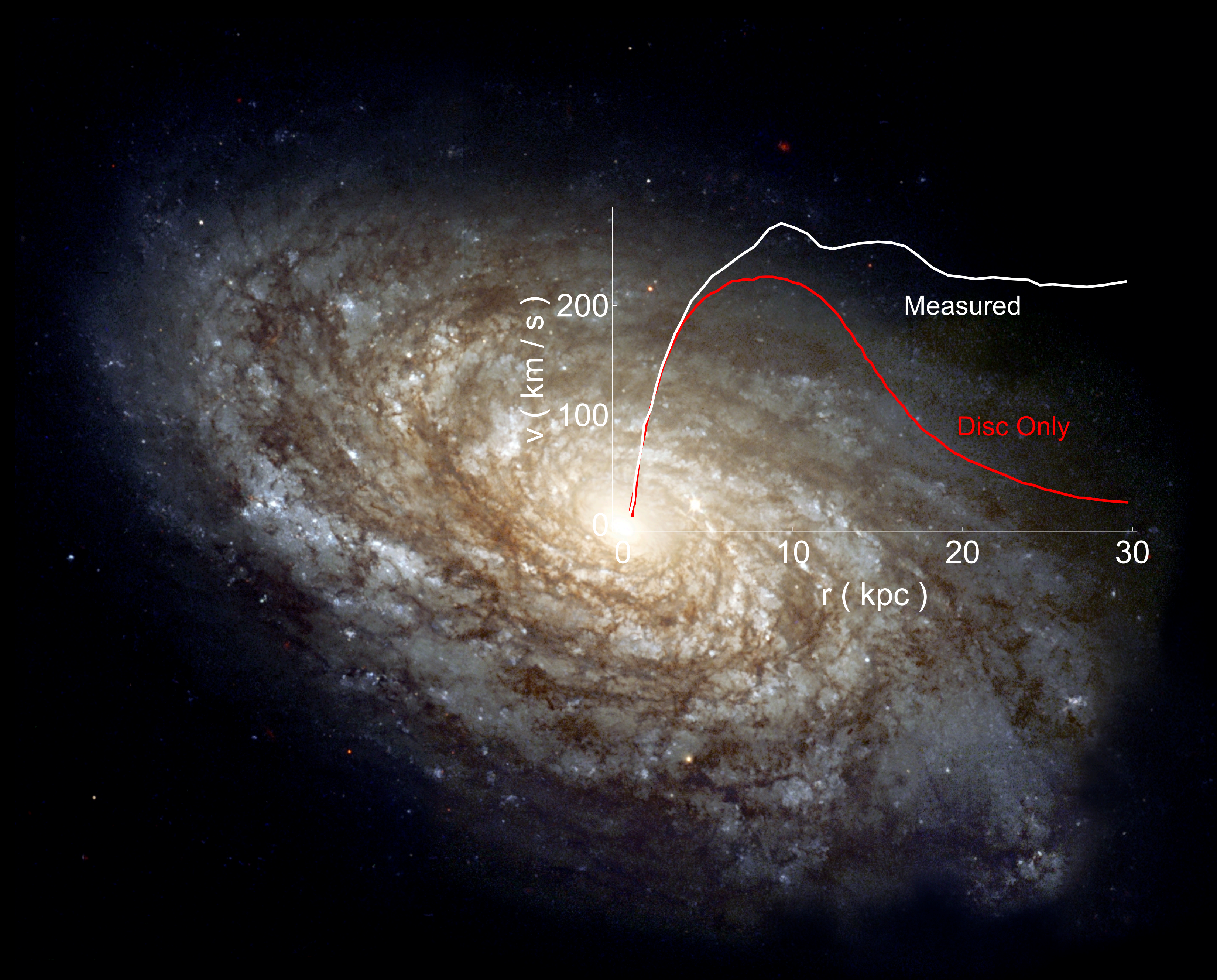
Dark Matter - Phonon Scattering

@ University of Michigan
10/09/2019

Simon Knapen
Institute for Advanced Study

There is missing mass in galaxies



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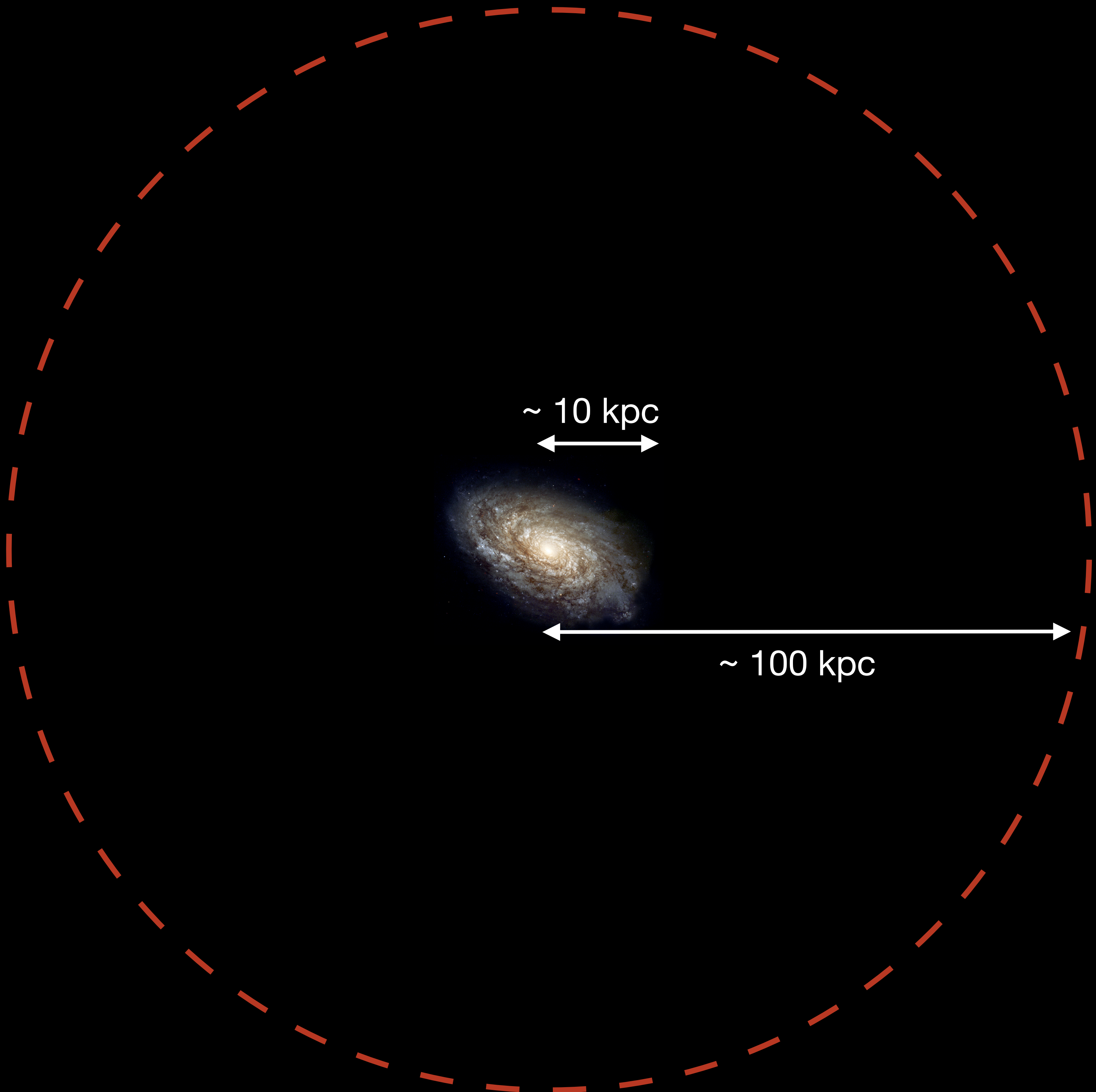
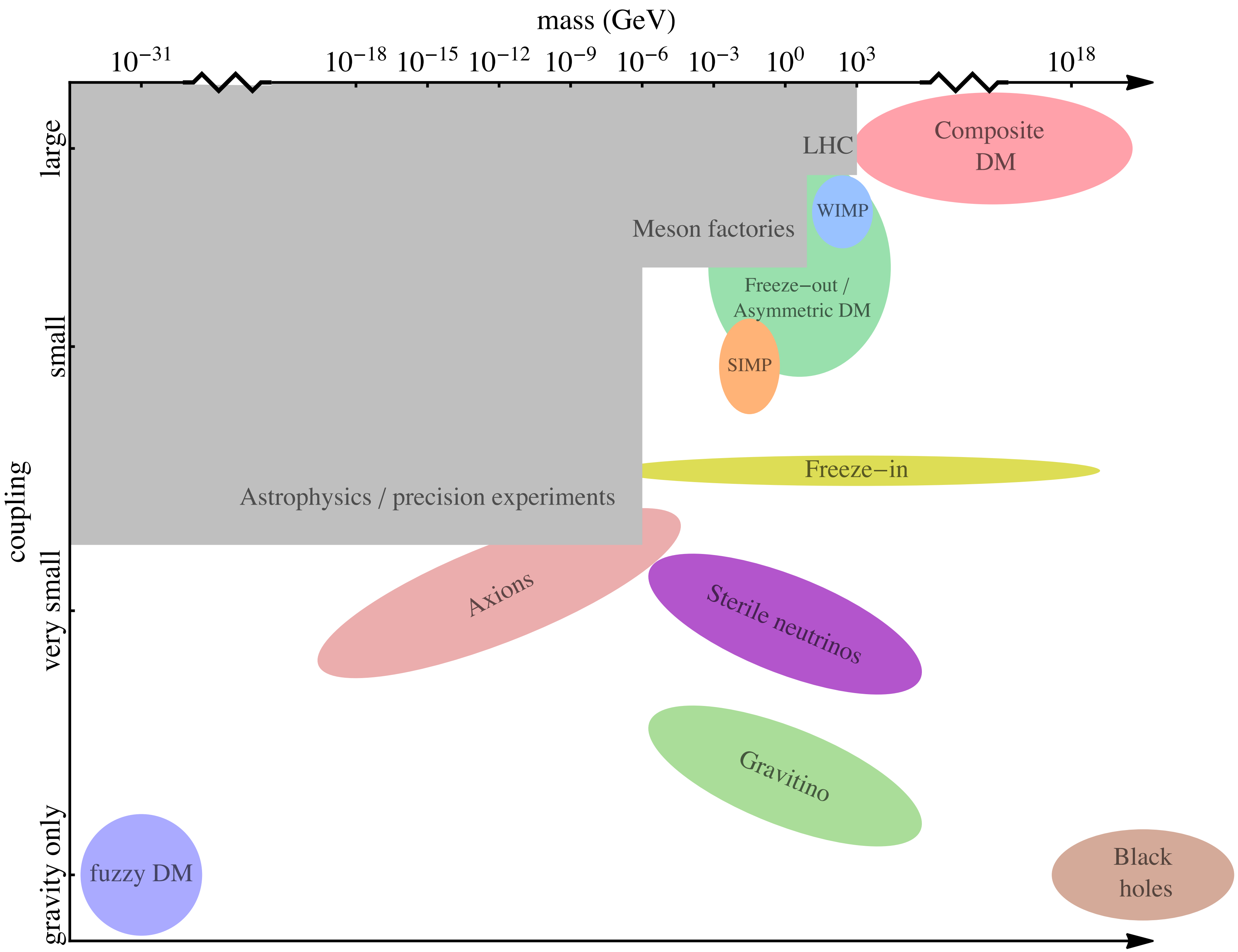


Dark Matter halo

Dark matter velocity

$$\langle v \rangle \sim \sqrt{\frac{G_N M_{halo}}{R_{halo}}} \sim 200\ \mathrm{km/s}$$

The dark matter landscape



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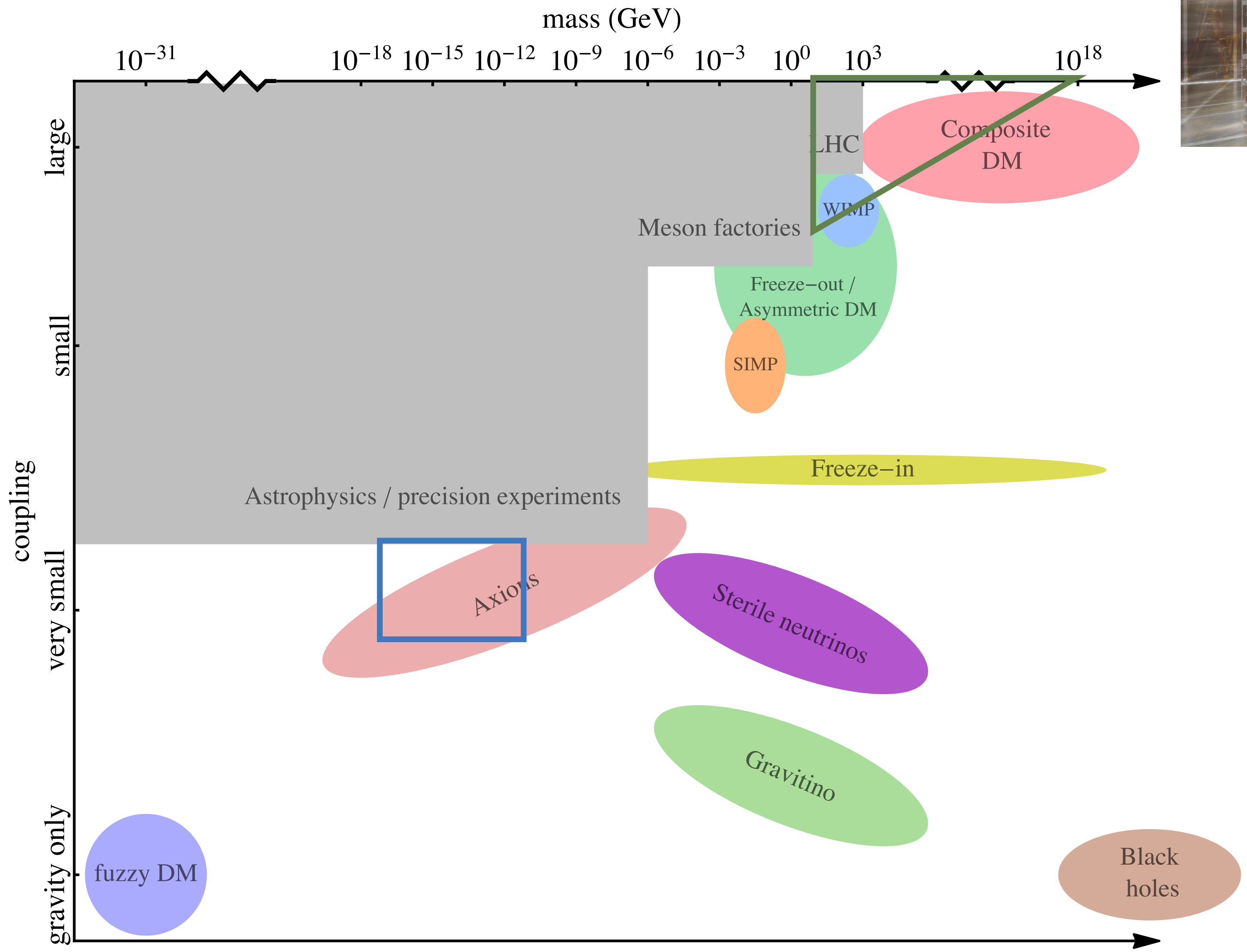

Nuclear recoils
(LUX, XENON1T, superCDMS, …)

Resonant cavities etc
(ADMX, MADMAX…)

The dark matter landscape

Nuclear recoils
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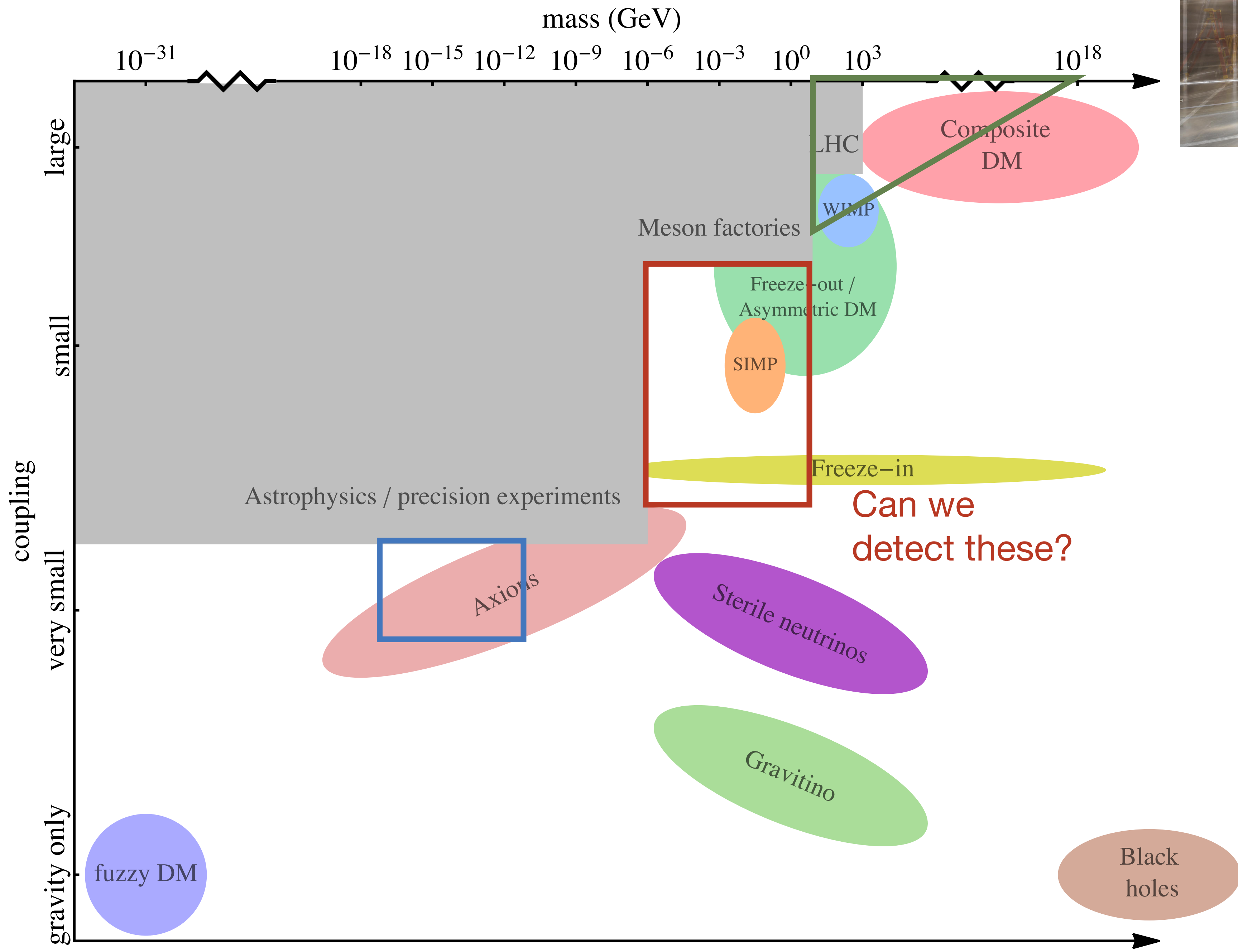
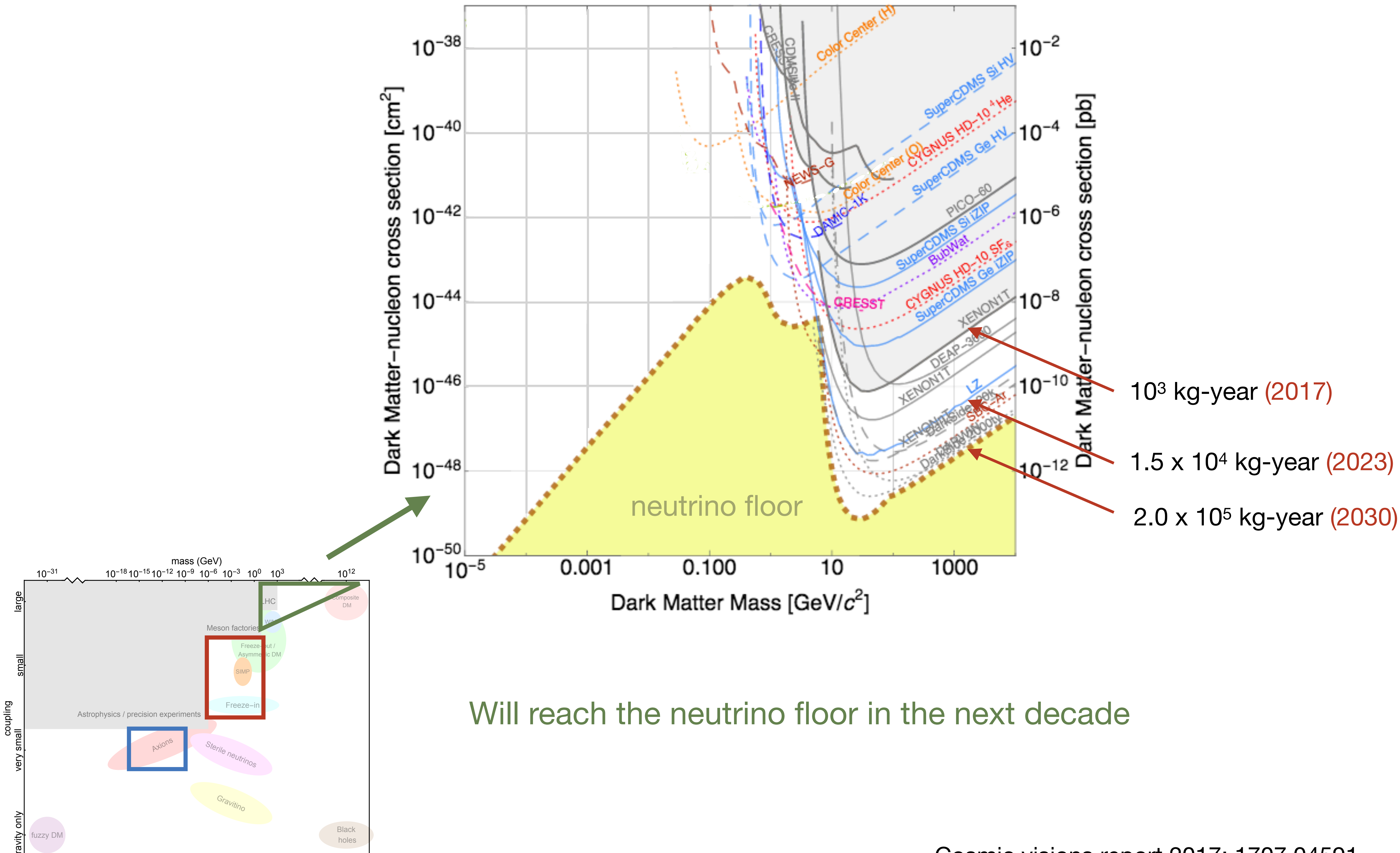


Can we detect these?

