AIM Qualifying Review Exam: Probability and Discrete Mathematics

January 3, 2022

There are five (5) problems in this examination.

There should be sufficient room in this booklet for all your work. But if you use other sheets of paper, be sure to mark them clearly and staple them to the booklet.

Problem 1

Suppose that a biased coin that lands on heads with probability p is flipped 10 times (independently). Given that a total of 6 heads results, what is the probability that the first three flips are heads, heads, tails?

Problem 1

Problem 1

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Problem 2

Suppose the joint density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{e^{-x/y}e^{-y}}{y}, & 0 < x, y \\ 0 & \text{otherwise.} \end{cases}$$

Find $P[X > 1 | Y = y]$.

Problem 2

Problem 2

Problem 2

Problem 3

A Grey Code of order n is a list of all binary strings of length n listed in an order so that cyclically consecutive strings differ in exactly one bit.

(a) Give a Grey code of order 3.

(b) Describe how to form a Grey code of any order.

Problem 3

Problem 3

Problem 3

Problem 4

There is available an unlimited number of pennies, nickels, dimes, quarters, and half-dollar pieces, worth 1, 5, 10, 25, and 50 cents each. Let h_n denote the number of ways to form n cents. Determine the generating function $g(x) = h_0 + h_1 x + h_2 x^2 + \cdots$ in closed form.

Problem 4

Problem 4

Problem 4

Problem 5

A Directed Graph is a set of nodes and set of directed edges (u, v), where u and v are nodes, and (u, v) is said to go from u to v. A Directed Acyclic Graph (DAG) is a directed graph with no directed cycles, i.e., no sequence $u_1, u_2, u_3 \ldots, u_k$ with all directed edges $(u_1, u_2), (u_2, u_3), \ldots, (u_{k-1}, u_k)$ and (u_k, u_1) present.

A topological ordering of a directed graph is a sequence of all nodes $v_1, v_2, v_3, \ldots$ such that if there is a directed edge (v_i, v_j) then $i < j$. That is, all edges point left to right in the the topological ordering.

Show that a directed graph G has a topological ordering iff it is acyclic.

Problem 5

Problem 5

Problem 5