math 115-116-215 is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof.

### **Content:**

Topics include vector algebra and vector functions; analytic geometry of planes, surfaces, and solids; functions of several variables and partial differentiation; line, surface, and volume integrals and applications; vector fields and integration; Green's Theorem, Stokes' Theorem, and Gauss' Theorem. There is a weekly computer lab using MAPLE.

### **Alternatives:**

Math 285 (Honors Calculus III) is a somewhat more theoretical course which covers the same material. Math 255 (Applied Honors Calculus III) is also an alternative.

### **Subsequent Courses:**

For students intending to concentrate in mathematics or who have some interest in the theory of mathematics as well as its applications, the appropriate sequel is Math 217 (Linear Algebra). Students who intend to take only one further mathematics course and need differential equations (respectively, linear algebra) should take Math 216 (respectively, Math 214).

## Math 216 - Introduction to Differential Equations

**Prerequisites:** Math 116, 156, 176, 186, or 296

**Frequency:** Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

**Student Body:** Sophomore engineering students

**Credit:** 4 Credits. 2 credits granted to those who have completed or are enrolled in Math 214. Credit is granted for only one course among Math 216, 256, 286, and 316.

**Recent Texts:** Differential Equations: Computing and Modeling (4th edition) by Edwards and Penney

### **Background and Goals:**

For a student who has completed the calculus sequence, there are two sequences which deal with linear algebra and differential equations: 216&417 (or 419) and 217&316. The sequence 216&417 emphasizes problem-solving and applications and is intended for students of engineering and the sciences. Mathematics concentrators and other students who have some interest in the theory of mathematics should elect the sequence 217&316.

### **Content:**

Math 216 is a basic course on differential equations, intended for engineers and other scientists who need to apply the techniques in their work. The lectures are accompanied by a computer lab and recitation section where students have the opportunity to discuss problems and work through computer experiments to further develop their understanding of the concepts of the class. Topics covered include some material on complex numbers and matrix algebra, first and second order linear and non-linear systems with applications, introductory numerical methods, and elementary Laplace transform techniques.

### **Alternatives:**

Math 286 (Honors Differential Equations) covers much of the same material, but with much more emphasis on theory. The sequence Math 217&316 covers all of this material and substantially more at greater depth and with greater emphasis on the theory. Math 256 (Applied Honors Calculus IV) is also an alternative.

### **Subsequent Courses:**

Math 404 (Intermediate Diff. Eq.) covers further material on differential equations. Math 217 (Linear Algebra) and Math 417 (Matrix Algebra I) cover further material on linear algebra. Math 371 (Engin. 303—Numerical Methods) and Math 471/472 (Intro. to Numerical Methods) cover additional material on numerical methods.

## Math 217 - Linear Algebra

**Prerequisites:** Math 215, 255, or 285

**Frequency:** Fall (I), Winter (II), Spring (IIIa)

**Student Body:** Sophomore prospective mathematics concentrators

**Credit:** 4 Credits. No credit granted to those who have completed or are enrolled in Math 419 or 420. 2 credits granted to those who have completed Math 214 or 417.

**Recent Texts:** Linear Algebra and Its Applications (3rd updated edition) by Lay

### **Background and Goals:**

For a student who has completed the calculus sequence, there are two sequences which deal with linear algebra and differential equations: 216&417 (or 419) and 217&316. The sequence 216&417 emphasizes problem-solving and applications and is intended for students of Engineering and the sciences. Mathematics concentrators and other students who have some interest in the theory of mathematics should elect the sequence 217&316. These courses are explicitly designed to introduce the student to both the concepts and applications of their subjects and to the methods by which the results are proved.

### **Content:**

The topics covered include: systems of linear equations; matrix algebra; vectors, vector spaces, and subspaces; geometry of  $\mathbf{R}^n$ ; linear dependence, bases, and dimension; linear transformations; Eigenvalues and Eigenvectors; diagonalization; inner products. Throughout there will be an emphasis on the concepts, logic, and methods of theoretical mathematics.

### **Alternatives:**

Math 214, 417, and 419 cover similar material with more emphasis on computation and applications and less emphasis on proofs.

### **Subsequent Courses:**

The intended course to follow Math 217 is Math 316 (Differential Equations). Math 217 is also prerequisite for Math 312 (Applied Modern Algebra), Math 412 (Introduction to Modern Algebra), and all of the more advanced courses in mathematics.

## Math 255 - Applied Honors Calculus III

**Prerequisites:** Math 156, or permission of instructor

**Frequency:** Winter (II)

**Student Body:** First-year students

**Credit:** 4 Credits. Credit is granted for only one course among Math 215, 255, and 285.

**Recent Texts:** Multivariable Calculus (5th edition) by Stewart

### **Background and Goals:**

Math 255 is part of the applied honors calculus sequence for engineering and science concentrators. Applications and concepts receive equal treatment. Theorems are stated precisely and are derived, but technical details are omitted. Examples are given to illustrate the theory. Critical thinking and class participation are encouraged. The goal is to provide students with the solid background needed for subsequent courses in mathematics, engineering, or science.

### **Content:**

Analytic geometry of lines and planes using vector notation, parametric representation of curves and surfaces, multivariable calculus, line surface and volume integrals, vector fields, Green's theorem, Stokes' theorem, divergence theorem, applications (e.g. electromagnetic fields, fluid dynamics). MAPLE will be used throughout.

### **Alternatives:**

Math 215 (Calculus III) or Math 285 (Honors Calculus III).

### **Subsequent Courses:**

Math 256 (Applied Honors Calculus IV) is the natural sequel.

## Math 256 - Applied Honors Calculus IV

**Prerequisites:** Math 255

**Frequency:** Fall (I)

**Student Body:** Sophomores

**Credit:** 4 Credits. Credit is granted for only one course among Math 216, 256, 286, and 316.

**Recent Texts:** Elementary Differential Equations and Boundary Value Problems (9th edition) by Boyce and DiPrima

### **Background and Goals:**

Math 256 is part of the applied honors calculus sequence for engineering and science concentrators. Applications and concepts receive equal treatment. Theorems are stated precisely and are derived, but technical details are omitted. Examples are given to illustrate the theory. Critical thinking and class participation are encouraged. The goal is to provide students with the solid background needed for subsequent courses in mathematics, engineering, or science.

### **Content:**

Linear algebra, matrices, systems of differential equations, initial value problems, qualitative theory of dynamical systems (e.g. equilibria, phase space, stability, bifurcations), nonlinear equations, numerical methods. Optional: boundary value problems. Students will learn to use MATLAB for computer simulations.

### **Alternatives:**

Math 216 (Intro. to Differential Equations) or Math 286 (Honors Differential Equations).

### **Subsequent Courses:**

Many upper-level courses

## Math 285 - Honors Calculus III

**Prerequisites:** Math 156, 176, 186, or permission of instructor

**Frequency:** Fall (I)

**Student Body:** Sophomores and first-year students with suitable background

**Credit:** 4 Credits. Credit is granted for only one course among Math 215, 255, and 285.

**Recent Texts:** Multivariable Calculus (6th edition) by Stewart

### **Background and Goals:**

The sequence Math 185-186-285-286 is an introduction to calculus at the honors level. It is taken by students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields who want a somewhat more theoretical approach. Although much attention is paid to concepts and solving problems, the underlying theory and proofs of important results are also included. This sequence is not restricted to students enrolled in the LS&A Honors Program.

### **Content:**

Topics include vector algebra and vector functions; analytic geometry of planes, surfaces, and solids; functions of several variables and partial differentiation, maximum-minimum problems; line, surface, and volume integrals and applications; vector fields and integration; curl, divergence, and gradient; Green's Theorem and Stokes' Theorem. Additional topics may be added at the discretion of the instructor.

### **Alternatives:**

Math 215 (Calculus III) is a less theoretical course which covers the same material. Math 255 (Applied Honors Calc. III) is an applications-oriented honors course which covers much of the same material.

### **Subsequent Courses:**

Math 216 (Intro. to Differential Equations), Math 286 (Honors Differential Equations), or Math 217 (Linear Algebra).



## Math 286 - Honors Differential Equations

**Prerequisites:** Math 285 or permission of instructor

**Frequency:** Winter (II)

**Student Body:** Sophomores

**Credit:** 3 Credits. Credit is granted for only one course among Math 216, 256, 286, and 316.

**Recent Texts:** Elementary Differential Equations and Boundary Value Problems (9th edition) by Boyce and DiPrima

### **Background and Goals:**

The sequence Math 185-186-285-286 is an introduction to calculus at the honors level. It is taken by students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields who want a somewhat more theoretical approach. Although much attention is paid to concepts and solving problems, the underlying theory and proofs of important results are also included. This sequence is not restricted to students enrolled in the LS&A Honors Program.

### **Content:**

Topics include first-order differential equations, higher-order linear differential equations with constant coefficients, an introduction to linear algebra, linear systems, the Laplace Transform, series solutions, and other numerical methods (Euler, Runge-Kutta). If time permits, Picard's Theorem will be proved.

### **Alternatives:**

Math 216 (Intro. to Differential Equations) and Math 316 (Differential Equations) cover much of the same material. Math 256 (Applied Honors Calculus IV) is also an alternative.

### **Subsequent Courses:**

Math 471 (Intro. to Numerical Methods) and/or Math 572 (Numer. Meth. for Sci. Comput. II) are natural sequels in the area of differential equations, but Math 286 is also preparation for more theoretical courses such as Math 451 (Advanced Calculus I).

## Math 289 - Problem Solving

**Prerequisites:** Permission

**Frequency:** Fall (I), Winter (II)

**Student Body:** Undergraduate students interested in learning to solve problems.

**Credit:** 1 Credit. May be repeated for credit

**Recent Texts:** None

### **Background and Goals:**

One of the better ways to develop mathematical abilities is by solving problems using a variety of methods. Familiarity with numerous methods is a great asset to the developing student of mathematics. Methods learned in attacking a specific problem frequently find application in many other areas of mathematics. In many instances an interest in and appreciation of mathematics is better developed by solving problems than by hearing formal lectures on specific topics. This course is intended for superior students who have exhibited both ability and interest in doing mathematics, but it is not restricted to honors students. This course is excellent preparation for the Putnam competition.

### **Content:**

Students and one or more faculty and graduate student assistants will meet in small groups to explore problems in many different areas of mathematics. Problems will be selected according to the interests and background of the students.

### **Alternatives:**

None

### **Subsequent Courses:**

This course may be repeated for credit.

## Math 295 - Honors Mathematics I

**Prerequisites:** Permission of honors advisor

**Frequency:** Fall (I)

**Student Body:** First-year students

**Credit:** 4 Credits.

**Recent Texts:** Calculus (4th edition) by M. Spivak

### **Background and Goals:**

Math 295-296-395-396 is the most theoretical and demanding honors math sequence. The emphasis is on concepts, problem solving, as well as the underlying theory and proofs of important results. It provides an excellent background for advanced courses in mathematics. The expected background is high school trigonometry and algebra (previous calculus is not required, but is helpful.) This sequence is **not** restricted to students enrolled in the LS&A Honors program. Math 295 and 296 may be substituted for any Math 451 requirement. Math 296 and 395 may be substituted for any Math 217 requirement.

### **Content:**

Axioms of the real numbers, completeness and connectedness in the real line. Functions of a real variable, limits and continuity, uniform continuity, extreme and intermediate value theorems, differentiation, integration, the fundamental theorem of calculus, Taylor's theorem with remainder.

### **Alternatives:**

Math 156 (Applied Honors Calc. II), Math 175 (Intro to Cryptology), and Math 185 (Honors Calculus I) are alternative honors courses.

### **Subsequent Courses:**

Math 296 (Honors Mathematics II)

## Math 296 - Honors Mathematics II

Prerequisites: Math 295

**Frequency:** Winter (II)

**Student Body:** First-year students

**Credit:** 4 Credits.

**Recent Texts:** Linear Algebra (2nd edition) by Kunze & Hoffman

### **Background and Goals:**

Math 295-296-395-396 is the most theoretical and demanding honors calculus sequence. The emphasis is on concepts, problem solving, as well as the underlying theory and proofs of important results. It provides an excellent background for advanced courses in mathematics. The expected background is high school trigonometry and algebra (previous calculus is not required, but is helpful.) This sequence is **not** restricted to students enrolled in the LS&A Honors program. Math 295 and 296 may be substituted for any Math 451 requirement. Math 296 and 395 may be substituted for any Math 217 requirement.

### **Content:**

Sequences and series of functions, power series, uniform convergence, real analytic functions. Vector spaces, bases, linear transformations, dual spaces, determinants, traces, eigenvalues, inner-product spaces, spectral theory. Limits and continuity in Euclidean space, derivative as a linear map, Chain Rule, inverse/implicit function theorems.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 395 (Honors Analysis I)

## Math 310 - Elementary Topics: Choice and Chance

**Prerequisites:** Sophomore standing and one previous university math course

**Frequency:** Winter (II)

**Student Body:**

**Credit:** 3 Credits.

**Recent Texts:** None

### **Background and Goals:**

The Elementary Topics course may focus on any one of several topics. The material is presented at a level appropriate for sophomores and juniors without extensive coursework in math. The current offering of the course focuses on game theory.

### **Content:**

Every day the media showers us with news, analysis, and op-eds, which use and misuse numbers to arrive at various far-reaching conclusions. The objective of the course is to help students to acquire some basic mathematical skills to navigate this sea of numbers. Often, this boils down to understanding a few fundamental, ancient, and deep concepts: randomness, fairness, coincidence, and bias. We will study what "probability", "events", and "independence" mean, how to compute some basic probabilities and why it can be costly to assume that events are independent when in fact they are not, as illustrated by recent and not so recent events in the insurance industry and the stock market. We will also discuss why randomized strategies in games can be quite helpful.

### **Alternatives:**

None

### **Subsequent Courses:**

None

## Math 312 - Applied Modern Algebra

**Prerequisites:** Math 217

**Frequency:** Fall (I)

**Student Body:** Sophomore and Junior mathematics and computer science concentrators

**Credit:** 3 Credits. No credit for those who have completed or are enrolled in Math 412.

**Recent Texts:** A Concrete Introduction to Higher Algebra (3rd edition) by Childs

### **Background and Goals:**

One of the main goals of the course (along with every course in the algebra sequence) is to expose students to rigorous, proof-oriented mathematics. Students are required to have taken Math 217, which should provide a first exposure to this style of mathematics. A distinguishing feature of this course is that the abstract concepts are not studied in isolation. Instead, each topic is studied with the ultimate goal being a real-world application.