Resonant cavities etc
(ADMX, MADMAX…)

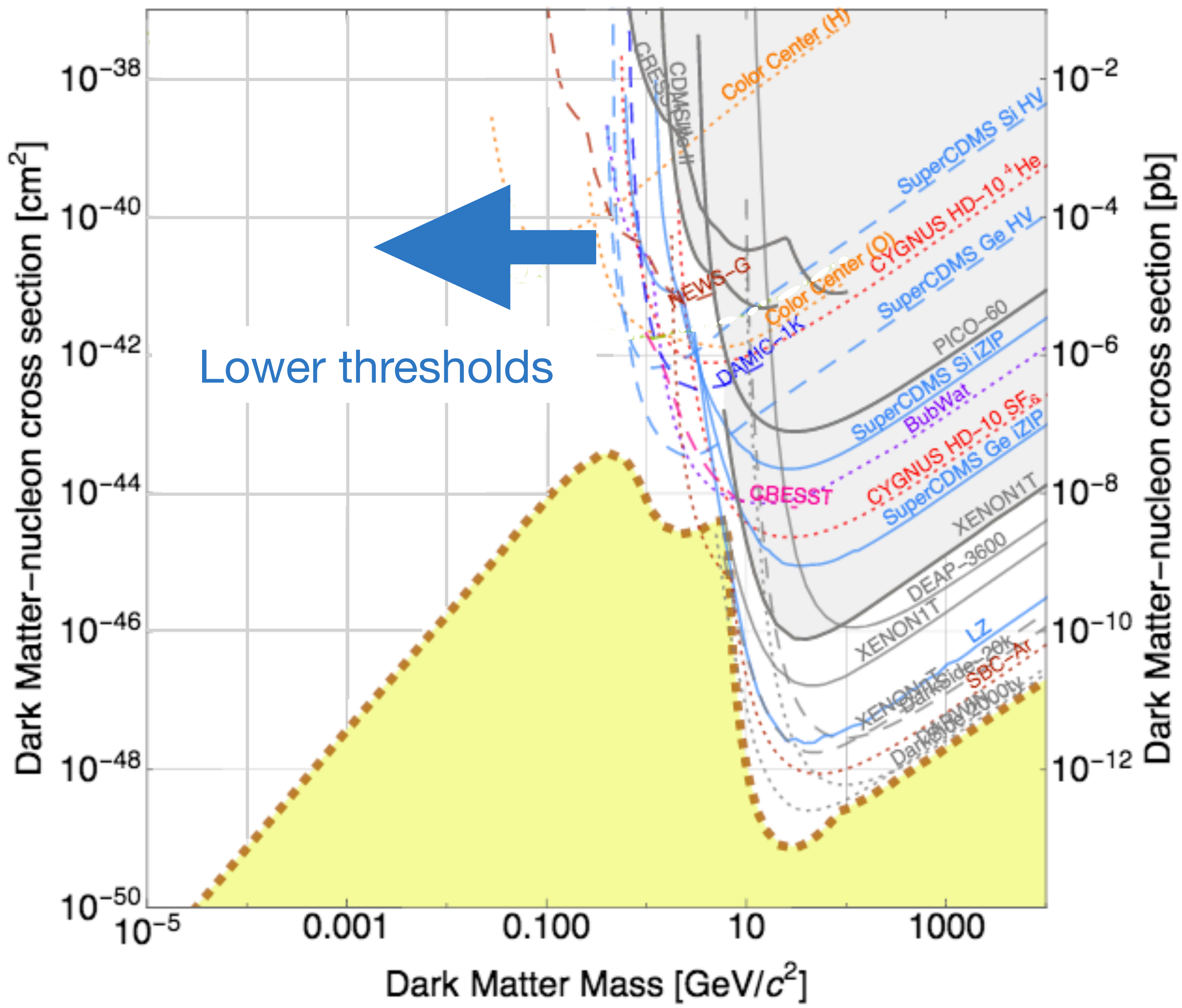
Experimental status



Will reach the neutrino floor in the next decade

Cosmic visions report 2017: 1707.04591

Low mass dark matter detection



What do we need?

Experiment:

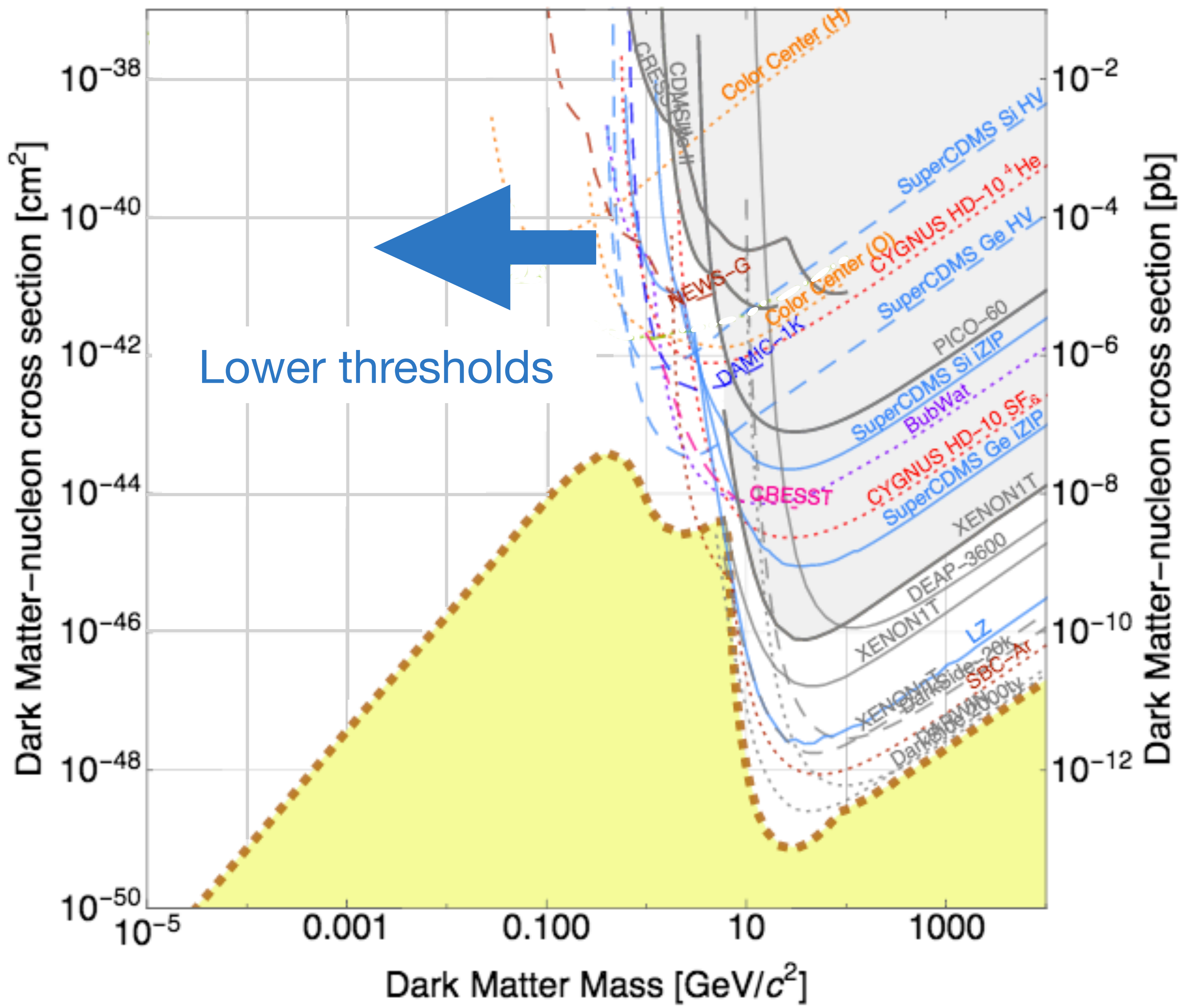
1. Low target mass materials:

$$q < 2m_\chi v_\chi, \qquad v_\chi \approx 10^{-3}$$

$$E_R = \frac{q^2}{2m_N} < 10^{-6} \times \frac{m_\chi^2}{m_N}$$

2. Ultra-sensitive calorimeters with low dark counts

Low mass dark matter detection



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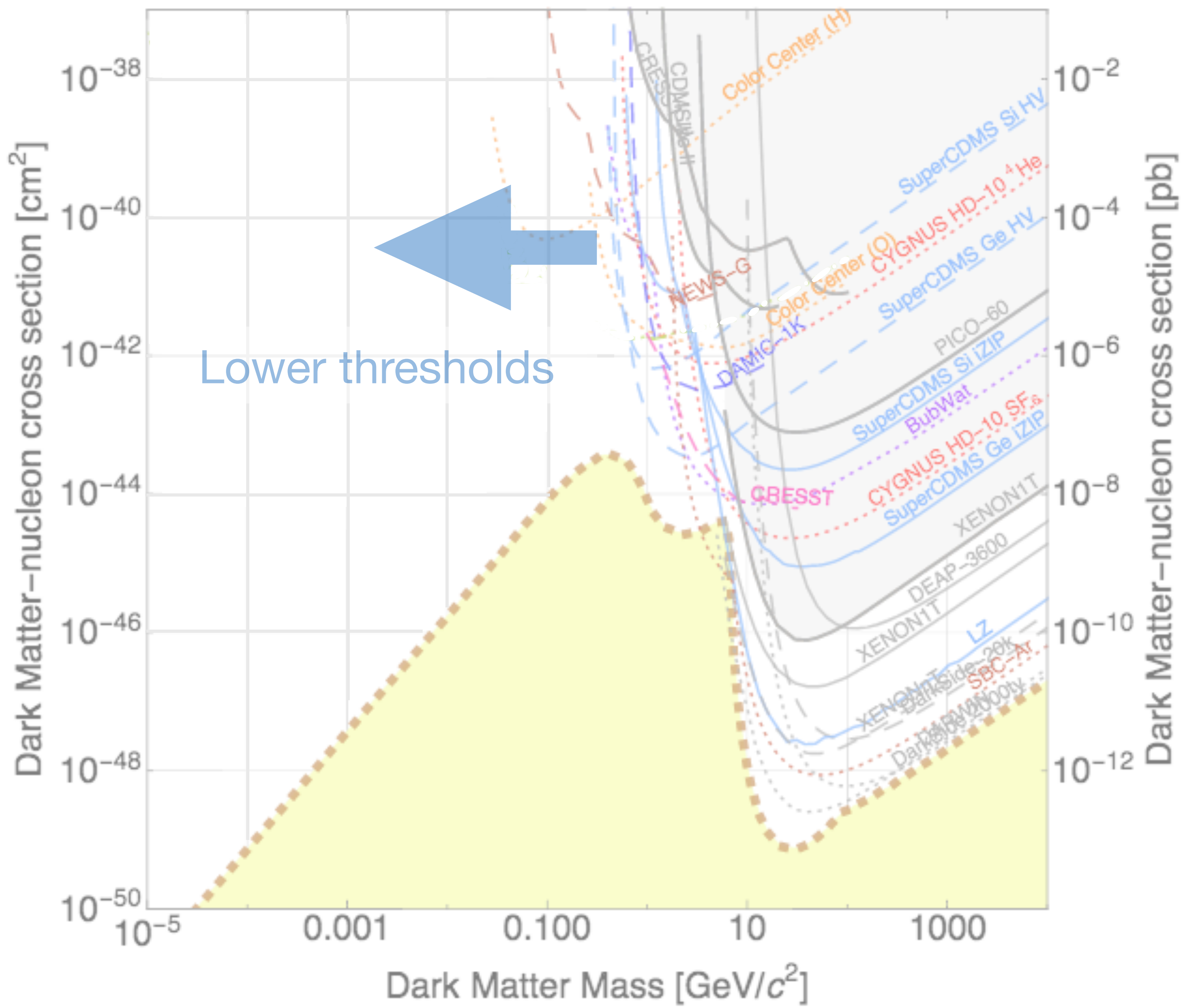
2. Ultra-sensitive calorimeters with low dark counts

Theory:

1. The mediator is important, independent set of constraints

2. Beyond “billiard ball” scattering: structure effects are critical!

Low mass dark matter detection



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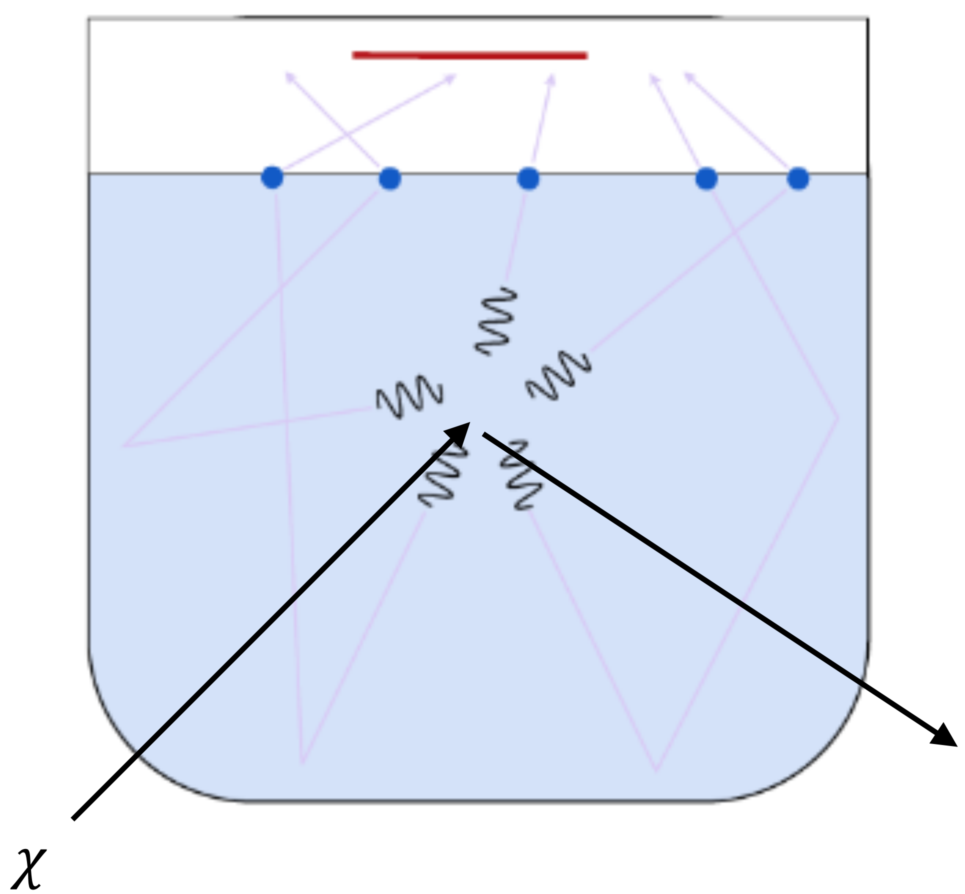
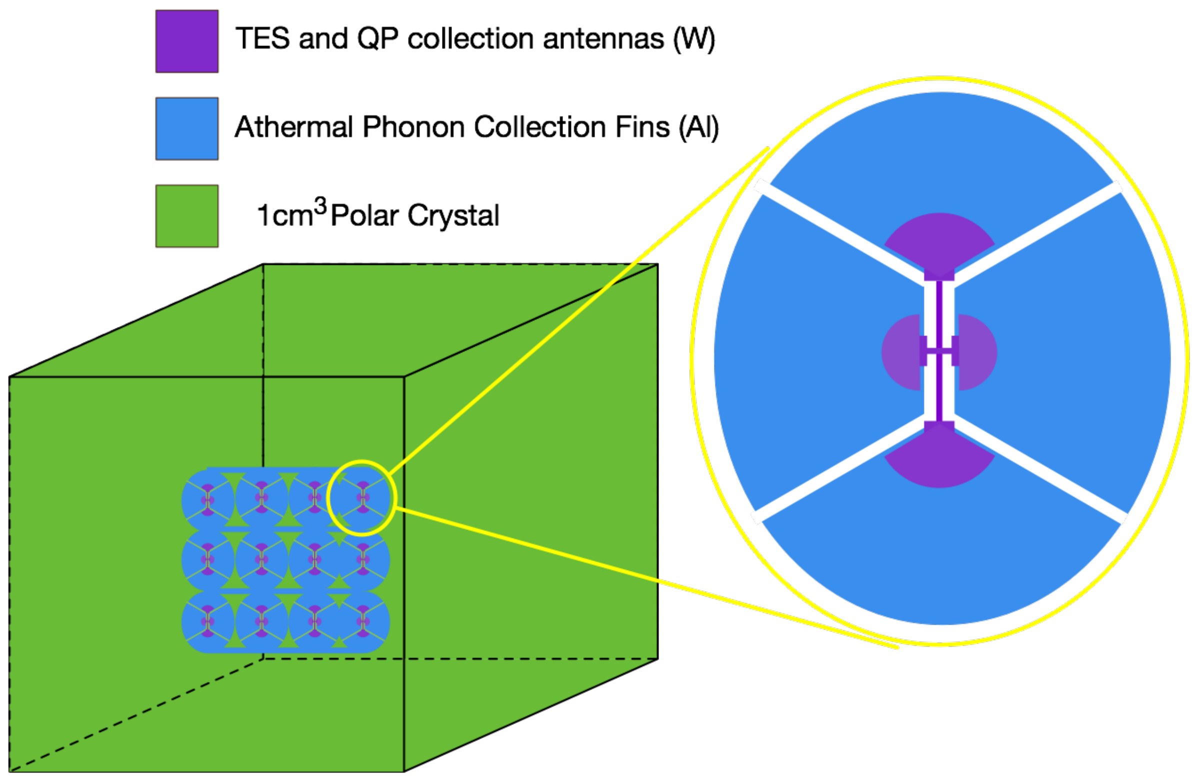
Experiments under development

HERALD experiment
(superfluid He)



W. Guo, D. McKinsey: 1302.0534

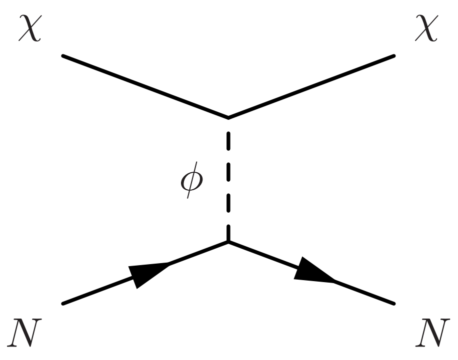
SPICE experiment
(GaAs, sapphire)



M. Pyle et. al.

Mediators matter!

<u>Example:</u> light scalar mediator with coupling to hadrons



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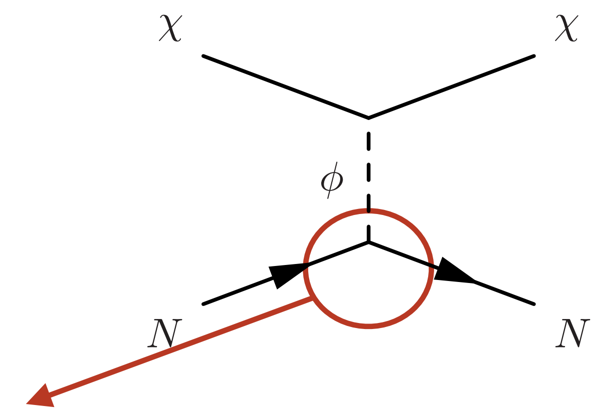

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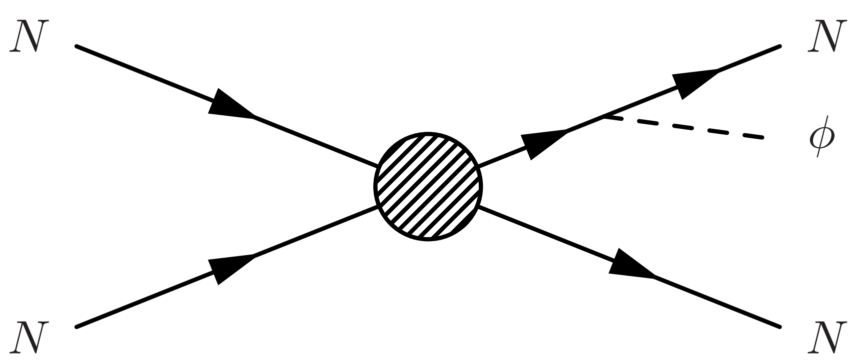

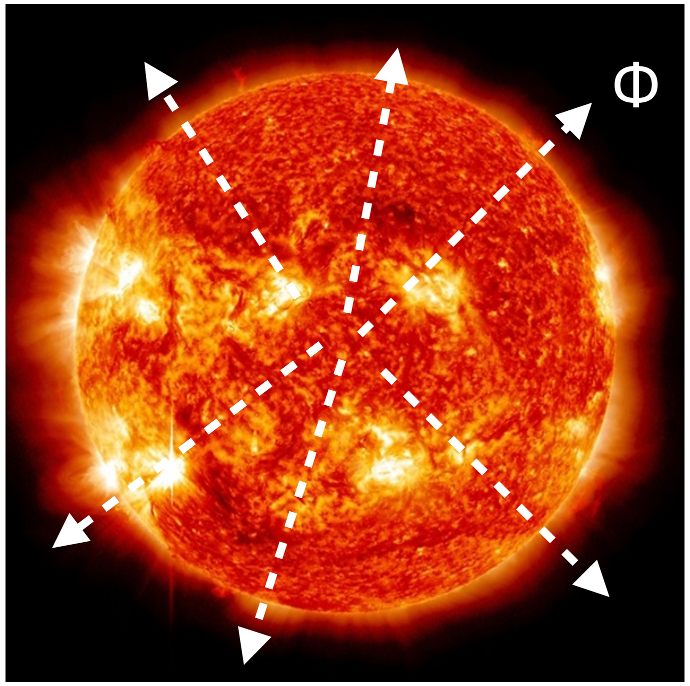

Anomalous cooling of stars

Mediators matter!

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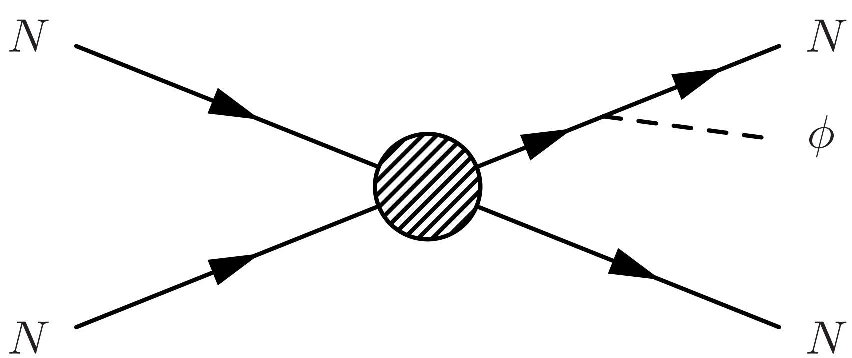
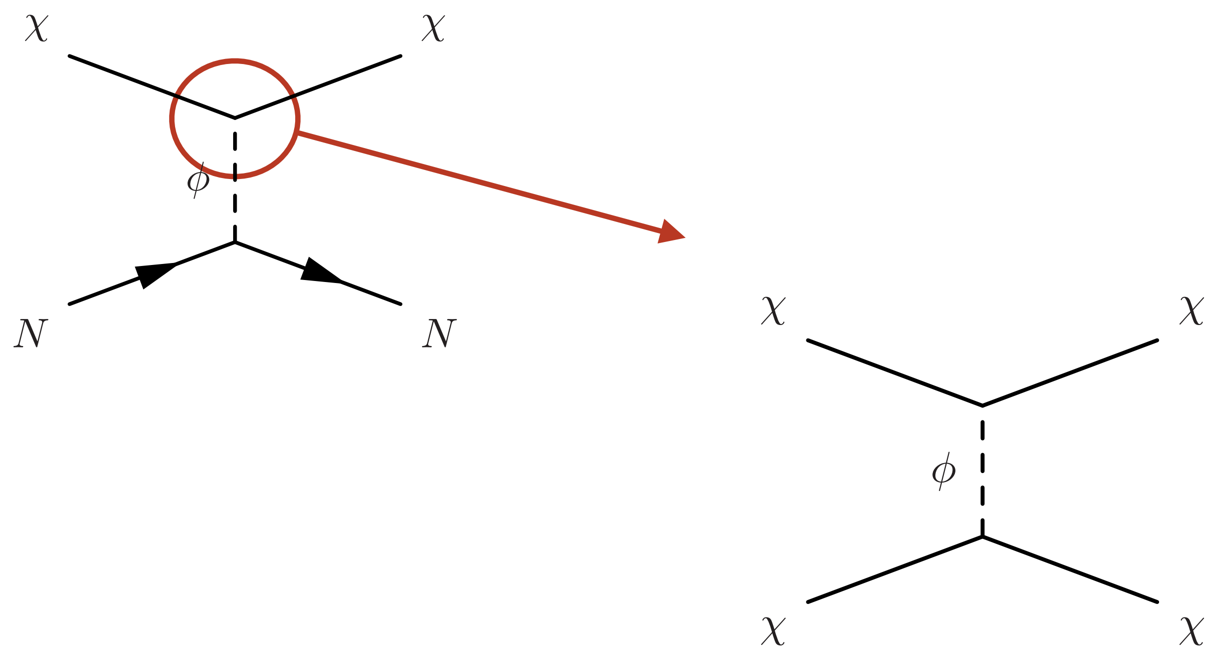
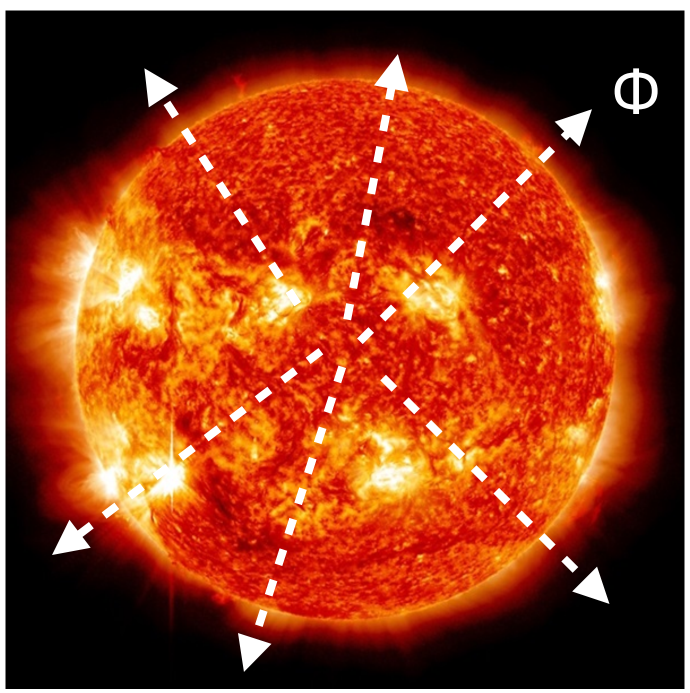




