Planimeter and spherimeter

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A planimeter is a mechanical device that measures, exactly or approximately, the area of a planar region by tracing the boundary of the region with a pointer once, counterclockwise, and reading the area off of a dial. A spherimeter (not spherometer!), in Mark Levi's book, chapter 10, similarly measures the area of a region of the surface of a sphere. These work on principles of Holomony and parallel transport, which can be seen in the differential geometer's salute: Outstretch your right arm forward horizontally, with hand in a fist, and thumb left. With the wrist locked, rotate the forearm counterclockwise in the horizontal plane pivoting at the elbow (as in future rotations, too), so the right forearm now points left and the thumb is back. Now rotate the forearm clockwise 90 degrees so the forearm points up and the thumb points back. Finally, rotate the forearm back to its initial position pointing forward, but note that the thumb now points up. The forearm has traced the boundary of an octant on the unit sphere, which has area $\frac{\pi}{2}$; this can be read off the rotation of $\pm\frac{\pi}{2}$ of the thumb.

We will plan and build some of the types of planimeters and the spherimeter and explore related mathematics, such as the Gauss-Bonnet theorem.