### **Content:**

Sets and functions, relations and graphs, rings, Boolean algebras, semigroups, groups, and lattices. Applications from areas such as switching, automata, and coding theory, and may include finite and minimal state machines, algebraic decompositions of logic circuits, semigroup machines, binary codes, and series and parallel decomposition of machines.

### **Alternatives:**

Math 412 (Introduction to Modern Algebra) is a more abstract and proof-oriented course with less emphasis on applications and is better preparation for most subsequent mathematics courses. EECS 203 (Discrete Structures) covers some of the same topics with a more applied approach.

### **Subsequent Courses:**

Math 312 is one of the alternative prerequisites for Math 416 (Theory of Algorithms), and several advanced EECS courses make substantial use of the material of Math 312. Another good follow-up course is Math 475 (Elementary Number Theory).

## Math 316 - Differential Equations

**Prerequisites:** Math 215, 255, or 285; and 217

**Frequency:** Fall (I), Winter (II)

**Student Body:** A mix of undergraduate mathematics, science, and economics concentrators

**Credit:** 3 Credits. Credit is granted for only one course among Math 216, 256, 286, and 316.

**Recent Texts:** Elementary Differential Equations and Boundary Value Problems (9th edition) by Boyce and DiPrima

### **Background and Goals:**

This is an introduction to differential equations for students who have studied linear algebra (Math 217). It treats techniques of solution (exact and approximate), existence and uniqueness theorems, some qualitative theory, and many applications. Proofs are given in class; homework problems include both computational and more conceptually oriented problems.

### **Content:**

First-order equations: solutions, existence and uniqueness, and numerical techniques; linear systems: eigenvector-eigenvalue solutions of constant coefficient systems, fundamental matrix solutions, nonhomogeneous systems; higher-order equations, reduction of order, variation of parameters, series solutions; qualitative behavior of systems, equilibrium points, stability. Applications to physical problems are considered throughout.

### **Alternatives:**

Math 216 covers somewhat less material without presupposing linear algebra and with less emphasis on theory. Math 286 (Honors Differential Equations) is the honors version of Math 316.

### **Subsequent Courses:**

Math 471 (Intro. to Numerical Methods) and/or Math 572 (Numer. Meth. for Sci. Comput. III) are natural sequels in the area of differential equations, but Math 316 is also preparation for more theoretical courses such as Math 451 (Advanced Calculus I).

## Math 327 - Evolution of Mathematical Concepts

**Prerequisites:** Math 116 or Math 186

**Frequency:** Sporadically

**Student Body:** Juniors and Seniors interested in mathematics and the history of science

**Credit:** 3 credits.

**Recent Texts:** None

### **Background and Goals:**

This course examines the evolution of major mathematical concepts from mathematical and historical points of view. The course's goal is to throw light on contemporary mathematics by retracing the history of some of the major mathematical discoveries.

### **Content:**

This course follows the evolution of three fundamental mathematical ideas in geometry, analysis and algebra. Typical choices of subject are: Euclid's parallel postulate and the development of non-Euclidean geometries, the notions of limit and infinitesimals, and the development of the theory of equations culminating with Galois theory.

### **Alternatives:**

None

### **Subsequent Courses:**

None

## Math 351 - Principles of Analysis

**Prerequisites:** Math 215 and 217 or permission of instructor.

**Frequency:** Fall (I), Winter (II)

**Student Body:** Sophomores and Juniors

**Credit:** 3 Credits. No credit granted to those who have completed or are enrolled in Math 451.

**Recent Texts:** Understanding Analysis by S. Abbott

**Area:** Analysis

### **Background and Goals:**

The course content is similar to that of Math 451, but Math 351 assumes less background. This course covers topics that might be of greater use to students considering a Mathematical Sciences concentration or a minor in Mathematics.

### **Content:**

Analysis of the real line, rational and irrational numbers, infinity—limits, convergence, infinite sequences and series, continuous functions, power series, and differentiation.

### **Alternatives:**

Math 451 (Advanced Calculus I) covers similar topics while assuming more background than 351.

### **Subsequent Courses:**

None

## Math 354 - Fourier Analysis and its Applications

**Prerequisites:** Math 216, 256, 286, or 316

**Frequency:** Fall (I), Winter (II)

**Student Body:** Junior and Senior math and non-math concentrators

**Credit:** 3 Credits. No credit granted to those who have completed or are enrolled in Math 450 or 454.

**Recent Texts:** Fourier Series and Orthogonal Polynomials by Jackson

**Area:** Analysis

### **Background and Goals:**

This course is an introduction to Fourier analysis with emphasis on applications. The course also can be viewed as a way of deepening one's understanding of the 100-and 200-level material by applying it in interesting ways.

### **Content:**

This is an introduction to Fourier analysis at an elementary level, emphasizing applications. The main topics are Fourier series, discrete Fourier transforms, and continuous Fourier transforms. A substantial portion of the time is spent on both scientific/technological applications (e.g. signal processing, Fourier optics), and applications in other branches of mathematics (e.g. partial differential equations, probability theory, number theory). Students will do some computer work, using MATLAB, an interactive programming tool that is easy to use, but no previous experience with computers is necessary.

### **Alternatives:**

Math 454 (Bound Val. Probs. for Part. Diff. Eq.) covers some of the same material with more emphasis on partial differential equations.

### **Subsequent Courses:**

This course is good preparation for Math 451 (Advanced Calculus I), which covers the theory of calculus in a mathematically rigorous way.

### **Math 371 (Engin 371) - Numerical Methods**

**Prerequisites:** Engin 101; and one of Math 216, 256, 286, or 316; and one of Math 214, 217, 417, or 419.

**Frequency:** Fall (I), Winter (II)

**Student Body:** Sophomore, Junior, and Senior engineering students

**Credit:** 3 Credits. No credit after Math 471.

**Recent Texts:** A Friendly Introduction to Numerical Analysis by B. Bradie

**Area:** Applied/NA

#### **Background and Goals:**

This is a survey course of the basic numerical methods which are used to solve practical scientific problems. Important concepts such as accuracy, stability, and efficiency are discussed. The course provides an introduction to MATLAB, an interactive program for numerical linear algebra, and may provide practice in FORTRAN programming and the use of software library subroutines. Convergence theorems are discussed and applied, but the proofs are not emphasized.

#### **Content:**

Floating point arithmetic, Gaussian elimination, polynomial interpolation, spline approximations, numerical integration and differentiation, solutions to non-linear equations, ordinary differential equations, polynomial approximations. Other topics may include discrete Fourier transforms, two-point boundary-value problems, and Monte-Carlo methods.

#### **Alternatives:**

Math 471 (Numerical Analysis) provides a more in-depth study of the same topics, with a greater emphasis on analyzing the accuracy and stability of the numerical methods. Math 571 (Numerical Linear Algebra) is a detailed study of the solution of systems of linear equations and eigenvalue problems, with some emphasis on large-scale problems. Math 572 (Numerical Methods for Differential Equations) covers numerical methods for both ordinary and partial differential equations. (Math 571 and 572 can be taken in either order).

#### **Subsequent Courses:**

This course is basic for many later courses in science and engineering. It is good background for 571-572.

### **Math 385 - Math for Elementary School Teachers**

**Prerequisites:** One year each of HS algebra and geometry

**Frequency:** Fall (I)

**Student Body:** Undergraduate concentrators in the Teaching Certificate Program

**Credit:** 3 Credits. No credit granted to those who have taken or are enrolled in Math 485.

**Recent Texts:** Mathematics for Elementary School Teachers (4th edition) by Bassarear

**Area:** Teaching

#### **Background and Goals:**

This course, together with its sequel Math 489, provides a coherent overview of the mathematics underlying the elementary and middle school curriculum. It is required of all students intending to earn an elementary teaching certificate and is taken almost exclusively by such students. Concepts are heavily emphasized with some attention given to calculation and proof. The course is conducted using a discussion format. Class participation is expected and constitutes a significant part of the course grade. Enrollment is limited to 30 students per section. Although only two years of high school mathematics are required, a more complete background including pre-calculus or calculus is desirable.

#### **Content:**

Topics covered include problem solving, sets and functions, notation systems, whole numbers (including some number theory), and integers. Each number system is examined in terms of its algorithms, its applications, and its mathematical structure.

#### **Alternatives:**

None

#### **Subsequent Courses:**

Math 489 (Math for Elem. and Middle School Teachers) is the sequel.

## Math 389 - Explorations in Mathematics

**Prerequisites:** None formally; see instructor beforehand.

**Frequency:** Winter (II)

**Student Body:** Undergraduates

**Credit:** 3 Credits.

**Recent Texts:** None

### **Background and Goals:**

The course is designed to show you how new mathematics is actually created: how to take a problem, make models and experiment with them, and search for underlying structure. The format involves little formal lecturing, much laboratory work, and student presentations discussing partial results and approaches. Course website: <http://www.math.lsa.umich.edu/courses/389/>

### **Content:**

Problems for projects are drawn from a wide variety of mathematical areas, pure and applied. Problems are chosen to be accessible to undergraduates.

### **Alternatives:**

None

### **Subsequent Courses:**

After this course students should be ready for a variety of courses and research experiences.

## Math 395 - Honors Analysis I

**Prerequisites:** Math 296

**Frequency:** Fall (I)

**Student Body:** First-year students and sophomores

**Credit:** 4 Credits.

**Recent Texts:** Analysis on Manifolds by Munkres

### **Background and Goals:**

This course is a continuation of the sequence Math 295-296 and has the same theoretical emphasis. Students are expected to understand and construct proofs.

### **Content:**

Inverse/implicit function theorems, immersion/submersion theorems. Quotient and dual spaces, inner product spaces, spectral theory. Metric spaces, basic point-set topology. Integration in Euclidean space, Fubini's theorem, change of variables formula. Topics in linear algebra: tensor products, exterior and symmetric powers, Jordan and rational canonical forms.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 396 (Honors Analysis II)

## Math 396 - Honors Analysis II

**Prerequisites:** Math 395

**Frequency:** Winter (II)

**Student Body:** Sophomores

**Credit:** 4 Credits.

**Recent Texts:** Comprehensive Introduction to Differential Geometry (3rd ed) by Spivak

### **Background and Goals:**

This course is a continuation of Math 395 and has the same theoretical emphasis. Students are expected to understand and construct proofs.

### **Content:**

Submanifolds (with or without corners) of Euclidean space, abstract manifolds, tangent and cotangent spaces, immersion/submersion theorems. Partitions of unity, vector fields and differential forms on manifolds, exterior differentiation, integration of differential forms. Stokes' theorem. deRham cohomology, Riemannian metrics, Hodge star operator and the standard vector calculus versions of Stokes' theorem.

### **Alternatives:**

None

### **Subsequent Courses:**

Students who have successfully completed the sequence Math 295-396 are generally prepared to take a range of advanced undergraduate and graduate courses such as Math 493 (Algebraic Structures), Math 594 (Introduction to Linear Algebra), Math 525 (Probability Theory), Math 590 (Intro. to Topology), and many others.

## Math 404 - Intermediate Differential Equations

**Prerequisites:** Math 216, 256, 286, or 316

**Frequency:** Fall (I), Winter (II)

**Student Body:** Undergraduate and graduate students from engineering and LS&A

**Credit:** 3 Credits.

**Recent Texts:** Nonlinear Dynamics and Chaos: With Applications in Physics, Biology, Chemistry, and Engineering by Strogatz

**Area:** Applied/NA

### **Background and Goals:**

This course is an introduction to the modern qualitative theory of ordinary differential equations with emphasis on geometric techniques and visualization. Much of the motivation for this approach comes from applications. Examples of applications of differential equations to science and engineering are a significant part of the course. There are relatively few proofs.

### **Content:**

Geometric representation of solutions, autonomous systems, flows and evolution, linear systems and phase portraits, nonlinear systems, local and global behavior, linearization, stability, conservation laws, periodic orbits. Applications: free and forced oscillations, resonance, relaxation oscillations, competing species, Zeeman's models of heartbeat and nerve impulse, chaotic orbits, strange attractors.

### **Alternatives:**

Math 558 (Ordinary Differential Equations) covers some of the same material at a faster pace and includes additional topics.

### **Subsequent Courses:**

Math 454 (Boundary Value Problems for Partial Differential Equations) is a natural sequel.



## Math 412 - Introduction to Modern Algebra

**Prerequisites:** Math 215, 255, or 285; and Math 217

**Frequency:** Fall (I), Winter (II)

**Student Body:** Mainly undergraduate mathematics concentrators with some graduate students from other departments

**Credit:** 3 Credits. 1 credit after Math 312

**Recent Texts:** Introduction to Abstract Algebra (7th edition) by McCoy and Janusz

**Area:** Algebra

### **Background and Goals:**

This course is designed to serve as an introduction to the methods and concepts of abstract mathematics. A typical student entering this course has substantial experience in using complex mathematical (calculus) calculations to solve physical or geometrical problems, but is inexperienced at analyzing carefully the content of definitions and the logical flow of ideas which underlie and justify these calculations. Although the topics discussed here are quite distinct from those of calculus, an important goal of the course is to introduce the student to this type of analysis. Much of the reading, homework exercises, and exams consists of theorems (propositions, lemmas, etc.) and their proofs. Math 217, or equivalent, required as background.

### **Content:**

The initial topics include ones common to every branch of mathematics: sets, functions (mappings), relations, and the common number systems (integers, rational numbers, real numbers, complex numbers). These are then applied to the study of particular types of mathematical structures: groups, rings, and fields. These structures are presented as abstractions from many examples such as the common number systems together with the operations of addition or multiplication, permutations of finite and infinite sets with function composition, sets of motions of geometric figures, and polynomials. Notions such as generator, subgroup, direct product, isomorphism, and homomorphism are defined and studied.

### **Alternatives:**

Math 312 (Applied Modern Algebra) is a less abstract course which replaces some of the material on rings and fields of Math 412 with additional applications to areas such as switching and coding theory.