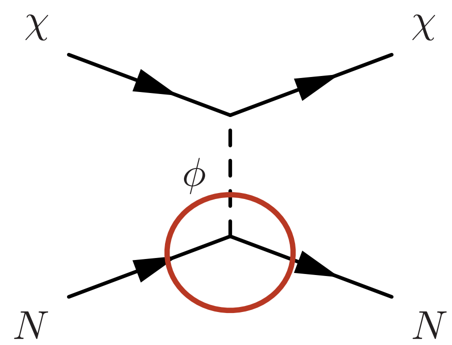


Anomalous cooling of stars

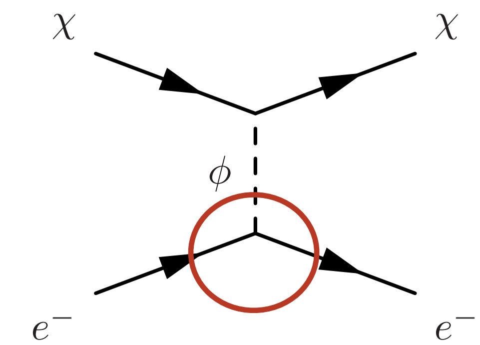
Dark Matter self-interactions
(Relaxed for subcomponent DM)

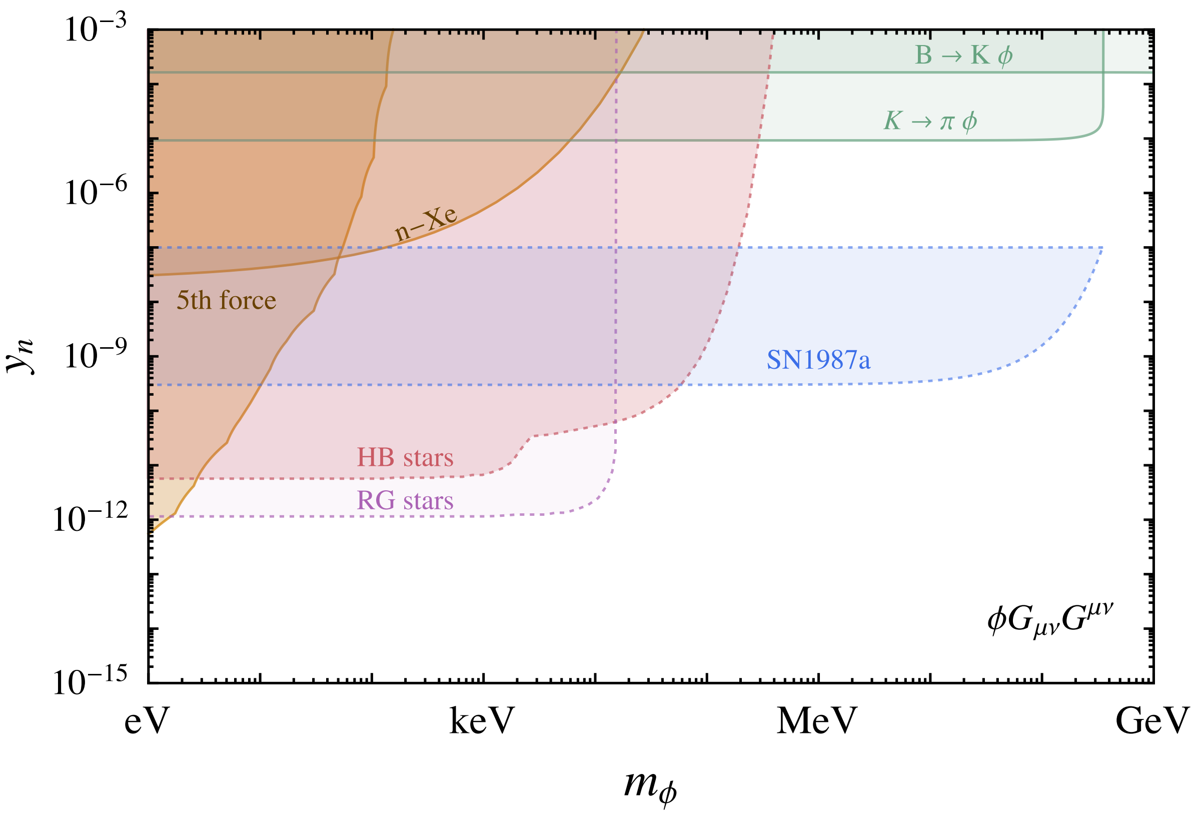
Mediators matter!

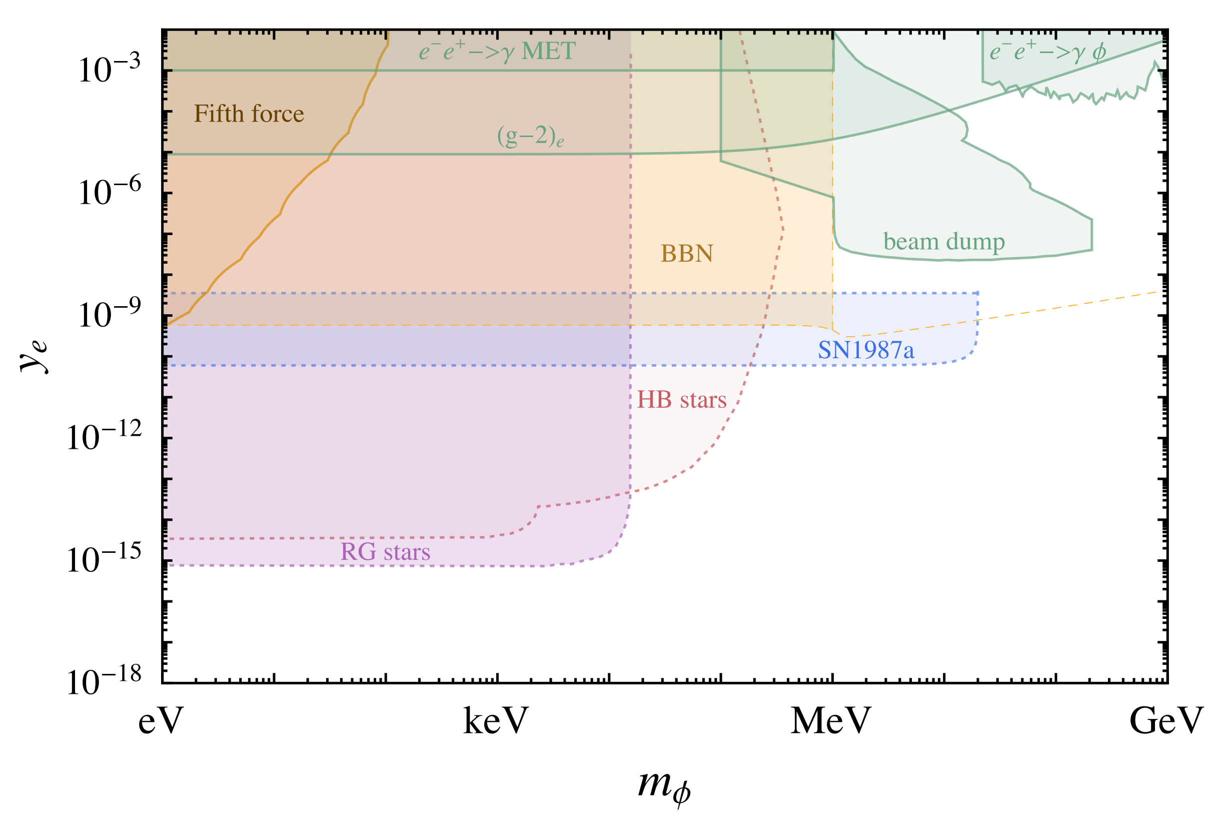
Coupling to nuclei:



Coupling to electrons:







Strong astrophysical & terrestrial constraints

SK, T. Lin, K. Zurek: 1709.07882

Mediators matter!

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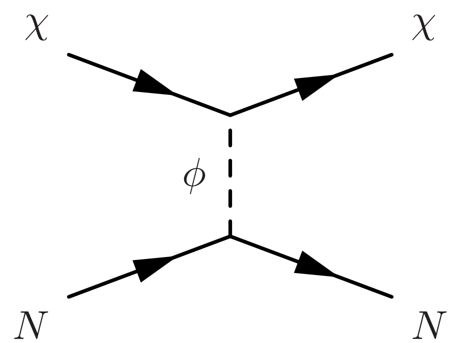

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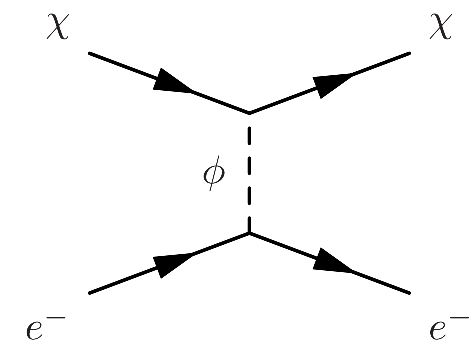
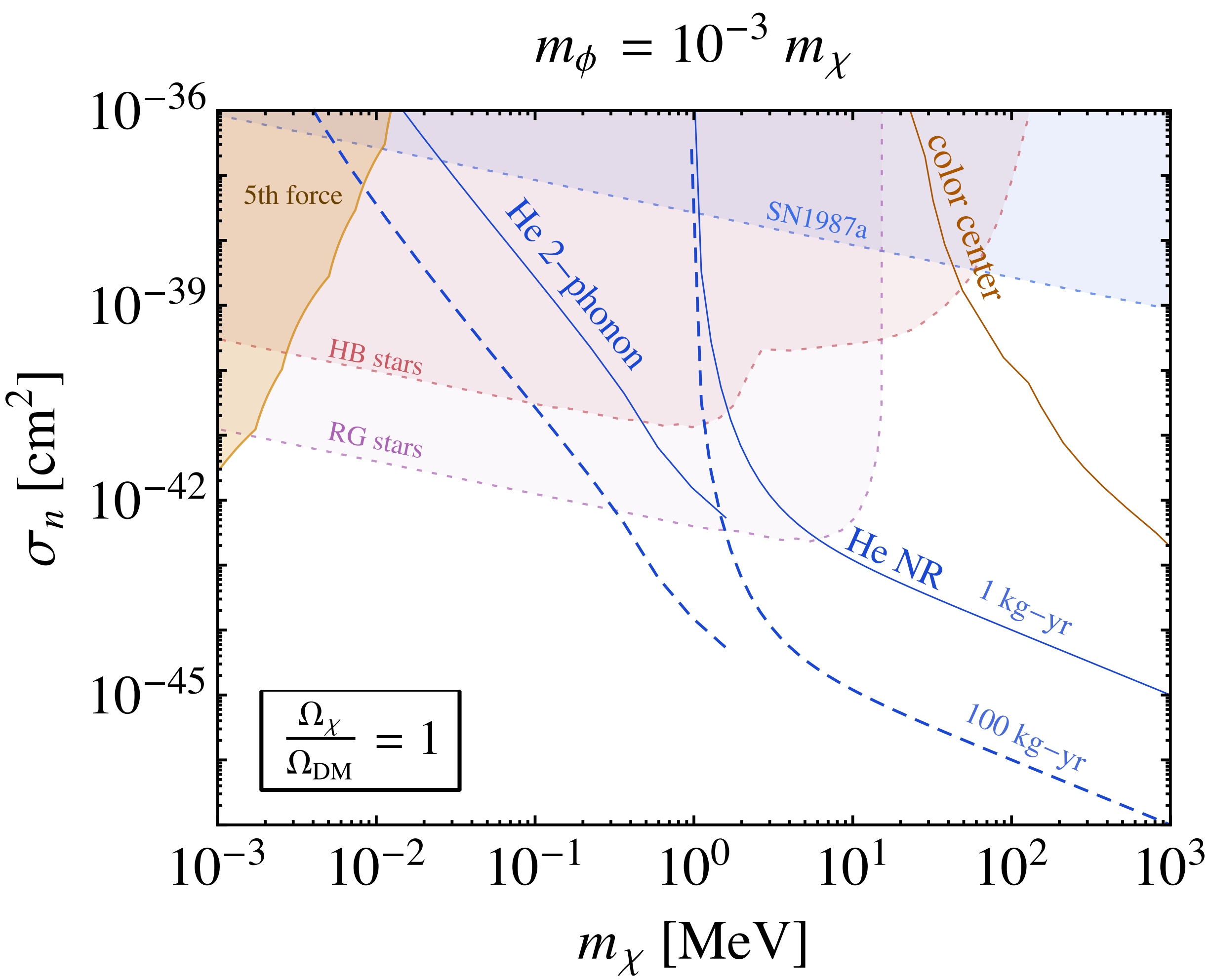
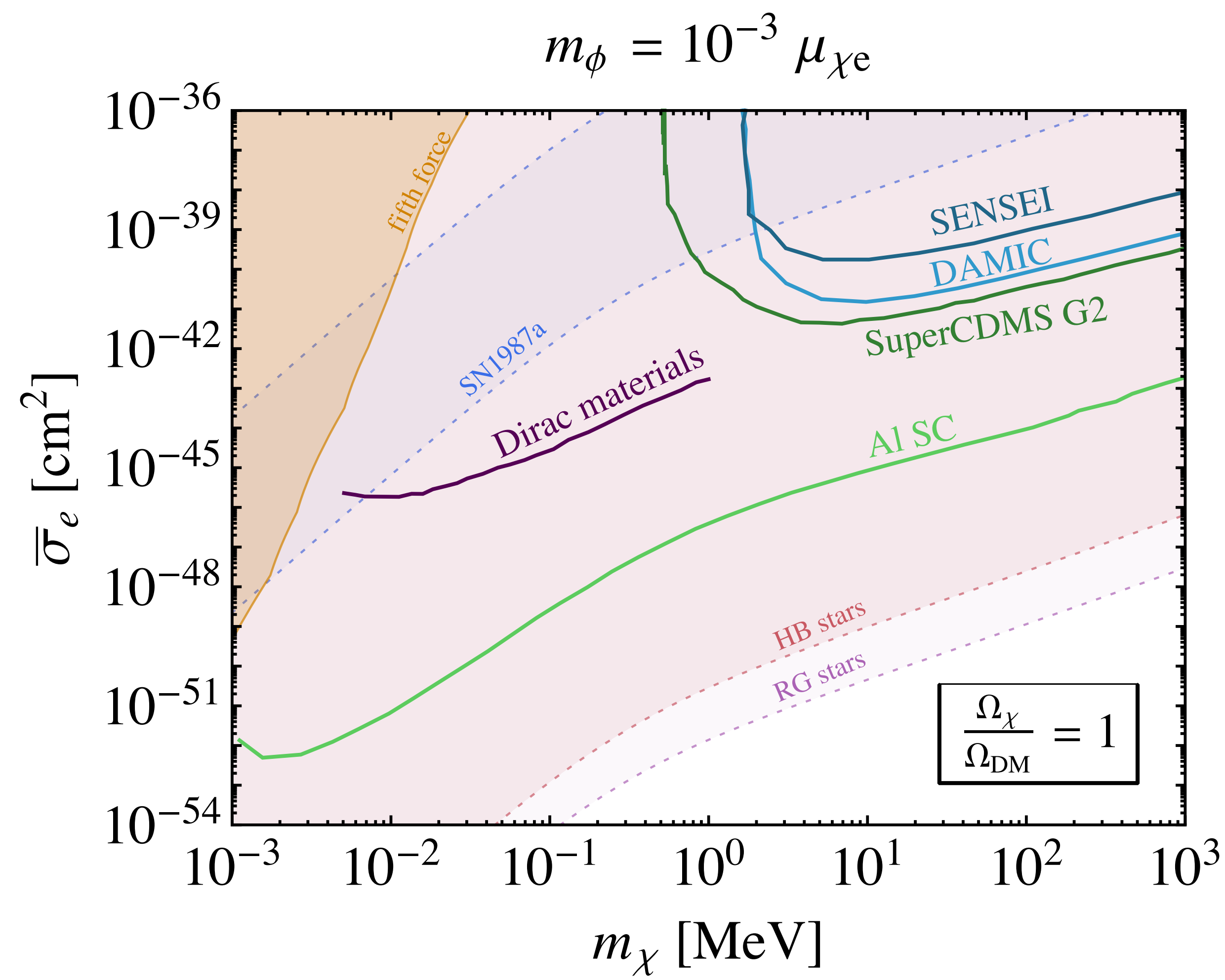
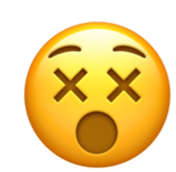






Experimentally viable if subcomponent DM

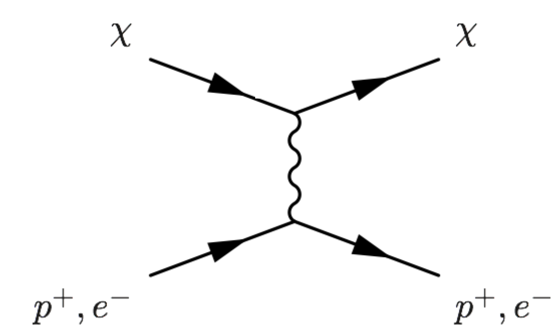


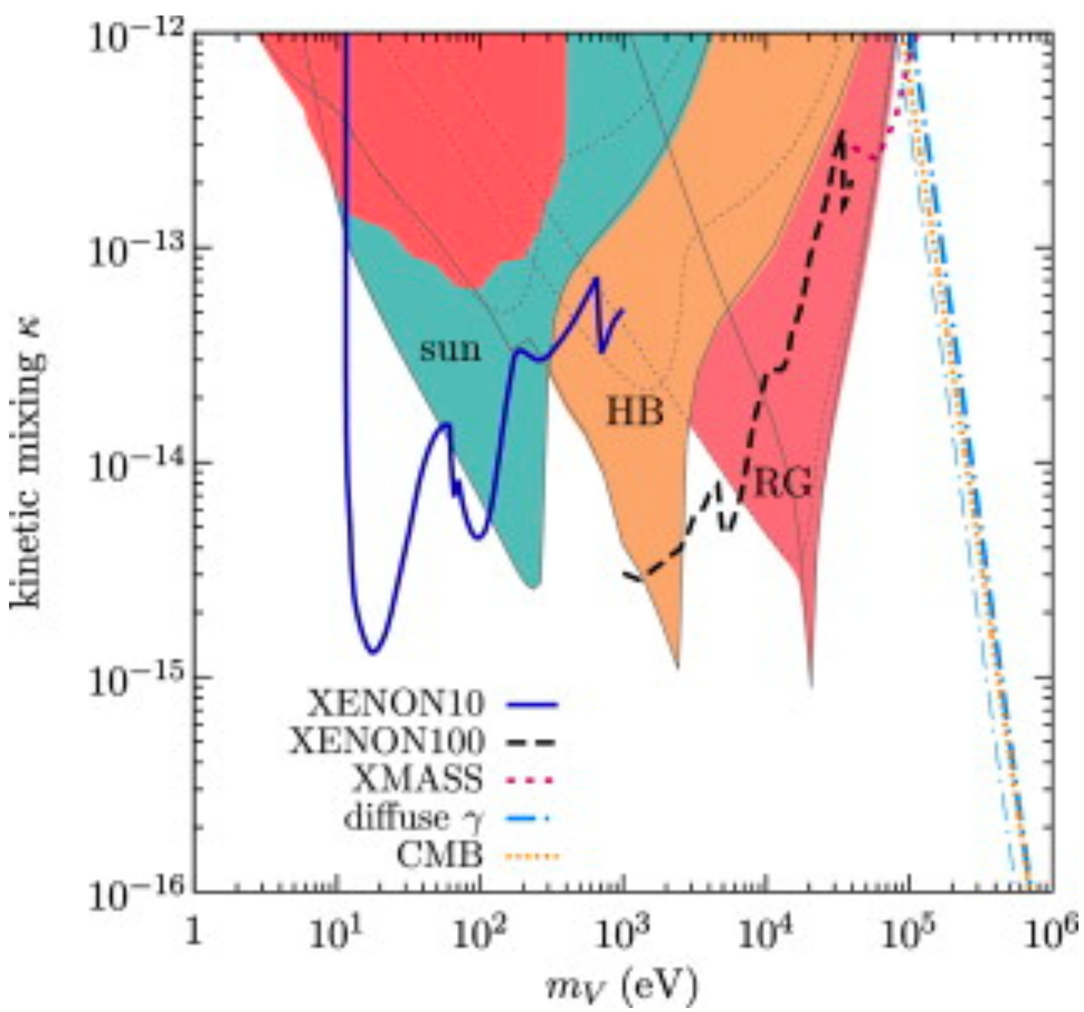
SK, T. Lin, K. Zurek: 1709.07882

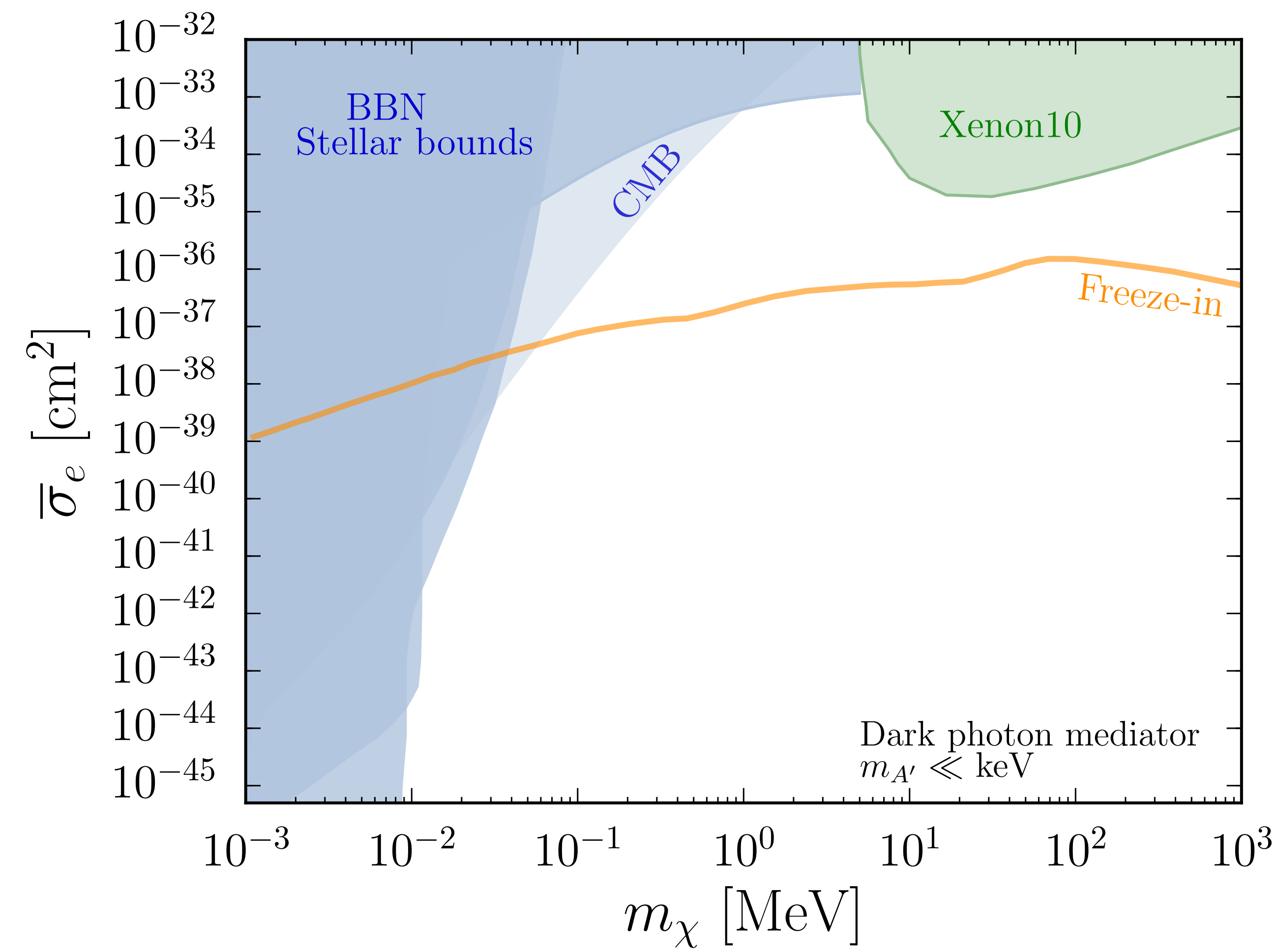
More freedom for mX > 1 MeV

An important special case

Very light dark photon mediator:







