Hyperbolicity of Fine Arc Graphs

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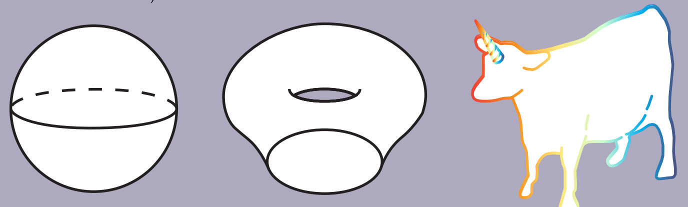
Question

This project aims to show that the fine arc graph of a given surface is a **δ**-hyperbolic metric space.

What is a surface?

- A topological surface, *S*, is a topological space is a space that locally looks like a plane or a half plane (on the boundary). (Sphere, Torus, Cow)

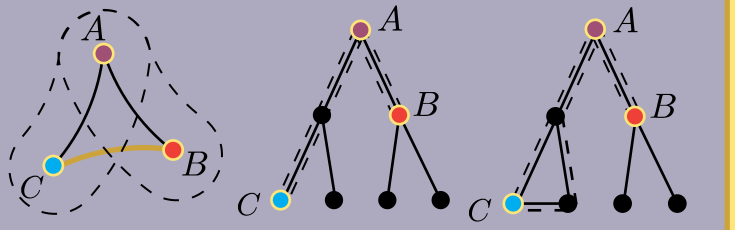


g=0 b=0 **g=1 b=1** **g=? b=?**

What is δ-hyperbolic?

- **Triangles are "slim". In other words, any segment BC in triangle ABC is contained in a neighborhood surrounding AB and AC (see "Slim Triangle" below).**

Slim Triangle: Baby Example: Another Example:



Why we care

Implications about the homeomorphisms of our surface (geometric group theory)

Key Concepts

S: orientable surface with boundary and arcs and curves embedded.
Arc graph $\mathcal{A}(S)$-Vertices: isotopy classes of essential arcs, meaning we can "wiggle" or "twist" an arc without letting them cross themselves, genus, and boundaries.
Edge: connects two vertices if their isotopy classes contain elements that are disjoint.

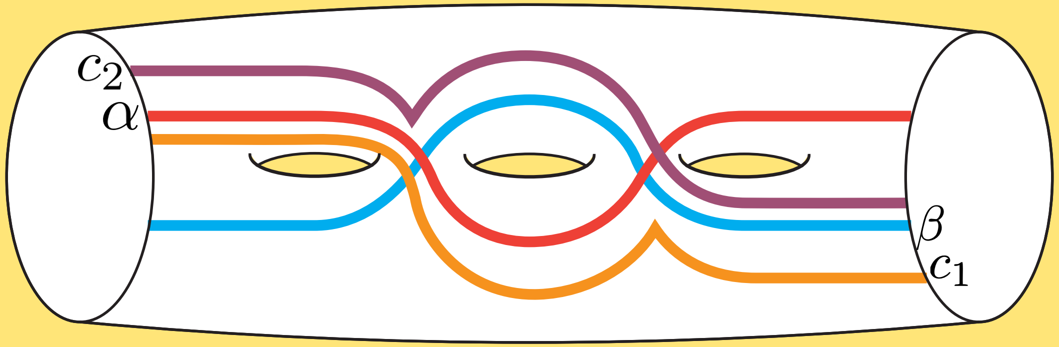
Fine arc graph $\mathcal{A}^{\dagger}(S)$: Refined $\mathcal{A}(S)$ by imposing additional restrictions

In these graphs, distance between two vertices is defined as the shortest number of edges to go through.

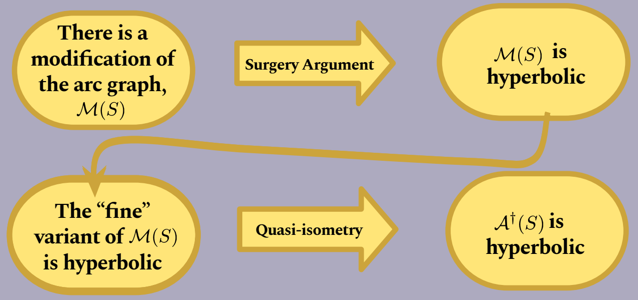
Hyperbolicity of Similar Graphs

Our argument is similar to the following:

- *nonseparating curve graph* is hyperbolic. [4]
- $\mathcal{C}^{\dagger}(S)$ is hyperbolic. [1]
- $\mathcal{A}(S)$ and $\mathcal{C}(S)$ are hyperbolic using unicorn paths. [2]
- Using surgery arguments from [3].

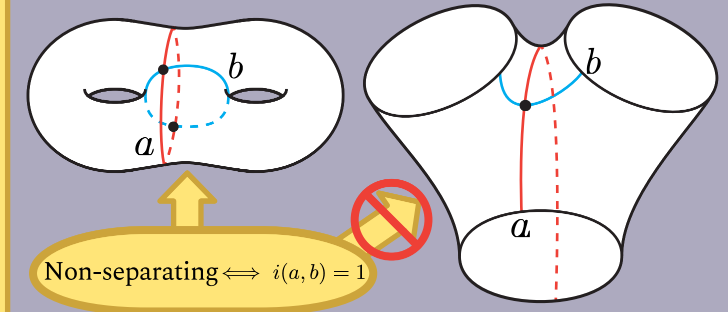


Outline of Proof



The Big Obstacle

Non-separating curves vs non-separating arcs.
A key property Rassmussen used to prove the non-separating curve graph is hyperbolic does not apply for arcs.



References

[1] Bowden, J., Hensel, S., & Webb, R. (2022). Quasi-morphisms on surface diffeomorphism groups. Journal of the American Mathematical Society, 35(1), 211-231.
[2] Hensel, S., Przytycki, P., & Webb, R. C. (2015). 1-slim triangles and uniform hyperbolicity for arc graphs and curve graphs. Journal of the European Mathematical Society, 17(4), 755-762.
[3] Masur, H. A., & Minsky, Y. N. (1998). Geometry of the complex of curves II: Hierarchical structure. arXiv preprint math/9807150.
[4] Rasmussen, A. (2020). Uniform hyperbolicity of the graphs of nonseparating curves via bicorn curves. Proceedings of the American Mathematical Society, 148(6), 2345-2357.