### **Subsequent Courses:**

A student who successfully completes this course will be prepared to take a number of other courses in abstract mathematics: Math 416 (Theory of Algorithms), Math 451 (Advanced Calculus I), Math 475 (Elementary Number Theory), Math 575 (Intro. to Theory of Numbers), Math 493-494, Math 481 (Intro. to Mathematical Logic), and Math 582 (Intro. To Set Theory). All of these courses will extend and deepen the student's grasp of modern abstract mathematics.

## Math 416 - Theory of Algorithms

**Prerequisites:** Math 312, 412; or EECS 280 and Math 465; or permission of instructor

**Frequency:** Sporadically

**Student Body:** Largely computer science concentrators with a few graduate students from other fields

**Credit:** 3 Credits.

**Recent Texts:** Algorithm Design by Kleinberg and Tardos

### **Background and Goals:**

Many common problems from mathematics and computer science may be solved by applying one or more algorithms — well-defined procedures that accept input data specifying a particular instance of the problem and produce a solution. Students entering Math 416 typically have encountered some of these problems and their algorithmic solutions in a programming course. The goal here is to develop the mathematical tools necessary to analyze such algorithms with respect to their efficiency (running time) and correctness. Different instructors will put varying degrees of emphasis on mathematical proofs and computer implementation of these ideas.

### **Content:**

Typical problems considered are: sorting, searching, matrix multiplication, graph problems (flows, travelling salesman), and primality and pseudo-primality testing (in connection with coding questions). Algorithm types such as divide-and-conquer, backtracking, greedy, and dynamic programming are analyzed using mathematical tools such as generating functions, recurrence relations, induction and recursion, graphs and trees, and permutations. The course often includes a section on abstract complexity theory including NP completeness.

### **Alternatives:**

This course has substantial overlap with EECS 586 (Design and Analysis of Algorithms) — more or less depending on the instructors. In general, Math 416 will put more emphasis on the analysis aspect in contrast to the design of algorithms aspect.

### **Subsequent Courses:**

EECS 574 (Computational Complexity) and 575 (Advanced Cryptography) include some topics which follow those of this course.

## Math 417 - Matrix Algebra I

**Prerequisites:** Three mathematics courses beyond Math 110

**Frequency:** Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

**Student Body:** Largely engineering and science students, both undergraduate and graduate

**Credit:** 3 Credits. No credit granted to those who have completed or are enrolled in Math 214, 217, 419, or 420.

**Recent Texts:** Linear Algebra with Applications (4th edition) by Bretscher

**Area:** Algebra

### **Background and Goals:**

Many problems in science, engineering, and mathematics are best formulated in terms of matrices — rectangular arrays of numbers. This course is an introduction to the properties of and operations on matrices with a wide variety of applications. The main emphasis is on concepts and problem-solving, but students are responsible for some of the underlying theory. Diversity rather than depth of applications is stressed. This course is not intended for mathematics concentrators, who should elect Math 217, and/or 493-494 if pursuing the honors concentration.

### **Content:**

Topics include matrix operations, echelon form, general solutions of systems of linear equations, vector spaces and subspaces, linear independence and bases, linear transformations, determinants, orthogonality, characteristic polynomials, eigenvalues and eigenvectors, and similarity theory. Applications include linear networks, least squares method (regression), discrete Markov processes, linear programming, and differential equations.

### **Alternatives:**

Math 419 (Lin. Spaces and Matrix Thy.) is an enriched version of Math 417 with a somewhat more theoretical emphasis. Math 217 (Linear Algebra) is also a more theoretical course which covers much of the material of 417 at a deeper level (despite its lower number).

### **Subsequent Courses:**

This course serves as prerequisite to several courses: Math 452 (Advanced Calculus II), Math 462 (Mathematical Models), Math 420 (Advanced Linear Algebra), and Math 571 (Numer. Meth for Sci. Comput. I)

## Math 419 - Linear Spaces and Matrix Theory

**Prerequisites:** 4 courses beyond Math 110

**Frequency:** Fall (I), Winter (II)

**Student Body:** Largely engineering graduate students and undergraduates; some mathematics undergraduates

**Credit:** 3 Credits. Credit is granted for only one course among Math 214, 217, 417, and 419. No credit granted to those who have completed or are enrolled in Math 420.

**Recent Texts:** Linear Algebra and its Applications (4th ed) by Strang

**Area:** Algebra

### **Background and Goals:**

Math 419 covers much of the same ground as Math 417 (Matrix Algebra I) but presents the material in a somewhat more abstract way in terms of vector spaces and linear transformations instead of matrices. There is a mix of proofs, calculations, and applications with the emphasis depending somewhat on the instructor. A previous proof-oriented course is helpful but by no means necessary.

### **Content:**

Basic notions of vector spaces and linear transformations: spanning, linear independence, bases, dimension, matrix representation of linear transformations; determinants; eigenvalues, eigenvectors, Jordan canonical form, inner-product spaces; unitary, self-adjoint, and orthogonal operators and matrices, applications to differential and difference equations.

### **Alternatives:**

Math 417 (Matrix Algebra I) is less rigorous and theoretical and more oriented to applications. Math 217 (Linear Algebra) is similar to Math 419 but more proof-oriented. Math 420 (Advanced Linear Algebra) is much more abstract and sophisticated.

### **Subsequent Courses:**

This course serves as prerequisite to several courses: Math 420 (Advanced Linear Algebra), Math 452 (Advanced Calculus II), Math 462 (Mathematical Models), Math 561 (Linear Programming I), and Math 571 (Numer. Meth. for Sci. Comp. I)

## Math 420 - Advanced Linear Algebra

**Prerequisites:** Math 214, 217, 417, or 419 and one of 296, 412, or 451

**Frequency:** Winter term each year.

**Student Body:** Graduate students who pursue, or may pursue, the Mathematics masters program and undergraduate mathematics concentrators. This is a required course for MS degree in mathematics.

**Credit:** 3 Credits.

**Recent Texts:** Linear Algebra (2nd Edition), Kenneth M Hoffman (Author), Ray Kunze (Author)

**Area:** Algebra

### Background and Goals:

This is an introduction to the formal theory of abstract vector spaces and linear transformations. It is expected that students have complete at least one prior linear algebra course. The emphasis is on concepts and proofs with some calculations to illustrate the theory. Students should have significant mathematical maturity, at the level of Math 412 or 451. In particular, students should expect to work with and be tested on formal proofs.

### Content:

Topics are selected from: vector spaces over arbitrary fields (including finite fields); linear transformations, bases, and matrices; inner product spaces, duals and spaces of linear transformations, theory of determinants, eigenvalues and eigenvectors; applications to linear differential equations; bilinear and quadratic forms; spectral theorem; Jordan Canonical Form, least squares, singular value theory.

### Alternatives:

None.

### Subsequent Courses:

The natural sequel to Math 420 is Math 593 (Algebra I).

## Math 422 (BE 440) - Risk Management and Insurance

**Prerequisites:** Math 115, Junior standing, and permission of instructor

**Frequency:** Winter (II)

**Student Body:** Junior and Senior mathematics concentrators; some business undergraduates

**Credit:** 3 Credits. Satisfies the Upper Level Writing Requirement (URLW).

**Recent Texts:** Introduction to Risk Management and Insurance (9th edition) by Dorfman

**Area:** Actuarial & Financial

### Background and Goals:

This course is designed to allow students to explore the insurance mechanism as a means of replacing uncertainty with certainty. A main goal of the course is to explain, using mathematical models from the theory of interest, risk theory, credibility theory, and ruin theory, how mathematics underlies many important individual and societal problems.

### Content:

We will explore how much insurance affects the lives of students (automobile insurance, social security, health insurance, theft insurance) as well as the lives of other family members (retirements, life insurance, group insurance). While the mathematical models are important, an ability to articulate why the insurance options exist and how they satisfy the consumer's needs are equally important. In addition, there are different options available (e.g., in social insurance programs) that offer the opportunity of discussing alternative approaches. This course may be used to satisfy the LS&A upper-level writing requirement.

### Alternatives:

None

### Subsequent Courses:

None

## Math 423 - Mathematics of Finance

**Prerequisites:** Math 217, Math 425, and EECS 183 or equivalents

**Frequency:** Fall (I), Winter (II), Spring (IIIa)

**Student Body:** Junior and Senior mathematics concentrators; some business undergraduates

**Credit:** 3 Credits.

**Recent Texts:** Mathematics for Finance: An Introduction to Financial Engineering by Capinski and Zastawniak

**Area:** Actuarial & Financial

### **Background and Goals:**

This course is an introduction to the mathematical models used in finance and economics with particular emphasis on models for pricing derivative instruments such as options and futures. The goal is to understand how the models derive from basic principles of economics and to provide the necessary mathematical tools for their analysis. A solid background in basic probability theory is necessary.

### **Content:**

Topics include risk and return theory, portfolio theory, the capital asset pricing model, the random walk model, stochastic processes, Black-Scholes Analysis, numerical methods, and interest rate models.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 523 (Risk Theory) and Math 623 (Computational Finance).

## Math 424 - Compound Interest and Life Insurance

**Prerequisites:** Math 215, 255, or 285

**Frequency:** Fall (I), Winter (II), Spring (IIIa)

**Student Body:** Undergraduate and graduate students in the Actuarial Mathematics Program, or students interested in exploring the concepts underlying the theory of interest.

**Credit:** 3 Credits.

**Recent Texts:** Mathematical Interest Theory (2nd edition) by Daniel and Vaaler

**Area:** Actuarial & Financial

### **Background and Goals:**

This course explores the concepts underlying the theory of interest and then applies them to concrete problems. The course also includes applications of spreadsheet software. The course is a prerequisite to advanced actuarial courses. It also helps students prepare for some of the professional actuarial exams.

### **Content:**

The course covers compound interest (growth) theory and its application to valuation of monetary deposits, annuities, and bonds. Problems are approached both analytically (using algebra) and geometrically (using pictorial representations). Techniques are applied to real-life situations: bank accounts, bond prices, etc. The text is used as a guide because it is prescribed for the professional examinations; the material covered will depend somewhat on the instructor.

### **Alternatives:**

Math 424 is required for students concentrating in actuarial mathematics; others may take Math 147 (Introduction to Interest Theory), which deals with the same techniques but with less emphasis on continuous growth situations.

### **Subsequent Courses:**

Math 520 (Life Contingencies I) applies the concepts of Math 424 and probability theory to the valuation of life contingencies (death benefits and pensions).

## Math 425 (Stats 425) - Introduction to Probability

**Prerequisites:** Math 215, 255, or 285

**Frequency:** Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

**Student Body:** About 80% undergraduate mathematics, engineering, and computer science concentrators with a few graduate students

**Credit:** 3 Credits.

**Recent Texts:** A First Course in Probability (8th edition) by Ross

**Area:** Analysis; Actuarial & Financial

### **Background and Goals:**

This course introduces students to both useful and interesting ideas from the mathematical theory of probability and to a number of applications of probability to a variety of fields including genetics, economics, geology, business, and engineering. The theory developed together with other mathematical tools such as combinatorics and calculus are applied to everyday problems. Concepts, calculations, and derivations are emphasized. The course will make essential use of the material of Math 116 and 215.

### **Content:**

Topics include the basic results and methods of both discrete and continuous probability theory: conditional probability, independent events, random variables, joint distributions, expectations, variances, and covariances. The culminating results are the Law of Large Numbers and the Central Limit Theorem. Beyond this, different instructors may add additional topics of interest.

### **Alternatives:**

Math 525 (Probability Theory) is a similar course at a faster pace and with deeper coverage. A stronger mathematical background is helpful for Math 525.

### **Subsequent Courses:**

Stats 426 (Introduction to Theoretical Statistics) is a natural sequel for students. Math 423 (Mathematics of Finance) and Math 523 (Risk Theory) include many applications of probability theory.

## Math 427 - Retirement Plans and Other Employee Benefits

**Prerequisites:** Math 115, Junior standing or permission of instructor

**Frequency:** Fall (I)

**Student Body:** Mainly Actuarial Mathematics students, but also some non-mathematics students

**Credit:** 3 Credits. Satisfies the Upper Level Writing Requirement (URLW).

**Recent Texts:** Pension Planning: Pensions, Profit-Sharing, and Other Deferred Compensation Plans by Allen et. al.

**Area:** Actuarial & Financial

### **Background and Goals:**

An overview of the range of employee benefit plans, the considerations (actuarial and others) which influence plan design and implementation practices, and the role of actuaries and other benefit plan professionals and their relation to decision makers in management and unions. \*Until January 2013: Each of the first five undergraduates to report this sentence to the Undergraduate Math Office will receive a free math t-shirt.\* This course is certified for satisfaction of the upper-level writing requirement.

### **Content:**

Particular attention will be given to government programs which provide the framework, and establish requirements, for privately operated benefit plans. Relevant mathematical techniques will be reviewed, but are not the exclusive focus of the course.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 521 (Life Contingencies II) and/or Math 522 (Act. Theory of Pensions and Soc. Sec.) (which can be taken independently of each other) provide more in depth examination of the actuarial techniques used in employee benefit plans.

## Math 429 - Internship

**Prerequisites:** Concentration in Mathematics

**Frequency:** Winter (II)

**Student Body:** International Mathematics Students with CTP requirements for internships

**Credit:** 1 Credit (Credit/No Credit grading scheme)

**Recent Texts:** None

### **Background and Goals:**

Credit is granted for a full-time internship of at least eight weeks that is used to enrich a student's academic experience and/or allows the student to explore careers related to his/her academic studies. Internship credit is not retroactive and must be prearranged.

### **Content:**

Course content is determined by student's internship.

### **Alternatives:**

None

### **Subsequent Courses:**

None

## Math 431 - Topics in Geometry for Teachers

**Prerequisites:** MATH 215, 255, or 285

**Frequency:** Fall (I)

**Student Body:** Mainly teaching certificate candidates; a few other mathematics and engineering students

**Credit:** 3 Credits.

**Recent Texts:** Euclidean and Non-Euclidean Geometry (4th edition) by Greenberg

**Area:** Teaching

### **Background and Goals:**

This course is a study of the axiomatic foundations of Euclidean and non-Euclidean geometry. Concepts and proofs are emphasized; students must be able to follow as well as construct clear logical arguments. For most students this is an introduction to proofs. A subsidiary goal is the development of enrichment and problem materials suitable for secondary geometry classes.

