

## Schedule of Capsule Research Talks August 26, 2019

Session I (4088 East Hall)  
Chair: Wei Ho

- 9:20–9:40 am: Eva Elduque **Alexander-type invariants in Singularities**  
9:45–10:05 am: Karol Koziol **The Local Langlands Conjectures**  
10:10–10:30 am: Yuan Liu **Heuristics on distributions of Galois groups of unramified extension**  
10:30–10:50: Coffee Break in 3096 East Hall  
10:50–11:10 am: Shizhang Li **An example of liftings with different Hodge numbers**  
11:15–11:35 am: Tim Ryan **The geometry of (nested) Hilbert schemes of points on surfaces**  
11:40–12:00 am: Jakub Witaszek **The geometry of mixed characteristic varieties**

Session II (3088 East Hall)  
Chair: Ralf Spatzier

- 8:55–9:15 am: Alexander Garver **Reverse plane partitions via representations of quivers**  
9:20–9:40 am: Zachary Norwood **Colorings and infinite games**  
9:45–10:05 am: Thang Nguyen **The Cheeger-Gromoll splitting theorem for groups**  
10:10–10:30 am: Chaya Norton **Symplectic Structures on the Moduli Space of Projective Connections**  
10:30–10:50: Coffee Break in 3096 East Hall  
10:50–11:10 am: Rebecca Patrias **Symmetric functions and tableaux combinatorics**  
11:15–11:45 am: Thomas Silverman **A Non-archimedean  $\lambda$ -Lemma**  
11:40–12:00 am: Yingchun Zhang **Gromov-Witten Theory and Modular Forms**

Session III(4096 East Hall)  
Chairs: Peter Miller (9:20–10:30) and Erhan Bayraktar (10:50–12:00)

- 9:20–9:40 am: Andrei Prokhorov **Painlevé equations and Riemann-Hilbert problems**  
9:45–10:05 am: Jonathan Tyler **Mathematical Modeling of Biological Clocks**  
10:10–10:30 am: Baole Wen **Modeling of convective CO<sub>2</sub> dissolution in a closed porous media system**  
10:30–10:50: Coffee Break in 3096 East Hall  
10:50–11:10 am: Thomas Bernhardt **Tontines, and the law of large numbers**  
11:15–11:35 am: Shuoqing Deng **Super-replication with model uncertainty**  
11:40–12:00 am: Dominykas Norgilas **Mathematical finance beyond classical models**

## Abstracts:

### **Thomas Bernhardt:** Tontines, and the law of large numbers

Combining the best of drawdown and annuity, the investment returns and the longevity credits, tontines offer a great alternative to current pension products. Instead of guaranteeing an income for life, tontines offer an income that fluctuates with realised mortality rates. Promoting tontines, we answer the question of how many members are needed in a tontine to ensure a stable income for life. Using tools from statistics, we give a precise answer that is independent of the underlying mortality distribution. In the end, we give a theoretical justification of the 1000-member rule derived in numerical studies.

### **Shuoqing Deng:** Super-replication with model uncertainty

We shall first discuss the robust finance approach and its relationship with Skorokhod embedding problem(SEP) and martingale optimal transport(MOT). Then we shall consider the more specific problems of super-replication of American options and super-replication with transaction cost, both under model uncertainty.

### **Eva Elduque:** Alexander-type invariants in Singularities

My research interests lie in the intersection of Algebraic Geometry and Topology. In particular, I study complex algebraic varieties, and I care about techniques and invariants that can help us understand aspects of their analytic topology and their singularities. One such example are Alexander-type invariants, which originally appeared in knot theory, a field which is itself closely related to the study of plane curve singularities. In this talk, I will try to paint a general picture of what these invariants are and the kind of information that they are able to tell us.

### **Alexander Garver:** Reverse plane partitions via representations of quivers

A reverse plane partition is an order-reversing map from a fixed poset to the nonnegative integers. I will discuss an interpretation of reverse plane partitions defined on certain posets in terms of representations of quivers. This interpretation will allow us to prove that a well-studied action on reverse plane partitions is periodic. This is joint work with Rebecca Patrias and Hugh Thomas.

