Exploring numerical schemes for conservation laws

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1 Abstract

The **shallow water equations** [1] describe a thin layer of fluid of constant density in hydrostatic balance, bounded from below by a bottom topography, and from above by a free surface. For example, the propagation of a tsunami can be described accurately by the shallow-water equations (until the wave approaches the shore).

The shallow water equations are a set of non-linear hyperbolic partial differential equations and there is no general closed form solution. Thus, in order to study and describe the solution at some time t, we must solve these equations numerically, through **numerical schemes** [2].

In this project, we will use a well-known finite element method to derive a numerical scheme to solve these equations. Furthermore, we will study the discretisation error of the numerical scheme, as well as strategies to mitigate this error. Namely, we will look in detail on the notion of *well-balanced* and *asymptotically preserving* numerical schemes, which retain analytical properties of the continuous model at the discrete level.

Pre-requisites:

- Programming skills (matlab or python or fortran)
- Multivariate calculus
- Linear Algebra
- Differential equations
- Interest in fluid dynamics

References

[1] R. B. Kellogg. The shallow water wave equations: Formulation, analysis and application (i. kinnmark). *SIAM Review*, 30(3):517–518, 1988.

[2] E. Godlewski and P.A. Raviart. *Numerical Approximation of Hyperbolic Systems of Conservation Laws.* Number n.^{o} 118 in Applied Mathematical Sciences. Springer, 1996.