

FORMALIZING GEOMETRIC GROUP THEORY THROUGH LEAN

Groups often appear as “groups of symmetries” of geometric objects. Geometric group theory attempts to study groups themselves as geometric objects. A natural starting point is to equip a group with a metric, turning it into a metric space. However, this raises a question, how do we define such a metric? And, is there a “canonical” way to do this? It turns out not quite, but often any two metrics we put on the group are “coarsely” the same. We will begin this project by precisely defining all of these notions and working our way through some basic first properties.

Lean is a functional programming language often used in formalization of mathematics. There is an ongoing project “mathlib” intended to establish a usable database of formalized math in Lean. Currently, this library is lacking any information about geometric group theory. As a way to both obtain a deeper understanding of the math, and gain experience with this new programming language, we will be working to formalize the definitions and basic lemmas of geometric group theory in Lean. Depending on how the progress goes, the eventual goal will be to contribute to the mathlib project.

Prerequisites: Not a lot, some familiarity with groups and metric spaces will be useful to get started. No prior knowledge of Lean or functional programming will be necessary. Part of the goal of this project is to develop these skills. If you want to get a jump start on learning Lean, you can work through the “Natural Number Game” found at: <https://adam.math.hhu.de/#/>.