Mediator decouples from SM at low mass

No fifth force bounds (screening)

Parameter space is wide open

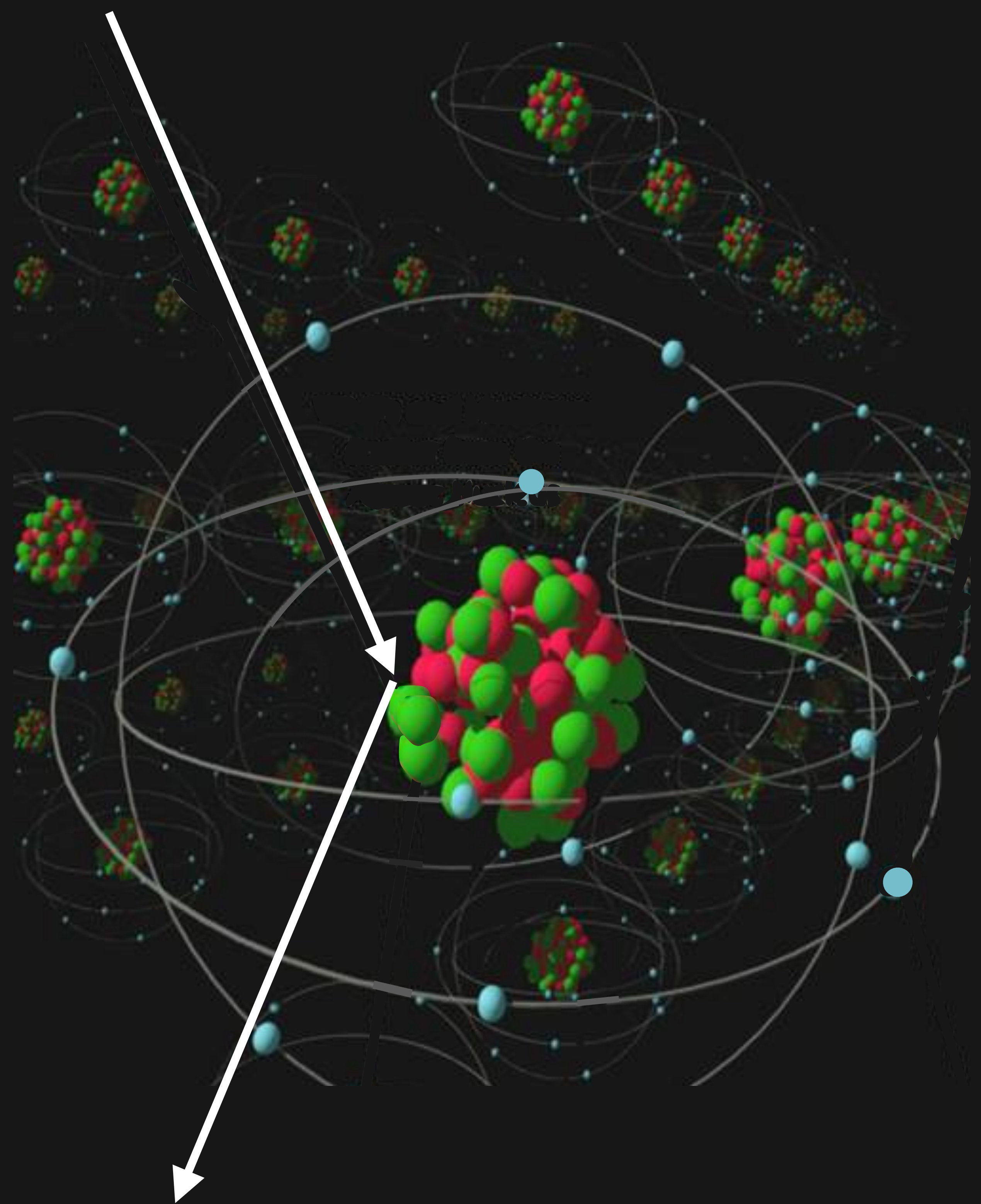
H. An, M. Pospelov, Josef Pradler, A.Ritz:1412.8378
J. Chang, R. Essig, S. McDermott: 1611.03864
…

R. Essig et. al.: 1509.01598
…

Beyond billiard ball scattering

Attempt to "match" target mass with dark matter mass

- $m_\chi > 1$ GeV
  → nuclear recoils
- 1 MeV < m_χ < 1 GeV
  → electron recoils
- m_χ < 1 MeV
  → $q \approx m_\chi v_\chi$ < keV ~ nm^{-1}



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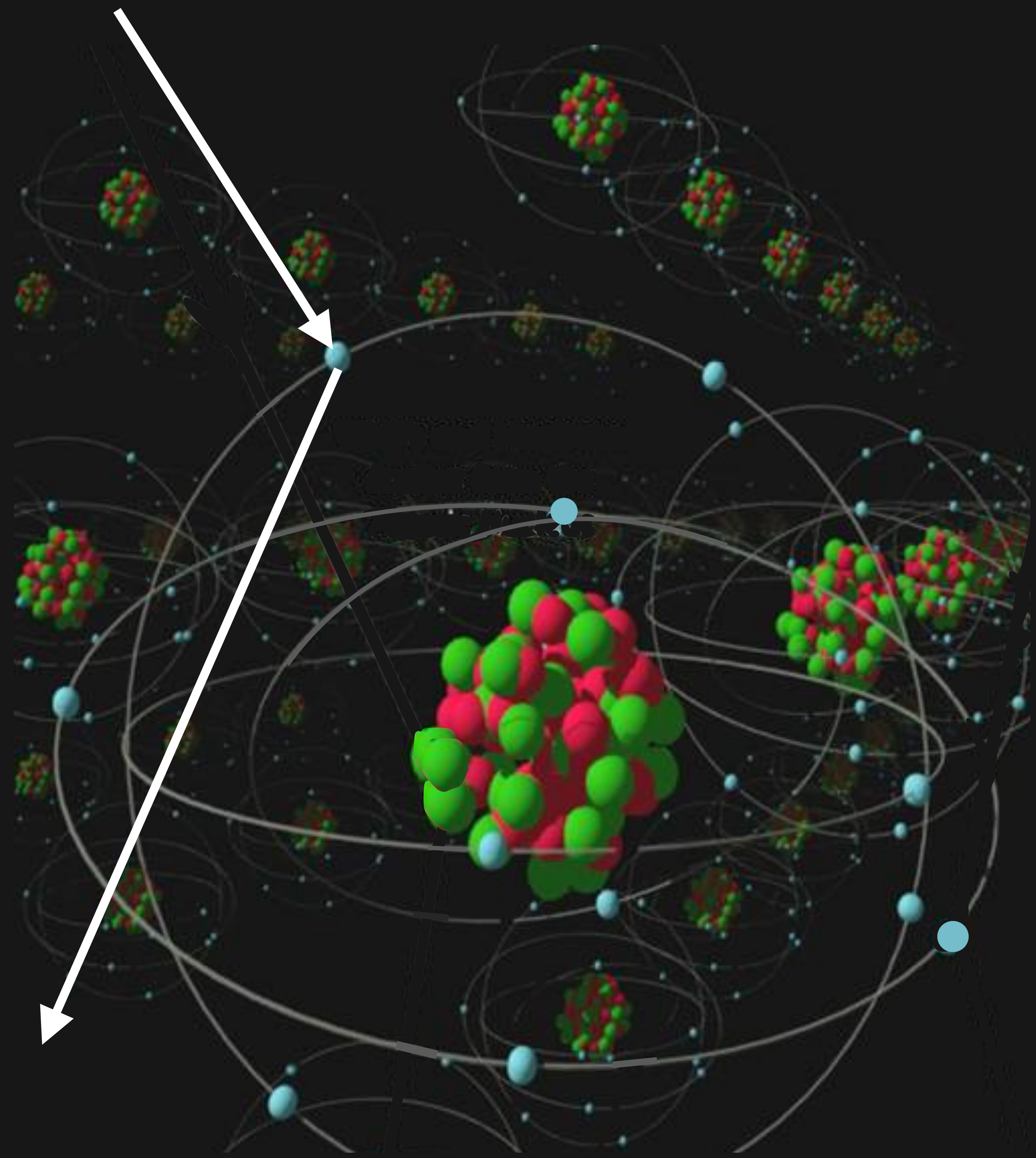

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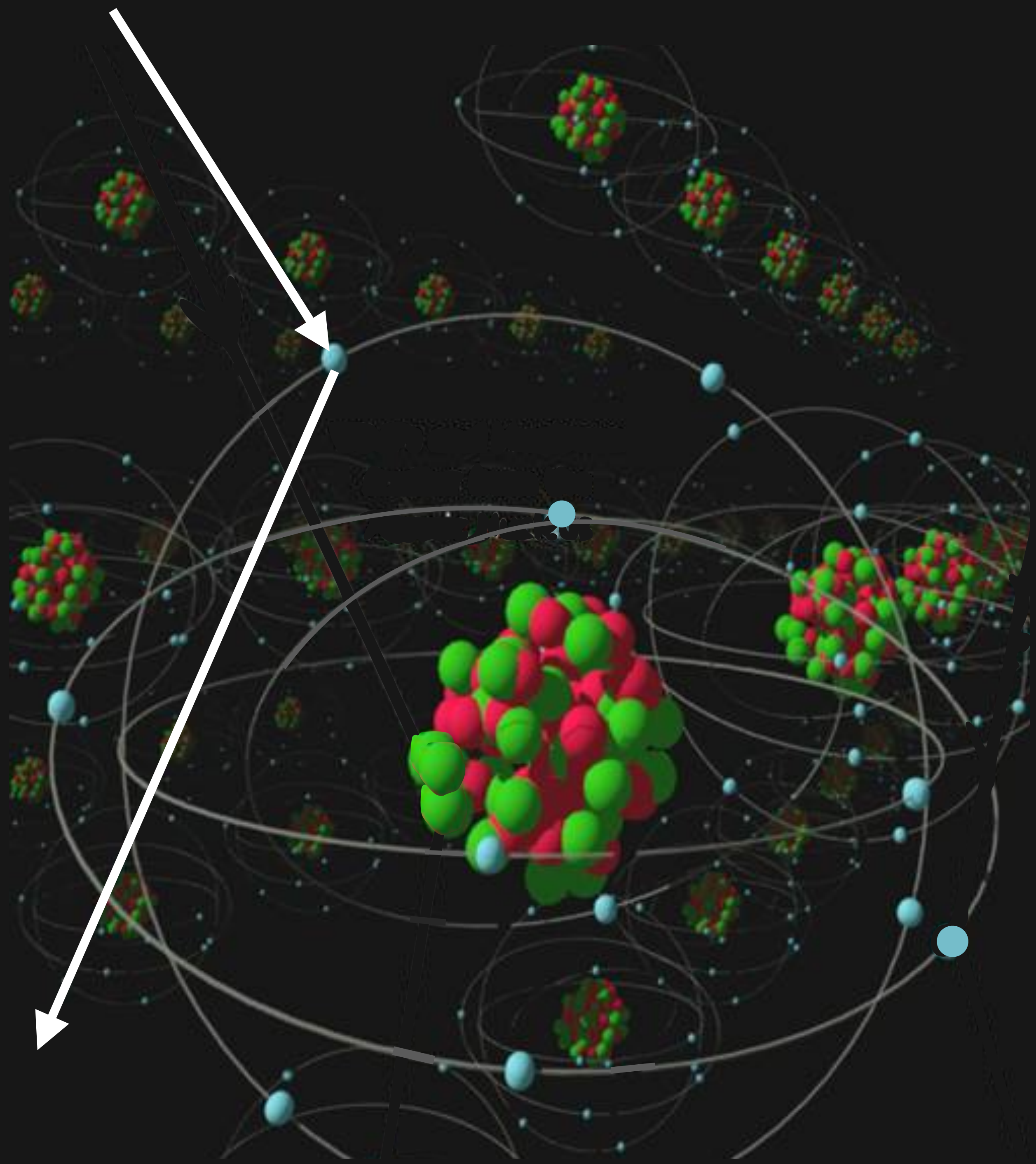

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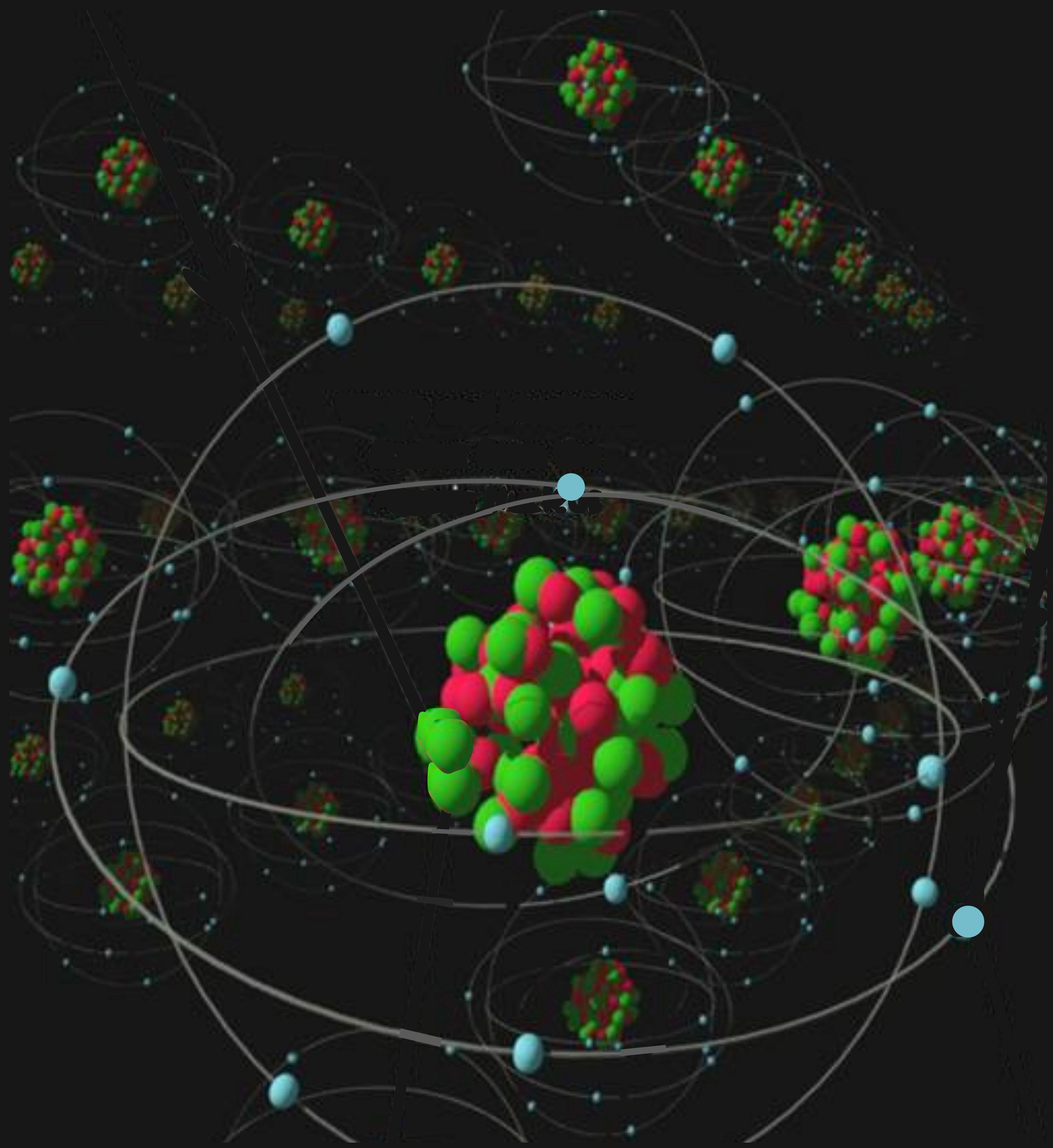

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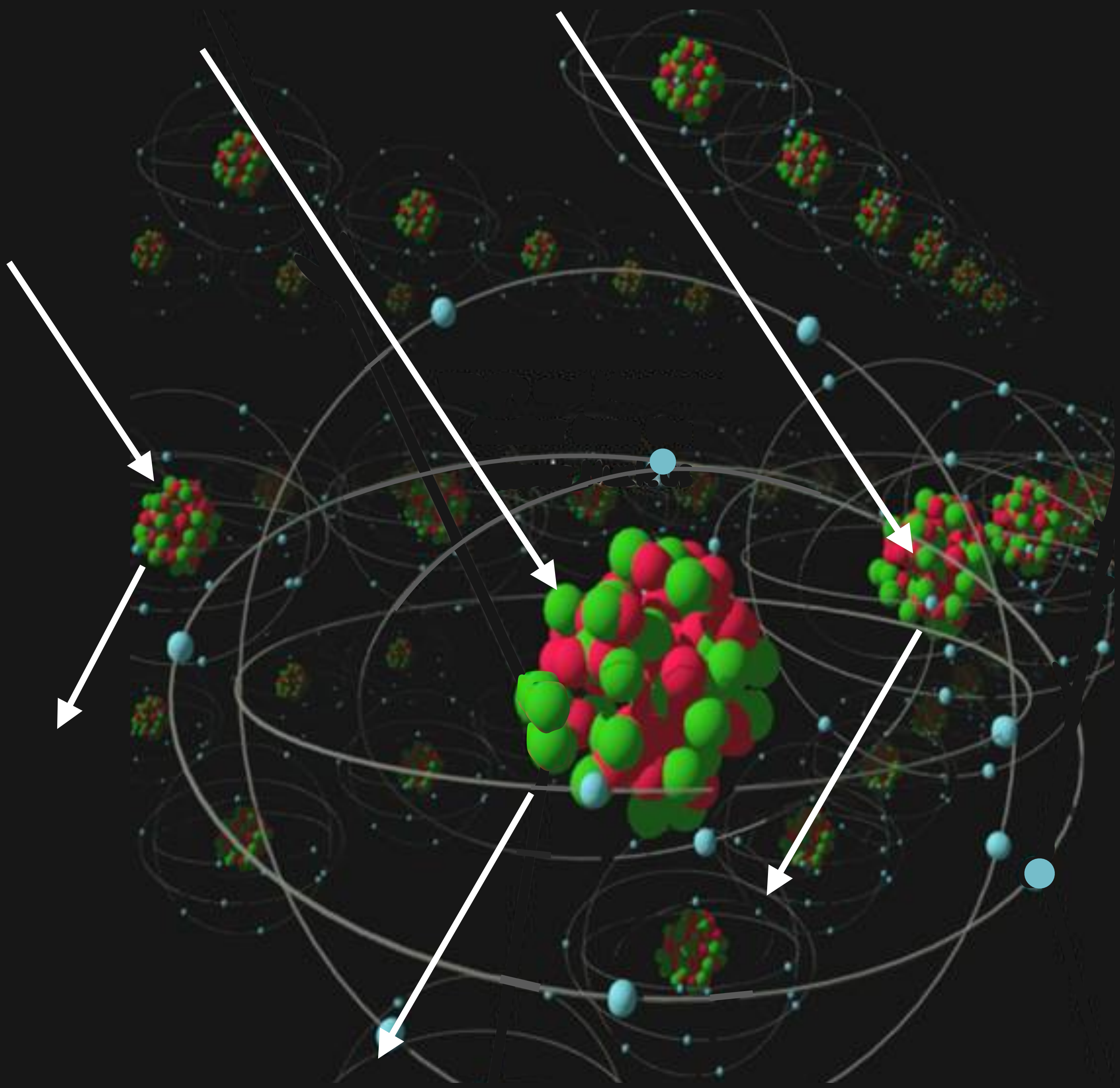
↓

Scatter directly off phonons

➔ strong material dependence!



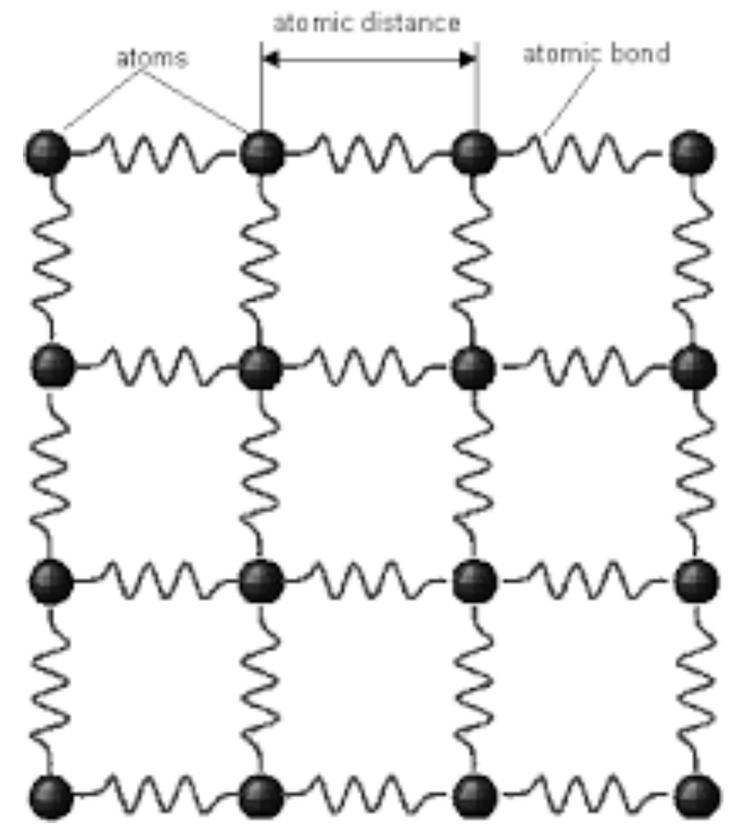
Phonon effective theory

Periodic potential (Hooke's law)

$$\mathcal{V} = \mathcal{V}^{(0)} + \sum_{\mathbf{l},j} \boldsymbol{\mathcal{V}}^{(1)}_{\mathbf{l},j} \cdot \mathbf{u}_{j,\mathbf{l}} + \frac{1}{2} \sum_{\mathbf{l},\mathbf{l}',j,j'} \mathbf{u}_{j,\mathbf{l}} \cdot \boldsymbol{\mathcal{V}}^{(2)}_{\mathbf{l},j,\mathbf{l}',j'} \cdot \mathbf{u}_{j',\mathbf{l}'} + \cdots$$

Atoms in unit cell

Lattice sites



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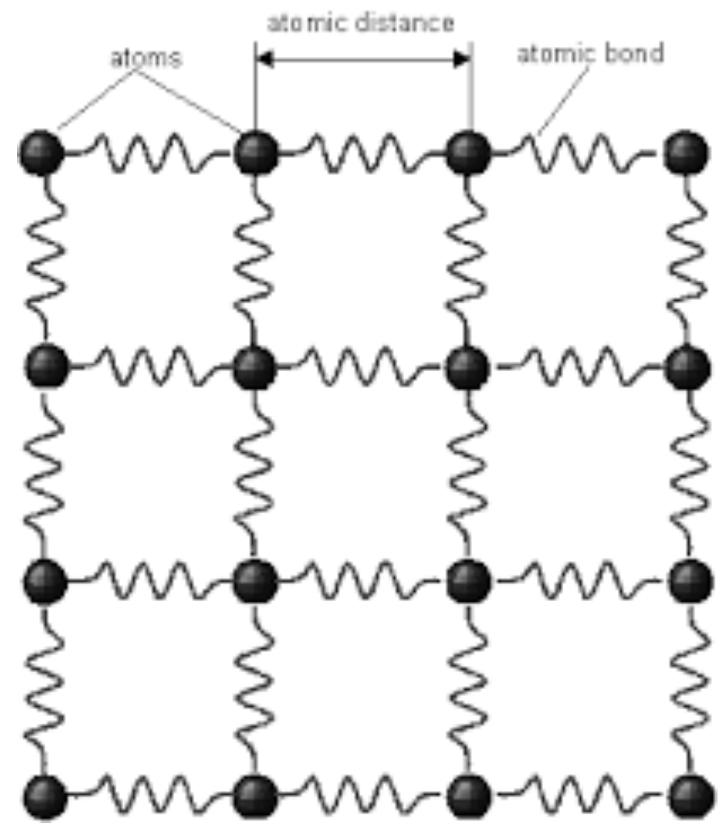

