LOG(M) PROJECT: HYPERBOLICITY OF FINE ARC GRAPHS

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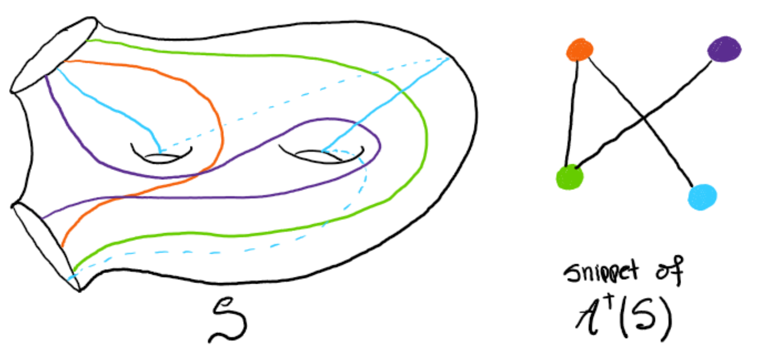
Given a surface (a 2-dimensional object), what are ways in which we can study this surface and groups related to it from the perspective of combinatorics?

Studying groups related to surfaces via their actions on graphs that encode topological information about said surfaces is a long-standing tradition in geometric group theory. However, for us to use a wide set of tools in geometric group theory, the graphs must have a certain geometric property: hyperbolicity.

At its core, (Gromov) hyperbolicity is a property that captures how distances work in a space and is modeled (to an extent) on the hyperbolic plane.

In this project, we will explore the hyperbolicity of *fine arc graphs* of surfaces. These graphs have as their vertices proper embeddings of the interval $[0, 1]$ into a surface S, while edges connect arcs which are disjoint. Some examples of arcs in a surface and how they translate into a snippet of the fine arc graph are included in the figure below.



There is a rich history of hyperbolicity of graphs related to fine arc graphs. We will study past results to see which proof methods can (or cannot) be translated into the setting of fine arc graphs.

Expectations. Participants will be expected to know how to write proofs and have a strong interest in doing research, especially in geometric group theory and combinatorial topology. Some knowledge of point-set topology (i.e. continuous maps, Cantor sets, open and closed sets) is recommended. Knowledge of group theory and graph theory is helpful but not required (interest and motivation are much more important). Proofs will often involve many drawings and plenty of inequalities.

This project will involve reading several papers, so motivation and desire to learn math is a must. The time commitment for research each week will be approximately the amount of time spent on a usual 3-credit math class.