THE UNIVERSITY OF MICHIGAN
DEPARTMENT OF MATHEMATICS

Qualifying Review examination in Algebraic Topology

August 2022

1. Consider the set $Z \subset \mathbb{R}^3$ of all triples (x_1, x_2, x_3) for which there exists $i \in \{1, 2, 3\}$ such that $|x_i| \leq 1$ and $|x_j| = 1$ for all $j \neq i$. Compute $\pi_1(\mathbb{R}^3 \smallsetminus Z)$.

2. Consider the set T of all 2×2 diagonalizable real matrices with determinant 0, with the subspace topology in $\mathbb{R}^4$. Is T a topological manifold (without boundary)? Prove or disprove.

3. Find, with proof, the minimal possible number of cells in a CW-decomposition of the torus $S^1 \times S^1$.

4. A *wedge sum* of two spaces $X \vee Y$ is obtained by taking the disjoint union $X \amalg Y$ and identifying one chosen point of X with one chosen point of Y, with the quotient topology. Describe the universal cover of the wedge sum $\mathbb{R}P^2 \vee \mathbb{R}P^2$.

5. (a) Is every continuous map $S^2 \to S^1 \times S^1$ homotopic to a constant map?

   (b) Is every continuous map $S^1 \times S^1 \to S^2$ homotopic to a constant map?

   Prove your answers.