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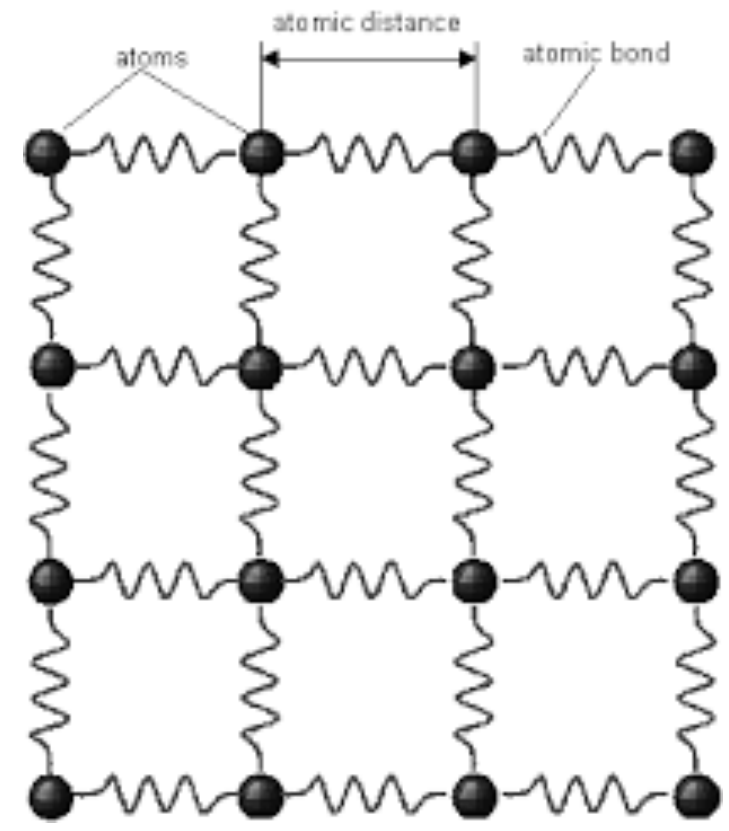

Displacement operator:

$$\mathbf{u}_{j,\mathbf{l}}(t) = \sum_{\nu}^{3n} \sum_{\mathbf{q}} \sqrt{\frac{1}{2N m_j \omega_{\nu,\mathbf{q}}}} \left(\mathbf{e}_{\nu,j,\mathbf{q}} \hat{a}_{\nu,\mathbf{q}} e^{i\mathbf{q}\cdot(\mathbf{l}+\mathbf{r}_j^0) - i\omega_{\nu,\mathbf{q}} t} + \mathbf{e}^*_{\nu,j,\mathbf{q}} \hat{a}^\dagger_{\nu,\mathbf{q}} e^{-i\mathbf{q}\cdot(\mathbf{l}+\mathbf{r}_j^0) + i\omega_{\nu,\mathbf{q}} t} \right)$$

ν : phonon branches

$\mathbf{q}$: momentum over Brioullin zone

j : atom in primitive cell

In the harmonic approximation, just quantize as harmonic oscillator

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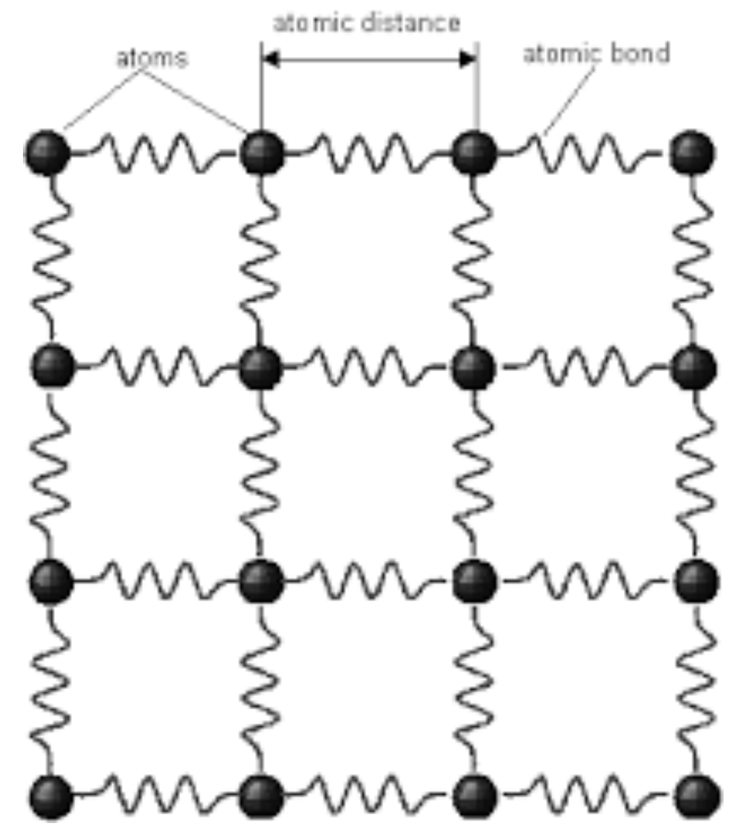

Displacement operator:

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creation/annihilation operators

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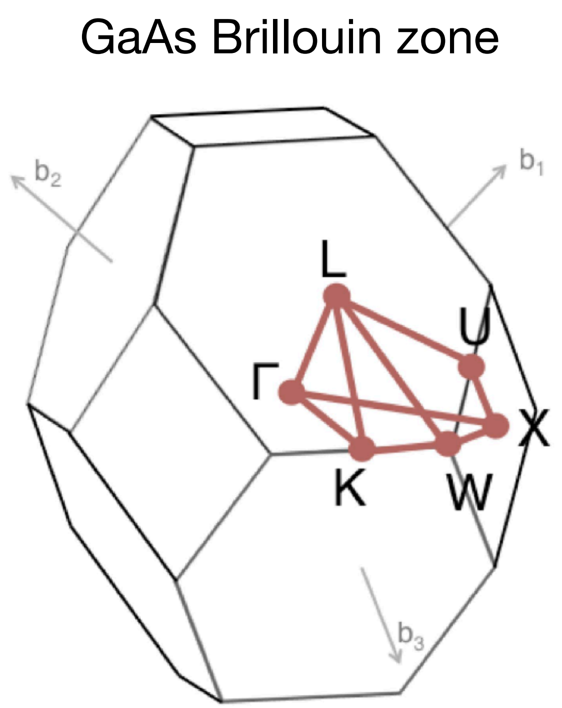
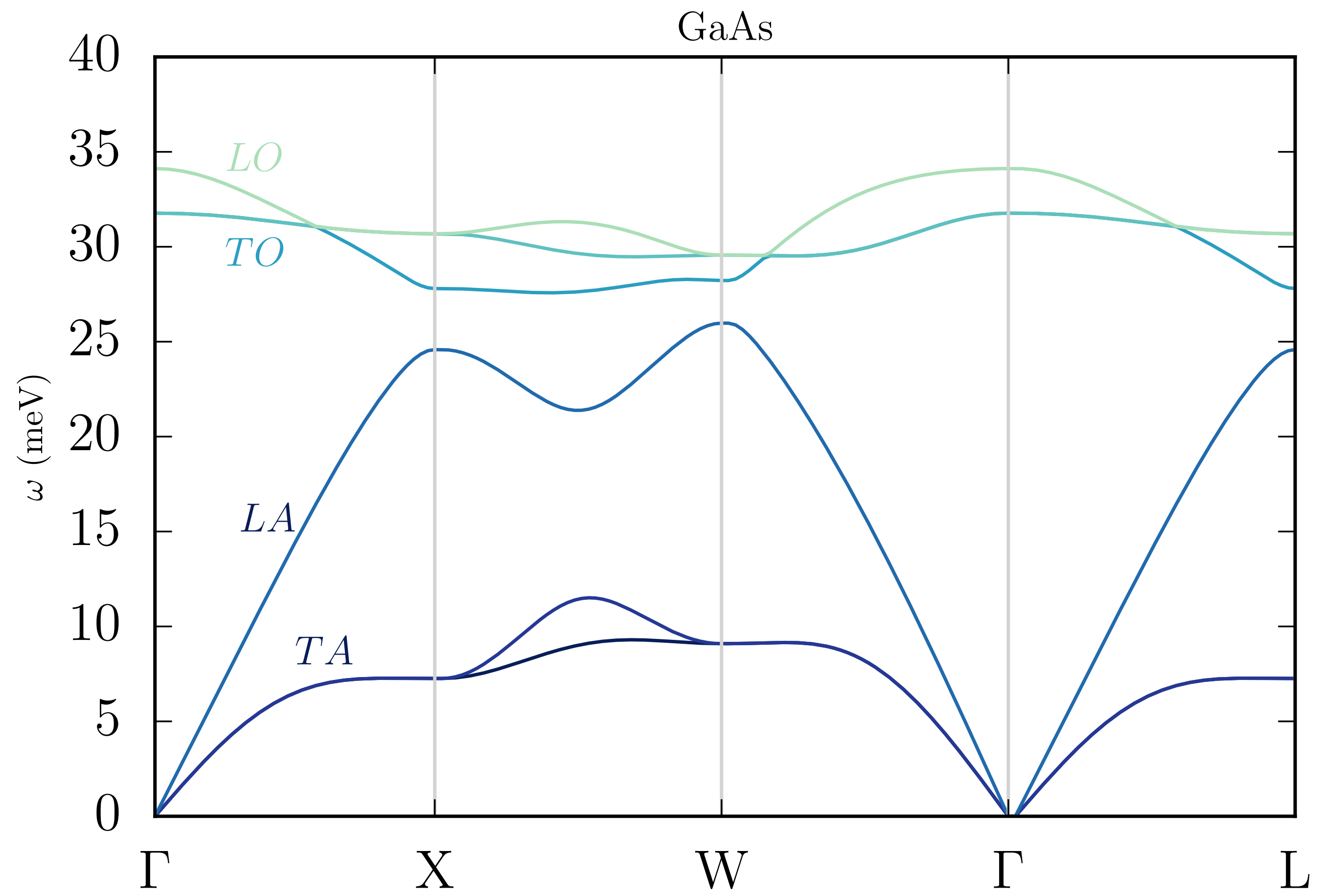
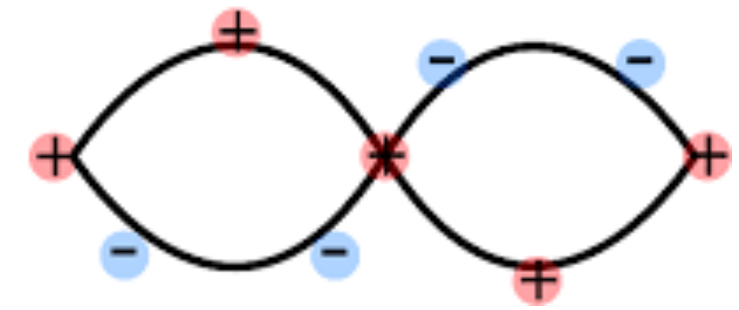
Types of phonons



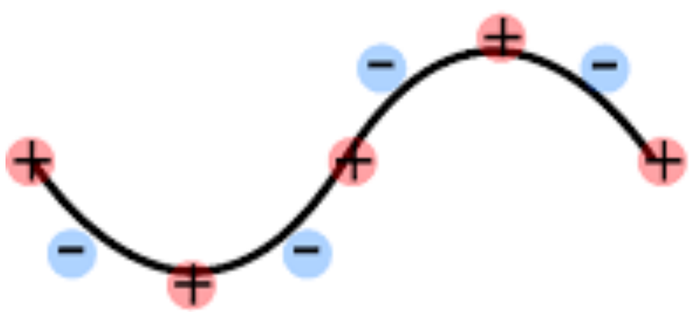


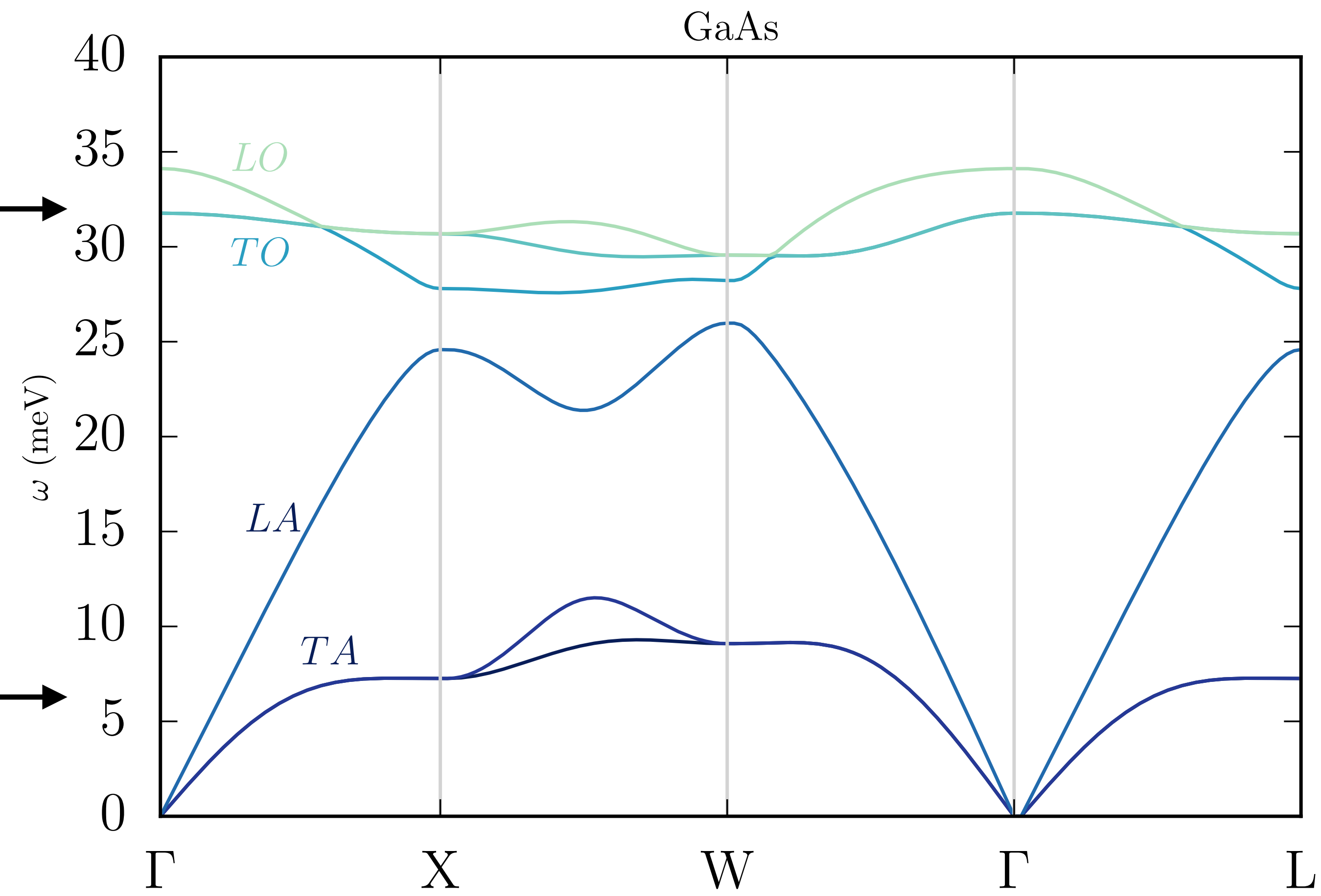
Types of phonons

Optical phonons



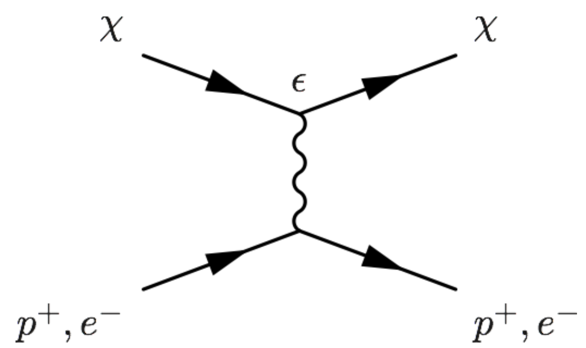
Acoustic phonons

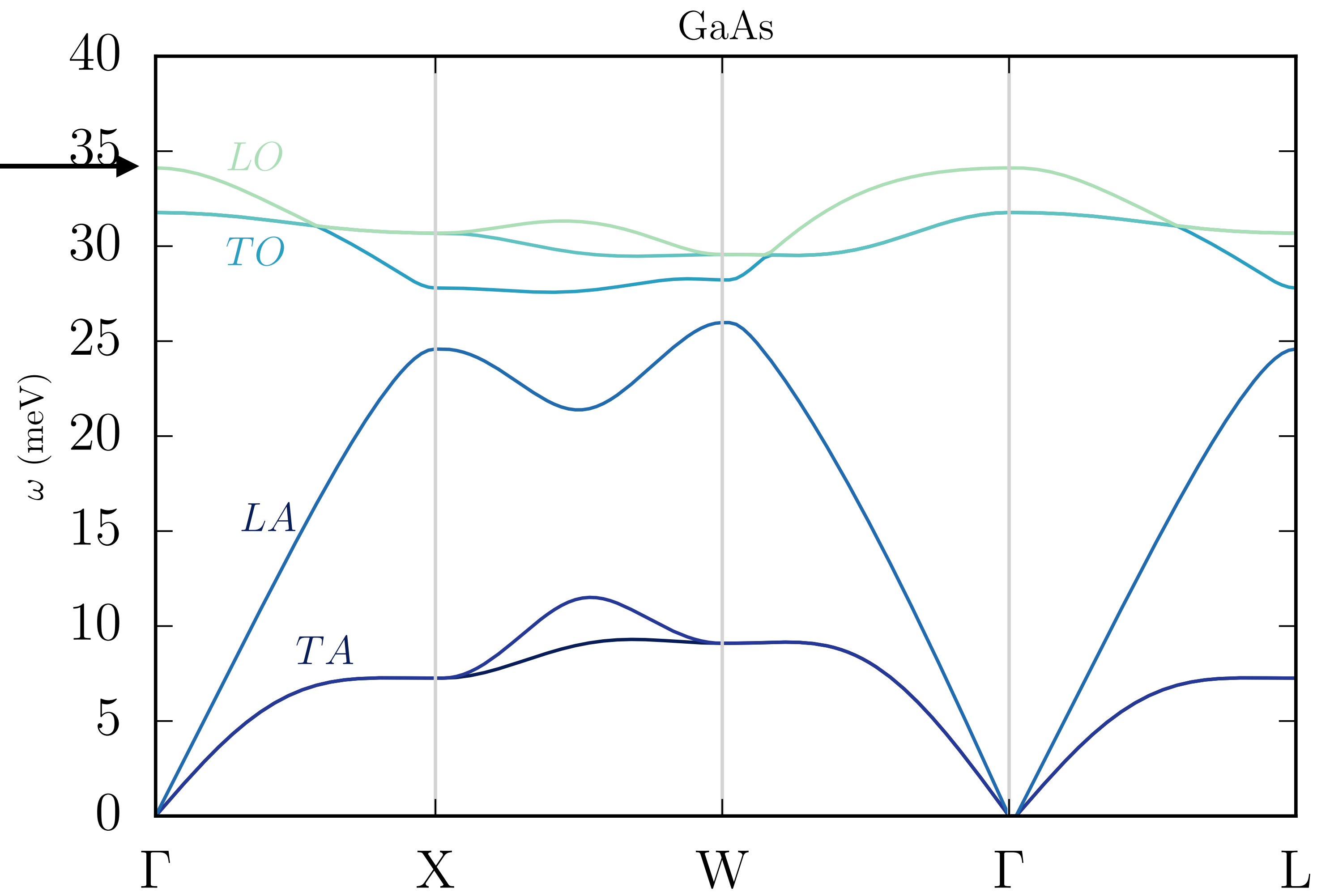


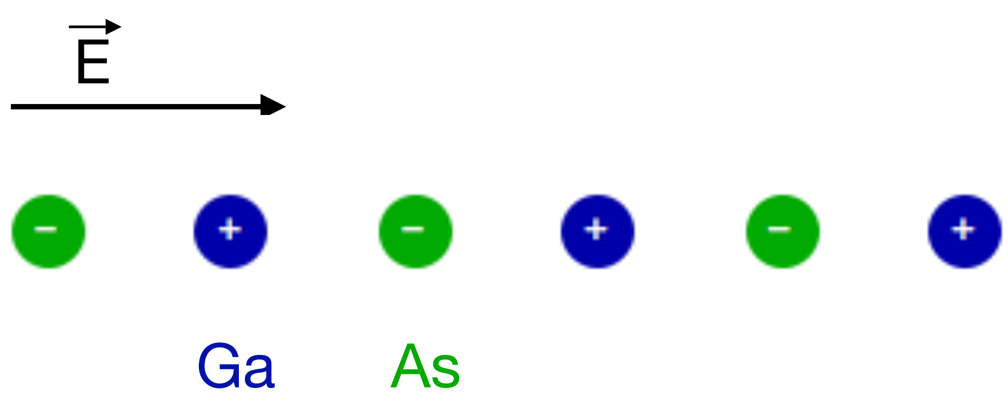


Dark Matter coupling to phonons

Coupling to charge







(in a “Polar” material)

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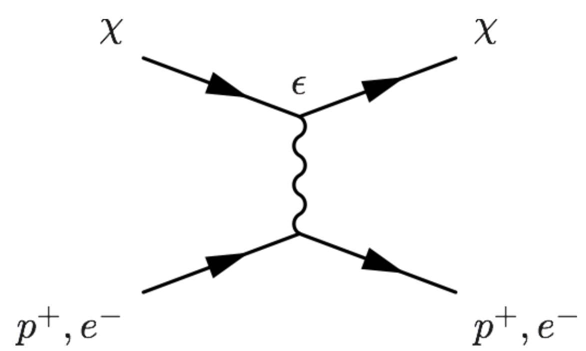

