LoG(M) proposal - Exploring domains of discontinuity in $\mathbb{R}^3$

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1 Background

In Euclidean space, the problem of tiling it with the same shape using simple rules is well understood - if the rules form a group, the shape has to be a (distorted) cube, and the group has to be abelian (imagine $\mathbb{R}^2$ tiled by an infinite square grid - all the tiles are the same square, and we can get to any square by a sequence of up-down and left-right motions).

What happens when we relax the notion of what it means to be the "same shape" - if we allow ourselves to distort the shape as we move it?

2 Affine actions

When we allow a three-dimensional shape to distort, the patterns it can make can be more wild. Instead of the instructions for passing between different tiles making an abelian group, they can make a free group. The tiles can cover all of $\mathbb{R}^3$, or just a part of it, with no way to make the shape bigger. The problem I would like us to work on is:

Suppose I have a set of instructions of how to move (a group acting on $\mathbb{R}^3$ by affine transformations), but I don't yet have the shape that would realize these instructions. How do I find the biggest shape, and what part of $\mathbb{R}^3$ does it cover? Can I draw or perhaps even 3d print this shape?

There are results in this direction, and when the shape can cover all of $\mathbb{R}^3$, we have a good understanding of what happens. I would like us to work on the case where it can't cover everything. There is a paper [KL24] that does this in more generality and with less geometric intuition; we could work on figuring out how to simplify the results and make them more geometric in the case of dimension three.

3 Prerequisites

It would be good if you have an understanding of linear algebra and some basic topology, but we can work up to it as well. I am not very good with computers, so we'll see how much I can help out with any coding.

References

[KL24] Michael Kapovich and Bernhard Leeb, *Domains of discontinuity of lorentzian affine group actions*, Geometriae Dedicata **218** (2024).