Department of Mathematics

Syllabus for Math 594: Algebra II.

1. Subgroups, cosets, homomorphisms, quotient groups, Lagrange's Theorem. Normal subgroups. Characteristic subgroups. Automorphisms and action on a normal subgroup by conjugation. Direct and semidirect products. Existence of non-split extensions. Examples.
2. Actions of groups on sets. Equivalence of a transitive representation to action on a coset space and applications of such actions. Orbits, transitivity, stabilizers. Normalizers and centralizers to symmetric and alternating groups. Conjugacy classes. Simplicity of alternating groups for $n > 4$. (Optional topic: Simplicity of groups; primitive actions.)
3. Sylow Theorems. (Optional topic: The Frattini argument.)
4. The commutator subgroup and the center. Series in groups. Solvable and nilpotent groups, nilpotence of finite p-groups. Groups with operators. Simple groups. Jordan Holder for groups with operators. Discussion of Krull-Schmidt Theorem and applications (proof optional).
5. Discussion of free groups and presentations (Existence proof for free groups optional.)
6. Field extensions: simple, algebraic, separable and purely inseparable extensions. Multiplicativity of degree. The theorem on the primitive element. Norm and trace.
7. Existence and (non)uniqueness of algebraic closures.
8. Galois theory: splitting fields, normal extensions, Galois extensions, Linear independence of field homomorphisms. The fundamental correspondence.
9. Further Galois theory: cyclotomic extensions of the rationals, including irreducibility of cyclotomic polynomials. Finite fields and their Galois groups. Solvability by radicals. Occurrence of the symmetric group on a prime number of elements as a Galois group over the rationals. (Optional topics: Ruler and compass constructions, Hilbert's Theorem 90, analysis of Galois groups reduction to characteristic p, and the general equation of degree n.)
10. Purely transcendental extensions. Transcendence bases and transcendence degree. (Optional topic: Lüroth's Theorem.)

References:

Dummit, D., and Foote, R., *Abstract Algebra.*
Hungerford: *Algebra.*
Jacobson: *Lectures in Abstract Algebra, Vol. III.*
Jacobson: *Basic Algebra.*
Lang: *Algebra.*
Rotman: *The Theory of Groups.*

Zariski and Samuel: *Commutative Algebra.*

Topics listed as optional will not be included on the Qualifying Review Examinations. Other topics may be included in the Qualifying Review Examination even when they are not covered in a particular course.

JSM 3/94