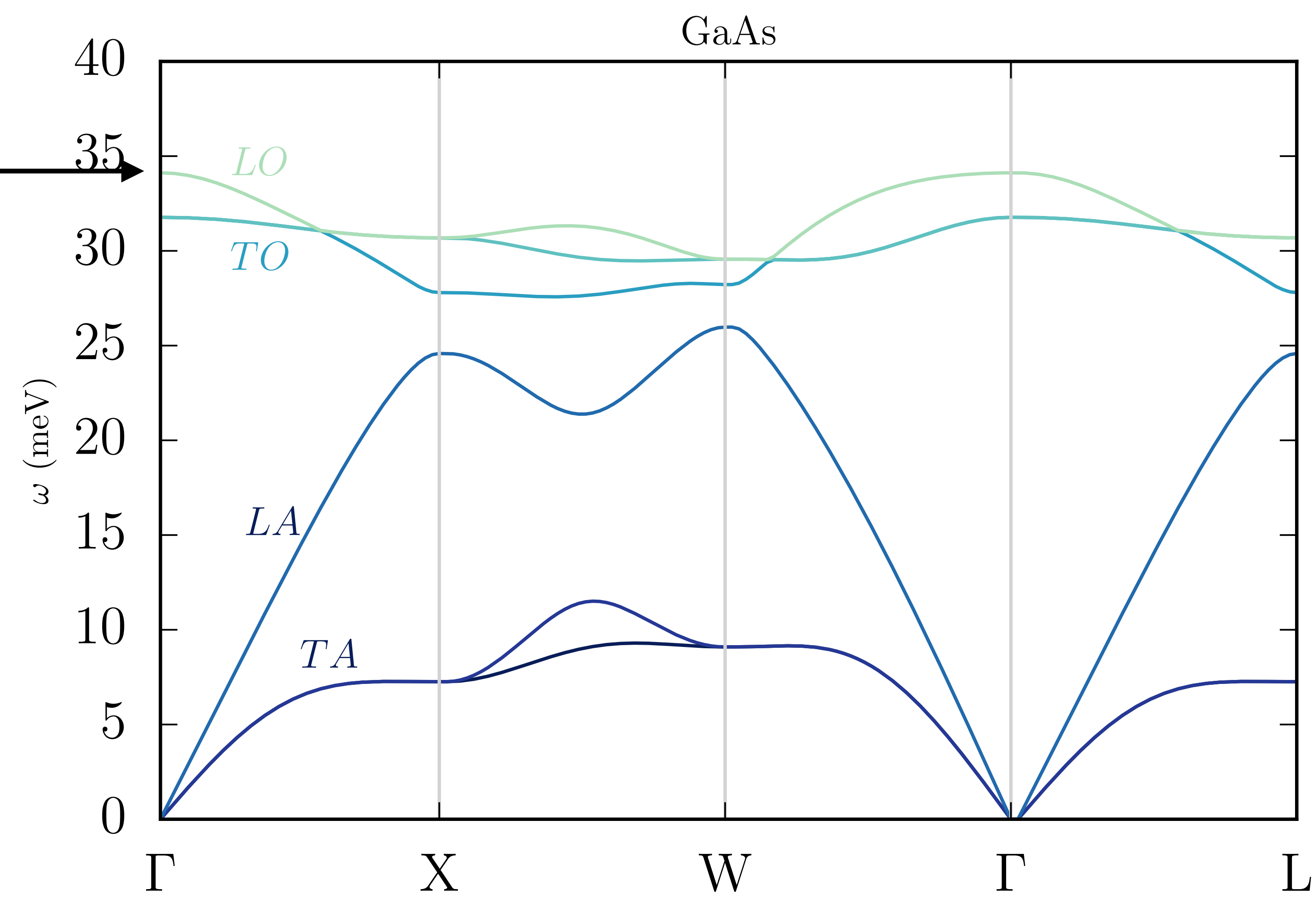

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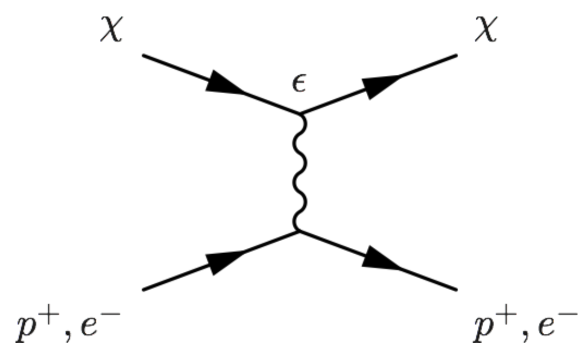
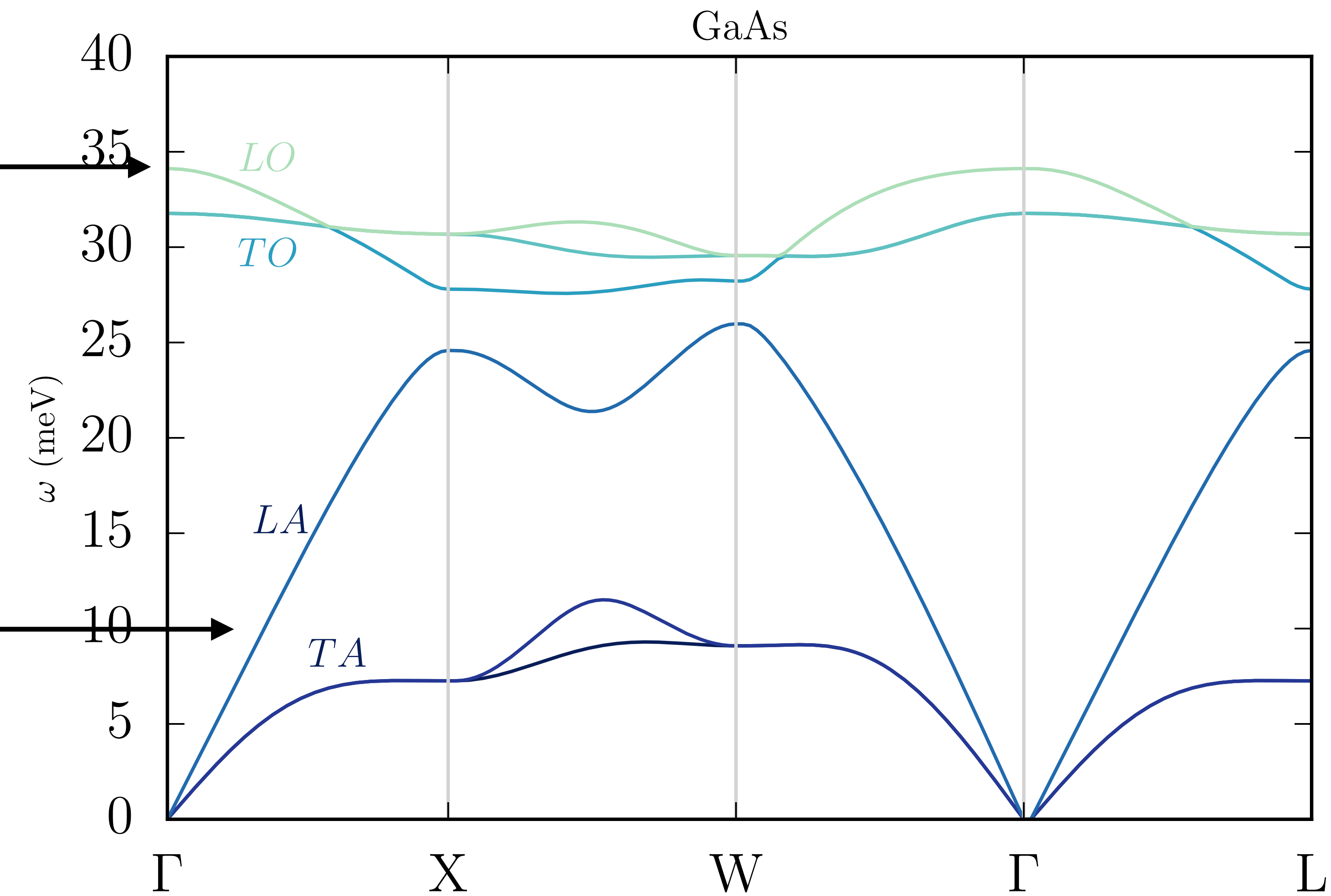
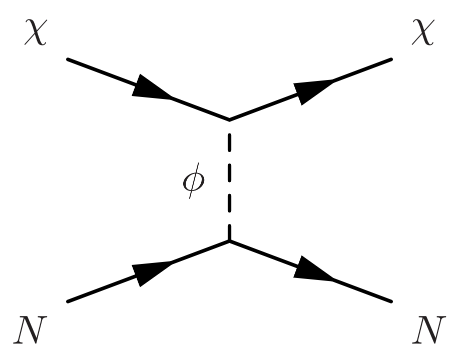




Coupling to mass



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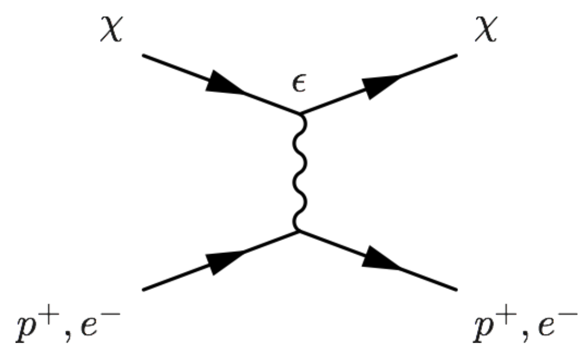
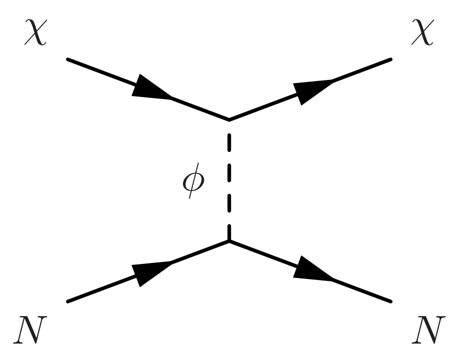
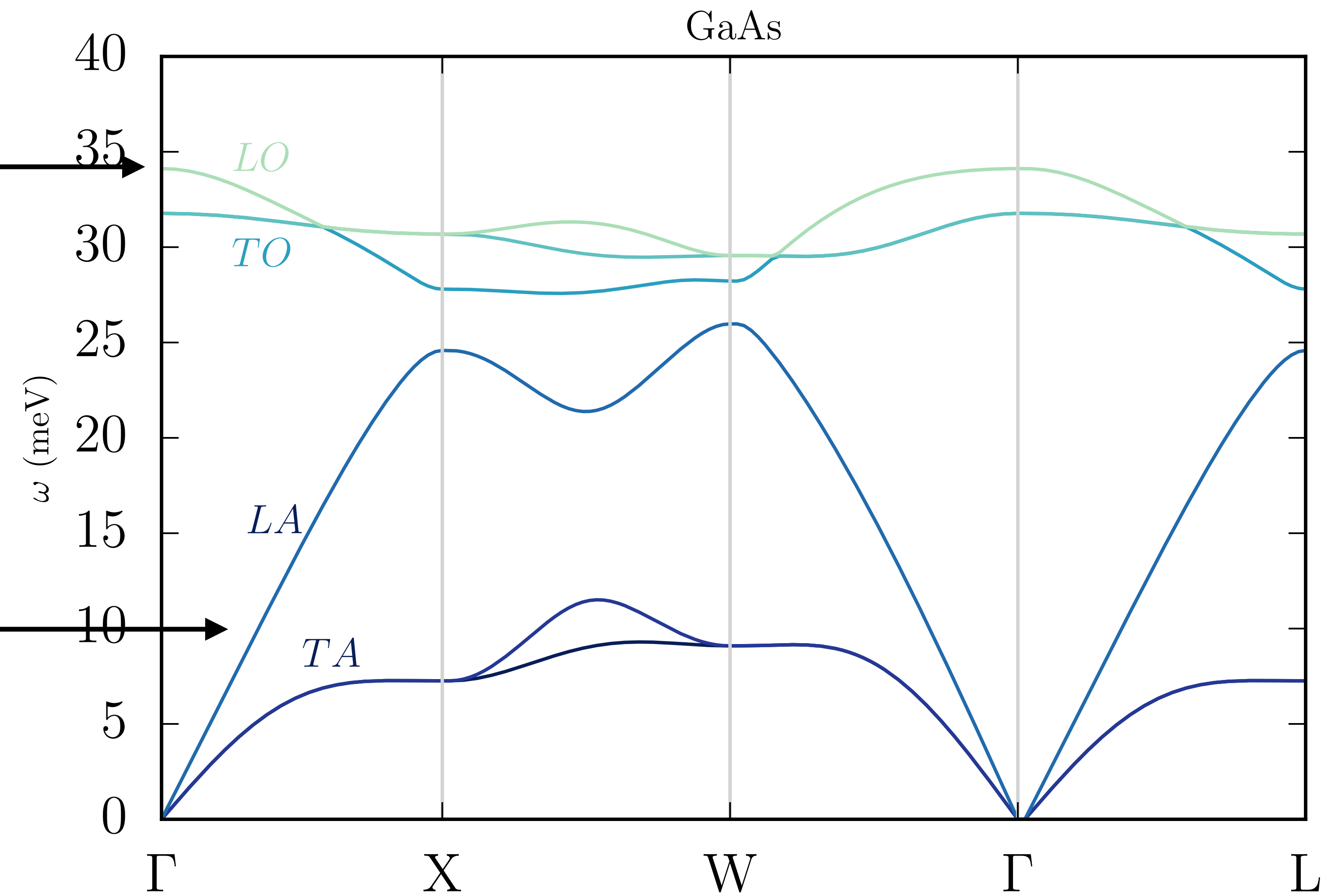
Dark Matter coupling to phonons

Coupling to charge



Coupling to mass





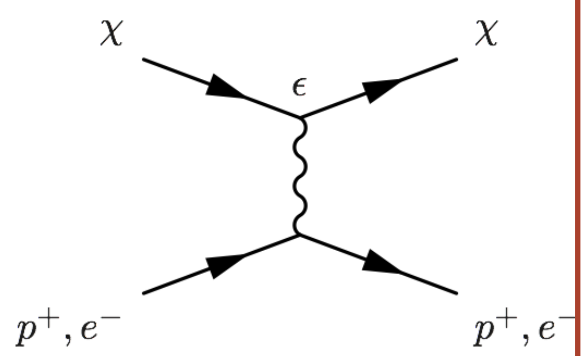
(in a “Polar” material)

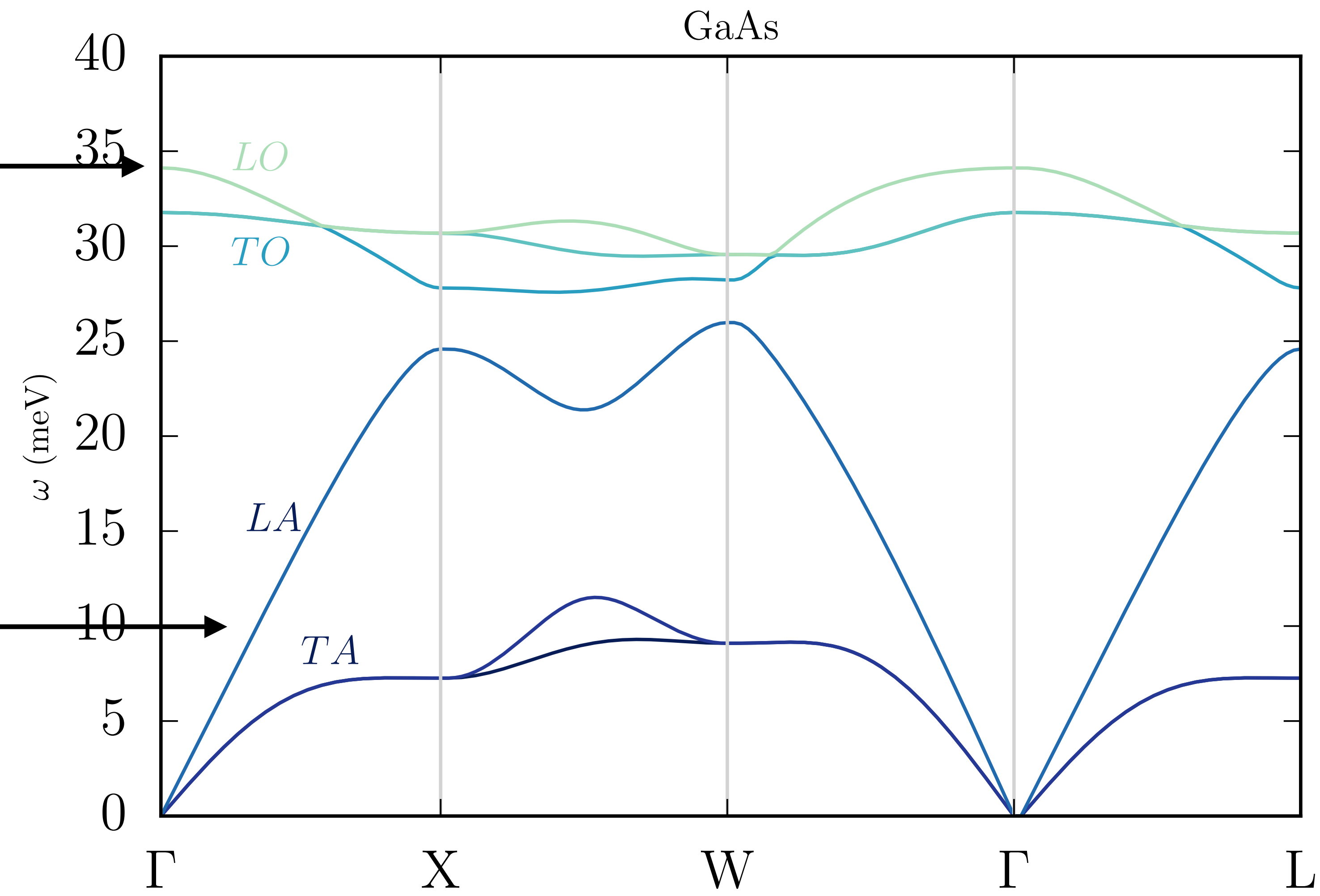
DM-phonon coupling depends strongly on underlying UV physics

Dark Matter coupling to phonons

Coupling

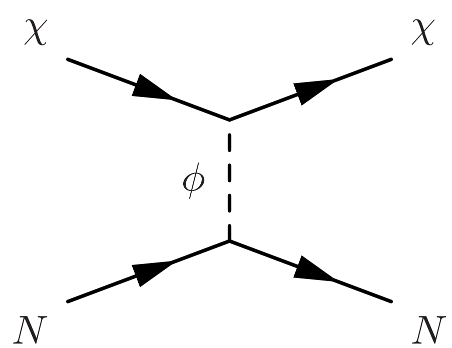
to charge





Coupling

to mass



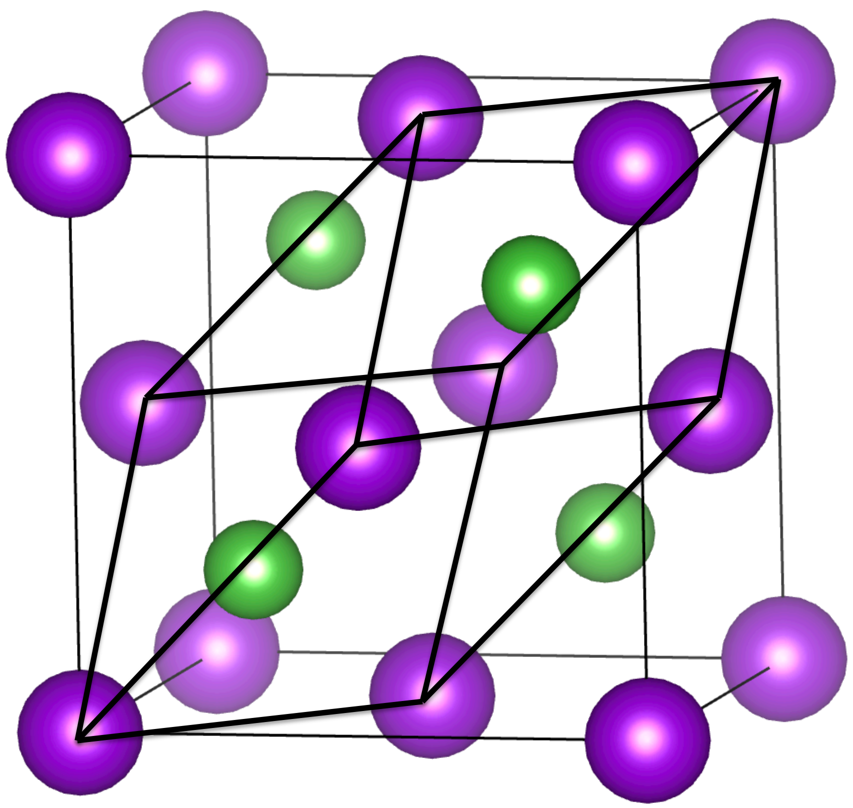
Ga As

(in a “Polar” material)

DM-phonon coupling depends strongly on underlying UV physics

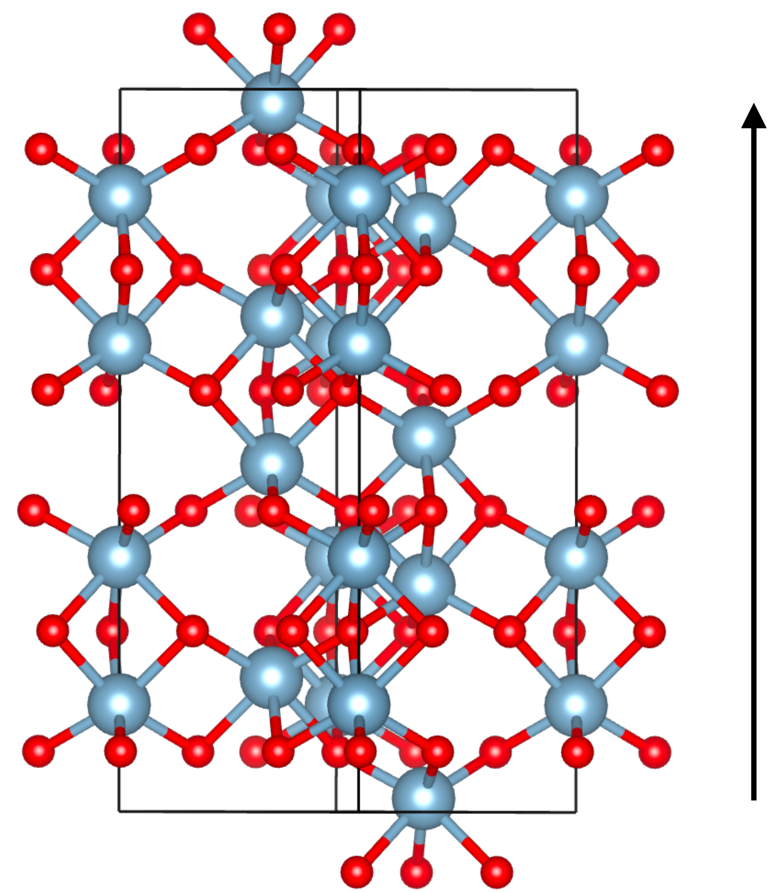
Examples of Polar Materials

GaAs



2 atoms in primitive cell

Al_2O_3 (Sapphire)

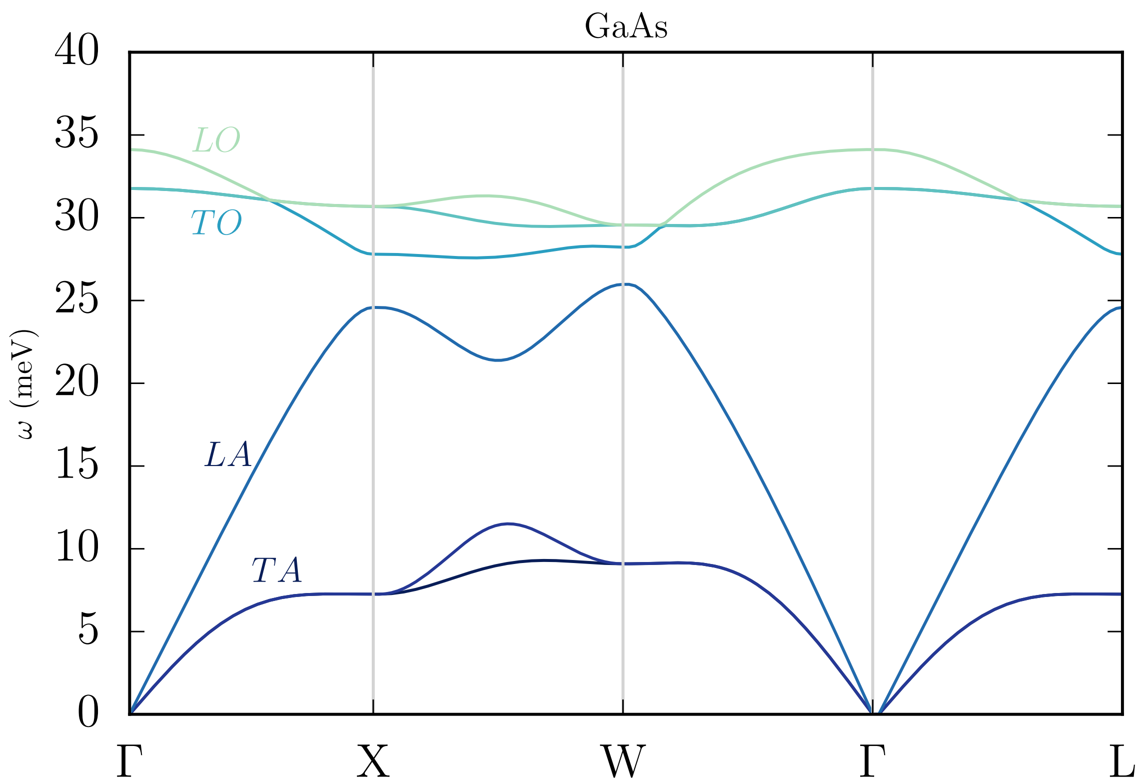


10 atoms in primitive cell

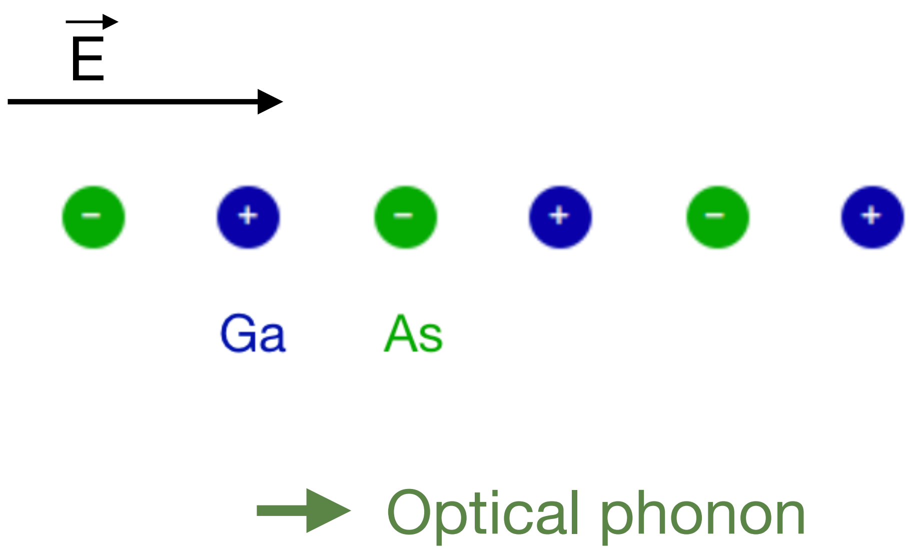
At least two *different* atoms in the unit cell

Why polar materials?

