ALGEBRAIC TOPOLOGY QR
MAY 2021

All maps below are assumed to be continuous.

(1) Let $i : \partial M \to M$ be the inclusion of the boundary of the Mobius strip M into M. Describe the induced map $\pi_1(i)$ on fundamental groups as a map of abelian groups.

(2) Let $f : X \to Y$ be a covering space of path-connected topological spaces. For each of the following constraints on f, X or Y, determine if such a covering space f exists. If one exists, construct it; if not, explain why.

(a) $Y = S^1 \times S^1 \times S^1$ and f is not a regular covering space. (Recall that *regular* covering spaces are sometimes also called *Galois* covering spaces.)

(b) $X = \mathbf{RP}^3$ and Y is homotopy-equivalent to a graph.

(3) Fix some $n \geq 1$. Assume we are given a continuous automorphism $f : \mathbf{CP}^n \to \mathbf{CP}^n$ of order 5. Show that f must have a fixed point.

(4) For any integer $g \geq 1$, let Σ_g be a compact oriented surface of genus g. Show that there are no covering spaces $f : \Sigma_4 \to \Sigma_3$.

(5) Let X be the space obtained by glueing two copies of S^3 together along a (smoothly embedded) closed submanifold diffeomorphic to the torus $T = S^1 \times S^1$, i.e., $X = S^3 \cup_T S^3$. Calculate $H_*(T)$.