### **Content:**

Topics selected depend heavily on the instructor but may include classification of isometries of the Euclidean plane; similarities; rosette, frieze, and wallpaper symmetry groups; tessellations; triangle groups; finite, hyperbolic, and taxicab non-Euclidean geometries.

### **Alternatives:**

An alternative geometry course at this level is Math 433 (Intro to Differential Geometry).

### **Subsequent Courses:**

None

## Math 433 - Introduction to Differential Geometry

**Prerequisites:** Math 215, 255, or 285; and Math 217

**Frequency:** Fall (I)

**Student Body:** Half undergraduate mathematics concentrators, half graduate students from EECS and physics

**Credit:** 3 Credits.

**Recent Texts:** Elements of Differential Geometry by Millman and Parker

**Area:** Geometry/Topology

### **Background and Goals:**

This course is about the analysis of curves and surfaces in 2- and 3-space using the tools of calculus and linear algebra. There will be many examples discussed, including some which arise in engineering and physics applications. Emphasis will be placed on developing intuition and learning to use calculations to verify and prove theorems. Students need a good background in multivariable calculus (215) and linear algebra (preferably 217). Some exposure to differential equations (216 or 316) is helpful but not absolutely necessary.

### **Content:**

Curves and surfaces in three-space using calculus. Curvature and torsion of curves. Curvature, covariant differentiation, parallelism, isometry, geodesics, and area on surfaces. Gauss-Bonnet Theorem. Minimal surfaces.

### **Alternatives:**

Math 437 is a substantially more advanced course which requires a strong background in topology (590), linear algebra, and advanced multivariable calculus (452). It treats some of the same material from a more abstract and topological perspective and introduces more general notions of curvature and covariant derivative for spaces of any dimension.

### **Subsequent Courses:**

Math 635 (Differential Geometry) and Math 636 (Topics in Differential Geometry) further study Riemannian manifolds and their topological and analytic properties. Physics courses in general relativity and gauge theory will use some of the material of this course.

## Math 437 - Introduction to Differentiable Manifolds

**Prerequisites:** Math 590 and 420

**Frequency:** Fall (I)

**Student Body:** Mainly graduate students in mathematics

**Credit:** 3 Credits.

**Recent Texts:** Differential Topology by Guillemin and Pollack; Differential Topology by Hirsch

**Area:** Geometry/Topology

### **Background and Goals:**

This course is an introduction to the theory of smooth manifolds. The prerequisites for this course are a basic knowledge of analysis, algebra, and topology.

### **Content:**

The following topics will be discussed: smooth manifolds and maps, tangent spaces, submanifolds, vector fields and flows, basic Lie group theory, group actions on manifolds, differential forms, de Rham cohomology, orientation and manifolds with boundary, integration of differential forms, Stokes' theorem.

### **Alternatives:**

Math 433 (Intro. to Differential Geometry) is an undergraduate version which covers less material in a less sophisticated way.

### **Subsequent Courses:**

Math 635 (Differential Geometry)

## Math 450 - Advanced Mathematics for Engineers I

**Prerequisites:** Math 215, 255, or 285 and Math 216, 256, 286, or 316

**Frequency:** Fall (I), Winter (II), Summer (IIIb)

**Student Body:** Mathematics and engineering students; engineering graduate students

**Credit:** 4 Credits. No credit after Math 354 or Math 454.

**Recent Texts:** Advanced Engineering Mathematics (2nd edition) by Greenberg

**Area:** Analysis

### **Background and Goals:**

This course is an introduction to some of the main mathematical techniques in engineering and physics. It is intended to provide some background for courses in those disciplines with a mathematical requirement that goes beyond calculus. Model problems in mathematical physics are studied in detail. Applications are emphasized throughout.

### **Content:**

Topics covered include: Fourier series and integrals; the classical partial differential equations (the heat, wave and Laplace's equations) solved by separation of variables; an introduction to complex variables and conformal mapping with applications to potential theory. A review of series and series solutions of ODEs will be included as needed. A variety of basic diffusion, oscillation, and fluid flow problems will be discussed.

### **Alternatives:**

This course overlaps with 454 and, to a much lesser extent, with 555. The coverage of PDEs in 450 is not as in-depth as 454; for example, in 450 coverage of special functions is reduced to the simplest Bessel functions. Those students needing a more thorough discussion of PDEs and boundary-value problems should take 454. On the other hand, 450 provides a broader introduction to applied methods.

### **Subsequent Courses:**

Math 555 (Complex Variables) and Math 556 (Methods of Applied Math I) are graduate-level courses that further develop both the theory and applications covered in 450.

## Math 451 - Advanced Calculus I

**Prerequisites:** A thorough understanding of Calculus and one of 217, 312, 412, or permission of instructor

**Frequency:** Fall (I), Winter (II), Spring (IIIa)

**Student Body:** Half undergraduate mathematics concentrators, half science and engineering graduate students

**Credit:** 3 Credits. No credit after 351.

**Recent Texts:** Elementary Analysis: The Theory of Calculus by K. Ross

**Area:** Analysis

### **Background and Goals:**

This course has two complementary goals: (1) a rigorous development of the fundamental ideas of Calculus, and (2) a further development of the student's ability to deal with abstract mathematics and mathematical proofs. The key words here are "rigor" and "proof;" almost all of the material of the course is geared toward understanding and constructing definitions, theorems (propositions, lemmas, etc.), and proofs. This is considered one of the more difficult among the undergraduate mathematics courses, and students should be prepared to make a strong commitment to the course. In particular, it is strongly recommended that some course which requires proofs (such as Math 412) be taken before Math 451.

### **Content:**

Topics covered include: logic and techniques of proofs; sets, functions, and relations; cardinality; the real number system and its topology; infinite sequences, limits, and continuity; differentiation; integration, the Fundamental Theorem of Calculus, infinite series; sequences and series of functions.

### **Alternatives:**

There is really no other course which covers the material of Math 451. Although Math 450 is an alternative prerequisite for some courses, the emphasis of the two courses is quite distinct. Math 351 covers similar topics with much less rigor.

### **Subsequent Courses:**

The natural sequel to Math 451 is 452, which extends the ideas considered here to functions of several variables. In a sense, Math 451 treats the theory behind Math 115-116, while Math 452 does the same for Math 215 and a part of Math 216. Math 451 is also a prerequisite for several other courses: Math 575, Math 590, Math 596, and Math 597.



## Math 452 - Advanced Calculus II

**Prerequisites:** Math 217, 417, 419 or 420 (may be concurrent) and Math 451

**Frequency:** Sporadically

**Student Body:** Mathematics undergraduates with some non-mathematics graduate students

**Credit:** 3 Credits.

**Recent Texts:** Advanced Calculus of Several Variables by Edwards

**Area:** Analysis

### **Background and Goals:**

This course gives a rigorous development of multivariable calculus and elementary function theory with some view towards generalizations. Concepts and proofs are stressed. This is a relatively difficult course, but the stated prerequisites provide adequate preparation.

### **Content:**

Topics include: (1) partial derivatives and differentiability; (2) gradients, directional derivatives, and the chain rule; (3) implicit function theorem; (4) surfaces, tangent planes; (5) max-min theory; (6) multiple integration, change of variable, etc.; (7) Greens' and Stokes' theorems, differential forms, exterior derivatives.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 452 is good general background for any of the more advanced courses in analysis (Math 596, 597) or differential geometry or topology (Math 537, 635).

## Math 454 - Boundary Value Problems for Partial Differential Equations

**Prerequisites:** Math 215, 255, or 285; and Math 216, 256, 286, or 316

**Frequency:** Fall (I), Winter (II), Spring (IIIa)

**Student Body:** Some mathematics undergraduates, but more non-mathematics graduate students

**Credit:** 3 Credits. 1 credit after Math 354. No credit granted to those who have completed or are enrolled in Math 450.

**Recent Texts:** Partial Differential Equations for Scientists by Farlow

**Area:** Applied/Numerical Analysis

### **Background and Goals:**

This course is devoted to the use of Fourier series and other orthogonal expansions in the solution of initial-value and boundary-value problems for second-order linear partial differential equations. Emphasis is on concepts and calculation. The official prerequisite is ample preparation.

### **Content:**

Classical representation and convergence theorems for Fourier series; method of separation of variables for the solution of the one-dimensional heat and wave equation; the heat and wave equations in higher dimensions; eigenfunction expansions; spherical and cylindrical Bessel functions; Legendre polynomials; methods for evaluating asymptotic integrals (Laplace's method, steepest descent); Laplace's equation and harmonic functions, including the maximum principle. As time permits, additional topics will be selected from: Fourier and Laplace transforms; applications to linear input-output systems, analysis of data smoothing and filtering, signal processing, time-series analysis, and spectral analysis; dispersive wave equations; the method of stationary phase; the method of characteristics.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 454 is prerequisite to Math 571 (Numer. Meth. For Sci. Comput. I) and Math 572 (Numer. Meth. For Sci. Comput. II). Although it is not a formal prerequisite, it is good background for Math 556 (Methods of Applied Math I).

## Math 462 - Mathematical Models

**Prerequisites:** Math 216, 256, 286, or 316; and Math 217, 417, or 419

**Frequency:** Winter (II)

**Student Body:** Mainly Junior and Senior mathematics concentrators; students from engineering, biology, physics, and medicine

**Credit:** 3 Credits. 1-3 credits after 463 depending on overlap

**Recent Texts:** Mathematical Models in Biology (2nd edition) by Edelstein-Keshet

**Area:** Applied/Numerical Analysis

### **Background and Goals:**

The focus of this course is the application of a variety of mathematical techniques to solve real-world problems. Students will learn how to model a problem in mathematical terms and use mathematics to gain insight and eventually solve the problem. Concepts and calculations, using applied analysis and numerical simulations, are emphasized.

### **Content:**

Construction and analysis of mathematical models in physics, engineering, economics, biology, medicine, and social sciences. Content varies considerably with instructor. Recent versions: Use and theory of dynamical systems (chaotic dynamics, ecological and biological models, classical mechanics), and mathematical models in physiology and population biology.

### **Alternatives:**

Students who are particularly interested in biology should consider Math 463 (Math Modeling in Biology).

### **Subsequent Courses:**

Any higher-level course in differential equations.

## Math 463 - Math Modeling in Biology

**Prerequisites:** Bioinf 463 or Biophys 463; and Math 214, 217, 417, or 419; and 216, 256, 286, or 316

**Frequency:** Fall (I)

**Student Body:** Juniors, Seniors, and first year graduate students (half engineering and half LS&A)

**Credit:** 3 Credits.

**Recent Texts:** Mathematical Models in Biology (5th ed) by L. Edelstein-Keshet; Mathematical Biology I (3rd ed) by J.D. Murray

**Area:** Applied/Interdisciplinary

### **Background and Goals:**

The complexities of the biological sciences make interdisciplinary involvement essential and the increasing use of mathematics in biology is inevitable as biology becomes more quantitative. Mathematical biology is a fast growing and exciting modern application of mathematics that has gained worldwide recognition. In this course, mathematical models that suggest possible mechanisms that may underlie specific biological processes are developed and analyzed. Another major emphasis of the course is illustrating how these models can be used to predict what may follow under currently untested conditions. The course moves from classical to contemporary models at the population, organ, cellular, and molecular levels. The goals of this course are: (i) Critical understanding of the use of differential equation methods in mathematical biology and (ii) Exposure to specialized mathematical and computational techniques which are required to study ordinary differential equations that arise in mathematical biology. By the end of this course students will be able to derive, interpret, solve, understand, discuss, and critique discrete and differential equation models of biological systems.

### **Content:**

This course provides an introduction to the use of continuous and discrete differential equations in the biological sciences. Biological topics may include single species and interacting population dynamics, modeling infectious and dynamic diseases, regulation of cell function, molecular interactions and receptor-ligand binding, biological oscillators, and an introduction to biological pattern formation. There will also be discussions of current topics of interest such as Tumor Growth and Angiogenesis, HIV and AIDS, and Control of the Mitotic Clock. Mathematical tools such as phase portraits, bifurcation diagrams, perturbation theory, and parameter estimation techniques that are necessary to analyze and interpret biological models will also be covered. Approximately one class period each week will be held in the mathematics computer laboratory where numerical techniques for finding and visualizing solutions of differential and discrete systems will be discussed.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 563

## Math 464 - Inverse Problems

**Prerequisites:** Math 217, 417, or 419 and Math 216, 256, 286, or 316

**Frequency:** Sporadically

**Student Body:** Upper-level undergraduates and graduate students in mathematics, science, and engineering

**Credit:** 3 Credits.

**Recent Texts:** None

**Area:** Applied/Numerical Analysis

### **Background and Goals:**

Solution of an inverse problem is a central component of fields such as medical tomography, geophysics, non-destructive testing, and control theory. The solution of any practical inverse problem is an interdisciplinary task. Each such problem requires a blending of mathematical constructs and physical realities. Thus, each problem has its own unique components; on the other hand, there is a common mathematical framework for these problems and their solutions. This framework is the primary content of the course. This course will allow students interested in the above-named fields to have an opportunity to study mathematical tools related to the mathematical foundations of said fields.

### **Content:**

The course content is motivated by a particular inverse problem from a field such as medical tomography (transmission, emission), geophysics (remote sensing, inverse scattering, tomography), or non-destructive testing. Mathematical topics include ill-posedness (existence, uniqueness, stability), regularization (e.g., Tikhonov, least squares, modified least squares, variation, mollification), pseudo-inverses, transforms (e.g., k-plane, Radon, X-ray, Hilbert), special functions, and singular-value decomposition. Physical aspects of particular inverse problems will be introduced as needed, but the emphasis of the course is investigation of the mathematical concepts related to analysis and solution of inverse problems.

### **Alternatives:**

None

### **Subsequent Courses:**

None

## Math 465 - Introduction to Combinatorics

**Prerequisites:** Linear Algebra (one of Math 214, 217, 296, 417, or 419) or permission of instructor

**Frequency:** Fall (I)

**Student Body:** Upper-level undergraduates and graduate students in mathematics, science, and engineering

**Credit:** 3 Credits. No credit granted to those who have completed or are enrolled in Math 565 or 566.

**Recent Texts:** Introductory Combinatorics (4th edition) by R. Brualdi

**Area:** Discrete Mathematics

### **Background and Goals:**

Combinatorics is the study of finite mathematical objects, including their enumeration, structural properties, design, and optimization. Combinatorics plays an increasingly important role in various branches of mathematics and in numerous applications, including computer science, statistics and statistical physics, operations research, bioinformatics, and electrical engineering. This course provides an elementary introduction to the fundamental notions, techniques, and theorems of enumerative combinatorics and graph theory.