1. Optical phonons for kinematic matching



2. Natural dipole in unit cell
(nanocharged DM sources tiny electric field)



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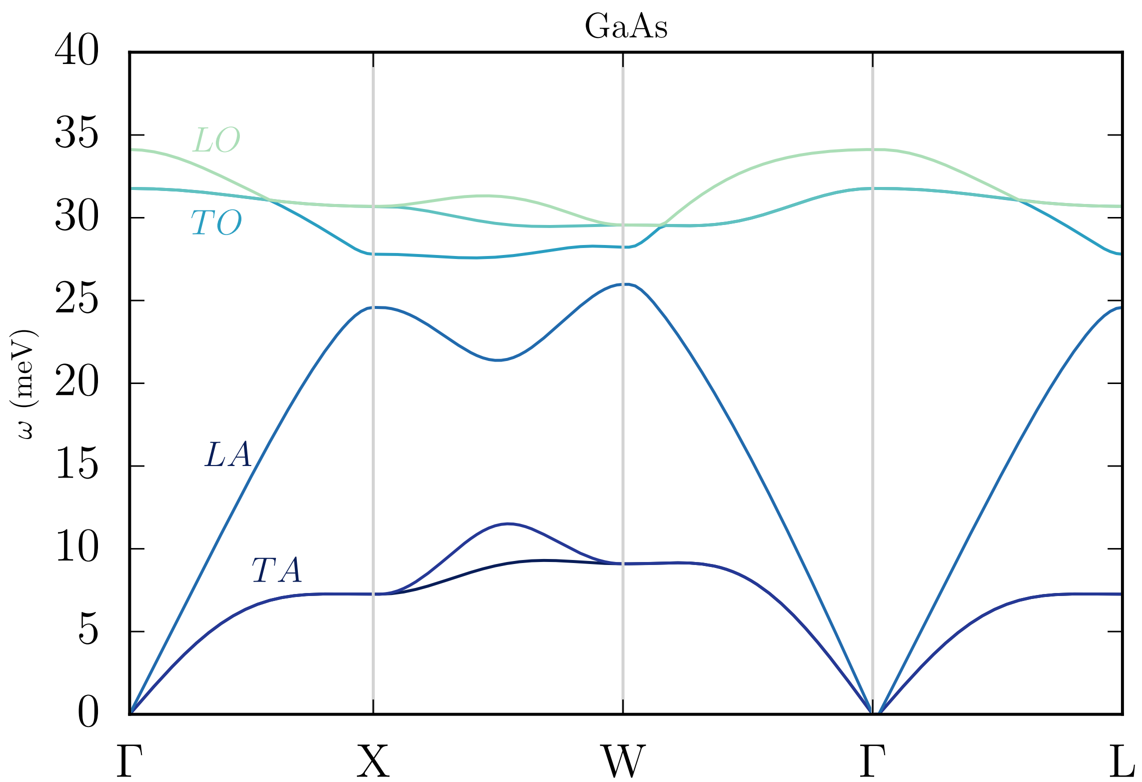

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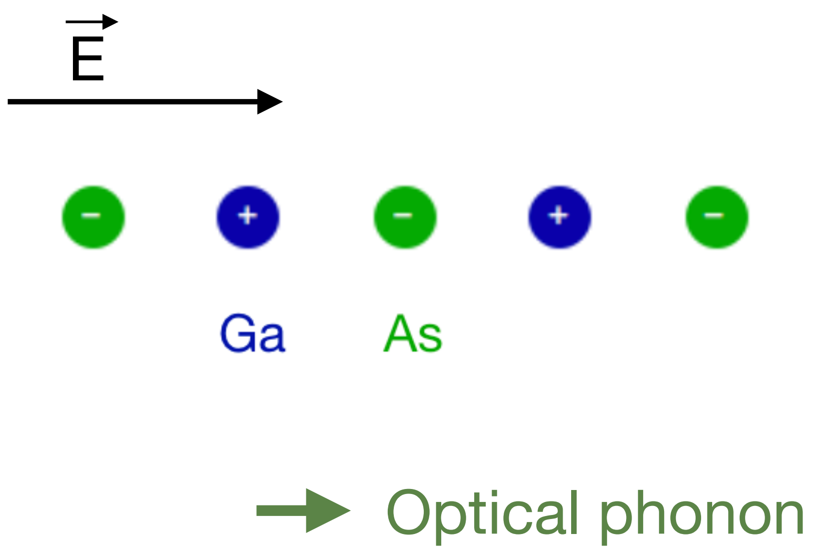

3. Semi-conductors or insulators: screening is small

4. Crystal axis allows for directional detection (daily modulation!)

5. Readily available now

SK, T. Lin, M. Pyle, K. Zurek: 1712.06598

Frölich Hamiltonian

H. Frölich, 1954
C. Verdi, F. Giustino, Phys. Rev. Lett. 115, 176401 (2015)

Electric dipole interacting with test charge: $$H \sim i\,e \sum_{\mathbf{q}} \frac{\mathbf{q}\cdot\mathbf{P}}{|\mathbf{q}|^2} e^{i\mathbf{q}\cdot\mathbf{r}}$$

S. Griffin, SK, T. Lin, M. Pyle, K. Zurek: 1807.10291

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Born effective charge tensor for each atom

phonon eigenvectors (atomic displacements)

$$H = i\frac{\kappa e^2}{V} \sum_{j,\nu,\mathbf{q}} \sum_{\mathbf{G}\neq\mathbf{q}} \frac{1}{\sqrt{2Nm_j\omega_{\nu,\mathbf{q}}}} \frac{(\mathbf{q}+\mathbf{G})\cdot\mathbf{Z}_j\cdot\mathbf{e}_{j,\nu}(\mathbf{q})}{(\mathbf{q}+\mathbf{G})\cdot\boldsymbol{\epsilon}_\infty\cdot(\mathbf{q}+\mathbf{G})} e^{i(\mathbf{q}+\mathbf{G})\cdot(\mathbf{r}+\boldsymbol{\tau}_j)}$$

Sum over:

- j atoms in unit cell
- ν phonon modes
- **q** 1st Brillouin zone
- **G** reciprocal lattice

phonon energy

high frequency dielectric tensor

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phonon energy

high frequency dielectric tensor

Calculation overview:

Crystal structure → **VASP** (DFT calculation) → Force constants, Born effective charges → **phonopy** (diagonalize force matrix) → Phonon energies & eigenvectors

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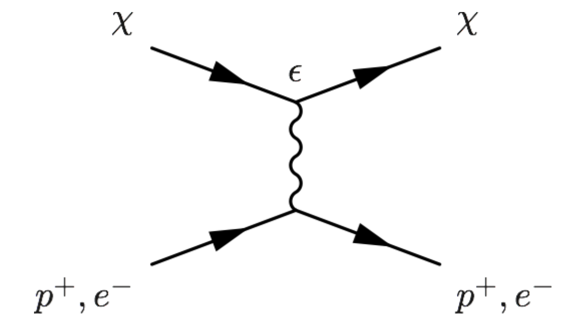
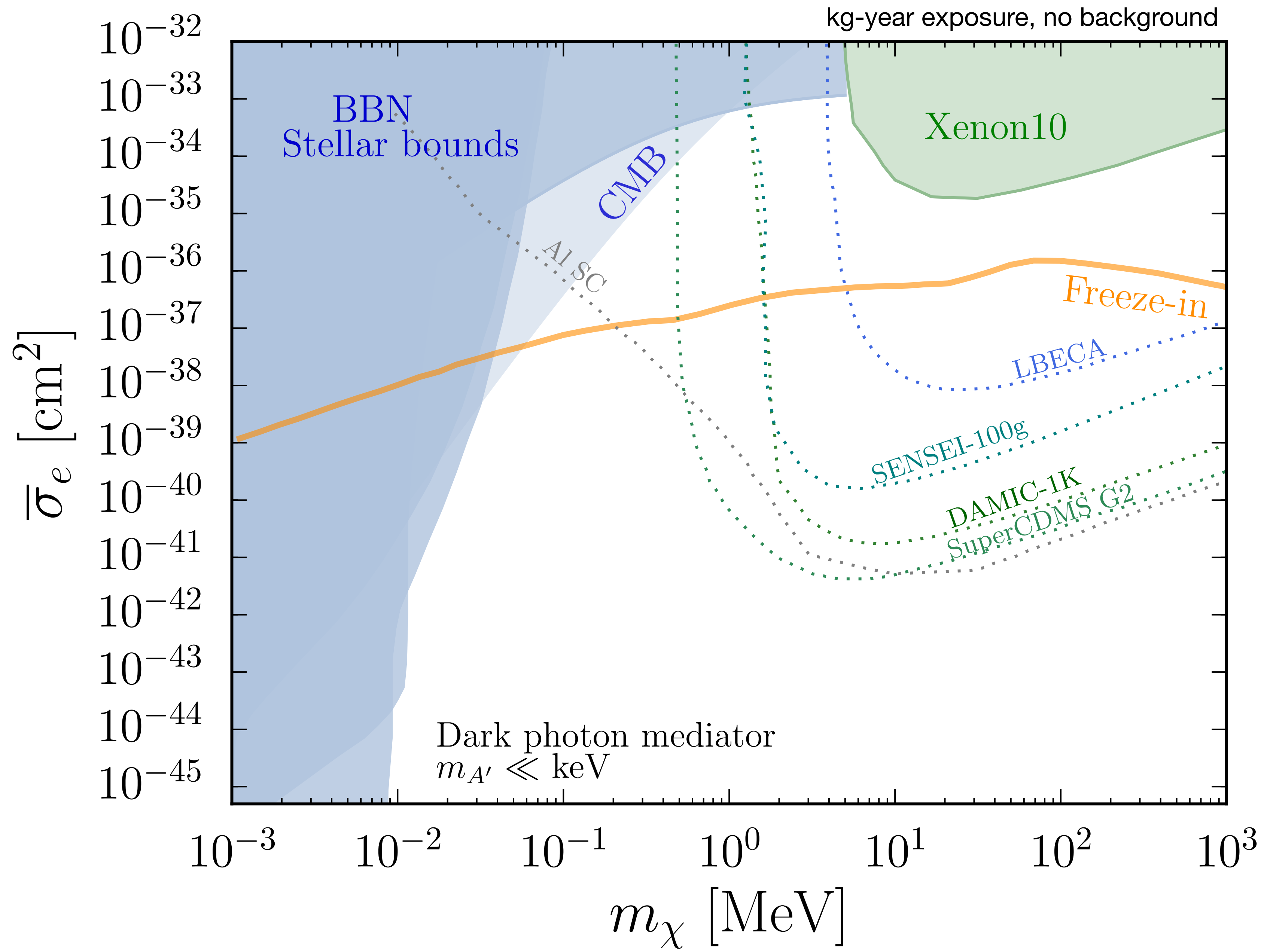
Reach



Both GaAs and Sapphire probe Dark Matter masses as low as 10 keV



Probe the new parameter space with milligram-day exposure

SK, T. Lin, M. Pyle, K. Zurek: 1712.06598
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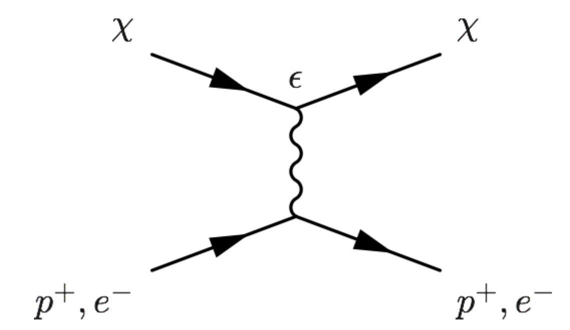

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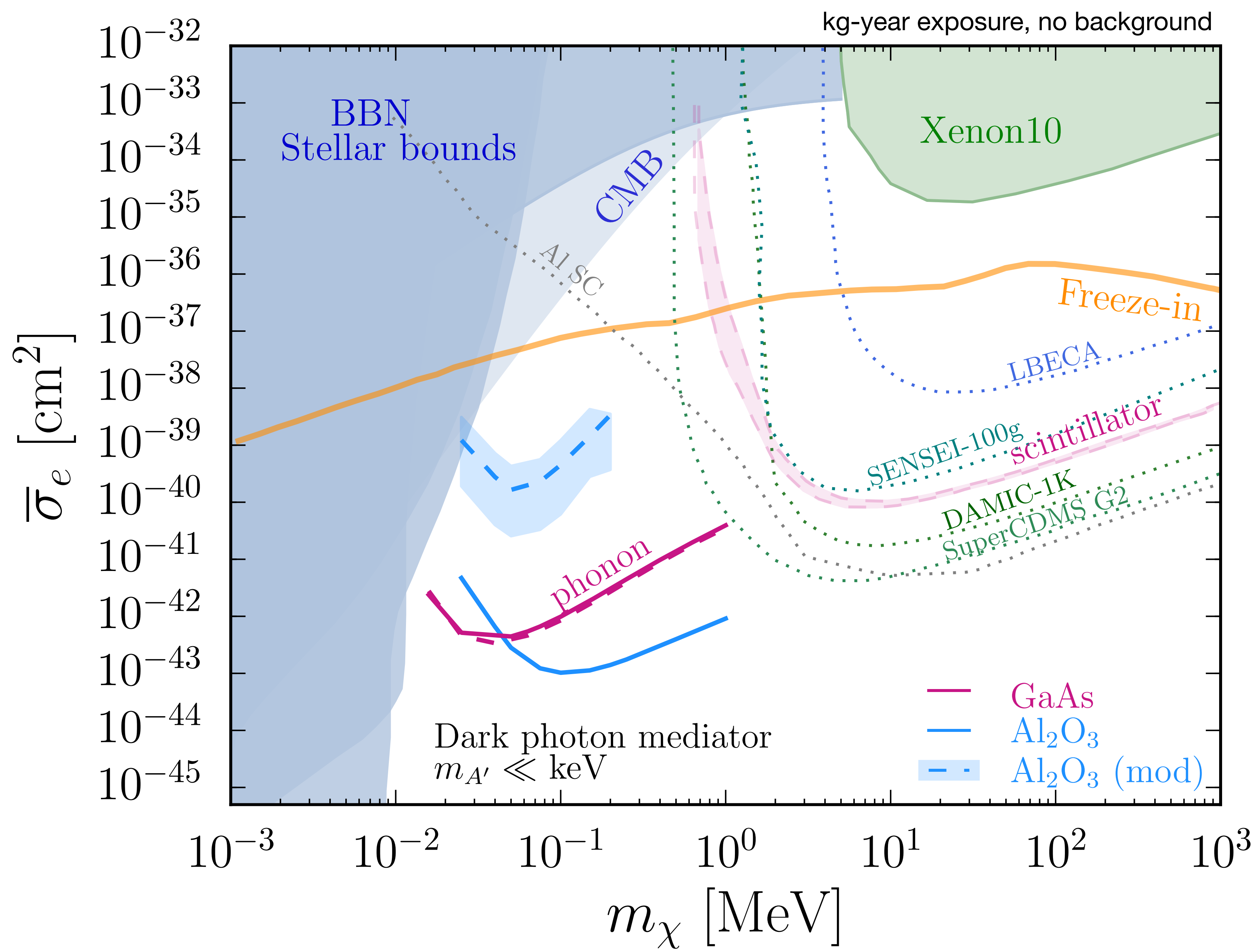

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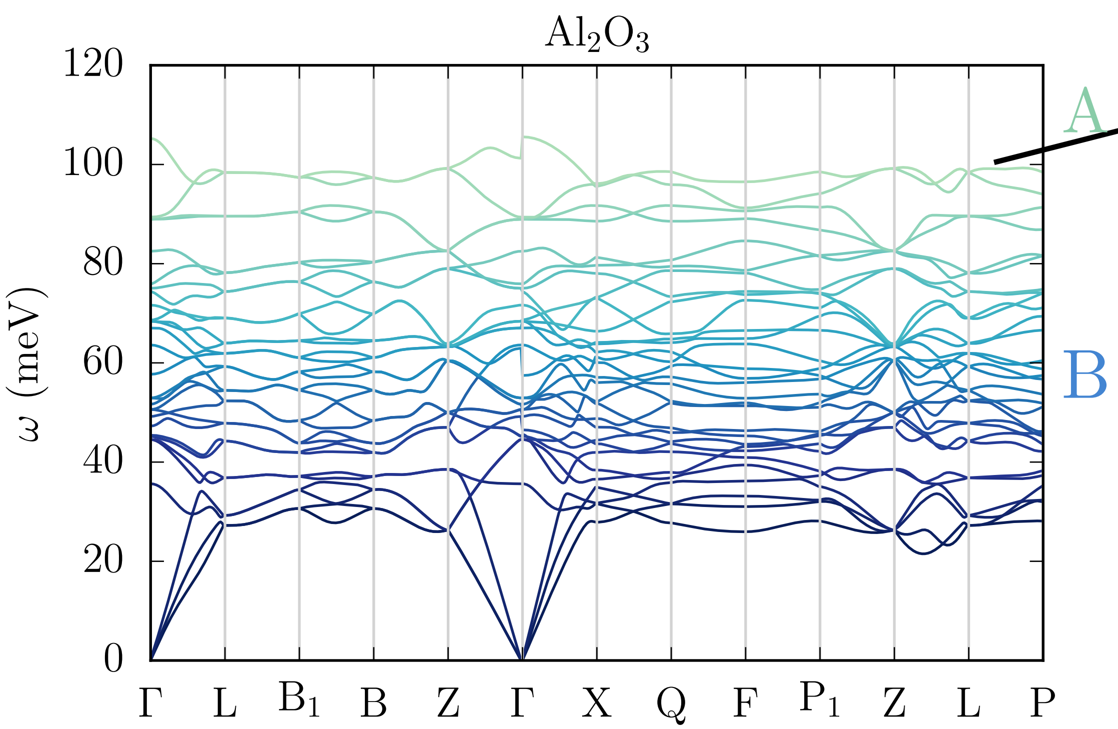
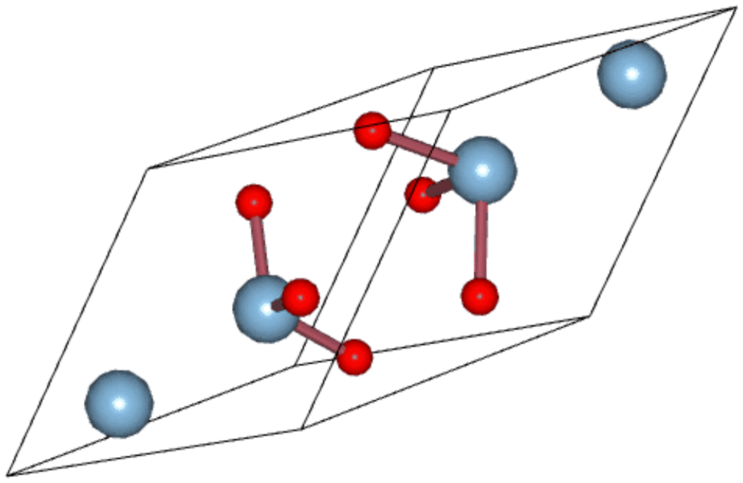
Sapphire in more detail

Most energetic mode dominates



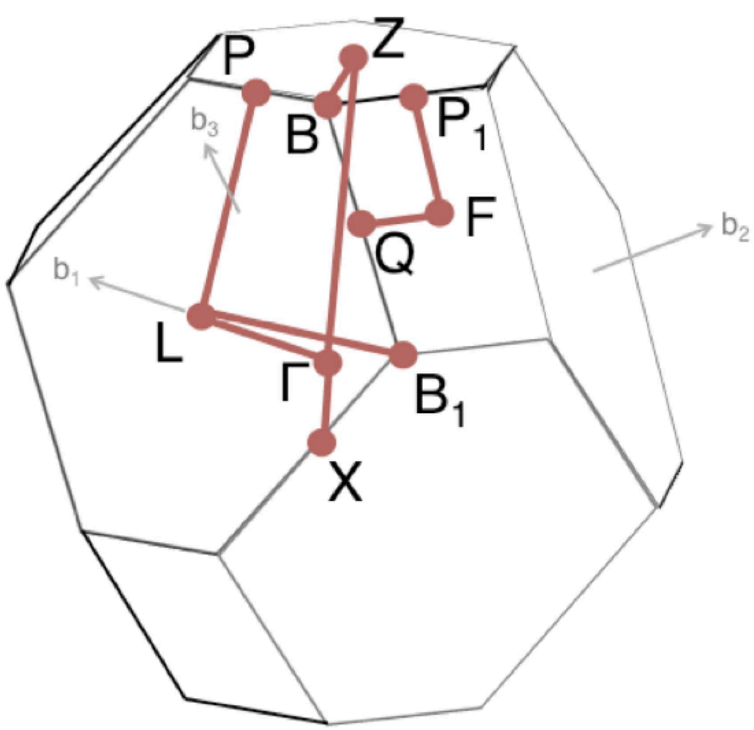


Aluminum atoms move in phase

↓

Large dipole

Al$_2$O$_3$ Brillouin zone



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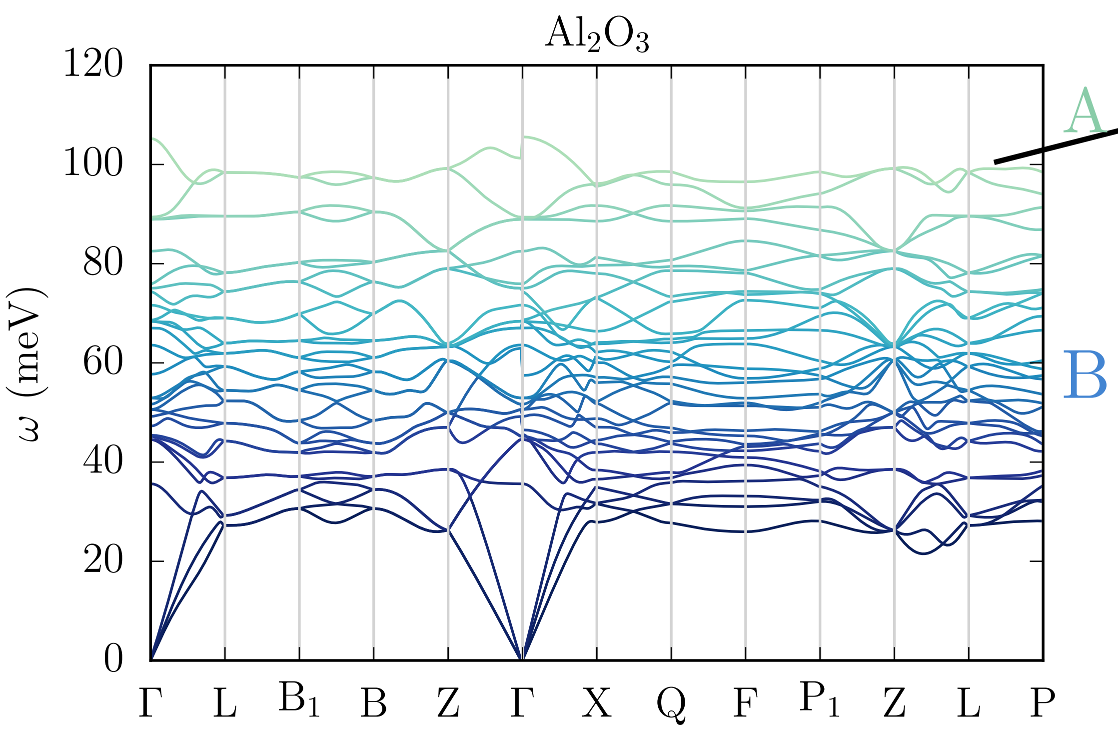