### **Karol Koziol:** The Local Langlands Conjectures

One of the crowning achievements of 20th century mathematics is Class Field Theory, which has its origins in Gauss' Law of Quadratic Reciprocity, and which (among other things) gives a description of all abelian field extensions of the rational numbers. This turns out to be the tip of a rather large iceberg known as the Langlands Conjectures, a vast program aimed at bridging the fields of Number Theory, Representation Theory, and Algebraic Geometry. I'll give an introduction to some "local" aspects of these conjectures, and try to indicate why one might be interested in a "mod-p" variant.

### **Yuan Liu:** Heuristics on distributions of Galois groups of unramified extensions

We will introduce several heuristics on the distributions of Galois groups of unramified extensions, which include the Cohen-Lenstra Heuristics regarding the class groups of quadratic fields and the Boston-Bush-Hajir Heuristics regarding the  $p$ -class tower groups of quadratic fields. We will discuss how these heuristics relate to reasonable random group models, and then explain a new conjecture on the distributions of the Galois groups of the maximal unramified extensions of Galois  $\Gamma$  number fields or function fields for a large family of finite groups  $\Gamma$ . Finally, we will give theorems in the function field case to support this new conjecture. This work uses techniques in number theory, group theory, probability and also arithmetic geometry. It's joint with Melanie Matchett Wood and David Zureick-Brown.

### **Shizhang Li:** An example of liftings with different Hodge numbers

Does a smooth proper variety in positive characteristic know the Hodge number of its liftings? The answer is "of course not. However, its not that easy to come up with a counter-example. In this talk, I will first introduce the background of this problem. Then I shall discuss some obvious constraints of constructing a counter-example. Lastly I will present such a counter-example and state a further question.

**Thang Nguyen:** The Cheeger-Gromoll splitting theorem for groups

Ricci curvature on Riemannian manifolds has been well studied while there is no satisfied parallel notion for graphs or more general metric spaces. We study a curvature notion defined on finitely generated groups by Bar-Natan - Duchin - Kropholler, which behaves like Ricci curvature in many senses. In a joint work with Shi Wang, we study how structures of groups depends on lower bounds of curvature. We obtain a version of the Cheeger-Gromoll splitting theorem. In the talk, I will sketch the curvature notions, examples and theorems. If time permits, I will quickly mention other projects and interest that I am working on.

**Dominykas Norgilas:** Mathematical finance beyond classical models

A light introduction to the robust mathematical finance is presented. Along the way, the connections to the Skorokhod Embedding Problem (SEP) and Martingale Optimal Transport (MOT) are discussed.

**Chaya Norton:** Symplectic Structures on the Moduli Space of Projective Connections

The affine bundle of projective connections over the moduli space of Riemann surfaces of genus  $g$  can be equipped with a symplectic structure by choosing an origin section. We study the induced symplectic structure(s) and show the monodromy map to the character variety is a symplectomorphism for various natural choices of origin section.

**Zachary Norwood:** Colorings and infinite games

Fix a set  $X$  of real numbers. Suppose that two players play a game in which they alternately play either a 0 or a 1. After infinitely many innings, they together have played the binary expansion of a real number, and Player 1 wins if and only if the number belongs to  $X$ . Must one or the other player have a winning strategy? Now suppose that every infinite subset of the integers is given a color, either red or blue. Must there be an infinite set whose subsets all receive the same color? The first question deals with the determinacy of infinite games, and the second with the Ramsey Property. I will give a brief introduction to these two topics and to their intersection.

**Rebecca Patrias:** Symmetric functions and tableaux combinatorics

I'll give a gentle introduction to the Schur function basis of the ring of symmetric functions and explain some connections to tableaux combinatorics and other areas of mathematics.

**Andrei Prokhorov:** Painlevé equations and Riemann-Hilbert problems

Painlevé equations are specific second order nonlinear differential equations. Their solutions have many applications in mathematics and physics and can be called special functions. Painlevé transcendents admit Riemann-Hilbert representation which is the analog of contour integral representation for classical special functions. The asymptotic of Painlevé transcendents can be found analyzing corresponding Riemann-Hilbert problems. The Painlevé equations can be interpreted as Hamiltonian systems. They describe the movement of particles in time-dependent potentials. One can ask different qualitative questions about movement of these particles which can be answered using asymptotic formulae for Painlevé transcendents. The exponents of integrals of corresponding Hamiltonians are called tau functions. The structure of their full expansions was conjectured in [2]. It was proved in some cases using Fredholm determinant representation for tau functions. Riemann-Hilbert problems allow to analyze different gap probabilities in random matrix theory, going beyond the framework of standard Painlevé equations. Recent works in this area include [4, 5] where coupled Painlevé equations appeared. My current work concerns the asymptotic of largest eigenvalue in matrix beta ensembles. This question is related to multicomponent Painlevé equations or Calogero-Painlevé equations.