### **Content:**

An introduction to combinatorics, covering basic counting techniques (inclusion-exclusion, permutations and combinations, generating functions) and fundamentals of graph theory (paths and cycles, trees, graph coloring). Additional topics may include partially ordered sets, recurrence relations, partitions, matching theory, and combinatorial algorithms.

### **Alternatives:**

565 (offered in the Fall) is significantly more demanding and proof-oriented. Math 566 is even more advanced.

### **Subsequent Courses:**

Math 565 and 566.

## Math 466 (EEB 466) - Mathematical Ecology

**Prerequisites:** Math 217, 417, or 419; Math 256, 286, or 316; and Math 450 or 451

**Frequency:** Sporadically

**Student Body:** The course is intended for graduate students and advanced undergraduates interested in the mathematical analysis and modeling of ecological systems.

**Credit:** 3 Credits.

**Recent Texts:** none

**Area:** Applied

### **Background and Goals:**

This course gives an overview of mathematical approaches to questions in the science of ecology. Topics include: formulation of deterministic and stochastic population models; dynamics of single-species populations; and dynamics of interacting populations (predation, competition, and mutualism), structured populations, and epidemiology. Emphasis is placed on model formulation and techniques of analysis.

### **Content:**

Why do some diseases become pandemic? Why does the introduction of certain species result in widespread invasions? Why do some populations grow while others decline and still others cycle rhythmically? How are all of these phenomena influenced by climate change? These and many other fundamental questions in the science of ecology are intrinsically quantitative and concern highly complex systems. To answer them, ecologists formulate and study mathematical models. This course is intended to provide an overview of the principal ecological models and the mathematical techniques available for their analysis. Emphasis is placed on model formulation and techniques of analysis. Although the focus is on ecological dynamics, the methods we discuss are readily applicable across the sciences. The course presumes mathematical maturity at the level of advanced calculus with prior exposure to ordinary differential equations, linear algebra, and probability.

### **Alternatives:**

none

### **Subsequent Courses:**

none

## Math 471 - Introduction to Numerical Methods

**Prerequisites:** Math 216, 256, 286, or 316; Math 214, 217, 417, or 419; and a working knowledge of one high-level computer language

**Frequency:** Fall (I), Winter (II), Summer (IIIb)

**Student Body:** Juniors, Seniors, and Master's level (half Engineering, half LS&A)

**Credit:** 3 Credits. No credit granted to those who have completed or are enrolled in Math 371 or 472.

**Recent Texts:** A Friendly Introduction to Numerical Analysis by Bradie

**Area:** Applied/Numerical Analysis

### **Background and Goals:**

This is a survey of the basic numerical methods which are used to solve scientific problems. The emphasis is evenly divided between the analysis of the methods and their practical applications. Some convergence theorems and error bounds are proved. The course also provides an introduction to MATLAB, an interactive program for numerical linear algebra, as well as practice in computer programming. One goal of the course is to show how calculus and linear algebra are used in numerical analysis.

### **Content:**

Topics may include computer arithmetic, Newton's method for non-linear equations, polynomial interpolation, numerical integration, systems of linear equations, initial value problems for ordinary differential equations, quadrature, partial pivoting, spline approximations, partial differential equations, Monte Carlo methods, 2-point boundary value problems, Dirichlet problem for the Laplace equation.

### **Alternatives:**

Math 371/Engin. 303 (Numerical Methods) is a less sophisticated version intended principally for Sophomore and Junior engineering students; the sequence Math 571—572 (Numer. Meth. for Sci Comput. I & II) is mainly taken by graduate students, but should be considered by strong undergraduates.

### **Subsequent Courses:**

Math 471 is good preparation for Math 571 and Math 572 (Numer. Meth. for Sci. Comput. I & II), although it is not prerequisite for these courses.

## Math 472 - Numerical Methods with Financial Applications

**Prerequisites:** Math 216, 256, 286, or 316; Math 214, 217, 417, or 419; and a working knowledge of one high-level computer language. Math 425 is recommended.

**Frequency:** Fall (I)

**Student Body:** Concentrators in the Actuarial Mathematics and Financial Mathematics programs.

**Credit:** 3 Credits. No credit granted to those who have completed or are enrolled in Math 371 or 471.

**Recent Texts:** Numerical Analysis by T. Sauer

**Area:** Actuarial & Financial

### **Background and Goals:**

This is a survey of the basic numerical methods which are used to solve scientific problems. The goals of the course are similar to those of Math 471, but the applications are chosen to be of interest to students in the Actuarial Mathematics and Financial Mathematics programs.

### **Content:**

Topics may include: Newton's method for non-linear equations, systems of linear equations, numerical integration, interpolation and polynomial approximation, ordinary differential equations, partial differential equations—in particular the Black-Scholes equation, Monte Carlo simulation, and numerical modeling.

### **Alternatives:**

Math 371/Engin 303 (Numerical Methods) is a less sophisticated version intended principally for Sophomore and Junior engineering students.

### **Subsequent Courses:**

None

## Math 475 - Elementary Number Theory

**Prerequisites:** No specific prerequisite

**Frequency:** Fall (I)

**Student Body:** Mainly mathematics undergraduates; some non-mathematics undergraduates and graduate students

**Credit:** 3 Credits.

**Recent Texts:** An Introduction to the Theory of Numbers (5th edition) by Niven, Zuckerman, and Montgomery

**Area:** Number Theory

### **Background and Goals:**

This is an elementary introduction to number theory, especially congruence arithmetic. Number Theory is one of the few areas of mathematics in which problems easily describable to a layman (is every even number the sum of two primes?) have remained unsolved for centuries. Recently some of these fascinating but seemingly useless questions have come to be of central importance in the design of codes and ciphers. In addition to strictly number-theoretic questions, concrete examples of structures such as rings and fields from abstract algebra are discussed. Concepts and proofs are emphasized, but there is some discussion of algorithms which permit efficient calculation. Students are expected to do simple proofs and may be asked to perform computer experiments. Although there are no special prerequisites and the course is essentially self-contained, most students have some experience in abstract mathematics and problem solving and are interested in learning proofs. At least three semesters of college mathematics are recommended. A Computational Laboratory (Math 476, 1 credit) will usually be offered as an optional supplement to this course.

### **Content:**

Topics usually include the Euclidean algorithm, primes and unique factorization, congruences, Chinese Remainder Theorem, Hensel's Lemma, Diophantine equations, arithmetic in polynomial rings, primitive roots, quadratic reciprocity, and quadratic fields. This material corresponds to Chapters 1-3 and selected parts of Chapter 5 of Niven, Zuckerman, and Montgomery.

### **Alternatives:**

Math 575 (Intro. to Theory of Numbers) moves much faster, covers more material, and requires more difficult exercises. There is some overlap with Math 412 (Introduction to Modern Algebra).

### **Subsequent Courses:**

Math 475 may be followed by Math 575 (Intro. to Theory of Numbers) and is good preparation for Math 412 (Introduction to Modern Algebra). All of the advanced number theory courses, Math 675, 676, 677, 678, and 679, presuppose the material of Math 575, although a good student may get by with Math 475.

## Math 476 - Computational Laboratory in Number Theory

**Prerequisites:** Prior or concurrent enrollment in Math 475 or 575

**Frequency:** Fall (I) sporadically

**Student Body:** Undergraduate mathematics concentrators

**Credit:** 1 Credit.

**Recent Texts:** Coursepack

**Area:** Number Theory

### **Background and Goals:**

Intended as a companion course to Math 475 or 575. Participation should boost the student's performance in either of those classes. Students in the Lab will see mathematics as an exploratory science (as mathematicians do).

### **Content:**

Students will be provided with software with which to conduct numerical explorations. No programming necessary, but students interested in programming will have the opportunity to embark on their own projects. Students will gain a knowledge of algorithms which have been developed for number theoretic purposes, e.g., for factoring.

### **Alternatives:**

None

### **Subsequent Courses:**

None

## Math 481 - Introduction to Mathematical Logic

**Prerequisites:** Math 412 or 451 or equivalent experience with abstract mathematics

**Frequency:** Fall (I)

**Student Body:** Undergraduate mathematics, philosophy, and computer science concentrators; a few non-math graduate students

**Credit:** 3 Credits.

**Recent Texts:** [A Mathematical Introduction to Logic](#) by Enderton

**Area:** Logic

### **Background and Goals:**

All of modern mathematics involves logical relationships among mathematical concepts. In this course we focus on these relationships themselves rather than the ideas they relate. Inevitably this leads to a study of the (formal) languages suitable for expressing mathematical ideas. The explicit goal of the course is the study of propositional and first-order logic; the implicit goal is an improved understanding of the logical structure of mathematics. Students should have some previous experience with abstract mathematics and proofs, both because the course is largely concerned with theorems and proofs and because the formal logical concepts will be much more meaningful to a student who has already encountered these concepts informally. No previous course in logic is prerequisite.

### **Content:**

In the first third of the course the notion of a formal language is introduced and propositional connectives ('and,' 'or,' 'not,' 'implies'), tautologies, and tautological consequence are studied. The heart of the course is the study of first-order predicate languages and their models. The new elements here are quantifiers ('there exists' and 'for all'). The study of the notions of truth, logical consequence, and probability leads to the completeness and compactness theorems. The final topics include some applications of these theorems, usually including non-standard analysis. This material corresponds to Chapter 1 and sections 2.0-2.5 and 2.8 of Enderton.

### **Alternatives:**

Math 681, the graduate introductory logic course, also has no specific logic prerequisite but does presuppose a much higher general level of mathematical sophistication. Philosophy 414 may cover much of the same material with a less mathematical orientation.

### **Subsequent Courses:**

Math 481 is not explicitly prerequisite for any later course, but the ideas developed have application to every branch of mathematics.

## Math 486 - Concepts Basic to Secondary School Math

**Prerequisites:** Math 215, 255, or 285

**Frequency:** Winter (II)

**Student Body:** Undergraduate concentrators in the Teaching Certificate Program and “minors” in other teaching programs

**Credit:** 3 Credits.

**Recent Texts:** Mathematics for High School Teachers: An Advanced Perspective by Peressini, Usiskin, Marchisotto, and Stanley

**Area:** Teaching

### **Background and Goals:**

This course is designed for students who intend to teach junior high or high school mathematics. It is advised that the course be taken relatively early in the program to help the student decide whether or not this is an appropriate goal. Concepts and proofs are emphasized over calculation. The course is intended to give students a strong theoretical background in high school-level topics such as introductory calculus and algebra, giving them the deeper insight necessary to teach these subjects to others.

### **Content:**

Topics covered have included number systems and their axiomatics; number theory, particularly a study of divisibility, primes, and prime factorizations; the abstract theory of sets, operators, and functions; and the epsilon-delta underpinnings of limits and derivatives.

### **Alternatives:**

There is no real alternative, but the Teaching Certificate requirement of Math 486 may be waived for strong students who intend to do graduate work in mathematics.

### **Subsequent Courses:**

Prior completion of Math 486 may be of use for some students planning to take Math 312, 412, or 425.

## Math 489 - Math for Elem. and Middle School Teachers

**Prerequisites:** Math 385 or permission of instructor

**Frequency:** Winter (II)

**Student Body:** Undergraduates in the Elementary Teaching Certificate Program

**Credit:** 3 Credits.

**Recent Texts:** Mathematics for Elementary School Teachers by Bassarear

**Area:** Teaching

### **Background and Goals:**

This course, together with its predecessor, Math 385, provides a coherent overview of the mathematics underlying the elementary and middle school curriculum. It is required of all students intending to earn an elementary teaching certificate and is taken almost exclusively by such students. Concepts are heavily emphasized with some attention given to calculation and proof. The course is conducted using a discussion format. Class participation is expected and constitutes a significant part of the course grade. Enrollment is limited to 30 students per section. Although only two years of high school mathematics is required, a more complete background including pre-calculus or calculus is desirable.

### **Content:**

Topics covered include fractions and rational numbers, decimals and real numbers, probability and statistics, geometric figures, and measurement. Algebraic techniques and problem-solving strategies are used throughout the course.

### **Alternatives:**

There is no alternative course.

### **Subsequent Courses:**

Math 497 (Topics in Elementary Mathematics).

## Math 490 - Introduction to Topology

**Prerequisites:** Math 451 or equivalent experience with abstract mathematics

**Frequency:** Fall (I)

**Student Body:** Mathematics, math-education, science and engineering

**Credit:** 3 Credits.

**Recent Texts:** None

**Area:** Geometry/Topology

### **Background and Goals:**

Topology is the study of a class of interesting spaces, geometric examples of which are knots and surfaces. We focus on those properties of such spaces which don't change if the space is deformed. Much of the course is devoted to understanding particular spaces, such as Moebius strips and Klein bottles. The material in this course has a wide range of applications. Most of the material is theoretical, but it is well-suited for developing intuition and giving convincing proofs which are pictorial or geometric rather than completely rigorous.

### **Content:**

Knots, orientable and non-orientable surfaces, Euler characteristic, open sets, connectedness, compactness, metric spaces. The topics covered are fairly constant but the presentation and emphasis will vary significantly with the instructor.

### **Alternatives:**

Math 590 (Intro. to Topology) is a deeper and more difficult presentation of much of the same material. Math 433 (Intro. to Differential Geometry) is a related course at about the same level.

### **Subsequent Courses:**

Math 490 is not prerequisite for any later course but provides good background for Math 591 (General and Differential Topology) or any of the other courses in geometry or topology.

## Math 493 - Honors Algebra I

**Prerequisites:** Math 296, 412, 451, or permission of instructor

**Frequency:** Fall (I)

**Student Body:** Mainly undergraduate mathematics concentrators with a few graduate students from other fields

**Credit:** 3 Credits.

**Recent Texts:** Algebra by Artin

**Area:** Algebra

### **Background and Goals:**

Math 493-494 is one of the more abstract and difficult sequences in the undergraduate program. It is frequently elected by students who have completed the 295-296 sequence. Its goal is to introduce students to the basic structures of modern abstract algebra (groups, rings, fields, and modules) in a rigorous way.

