Syllabus for Math 597: Real Analysis

- *Abstract measure spaces.*
  Algebras and sigma-algebras of sets, outer measure, Lebesgue-Caratheodory theorem, product measures, Fubini and Tonelli theorems.
- *Lebesgue measure in* $\mathbb{R}^n$*.*
  Lebesgue measure on the real line, measurable sets, approximation of a measurable set by open and compact sets, Borel sets, existence of non-measurable sets, Lebesgue measure in $\mathbb{R}^n$.
- *Measurable functions.*
  Approximation of measurable functions by simple functions, convergence of measurable functions, Egoroff's and Lusin's theorems.
- *Integration.*
  Lebesgue integral, monotone and dominated convergence theorems, Fatou's lemma, change of variables formula for the Lebesgue integral in $\mathbb{R}^n$, functions of bounded variation.
- L^p *spaces.*
  Holder and Minkowski inequalities, L^p and L^∞ spaces, dual of L^p spaces for $p < \infty$.
- *Signed and complex measures.*
  Hahn and Jordan decompositions, variation of a complex measure, Radon-Nikodym theorem.
- *Differentiation.*
  Hardy-Littlewood maximal function, Hardy-Littlewood theorem, Lebesgue differentiation theorem in $\mathbb{R}^n$, fundamental theorem of calculus.

References.

- Folland: Real Analysis;
- Royden: Real Analysis;
- Stein and Shakarchi: Real Analysis;
- Tao: An Introduction to measure theory.