Dynamic Update Anaphora Logic (DUAL)

Ezra Keshet, University of Michigan

DUAL is a new dynamic semantic system that covers the same empirical ground as Dynamic Plural Logic (DPlL, van den Berg 1996) plus a new domain, paycheck pronouns (Karttunen 1969), in a system better capturing parallels between simple and quantified sentences. Besides e and t, DUAL assumes the following types:

(1) $\boldsymbol{\nu}$: individual variables $a, a', \ldots, z, z'$; $\boldsymbol{g}$ in $\mathrm{D}_\nu \to \mathrm{D}_e$: individual assignments; $\boldsymbol{u}$: subsets of $\mathrm{D}_g \times \mathrm{D}_g$ ("updates"); $\hat{\boldsymbol{\nu}}$: update variables $\hat{a}, \hat{a}' \ldots, \hat{z}, \hat{z}'$; $\hat{\boldsymbol{g}}$ in $\mathrm{D}_{\hat{\nu}} \to \mathrm{D}_u$: update assignments; $\boldsymbol{c}$: assignments $g_c \cup \hat{g}_c$ ("states")

DUAL's main innovation over, e.g., Dynamic Predicate Logic (DPL, Groenendijk and Stokhof 1991) is a new class of variables (with hats) to store DPL-style denotations, here named **updates**[1] (type u). A DUAL formula $(\hat{w} : woman(w); [b]; book(b); bought(w, b))$ essentially stores the DPL denotation $[\![[w]; woman(w); [b]; book(b); bought(w, b)]\!]$ in the variable $\hat{w}$ (notice the implied "$[w]$").[2] Thereafter, the formula "$\hat{w}(w')$" retrieves a **restricted update**, one where $w{=}w'$, and applies this update to the current local context; so, $(\hat{w}' : \hat{w}(w'); read(w', b))$ stores $[\![[w]; woman(w); [b]; book(b); bought(w, b); [w']; w'{=}w; read(w', b)]\!]$ in $\hat{w}'$. (Restricted updates let us maintain one distinguished individual variable ν per update variable $\hat{\nu}$; they also help translate strong donkey pronouns.)

Next, the compound term $\hat{w}.w$ represents all values w may have in $\hat{w}$, i.e., all women who bought a book; similarly, $\hat{w}.b$ represents all the books women bought. With these tools, DUAL handles generalized quantifiers, avoiding the proportion problem, in a way that parallels simple sentence sequences (cf. "Jane^w bought a^b book. She$_w$ read it$_b$."). A quantifier compares the compound terms comprising its update variable arguments and their distinguished variables (so, $\hat{w}'.w'$ and $\hat{w}.w$ here):

(2) Most [women who bought a^b book]$^{\hat{w}}$ [read it$_b$]$^{\hat{w}'}$ $\rightsquigarrow$
$$\Big(\hat{w} : woman(w); [b]; book(b); bought(w, b)\Big); \Big(\hat{w}' : w(w'); read(w', b)\Big); |\hat{w}'.w'| {>} \tfrac{1}{2} |\hat{w}.w|$$

Discourse plurals (3a) are captured. Quantificational subordination (3b) uses an update variable to quantify over a subset of a previous quantification. Telescoping (3c) requires a covert quantifier; notice that the superscript b allows counting books instead of women.

(3) a. They$_{\hat{w}'.b}$ are in that pile. $\rightsquigarrow in{-}pile(\hat{w}'.b)$
b. Some$_{\hat{w}'}$ [read it$_b$ twice]$^{\hat{r}}$. $\rightsquigarrow (\hat{r} : \hat{w}'(r); read{-}twice(r, b)); |\hat{r}.r| > 1$
c. $\langle every^b_{\hat{w}'}\rangle$ [It$_b$ was scary.]$^{\hat{s}}$ $\rightsquigarrow (\hat{s} : \hat{w}'(s); scary(b)); |\hat{s}.b| {=} |\hat{w}'.b|$