Emphasis is on concepts and proofs; calculations are used to illustrate the general theory. Exercises tend to be quite challenging. Students must have some previous exposure to rigorous proof-oriented mathematics and be prepared to work hard.

Students from the Math 185-286 and Math 156-256 sequences are strongly advised to take both Math 217 and some 400-500 level course (for examples, Math 451) prior to attempting Math 493.

### **Content:**

The course covers basic definitions and properties of groups, fields, and vector spaces including homomorphisms, isomorphisms, subgroups, and bilinear forms.

Further topics are selected from: Sylow theorems; structure theorem for finitely-generated abelian groups; permutation representation; the symmetric and alternating groups; vector spaces over arbitrary fields; spectral theorem; and linear groups.

### **Alternatives:**

Math 412 (Introduction to Modern Algebra) is a substantially lower level course which covers about half of the material of Math 493. The sequence Math 593-594 covers about twice as much Group and Field Theory as well as several other topics and presupposes that students have had a previous introduction to these concepts at least at the level of Math 412.

### **Subsequent Courses:**

Math 494 (Honors Algebra II)



## Math 494 - Honors Algebra II

**Prerequisites:** Math 493

**Frequency:** Winter (II)

**Student Body:** Mainly undergraduate mathematics concentrators with a few graduate students from other fields.

**Credit:** 3 Credits.

**Recent Texts:** Algebra by Artin

**Area:** Algebra

### **Background and Goals:**

Math 493-494 is one of the more abstract and difficult sequences in the undergraduate program. It is frequently elected by students who have completed the 295-296 sequence. Its goal is to introduce students to the basic structures of modern abstract algebra (groups, rings, fields, and modules) in a rigorous way.

Emphasis is on concepts and proofs; calculations are used to illustrate the general theory. Exercises tend to be quite challenging. Students must have some previous exposure to rigorous proof-oriented mathematics and be prepared to work hard.

### **Content:**

This course is a continuation of Math 493. It covers basic definitions and properties of rings and modules including quotients, ideals, factorization, and field extensions.

Further topics are selected from: representation theory; structure theory of modules over a PID; Jordan canonical form; Galois theory, Nullstellensatz; finite fields; Euclidean, Principal ideal, and unique factorization domains; polynomial rings in one and several variables; and algebraic varieties.

### **Alternatives:**

None

### **Subsequent Courses:**

The natural sequel to Math 494 is Math 593.

## Math 497 - Topics in Elementary Mathematics

**Prerequisites:** Math 489 or permission of instructor

**Frequency:** Fall (I)

**Student Body:** Undergraduates in the Elementary Teaching Certificate Program

**Credit:** 3 Credits.

**Recent Texts:** None

**Area:** Teaching

### **Background and Goals:**

This is a required course for elementary teaching certificate candidates that extends and deepens the coverage of mathematics begun in the required two-course pair Math 385&489. Topics are chosen from geometry, algebra, computer programming, logic, and combinatorics. Applications and problem-solving are emphasized. The class meets three times per week in recitation sections. Grades are based on class participation, two one-hour exams, and a final exam.

### **Content:**

Selected topics in geometry, algebra, computer programming, logic, and combinatorics for prospective and in-service elementary, middle, or junior-high school teachers. Content will vary from term to term.

### **Alternatives:**

None

### **Subsequent Courses:**

None

## Math 498 - Topics in Modern Mathematics

**Prerequisites:** Junior or Senior standing

**Frequency:** Sporadically

**Student Body:** Junior and Senior students from mathematics and other fields

**Credit:** 3 Credits.

**Recent Texts:** Mathematics for High School Teachers : An Advanced Perspective by Zalman Usiskin

### **Background and Goals:**

As a topics course, this course will vary greatly from term to term. In one recent offering, the aim of the course was to introduce, at an elementary level, the basic concepts of the theory of dynamical systems.

### **Content:**

varies

### **Alternatives:**

None

### **Subsequent Courses:**

No specific sequels

## Math 520 - Life Contingencies I

**Prerequisites:** Math 424 and 425, both with a minimum grade of C-, and permission of instructor

**Frequency:** Fall (I)

**Student Body:** Undergraduate students of actuarial mathematics

**Credit:** 3 Credits.

**Recent Texts:** Actuarial Mathematics (2nd ed) by N.L. Bowers et al.

**Area:** Actuarial & Financial

### **Background and Goals:**

Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of this course is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. In addition to actuarial students, this course is appropriate for anyone interested in mathematical modeling outside of the sciences.

### **Content:**

The main topics are the development of (1) probability distributions for the future lifetime random variable; (2) probabilistic methods for financial payments on death or survival; and (3) mathematical models of actuarial reserving.

### **Alternatives:**

Math 523 (Risk Theory) is a complementary course covering the application of stochastic process models.

### **Subsequent Courses:**

Math 520 is prerequisite to all succeeding actuarial courses. Math 521 (Life Contingencies II) extends the single decrement and single life ideas of 520 to multi-decrement and multiple-life applications directly related to life insurance and pensions. The sequence 520-521 covers the syllabus of the Course 3 examination of the Casualty Actuarial Society and covers the syllabus of the Exam MLC examination of the Society of Actuaries. Math 522 (Act. Theory of Pensions and Soc. Sec.) applies the models of 520 to funding concepts of retirement benefits such as social insurance, private pensions, retiree medical costs, etc.

### **Math 521 - Life Contingencies II**

**Prerequisites:** Math 520, with a minimum grade of C-, is the enforced prerequisite

**Frequency:** Winter (II)

**Student Body:** Undergraduate students of actuarial mathematics

**Credit:** 3 Credits.

**Recent Texts:** Actuarial Mathematics (2nd edition) by Bowers et al.

**Area:** Actuarial & Financial

#### **Background and Goals:**

This course extends the single decrement and single life ideas of Math 520 to multi-decrement and multiple-life applications directly related to life insurance. The sequence 520-521 covers the syllabus of the Course 3 examination of the Casualty Actuarial Society and covers the syllabus of the Exam MLC examination of the Society of Actuaries. Concepts and calculation are emphasized over proof.

#### **Content:**

Topics include multiple life models—joint life, last survivor, contingent insurance; multiple decrement models—disability, withdrawal, retirement, etc.; and reserving models for life insurance. This corresponds to chapters 7-10, 14, and 15 of Bowers et. al.

#### **Alternatives:**

Math 522 (Act. Theory of Pensions and Soc. Sec.) is a parallel course covering mathematical models for prefunded retirement benefit programs.

#### **Subsequent Courses:**

None

### **Math 522 - Actuarial Theory of Pensions and Social Security**

**Prerequisites:** Math 520, with a minimum grade of C-, or permission

**Frequency:** Sporadically

**Student Body:** Undergraduate students of actuarial mathematics

**Credit:** 3 Credits.

**Recent Texts:** Fundamentals of Pension Mathematics by B. Berin; Pension Mathematics by A. Anderson (as a reference)

**Area:** Actuarial & Financial

#### **Background and Goals:**

This course develops the mathematical models for pre-funded retirement benefit plans. Concepts and calculation are emphasized over proofs.

#### **Content:**

Mathematical models for (1) retirement income, (2) retiree medical benefits, (3) disability benefits, and (4) survivor benefits. There is some coverage of how accounting theory and practice can be explained by these models and of the U.S. laws and regulations that give rise to the models used in practice.

#### **Alternatives:**

Math 521 (Life Contingencies II) is a parallel course covering models for insurance rather than retirement benefits.

#### **Subsequent Courses:**

None

## Math 523 - Risk Theory

**Prerequisites:** Math 425 or equivalent

**Frequency:** Fall (I), Winter (II)

**Student Body:** Undergraduate students of financial and actuarial mathematics

**Credit:** 3 Credits.

**Recent Texts:** Loss Models: From Data to Decisions (3rd edition) by Klugman, Panjer, and Willmot

**Area:** Actuarial & Financial

### **Background and Goals:**

Risk management is of major concern to all financial institutions and is an active area of modern finance. This course is relevant for students with interests in finance, risk management, or insurance, and it provides background for the professional examinations in Risk Theory offered by the Society of Actuaries and the Casualty Actuary Society. Students should have a basic knowledge of common probability distributions (Poisson, exponential, gamma, binomial, etc.) and have at least Junior standing. Two major problems will be considered: (1) modeling of payouts of a financial intermediary when the amount and timing vary stochastically over time, and (2) modeling of the ongoing solvency of a financial intermediary subject to stochastically varying capital flow. These topics will be treated historically beginning with classical approaches and proceeding to more dynamic models.

### **Content:**

Classical approaches to risk including the insurance principle and the risk-reward tradeoff. Review of probability. Compound Poisson process. Modeling of individual losses that arise in a loss aggregation process, modeling the frequency of losses, and credibility theory.

### **Alternatives:**

None

### **Subsequent Courses:**

None

## Math 525 (Stats 525) - Probability Theory

**Prerequisites:** Math 451 (strongly recommended). Math 425/Stats 425 would be helpful.

**Frequency:** Fall (I), Winter (II)

**Student Body:** A mix of undergraduate and graduate students, drawn largely from mathematics, statistics, and engineering

**Credit:** 3 Credits.

**Recent Texts:** Introduction to Probability Models (10th edition) by Ross

**Area:** Analysis

### **Background and Goals:**

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

### **Content:**

The following topics will be covered: sample space and events, random variables, concept and definition of probability and expectation, conditional probability and expectation, independence, moment generating functions, Law of large numbers, Central limit theorem, Markov chains, Poisson process and exponential distribution.

### **Alternatives:**

EECS 501 also covers some of the same material with less emphasis on mathematical rigor. Math/Stats 425 (Intro. To Probability) treats similar topics, but is accessible with less mathematical background.

### **Subsequent Courses:**

Math 526 (Discr. State Stoch. Proc.), Stats 426 (Intro. to Math Stats), and the sequence Stats 510-511 (Mathematical Statistics I & II) are natural sequels.

## Math 526 (Stats 526) - Discrete State Stochastic Processes

**Prerequisites:** Math 525 or EECS 501

**Frequency:** Fall (I), Winter (II)

**Student Body:** Half undergraduates, half graduate students from mathematics and many other fields

**Credit:** 3 Credits.

**Recent Texts:** Introduction to Probability Models (10th edition) by Ross

**Area:** Analysis

### **Background and Goals:**

This is a course on the theory and applications of stochastic processes, mostly on discrete state spaces. It is a second course in probability which should be of interest to students of mathematics and statistics as well as students from other disciplines in which stochastic processes have found significant applications.

### **Content:**

The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include generating functions; recurrent events and the renewal theorem; random walks; Markov chains; branching processes; limit theorems; Markov chains in continuous time with emphasis on birth and death processes and queueing theory; an introduction to Brownian motion; stationary processes and martingales.

### **Alternatives:**

This course is similar to EECS 502 and IOE 515, although the latter course tends to be somewhat more oriented to applications. It is cross-listed as Statistics 526.

### **Subsequent Courses:**

The next courses in probability are Math 625 and 626, which presuppose substantial additional background (e.g., Math 597).

## Math 528 - Topics in Casualty Insurance

**Prerequisites:** Math 217, 417, 419, or permission of instructor

**Frequency:** Winter (II), generally in even years

**Student Body:** Undergraduate students of actuarial mathematics and insurance concentrators in business

**Credit:** 3 Credits.

**Recent Texts:** Foundations of Casualty Actuarial Science (4th ed)

**Area:** Actuarial & Financial

### **Background and Goals:**

Historically the Actuarial Program has emphasized life, health, and pension topics. This course will provide background in casualty topics for the many students who take employment in this field. Guest lecturers from the industry will provide some of the instruction. Students are encouraged to take the Casualty Actuarial Society's Course 3 examination at the completion of the course.

### **Content:**

The insurance policy is a contract describing the services and protection which the insurance company provides to the insured. This course will develop an understanding of the nature of the coverages provided and the bases of exposure and principles of the underwriting function, how products are designed and modified, and the different marketing systems. It will also look at how claims are settled, since this determines losses which are key components for insurance ratemaking and reserving. Finally, the course will explore basic ratemaking principles and concepts of loss reserving.

### **Alternatives:**

None

### **Subsequent Courses:**

None

## Math 547 - Biological Sequence Analysis

**Prerequisites:** Flexible. Basic probability (level of Math/Stats 425) or molecular biology (level of Biology 427) or biochemistry (level of Chem/BioChem 451) or basic programming skills desirable; or permission of instructor.

**Frequency:** Sporadically

**Student Body:** Interdisciplinary: mainly mathematics, statistics, bio-statistics, and bioinformatics students; also biology, biomedical and engineering students.

**Credit:** 3 Credits.

**Recent Texts:** Biological Sequence Analysis by R. Durbin, et. al.; Computational Molecular Biology by Clote and Backofen

**Area:** Analysis

### **Background and Goals:**

#### **Content:**

Probabilistic models of proteins and nucleic acids. Analysis of DNA/RNA and protein sequence data. Algorithms for sequence alignment, statistical analysis of similarity scores, hidden markov models, neural networks, training, gene finding, protein family profiles, multiple sequence alignment, sequence comparison, and structure prediction. Analysis of expression array data.

#### **Alternatives:**

Bioinformatics 526

#### **Subsequent Courses:**

Bioinformatics 551 (Proteome Informatics)

## Math 550 (CMPLXSYS 510) - Introduction to Adaptive Systems

**Prerequisites:** Math 215, 255, or 285; Math 217; and Math 425

**Frequency:** Sporadically

**Student Body:** Graduate and undergraduate students from many disciplines

**Credit:** 3 Credits.

**Recent Texts:** Essential Mathematical Biology by N. Britton

### **Background and Goals:**

This course centers on the construction and use of agent-based adaptive models to study phenomena which are prototypical in the social, biological, and decision sciences. These models are “agent-based” or “bottom-up” in that the structure is placed at the level of the individuals as basic components; they are “adaptive” in that individuals often adapt to their environment through evolution or learning. The goal of these models is to understand how the structure at the individual or micro level leads to emergent behavior at the aggregate or macro level. Often the individuals are grouped into sub-populations or interesting hierarchies, and the researcher may want to understand how the structure or development of these populations affects macroscopic outcomes.

#### **Content:**

The course will start with classical differential equation and game theory approaches. It will then focus on the theory and application of particular models of adaptive systems such as models of neural systems, genetic algorithms, classifier systems, and cellular automata. Time permitting, we will discuss more recent developments such as sugarscape and echo.

