Digital Spherimeter: Implementation of a Thought Experiment

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MATH

Log(M)

Introduction

The Spherimeter was originally introduced as a thought experiment in differential geometry by Mark Levi **[1]**, a device that measures area on a curved surface. It was inspired by the Planimeter, an instrument that measures planar area by tracing a boundary curve. They both take principles from differentials of recording motion along the boundary, which then calculates the area of the figure.

Central questions:

- Why does rotation along a closed path encode the enclosed spherical area?
- How can this be turned into a discrete computational algorithm?
- How can this be applied to a digital spherimeter by sampling boundary points?

Planimeter

A Planimeter (Fig. 1) is a mechanical instrument that measures the area enclosed by a simple closed curve by tracing its boundary. Its operation comes from Green's Theorem **[2]**.

$$\oint_C P\,dx + Q\,dy \;=\; \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$$

The wheel of the planimeter records boundary motion that exactly corresponds to the right-hand side of Green's Theorem, yielding the area**[3]**.

Key Idea: Area can be computed from boundary information alone.

Planimeter (flat surface) → Spherimeter (curved surface)

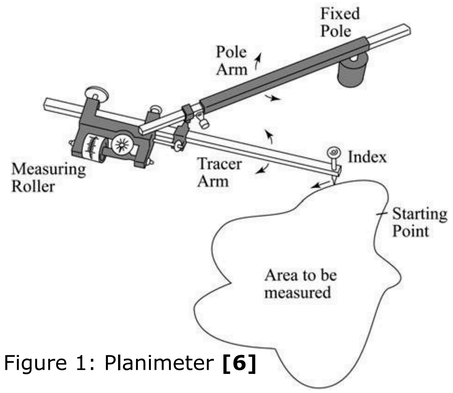


Figure 1: Planimeter **[6]**

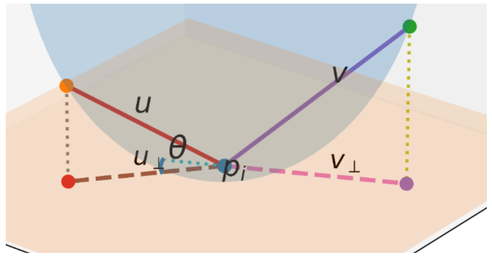


Figure 2: Tangent Plane Projection

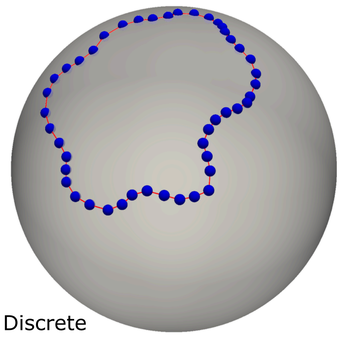


Figure 3: Discrete boundary points

Gauss–Bonnet

$$\int_M K\,dA + \int_{\partial M} k_g\,ds + \sum_i \theta_i = 2\pi\chi(M)$$

Approximate a spherical shape by a spherical polygon and compute its area from the turning angles **[4,5]** ($\chi(M) = 1, R = 1, K = \frac{1}{R^2} = 1$) :

$$\mathrm{Area}(M) = 2\pi - \sum_i \theta_i$$

Each turning angle is equivalent to the angle between two adjacent edges projected onto the tangent plane (Fig. 2), denoted $u_\perp, v_\perp$.

$$\theta_i = \arccos\left(\frac{u_\perp \cdot v_\perp}{\|u_\perp\|\|v_\perp\|}\right)$$

Use the radius vector r to compute the sign, indicating the turning direction (clockwise or counterclockwise) relative to the surface normal.

$$\mathrm{sign}_i = \mathrm{sgn}\left((u_\perp \times v_\perp) \cdot r\right)$$

Digital Spherimeter

Sample discrete boundary points → Compute edge vectors → Project to tangent plane → Compute turning angles → Sum & output area

We developed a Digital Spherimeter, where the user inputs discrete points on a sphere, and then our calculations are applied to calculate the area. To test the code, we took sampled points of the borders of all the countries in the world, used our program to calculate the area, and compared it with the accepted area.

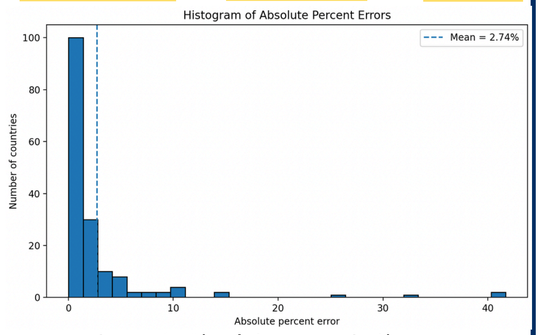


Figure 4: Absolute errors in the area calculations for every country

References: **[1]** Mark Levi. The mathematical mechanic: using physical reasoning to solve problems. 2023. **[2]** Yuteng Wang. Planimeter: A magical tool to calculate area. Highlights in Science, Engineering and Technology, 15:245–251, Nov. 2022. **[3]** Claudia Vela Urrego, Edilberto Sarmiento Sarmiento, Edier Hernan Bustos Velazco, et al. Analytical foundation of the planimeter. Visi´on electr´onica, 11(2):311–317, 2017. **[4]** Wikipedia contributors. Euler characteristic — Wikipedia, the free encyclopedia, 2025. **[5]** Wikipedia contributors. Gauss–Bonnet theorem — Wikipedia, the free encyclopedia, 2025. **[6]** Kiyohide Ito, Y. Fujimoto, Makoto Okamoto, J. Akita, A. Masatani, and T. Ono. Futurebody-finger: A novel alternative aid for visually impaired persons. International Journal on Advances in Life Sciences, 01 2015.

https://github.com/Marico0816/digital-spherimeter

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