DUAL also captures discourse plurals within quantificational contexts (4a), and mixed weak and strong donkey anaphora (4b), deficits of DPlL noted by Nouwen (2003) and Brasoveanu (2007). Strong donkey pronouns like her_d in (4b) use the same covert quantifier $\langle every\rangle$ as telescoping, with one innovation: the subscript is a restricted update, in this case $\hat{x}(x')$. The resulting clause "$|\hat{x}(x').d| {=} |\hat{i}.d|$" requires each parent x' to pick up each daughter of x' in some minivan of x' (or other).

[1]An anonymous reviewer points out that this move echoes one in Hardt (1999), who stores dynamic properties, in part to capture paycheck pronouns. Hardt does not extend his system to the other cases considered here. Also, note that the term *update* does not refer to Veltman's (1996) Update Semantics; updates here are relations, not functions.

[2]DUAL adopts the standard revisions to DPL to make it more like a programming language: "$[\nu]$" for random assignment and ';' instead of '$\wedge$' for conjunction.

(4) a. Everyone [with a^d daughter in the game]$^{\hat{x}}$ [watched them$_{\hat{x}.d}$ gather outside]$^{\hat{x}'}$.$\rightsquigarrow$
$(\hat{x} : [d]; \mathit{daughter\text{-}of}(d, x)); (\hat{w}' : \hat{x}(x'); \mathit{watched\text{-}gather}(x', \hat{x}.d)); |\hat{x}'.x'|{=}|\hat{x}.x|$

b. Everyone [with a^d daughter & a^m minivan]$^{\hat{x}}$ [$\langle every^d_{\hat{x}(x')}\rangle$ [drove her$_d$ in it$_m$]$^{\hat{i}}$]$^{\hat{x}'}$.$\rightsquigarrow$
$$\Big(\hat{x} : [d]; \mathit{daughter\text{-}of}(d, x); [m]; \mathit{minivan\text{-}of}(m, x)\Big);$$
$$\Big(\hat{x}' : \hat{x}(x'); (\hat{i} : \hat{x}(x'); \mathit{drove\text{-}in}(x, d, m)); |\hat{x}(x').d|{=}|\hat{i}.d|\Big); |\hat{x}'.x'|{=}|\hat{x}.x|$$

Finally, paycheck pronouns involve a stored update applied in a novel context:

(5) Most women [deposited their$_w$ paycheck$^{\hat{p}}$. Some$_{\hat{w}}$ [cashed it$_{\hat{p}.p}$]$^{\hat{w}'}$ though.

The variable $\hat{p}$ here is set via the formula $(\hat{p} : \mathit{paycheck}(p); \mathit{of}(p, w))$ with w free. In its original context, w ranges over the women who deposited their paychecks, but in the second sentence, w ranges over those who cashed their paychecks. (Note the singular *it*, since $\hat{p}.p$ will always be a singleton.) Systems based on DPlL cannot handle such pronouns without adding machinery effectively equivalent to that presented here.

The full semantics of DUAL is below. For any state variable c below, we use g_c to refer to the individual assignment subset of c and $\hat{g}_c$ for the update assignment; i.e., c abbreviates $g_c{\cup}\hat{g}_c$ and d abbreviates $g_d{\cup}\hat{g}_d$. As usual, a model $M = \langle D_e, I\rangle$ is understood, where I returns relational meanings for predicates over sets of individuals.