#### **Alternatives:**

Cross-listed as Complex Systems 510.

#### **Subsequent Courses:**

None

### **Math 555 - Introduction to Complex Variables**

**Prerequisites:** Math 450 or 451. Students who had 450 (or equivalent) but not 451 are encouraged to take 451 simultaneously with 555.

**Frequency:** Fall (I), Winter (II)

**Student Body:** Graduate students in mathematics, science, and engineering, and some advanced mathematics, science, and engineering undergraduates

**Credit:** 3 Credits.

**Recent Texts:** Complex Variables and Applications (8th edition) by Churchill and Brown

**Area:** Applied/Numerical Analysis

#### **Background and Goals:**

This course is an introduction to the analysis of complex-valued functions of a complex variable with substantial attention to applications in science and engineering. Concepts, calculations, and the ability to apply principles to problems are emphasized rather than the most general proofs of the fundamental theorems, but arguments are rigorous. The prerequisite of a course in advanced calculus is absolutely essential.

#### **Content:**

Differentiation and integration of complex-valued functions of a complex variable, series, mappings, residues, and applications including evaluation of improper real integrals and fluid mechanics. These topics correspond to the material in Chapters 1-9 of Churchill & Brown.

#### **Alternatives:**

Math 596 (Analysis I (Complex)) covers the theoretical material of Math 555 with an emphasis on proofs rather than applications.

#### **Subsequent Courses:**

Math 555 is prerequisite to many advanced courses in applied mathematics, including the sequence Math 556 and 557 (Methods of Applied Mathematics I and II), and in science and engineering.

### **Math 556 - Methods of Applied Mathematics I**

**Prerequisites:** Math 217, 419, or 420; Math 451; and Math 555

**Frequency:** Fall (I)

**Student Body:** Graduate students in mathematics, science, and engineering

**Credit:** 3 Credits.

**Recent Texts:** Applied Functional Analysis by Griffel

**Area:** Applied/Numerical Analysis

#### **Background and Goals:**

This is an introduction to methods of applied analysis with emphasis on Fourier analysis for differential equations. Initial and boundary value problems are covered. Students are expected to master both the proofs and applications of major results. The prerequisites include linear algebra, advanced calculus, and complex variables.

#### **Content:**

Topics may vary with the instructor but often include Fourier series, separation of variables for partial differential equations, heat conduction, wave motion, electrostatic fields, Sturm-Liouville problems, Fourier transform, signal analysis, sampling theorem, Green's functions, distributions, Hilbert space, complete orthonormal sets, integral operators, spectral theory for compact self-adjoint operators.

#### **Alternatives:**

Math 454 (Bound Val. Probs. for Part. Diff. Eq.) is an undergraduate course on the same topics.

#### **Subsequent Courses:**

Math 557 (Methods of Applied Math II), Math 558 (Ordinary Differential Equations), Math 656 (Partial Differential Equations), and Math 658 (Ordinary Differential Equations).

## Math 557 - Methods of Applied Mathematics II

**Prerequisites:** Math 217, 419, or 520; Math 451; and Math 555

**Frequency:** Winter (II)

**Student Body:** Graduate students in mathematics, science, and engineering

**Credit:** 3 Credits.

**Recent Texts:** Asymptotic Analysis by Murray

**Area:** Applied/Numerical Analysis

### **Background and Goals:**

This is an introduction to methods of asymptotic analysis including asymptotic expansions for integrals and solutions of ordinary and partial differential equations. The prerequisites include linear algebra, advanced calculus, and complex variables. Math 556 is not a prerequisite.

### **Content:**

Topics include stationary phase, steepest descent, characterization of singularities in terms of the Fourier transform, regular and singular perturbation problems, boundary layers, multiple scales, WKB method. Additional topics depend on the instructor but may include non-linear stability theory, bifurcations, applications in fluid dynamics (Rayleigh-Benard convection), combustion (flame speed).

### **Alternatives:**

None

### **Subsequent Courses:**

Math 656 (Partial Differential Equations) and 658 (Ordinary Differential Equations).

## Math 558 - Ordinary Differential Equations

**Prerequisites:** Math 451

**Frequency:** Fall (I)

**Student Body:** Graduate students in mathematics, science, and engineering

**Credit:** 3 Credits.

**Recent Texts:** Differential Equations, Dynamical Systems, and an Introduction to Chaos (2nd edition) by Hirsh, Smale, and Devaney

**Area:** Applied/Numerical Analysis

### **Background and Goals:**

This course is an introduction to Ordinary Differential Equations and Dynamical Systems with emphasis on qualitative analysis.

### **Content:**

The basic results on qualitative behavior, centered on themes of stability and phase plane analysis will be presented in a context that includes applications to a variety of classic examples. The proofs of the fundamental facts will be presented, along with discussions of examples.

### **Alternatives:**

Math 404 (Intermediate Differential Equations) is an undergraduate course on similar topics.

### **Subsequent Courses:**

Math 658 (Ordinary Differential Equations)



## Math 559 - Topics in Applied Mathematics

**Prerequisites:** Vary by topic, check with instructor

**Frequency:** Sporadically

**Student Body:** Undergraduate and graduate students in mathematics or science

**Credit:** 3 Credits.

**Recent Texts:** Varies

**Area:** Applied/NA

### **Background and Goals:**

This is an advanced topics course intended for students with strong interests in the intersection of mathematics and the sciences, but not necessarily experience with both applied mathematics and the application field. Mathematical concepts, as well as intuitions arising from the field of application, will be stressed.

### **Content:**

This course will focus on particular topics in emerging areas of applied mathematics for which the application field has been strongly influenced by mathematical ideas. It is intended for students with interests in mathematical, computational, and/or modeling aspects of interdisciplinary science, and the course will develop the intuitions of the field of application as well as the mathematical concepts. The applications considered will vary with the instructor and may come from physics, biology, economics, electrical engineering, and other fields. Recent examples have been: Nonlinear Waves, Mathematical Ecology, and Computational Neuroscience.

### **Alternatives:**

None

### **Subsequent Courses:**

Other courses in applied mathematics.

## Math 561 (OMS 518, IOE 510) - Linear Programming I

**Prerequisites:** Math 214, 217, 417, or 419

**Frequency:** Fall (I), Winter (II)

**Student Body:** Graduate and undergraduate students from many fields

**Credit:** 3 Credits.

**Recent Texts:** Numerical Linear Algebra by L.N. Trefethen et. al.

### **Background and Goals:**

The allocation of constrained resources such as funds among investment possibilities or personnel among production facilities is a fundamental problem which is very well-suited to mathematical analysis. Each such problem has as its goal the maximization of some positive objective such as investment return or the minimization of some negative objective such as cost or risk. Such problems are called Optimization Problems. Linear Programming deals with optimization problems in which both the objective and constraint functions are linear (the word "programming" means "planning" rather than computer programming). In practice, such problems involve thousands of decision variables and constraints, so a primary focus is the development and implementation of efficient algorithms. However, the subject also has deep connections with higher-dimensional convex geometry. A recent survey showed that most Fortune 500 companies regularly use linear programming in their decision making. This course will present both the classical and modern approaches to the subject and discuss numerous applications of current interest.

### **Content:**

Formulation of problems from the private and public sectors using the mathematical model of linear programming. Development of the simplex algorithm; duality theory and economic interpretations. Postoptimality (sensitivity) analysis; algorithmic complexity; the ellipsoid method; scaling algorithms; applications and interpretations. Introduction to transportation and assignment problems; special purpose algorithms and advanced computational techniques. Students have opportunities to formulate and solve models developed from more complex case studies and to use various computer programs.

### **Subsequent Courses:**

IOE 610 (Linear Programming II) and IOE 611 (Nonlinear Programming)

## Math 562 (IOE 511) - Continuous Optimization Methods

**Prerequisites:** Math 214, 217, 417, or 419

**Frequency:** Fall (I)

**Student Body:** Graduate students in engineering and LSA. Occasionally it attracts an undergraduate Math or Engineering student.

**Credit:** 3 Credits.

**Recent Texts:**

### **Background and Goals:**

Optimization is widely used in engineering and science models. The goal of this course is to give a rigorous background to the field. Examples are drawn from engineering and science.

### **Content:**

Survey of continuous optimization problems. Unconstrained optimization problems: unidirectional search techniques, gradient, conjugate direction, quasi-Newtonian methods; introduction to constrained optimization using techniques of unconstrained optimization through penalty transformation, augmented Lagrangians, and others; discussion of computer programs for various algorithms.

### **Alternatives:**

Cross-listed as IOE 511.

### **Subsequent Courses:**

This is not a prerequisite for any other course.

## Math 563 - Advanced Mathematical Methods for the Biological Sciences

**Prerequisites:** Math 217, 417, or 419 and Math 450 or 454

**Frequency:** Winter (II)

**Student Body:** Graduate students in mathematics, science, engineering, and medicine

**Credit:** 3 Credits.

**Recent Texts:** Mathematical Biology (3rd ed) by J.D. Murray

### **Background and Goals:**

Natural systems behave in a way that reflects an underlying spatial pattern. This course focuses on discovering the way in which spatial variation influences the motion, dispersion, and persistence of species. The concepts underlying spatially dependent processes and the partial differential equations which model them will be discussed in a general manner with specific applications taken from molecular, cellular, and population biology. This course is centered on modeling in three major areas i) Models of Motion: Diffusion, Convection, Chemotaxis, and Haptotaxis; ii) Biological Pattern Formation; and iii) Delay-differential Equations and Age-structured Models.

### **Content:**

This course will introduce and explore partial differential equation modeling in biological settings. Students should have some experience with solution techniques for partial differential equations as well as an interest in biomedical applications. There will also be a brief introduction to delay differential equations and age-structured models; however, no previous background in these areas is required. Mathematical topics covered include derivation of relevant PDEs from first principles; reduction of PDEs to ODEs under steady state, quasi-steady state, and traveling wave assumptions; solution techniques for PDEs and concepts of spatial stability and instability. These concepts will be introduced within the setting of classical and current problems in biology and the biomedical sciences such as cell motion, transport of biological substances, and biological pattern formation. Above all, this course aims to enhance the interdisciplinary training of advanced undergraduate and graduate students from mathematics and other disciplines by introducing fundamental properties of partial differential equations in the context of interesting biological phenomena. Grades will be based on the completion of a research project and weekly (or biweekly) homework assignments, computer lab assignments, and in class presentations.

### **Alternatives:**

None

### **Subsequent Courses:**

None

## Math 564 - Topics in Mathematical Biology

**Prerequisites:** Variable, permission of instructor

**Frequency:** Winter (II), not every year

**Student Body:** Juniors, Seniors, but mainly graduate students

**Credit:** 3 credits

**Area:** Applied/Numerical Analysis

Recent Texts: None

### **Background and Goals:**

This is an advanced course on further topics in mathematical biology. Topic will vary according to the instructor. Possible topics include modeling infectious diseases, cancer modeling, mathematical neurosciences or biological oscillators, among others. The sample description below is for a course in biological oscillators from Winter 2006.

### **Content:**

From sleeping patterns, heartbeats, locomotion, and firefly flashing to the treatment of cancer, diabetes, and neurological disorders, oscillations are of great importance in biology and medicine. Mathematical modeling and analysis are needed to understand what causes these oscillations to emerge, properties of their period and amplitude, and how they synchronize to signals from other oscillators or from the external world. The goal of this course will be to teach students how to take real biological data, convert it to a system of equations and simulate and/or analyze these equations. Models will typically use ordinary differential equations. Mathematical techniques introduced in this course include 1) the method of averaging, 2) harmonic balance, 3) Fourier techniques, 4) entrainment and coupling of oscillators, 4) phase plane analysis, and 5) various techniques from the theory of dynamical systems. Emphasis will be placed on primary sources (papers from the literature) particularly those in the biological sciences. Consideration will be given in the problem sets and course project to interdisciplinary student backgrounds. Teamwork will be encouraged.

### **Alternatives:**

None

### **Subsequent Courses:**

None

## Math 565 - Combinatorics and Graph Theory

**Prerequisites:** Math 412, 451, or equivalent experience with abstract mathematics

**Frequency:** Fall (I)

**Student Body:** Largely mathematics and EECS graduate students with a few mathematics undergraduates

**Credit:** 3 Credits

**Recent Texts:** A Course in Combinatorics (2nd edition) by J.H. van Lint and R.M. Wilson

**Area:** Combinatorics

### **Background and Goals:**

This course has two somewhat distinct halves devoted to (1) graph theory and (2) topics in the theory of finite partially ordered sets. Students should have taken at least one proof-oriented course.

### **Content:**

The first part of this course will be devoted to graph theory. A graph (in the combinatorial sense) is a finite set of points and a specification of which pairs of these points are deemed "adjacent." Despite the simplicity of the concept, it leads to numerous interesting theorems, problems, and applications. Topics in the graph theory part of the course include (if time permits) trees,  $k$ -connectivity, Eulerian and Hamiltonian graphs, tournaments, graph coloring, planar graphs, Euler's formula, the 5-Color theorem, Kuratowski's theorem, and the matrix-tree theorem.

The second part of the course will deal with topics in the theory of finite partially ordered sets. This will include material about Möbius functions, lattices, simplicial complexes, and matroids.

### **Alternatives:**

There is small overlap with Math 566 (Combinatorial Theory). Math 416 (Theory of Algorithms) is somewhat related but much more concerned with algorithms.

### **Subsequent Courses:**

Math 566 (Combinatorial Theory)

## Math 566 - Combinatorial Theory

**Prerequisites:** Math 412, 451, or equivalent experience with abstract mathematics

**Frequency:** Winter (II)

**Student Body:** Undergraduates and graduate students from mathematics, statistics, engineering, and other natural and social sciences

**Credit:** 3 Credits.

**Recent Texts:** None

**Area:** Combinatorics

### **Background and Goals:**

This course is a rigorous introduction to classical combinatorial theory. Concepts and proofs are the foundation, but there are copious applications to modern industrial problem-solving.

### **Content:**

Permutations, combinations, generating functions, and recurrence relations. The existence and enumeration of finite discrete configurations. Systems of representatives, Ramsey's Theorem, and extremal problems. Construction of combinatorial designs.

### **Alternatives:**

There is no real alternative, although there is some overlap with Math 565 (Combinatorics and Graph Theory).

