Fibonacci Links

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Project Description

A thin, flexible, chain hangs in the shape of a catenary, the hyperbolic cosine, $\cosh(x) = (e^x + e^{-x})/2$, and the derivative is the hyperbolic sine, $\sinh(x) = (e^x - e^{-x})/2$. These functions are related to the Fibonacci numbers 0, 1, 1, 2, 3, 5, 8, and Lucas numbers, 2, 1, 3, 4, 7, 11, ..., defined by the recursion that makes each number the sum of the previous two numbers (with different starting conditions). For example, the eighth Fibonacci number is given by the horrendous-looking

$$\frac{2\sinh(8\ln(\phi))}{\sqrt{5}}$$

and similarly for other even-indexed Fibonacci numbers, gotten by replacing 8 with some other even number while leaving the structural constants 2, 5, and the golden ratio ϕ. Type into google! Try replacing sinh with cosh and/or removing the $\sqrt{5}$ to get odd-indexed Fibonaccis and odd- and even-indexed Lucases.

In this project, we'll study the hyperbolic trigonometric formulas, Fibonacci numbers, and Lucas numbers. We will build a "discrete catenary" of inflexible links attached to each other at pivots, so that the chain will naturally hang in a shape that (after normalization) miraculously goes through integer-valued coordinates given by Fibonacci and Lucas numbers.

Prerequisites

Math 217