**Tim Ryan:** The geometry of (nested) Hilbert schemes of points on surfaces

Hilbert schemes of points on surfaces are some of the most classically studied varieties in algebraic geometry and are also important objects in representation theory, combinatorics, and symplectic geometry. One way to study these objects is to understand their birational geometry. In this talk, I will motivate this study and discuss some of my results in the area.

**Thomas Silverman:** A Non-archimedean  $\lambda$ -Lemma

In a celebrated paper published in 1983, R. Mañé, P. Sad, and D. Sullivan prove a result about holomorphic families of injections called the  $\lambda$ -Lemma with impressive applications to the complex dynamics of families of one-variable rational functions. In this talk, I will give a brief introduction to dynamics over a non-archimedean field and discuss a framework for studying the dynamics of families of one-variable rational functions parametrized by Berkovich spaces, including a suitable non-archimedean analogue of the  $\lambda$ -Lemma.

**Jonathan Tyler:** Mathematical Modeling of Biological Clocks

Biological clocks generate rhythms with periods from seconds to months in many organisms and control many processes that are critical to the survival of the organism. Arguably, the most important biological clock is the circadian clock, which coordinates many physiological processes in the 24 hour day. To better understand the circadian clock, mathematical and computational modeling is crucial to providing directions for experimental procedures, to testing new hypotheses quickly and efficiently, and to corroborating new biological revelations. In this talk, we will discuss two active areas of research in the mathematical modeling of biological clocks: the development and analysis of more comprehensive mathematical models of molecular clocks and parameter estimation given experimental data of biological clocks. Finally, we will discuss the implications of results from these research areas to the fields of molecular biology and medicine.

**Baole Wen:** Modeling of convective  $\text{CO}_2$  dissolution in a closed porous media system

Motivated by geological carbon dioxide ( $\text{CO}_2$ ) storage, many recent studies have investigated the fluid dynamics of solutal convection in porous media. Here we study the convective dissolution of  $\text{CO}_2$  in a *closed* porous media system, where the pressure in the gas declines as convection proceeds. This introduces a negative feedback that reduces the convective dissolution rate even before the brine becomes saturated. We analyze the case of an ideal gas with a solubility given by Henry's law, in the limits of very low and very high Rayleigh numbers. The equilibrium state in this system is determined by the dimensionless dissolution capacity,  $\Pi$ , which gives the fraction of the gas that can be dissolved into the underlying brine. Analytical approximations of the pure diffusion problem with  $\Pi > 0$  show that the diffusive base state is no longer self-similar and that diffusive mass transfer declines rapidly with time. Direct numerical simulations at high Rayleigh numbers show that no constant flux regime exists for  $\Pi > 0$ ; nevertheless, the quantity  $F/C_s^2$  remains constant, where  $F$  is the dissolution flux and  $C_s$  is the dissolved concentration at the top of the domain. Simple mathematical models are developed to predict the evolution of  $C_s$  and  $F$  for high-Rayleigh-number convection in closed systems. Finally, the modeling is extended to high-pressure & real-gas conditions and verified using laboratory experiments.

**Jakub Witaszek:** The geometry of mixed characteristic varieties

My research centres around algebraic varieties – geometric objects described by polynomial equations. There are three types of algebraic varieties: of characteristic zero (described by real or complex polynomials), of positive characteristic (described by polynomial congruences modulo a prime number  $p$ ), and of mixed characteristic (described by integer or  $p$ -adic polynomials). In my talk I will discuss some recent developments towards the study of the geometry of mixed characteristic varieties.

**Yingchun Zhang:** Gromov-Witten Theory and Modular Forms.

In this talk, I will briefly introduce the Gromov-Witten theory and some structural results of Gromov-Witten theory. I will also introduce some interaction of Gromov-Witten theory with other fields, for example, modular form. For some varieties, for example, toric Calabi-Yau 3-folds, the generating functions of Gromov-Witten invariants turn out to be quasi-modular forms.