### **Subsequent Courses:**

Sequels are Math 664-665 and Math 669.

## Math 567 - Introduction to Coding Theory

**Prerequisites:** Math 217, 417, 419, or 420

**Frequency:** Fall (I)

**Student Body:** Undergraduate mathematics concentrators and EECS graduate students

**Credit:** 3 Credits.

**Recent Texts:** Introduction to Coding Theory by Roth

**Area:** Algebra

### **Background and Goals:**

This course is designed to introduce mathematics concentrators to an important area of applications in the communications industry. Using linear algebra it will cover the foundations of the theory of error-correcting codes and prepare a student to take further EECS courses or gain employment in this area. For EECS students it will provide a mathematical setting for their study of communications technology.

### **Content:**

Introduction to coding theory focusing on the mathematical background for error-correcting codes. Topics include: Shannon's Theorem and channel capacity; review of tools from linear algebra and an introduction to abstract algebra and finite fields; basic examples of codes such as Hamming, BCH, cyclic, Melas, Reed-Muller, and Reed-Solomon; introduction to decoding starting with syndrome decoding and covering weight enumerator polynomials and the Mac-Williams Sloane identity. Further topics range from asymptotic parameters and bounds to a discussion of algebraic geometric codes in their simplest form.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 565 (Combinatorics and Graph Theory) and Math 556 (Methods of Applied Math I) are natural sequels or predecessors. This course also complements Math 312 (Applied Modern Algebra) by presenting direct applications of finite fields and linear algebra.

## Math 571 - Numerical Methods for Scientific Computing I

**Prerequisites:** Math 214, 217, 417, 419, or 420 and one of Math 450, 451, or 454; or permission of instructor

**Frequency:** Fall (I), Winter (II)

**Student Body:** Mathematics and engineering graduate students, strong undergraduates

**Credit:** 3 Credits.

**Recent Texts:** Numerical Linear Algebra by Trefethen and Bau

**Area:** Applied/NA

### **Background and Goals:**

This course is an introduction to numerical linear algebra, which is at the foundation of much of scientific computing. Numerical linear algebra deals with (1) the solution of linear systems of equations, (2) computation of eigenvalues and eigenvectors, and (3) least squares problems. We will study accurate, efficient, and stable algorithms for matrices that could be dense, or large and sparse, or even highly ill-conditioned. The course will emphasize both theory and practical implementation.

### **Content:**

Topics: (1) background, orthogonal matrices, vector and matrix norms, singular value decomposition; (2) least squares problems, QR factorization, normal equations, projection matrices, Gram-Schmidt orthogonalization, Householder triangularization; (3) stability, condition number, floating point arithmetic, backward error analysis; (4) iterative methods, classical iterative methods (Jacobi, Gauss-Seidel, SOR), conjugate gradient method, Lanczos iteration, Krylov subspace methods, Arnoldi iteration, GMRES, preconditioning; (5) direct methods, Gaussian elimination, LU factorization, pivoting, Cholesky factorization; (6) eigenvalues and eigenvectors, Schur factorization, reduction to Hessenberg and tridiagonal form, power method, QR algorithm.

### **Alternatives:**

Math 471 (Intro. to Numerical Methods) is a survey course in numerical methods at a more elementary level.

### **Subsequent Courses:**

Math 572 (Number. Meth. for Sci. Comput. II) covers initial value problems for ordinary and partial differential equations. Math 571 and 572 may be taken in either order. Math 671 (Analysis of Numerical Methods I) is an advanced course in numerical analysis with varying topics chosen by the instructor.

## Math 572 - Numerical Methods for Scientific Computing II

**Prerequisites:** Math 214, 217, 417, 419, or 420 and one of Math 450, 451, or 454; or permission of instructor

**Frequency:** Winter (II)

**Student Body:** Mathematics and engineering graduate students, strong undergraduates

**Credit:** 3 Credits.

**Recent Texts:** Finite Difference Methods for Ordinary and Differential Equations by LeVeque

**Area:** Applied/NA

### **Background and Goals:**

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. Graduate students from engineering and science departments and strong undergraduates are also welcome. The course is an introduction to numerical methods for solving ordinary differential equations and hyperbolic and parabolic partial differential equations. Fundamental concepts and methods of analysis are emphasized. Students should have a strong background in linear algebra and analysis, and some experience with computer programming.

### **Content:**

Content varies somewhat with the instructor. Numerical methods for ordinary differential equations; Lax's equivalence theorem; finite difference and spectral methods for linear time dependent PDEs: diffusion equations, scalar first order hyperbolic equations, symmetric hyperbolic systems.

### **Alternatives:**

There is no real alternative; Math 471 (Intro. to Numerical Methods) covers a small part of the same material at a lower level. Math 571 and 572 may be taken in either order.

### **Subsequent Courses:**

Math 671 (Analysis of Numerical Methods I) is an advanced course in numerical analysis with varying topics chosen by the instructor.

## Math 575 - Introduction to Theory of Numbers

**Prerequisites:** Math 451 and one of Math 420 or 494, or permission of instructor

**Frequency:** Fall (I)

**Student Body:** Roughly half honors mathematics undergraduates and half graduate students

**Credit:** 3 Credits. 1 credit after Math 475

**Recent Texts:** A Classical Introduction to Modern Number Theory by Ireland and Rosen

**Area:** Number Theory

### **Background and Goals:**

Many of the results of algebra and analysis were invented to solve problems in number theory. This field has long been admired for its beauty and elegance and recently has turned out to be extremely applicable to coding problems. This course is a survey of the basic techniques and results of elementary number theory. Students should have significant experience in writing proofs at the level of Math 451 and should have a basic understanding of groups, rings, and fields, at least at the level of Math 412 and preferably Math 493. Proofs are emphasized, but they are often pleasantly short.

### **Content:**

This is a first course in number theory. Topics covered include divisibility and prime numbers, congruences, quadratic reciprocity, quadratic forms, arithmetic functions, and Diophantine equations. Other topics may be covered as time permits or by request.

### **Alternatives:**

Math 475 (Elementary Number Theory) is a version of Math 575 which puts much more emphasis on computation and less on proof. Only the standard topics above are covered, the pace is slower, and the exercises are easier.

### **Subsequent Courses:**

All of the advanced number theory courses Math 675, 676, 677, 678, and 679 presuppose the material of Math 575. Each of these is devoted to a special subarea of number theory.

## Math 582 - Introduction to Set Theory

**Prerequisites:** Math 412 or 451 or equivalent experience with abstract mathematics

**Frequency:** Winter (II)

**Student Body:** Undergraduate mathematics (often honors) concentrators and some graduate students

**Credit:** 3 Credits.

**Recent Texts:** Discovering Modern Set Theory I: The Basics (8th edition) by Just and Weese

**Area:** Logic

### **Background and Goals:**

One of the great discoveries of modern mathematics was that essentially every mathematical concept may be defined in terms of sets and membership. Thus Set Theory plays a special role as a foundation for the whole of mathematics. One of the goals of this course is to develop some understanding of how Set Theory plays this role. The analysis of common mathematical concepts (e.g., function, ordering, infinity) in set-theoretic terms leads to a deeper understanding of these concepts. At the same time, the student will be introduced to many new concepts (e.g., transfinite ordinal and cardinal numbers, the Axiom of Choice) which play a major role in many branches of mathematics. The development of Set Theory will be largely axiomatic with the emphasis on proving the main results from the axioms. Students should have substantial experience with theorem-proof mathematics; the listed prerequisites are minimal and stronger preparation is recommended. No course in mathematical logic is presupposed.

### **Content:**

The main topics covered are set algebra (union, intersection), relations and functions, orderings (partial, linear, well), the natural numbers, finite and denumerable sets, the Axiom of Choice, and ordinal and cardinal numbers.

### **Alternatives:**

Some elementary set theory is typically covered in a number of advanced courses, but Math 582 is the only course which presents a thorough development of the subject.

### **Subsequent Courses:**

Math 582 is not an explicit prerequisite for any later course, but it is excellent background for many of the advanced courses numbered 590 and above.

## Math 590 - An Introduction to Topology

**Prerequisites:** Math 451

**Frequency:** Fall (I)

**Student Body:** Mathematics graduate students, some non-math graduate students, and mathematics undergraduates

**Credit:** 3 Credits.

**Recent Texts:** Topology (2nd edition) by Munkres

**Area:** Geometry/Topology

### **Background and Goals:**

The purpose of this course is to introduce basic concepts of topology. Most of the course will be devoted to the fundamentals of general (point set) topology.

### **Content:**

Topics include metric spaces, topological spaces, continuous functions and homeomorphisms, separation axioms, quotient and product topology, compactness, and connectedness. We will also cover a bit of algebraic topology (e.g., fundamental groups) as time permits.

### **Alternatives:**

Math 490 (Introduction to Topology) is a more gentle introduction that is more concrete, somewhat less rigorous, and covers parts of both Math 591 (General and Differential Topology) and Math 592 (Intro. to Algebraic Topology). Math 591 (General and Differential Topology) is a more rigorous course covering much of this material and more. Combinatorial and algebraic aspects of the subject are emphasized over the geometrical.

### **Subsequent Courses:**

Students who take Math 590 will be well prepared to continue with Math 591 (General and Differential Topology) and/or Math 437 (Differentiable Manifolds).

## Math 591 - General and Differential Topology

**Prerequisites:** Math 451

**Frequency:** Fall (I)

**Student Body:** Mainly mathematics graduate students, a few mathematics undergraduates and non-math graduate students

**Credit:** 3 Credits.

**Recent Texts:** Topology (2nd edition) by Munkres; Differential Topology by Guillemin and Pollack

**Area:** Geometry/Topology

### **Background and Goals:**

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

### **Content:**

Topological and metric spaces, continuity, subspaces, products and quotient topology, compactness and connectedness, extension theorems, topological groups, topological and differentiable manifolds, tangent spaces, vector fields, submanifolds, inverse function theorem, immersions, submersions, partitions of unity, Sard's theorem, embedding theorems, transversality, classification of surfaces.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 592 (An Introduction to Algebraic Topology) is the natural sequel.

## Math 592 - An Introduction to Algebraic Topology

**Prerequisites:** Math 591

**Frequency:** Winter (II)

**Student Body:** Largely mathematics graduate students

**Credit:** 3 Credits.

**Recent Texts:** Elements of Algebraic Topology by Munkres, Algebraic Topology by Hatcher

**Area:** Geometry/Topology

### **Background and Goals:**

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

### **Content:**

Fundamental group, covering spaces, simplicial complexes, graphs and trees, applications to group theory, singular and simplicial homology, Eilenberg-Steenrod axioms, Brouwer's and Lefschetz' fixed-point theorems, and other topics.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 695 (Algebraic Topology I).

## Math 593 - Algebra I

**Prerequisites:** Math 494; or Math 412, 420, and 451

**Frequency:** Fall (I)

**Student Body:** Largely mathematics graduate students

**Credit:** 3 Credits.

**Recent Texts:** Algebra (an approach via module theory) by Adkins and Weintraub

**Area:** Algebra

### **Background and Goals:**

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs. This course, together with Math 594, offer excellent preparation for the PhD Qualifying exam in algebra. Students should have had a previous course equivalent to Math 493 (Honors Algebra I).

### **Content:**

Topics include basics about rings and modules, including Euclidean rings, PIDs, UFDs. The structure theory of modules over a PID will be an important topic, with applications to the classification of finite abelian groups and to Jordan and rational canonical forms of matrices. The course will also cover tensor, symmetric, and exterior algebras, and the classification of bilinear forms with some emphasis on the field case.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 594 (Algebra II) and Math 614 (Commutative Algebra I).



## Math 594 - Algebra II

**Prerequisites:** Math 593

**Frequency:** Winter (II)

**Student Body:** Largely mathematics graduate students

**Credit:** 3 Credits.

**Recent Texts:** Algebra, A Graduate Course by Isaacs

**Area:** Algebra

### **Background and Goals:**

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

### **Content:**

Topics include group theory, permutation representations, simplicity of alternating groups for  $n > 4$ , Sylow theorems, series in groups, solvable and nilpotent groups, Jordan-Hölder Theorem for groups with operators, free groups and presentations, fields and field extensions, norm and trace, algebraic closure, Galois theory, and transcendence degree.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 612 (Algebra III), Math 613 (Homological Algebra), Math 614 (Commutative Algebra I), and various topics courses in algebra.

## Math 596 - Analysis I (Complex)

**Prerequisites:** Math 451

**Frequency:** Fall (I)

**Student Body:** Largely mathematics graduate students

**Credit:** 3 Credits. 2 credits after Math 555

**Recent Texts:** Complex Analysis (3rd edition) by Ahlfors

**Area:** Analysis

### **Background and Goals:**

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

### **Content:**

Review of analysis in  $\mathbf{R}^2$  including metric spaces, differentiable maps, Jacobians; analytic functions, Cauchy-Riemann equations, conformal mappings, linear fractional transformations; Cauchy's theorem, Cauchy integral formula; power series and Laurent expansions, residue theorem and applications, maximum modulus theorem, argument principle; harmonic functions; global properties of analytic functions; analytic continuation; normal families, Riemann mapping theorem.

### **Alternatives:**

Math 555 (Intro. to Complex Variables) covers some of the same material with greater emphasis on applications and less attention to proofs.

### **Subsequent Courses:**

Math 597 (Analysis II (Real)), Math 604 (Complex Analysis II), and Math 605 (Several Complex Variables).

## **Math 597 - Analysis II (Real)**

**Prerequisites:** Math 451 and 420

**Frequency:** Winter (II)

**Student Body:** Largely mathematics graduate students

**Credit:** 3 Credits.

**Recent Texts:** Real Analysis: Modern Techniques and Their Applications (2nd edition) by Folland

**Area:** Analysis

### **Background and Goals:**

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

### **Content:**

Topics include: Lebesgue measure on the real line; measurable functions and integration on  $\mathbf{R}$ ; differentiation theory, fundamental theorem of calculus; function spaces,  $L^p(\mathbf{R})$ ,  $C(K)$ , Hölder and Minkowski inequalities, duality; general measure spaces, product measures, Fubini's Theorem; Radon-Nikodym Theorem, conditional expectation, signed measures, introduction to Fourier transforms.

### **Alternatives:**

None

### **Subsequent Courses:**

Math 602 (Real Analysis II).