Optimal Stopping Problems with applications to Finance

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1 Introduction

The goal of this project is to introduce the students to the theory and applications of optimal stopping problems, with a specific focus on finance applications. Optimal stopping is a branch of stochastic control that involves deciding on the best time to take some particular actions to maximize the expected gain or minimize the expected loss. In the financial world, such problems often appear in asset pricing, evaluation of real options, and investment timing decisions. By the end of the project, students will have a solid understanding of optimal stopping problems both in discrete and continuous time, some concrete examples of financial contexts, along with numerical techniques for obtaining solutions.

2 Project objectives

1. **Stopping theory in discrete time**: We start by reviewing optimal stopping problems in discrete time, where students can learn about some important concepts, such as the dynamic programming principle and the backward induction method. Students will build some basic understanding of stopping problems and how to determine the best actions using a step-by-step approach.
2. **Stopping theory in continuous time**: We will then transition to continuous time analysis where we will review some of the key concepts such as Snell envelopes, free-boundary problems, and the *"guess and verify"* approach. Students will recall the Feynman-Kac theorem and understand the connection between the solution for optimal stopping problems and free-boundary problems.
3. **Real world financial examples**: Using the tools we learned, we will then look at some real-world financial examples, such as pricing American options and the optimal consumption problem, perhaps some exotic options if time permits. We will solve these examples explicitly and understand how to make timely decisions under uncertainty in finance.
4. **Numerical experiments**: Students will choose their favorite financial example and implement them using methods such as the binomial tree model, Monte Carlo simulation, as well as finite difference, and they will compare these different methods using their chosen example.

3 Assessment

Project report and presentation.

4 Prerequisites

Probability theory (525), basic knowledge of MATLAB or Python, and interest in finance. Mathematics of finance (473) and Stochastic Analysis with financial applications (474) are recommended but not required.