(6) Random Variable Assignment for States
$\boldsymbol{c[\bar{\nu}]d} := \forall\nu{\neq}\bar{\nu}(c(\nu){=}d(\nu))\ \&\ \forall\hat{\nu}{\neq}\bar{\nu}(c(\hat{\nu}){=}d(\hat{\nu}))$ [where $\bar{\nu} \in \mathrm{D}_\nu{\cup}\mathrm{D}_{\hat{\nu}}$]

(7) Terms
a. $[\![\boldsymbol{\nu}]\!]_c = \{c(\nu)\}$ b. $[\![\boldsymbol{\phi.\alpha}]\!]_c = \bigcup\{[\![\alpha]\!]_d : c[\![\phi]\!]d\}$

(8) Atomic Formulas
a. $c[\![\boldsymbol{P(\alpha_1, \ldots, \alpha_n)}]\!]d$ iff $c{=}d$ & $\langle[\![\alpha_1]\!]_c, \ldots, [\![\alpha_n]\!]_c\rangle{\in}I(P)\}$
b. $c[\![\boldsymbol{\alpha = \beta}]\!]d$ iff $c{=}d$ & $[\![\alpha]\!]_c{=}[\![\beta]\!]_c$
c. $c[\![\boldsymbol{\hat{\nu}}]\!]d$ iff $\hat{g}_c{=}\hat{g}_d$ & $\langle g_c, g_d\rangle{\in}c(\hat{\nu})$, [Abbreviation: $\boldsymbol{\hat{\nu}(\nu')} := (\hat{\nu}; \nu{=}\nu')$]

(9) Connectives
a. $c[\![\boldsymbol{\sim\phi}]\!]d$ iff $c{=}d$ & $\neg\exists d'\Big(c[\![\phi]\!]d'\Big)$ b. $[\![\boldsymbol{\phi; \psi}]\!] = [\![\phi]\!] \circ [\![\psi]\!]$

(10) Variable Assignments
a. $c[\![\boldsymbol{[\nu]}]\!]d$ iff $c[\nu]d$
b. $c[\![\boldsymbol{(\hat{\nu}{:}\phi)}]\!]d$ iff $g_c{=}g_d$ & $\exists e\Big(e[\hat{\nu}]d\ \&\ d(\hat{\nu}){=}\{\langle g, h\rangle : (g{\cup}\hat{g}_c)[\![[\nu]; \phi]\!](h{\cup}\hat{g}_e)\}\Big)$

(11) a. Contextual Truth: ϕ is true relative to c iff $\exists d(c[\![\phi]\!]d)$.
b. Absolute Truth: ϕ is true iff it is true relative to all c.

Notes: Terms (simple and compound) always denote sets of individuals. Compound terms (7b) may recurse, e.g., "$\hat{x}.\hat{y}.y$". Atomic formulas and connectives are the same as in DPL, except (8c), which applies an update variable to the local context. (The full paper gives a modal definition for negation that improves the explanation of modal subordination and quantified anaphora facts.) Individual random assignment (10a) is as in DPL. Update assignment (10b) is complicated in order to allow ':' clauses within ':' clauses: the intermediate state e captures any embedded update assignments and the output state d stores the DPL-denotation value for $\hat{\nu}$, including the implicit "$[\nu]$" for resetting $\hat{\nu}$'s distinguished individual variable ν.

References

van den Berg, M. H.: 1996, *Some aspects of the internal structure of discourse. The dynamics of nominal anaphora*, PhD thesis, University of Amsterdam.

Brasoveanu, A.: 2007, *Structured nominal and modal reference*, PhD thesis, Rutgers, The State University of New Jersey.

Groenendijk, J. and Stokhof, M.: 1991, Dynamic predicate logic, *Linguistics and philosophy* **14**(1), 39–100.

Hardt, D.: 1999, Dynamic interpretation of verb phrase ellipsis, *Linguistics and Philosophy* **22**(2), 187–221.

Karttunen, L.: 1969, Discourse referents, *Proceedings of the 1969 conference on Computational linguistics*, Association for Computational Linguistics, pp. 1–38.

Nouwen, R.: 2003, *Plural pronominal anaphora in context*, PhD thesis, University of Utrecht.

Veltman, F.: 1996, Defaults in update semantics, *Journal of philosophical logic* **25**(3), 221–261.