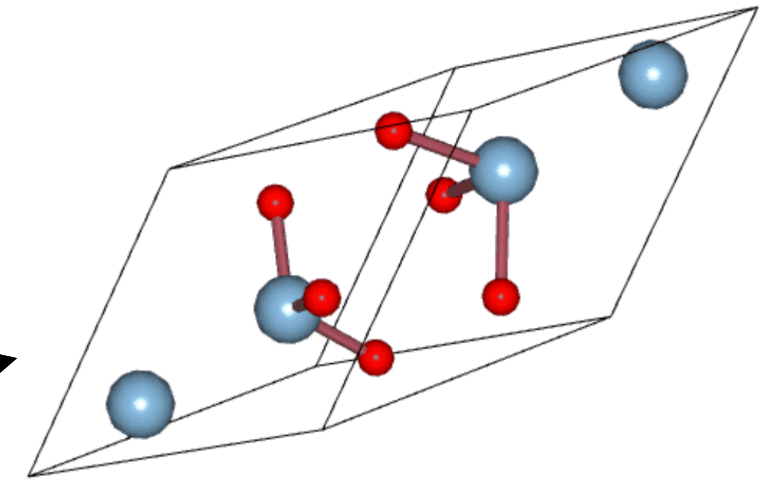

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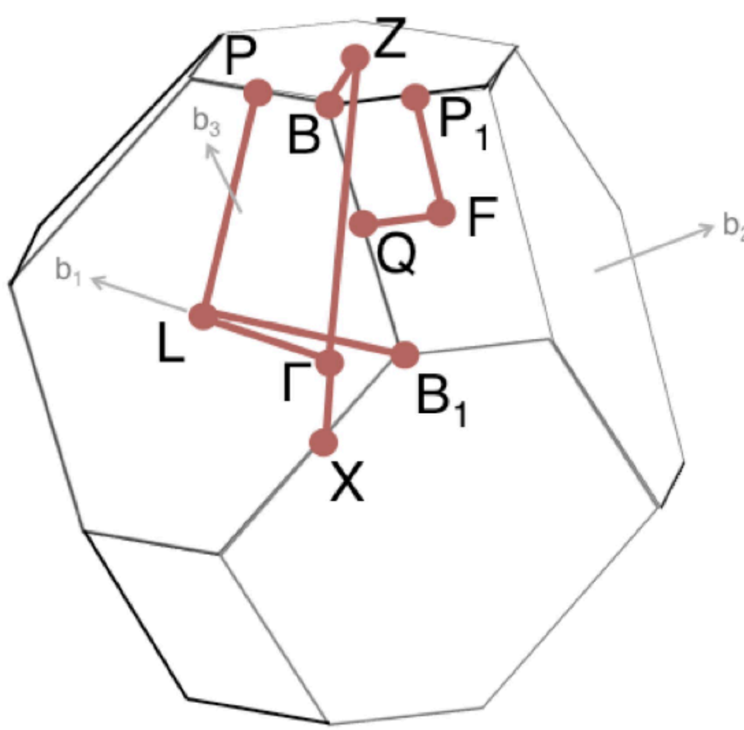

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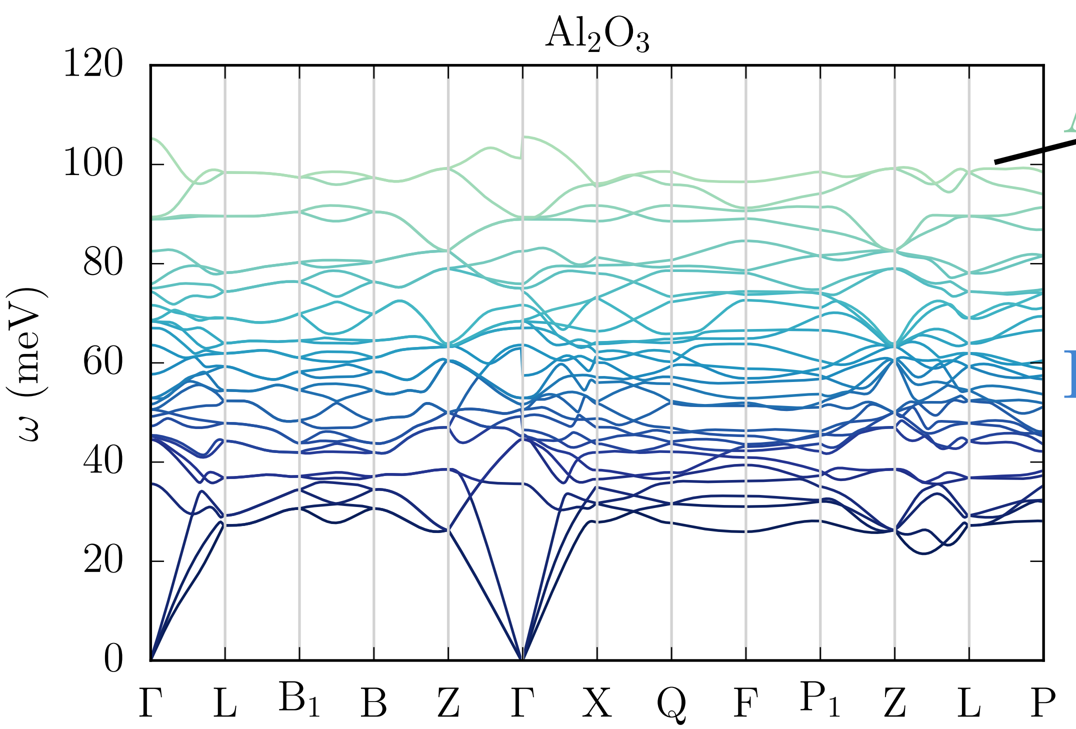

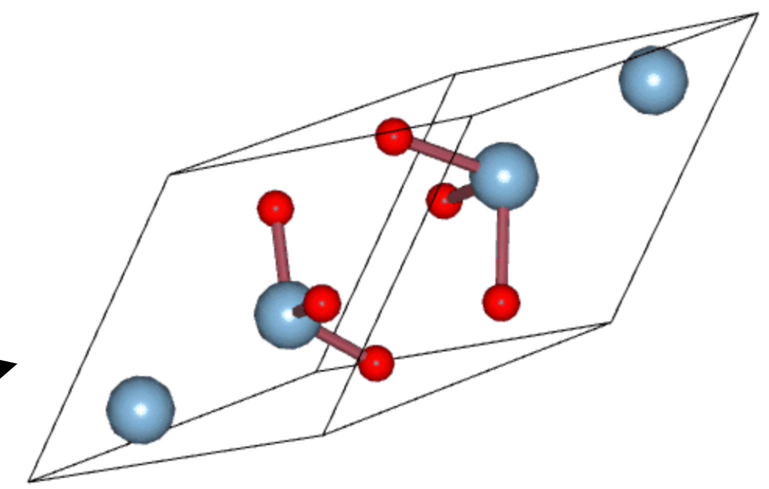

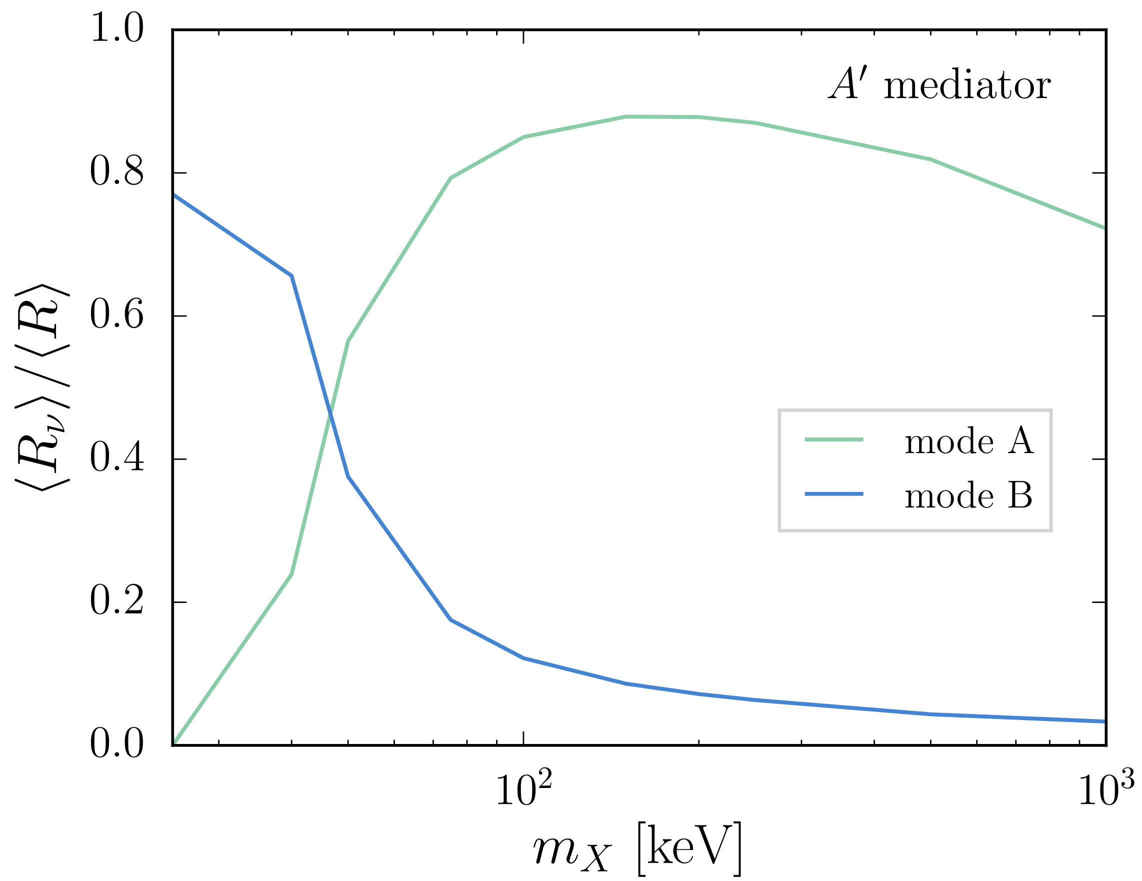

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Large dipole

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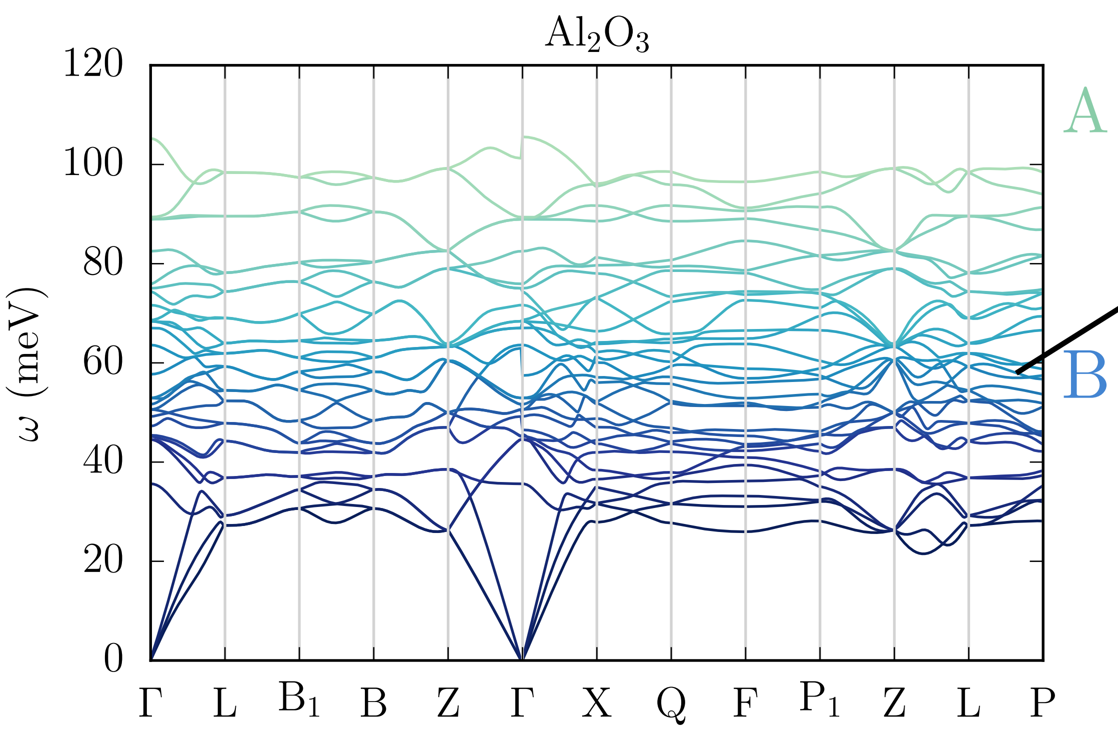
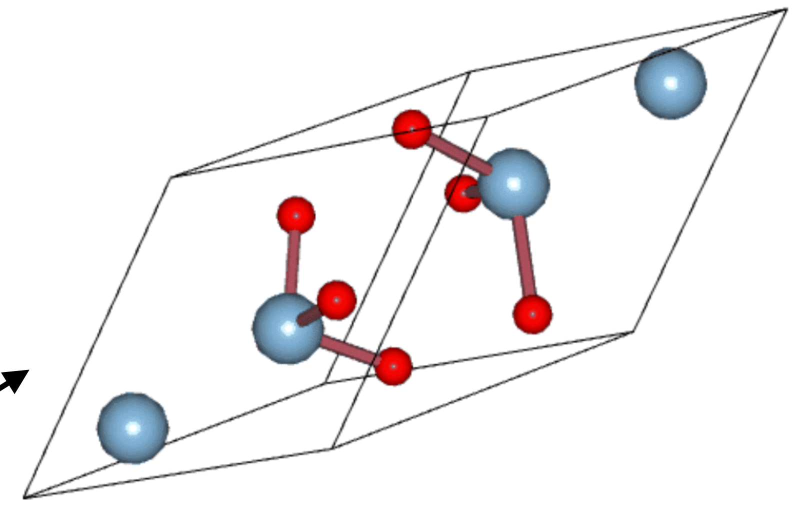
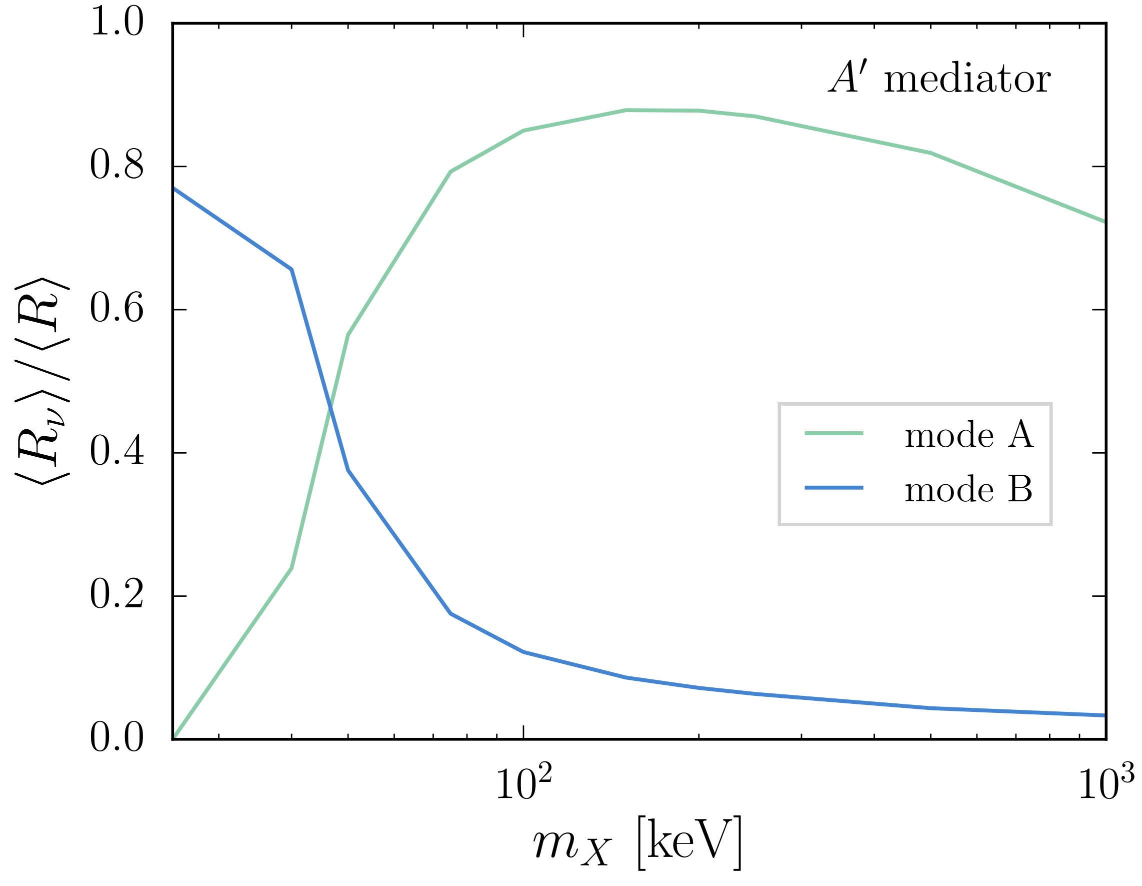
Sapphire in more detail

For low m_X, a lower mode dominates







Aluminum atoms still move in phase, but smaller amplitude

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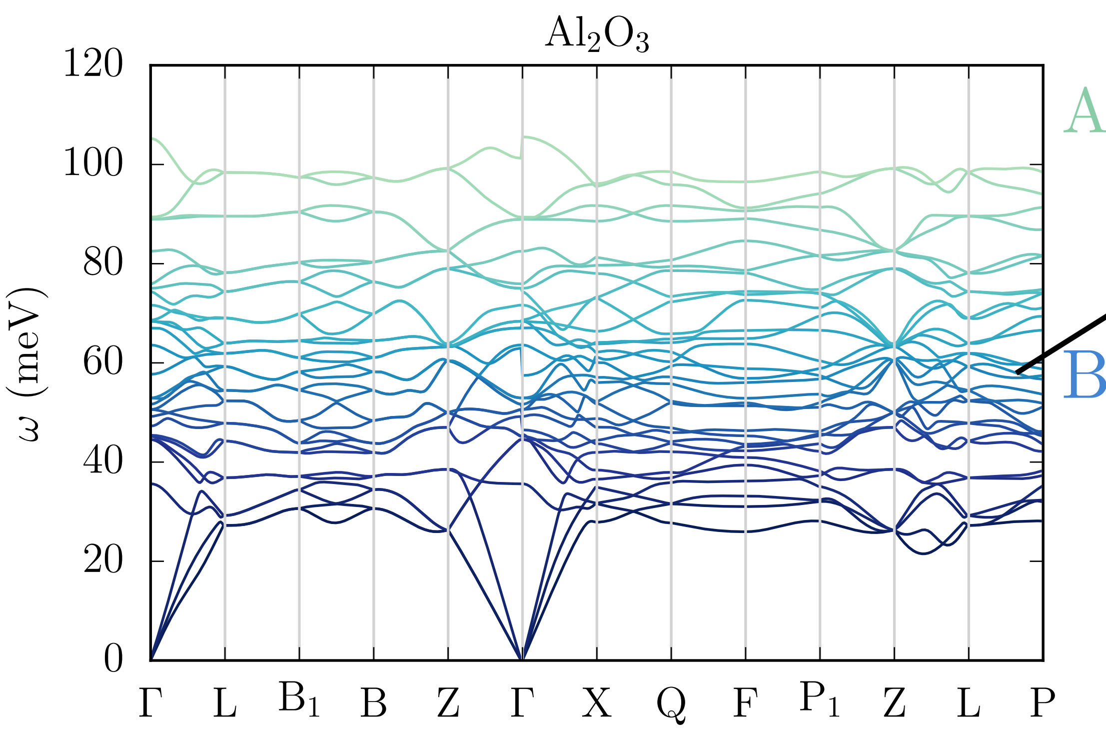

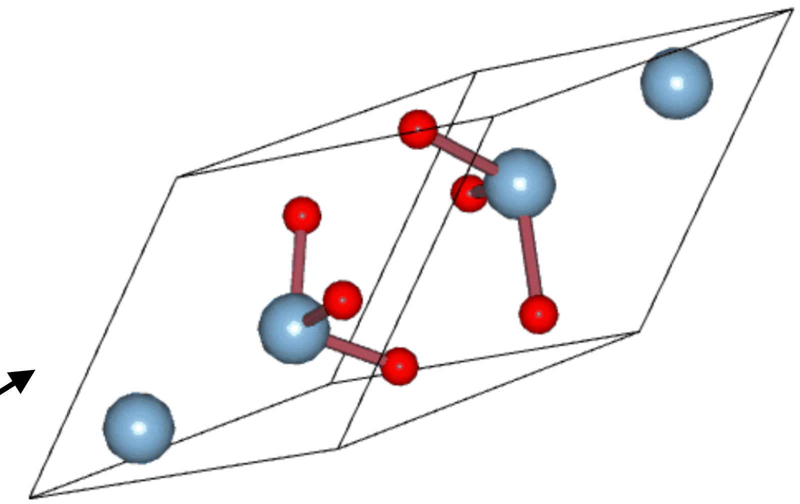

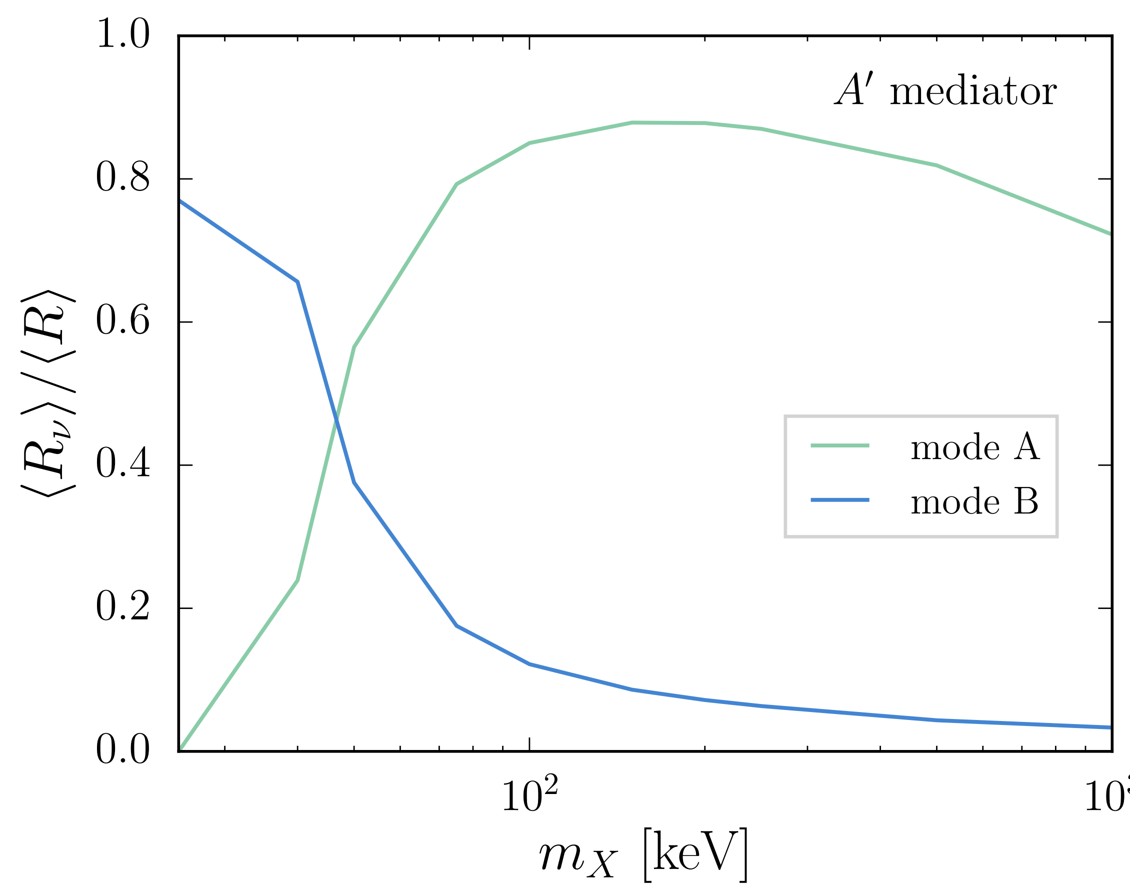

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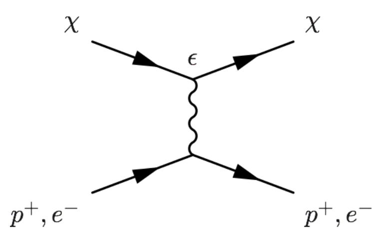
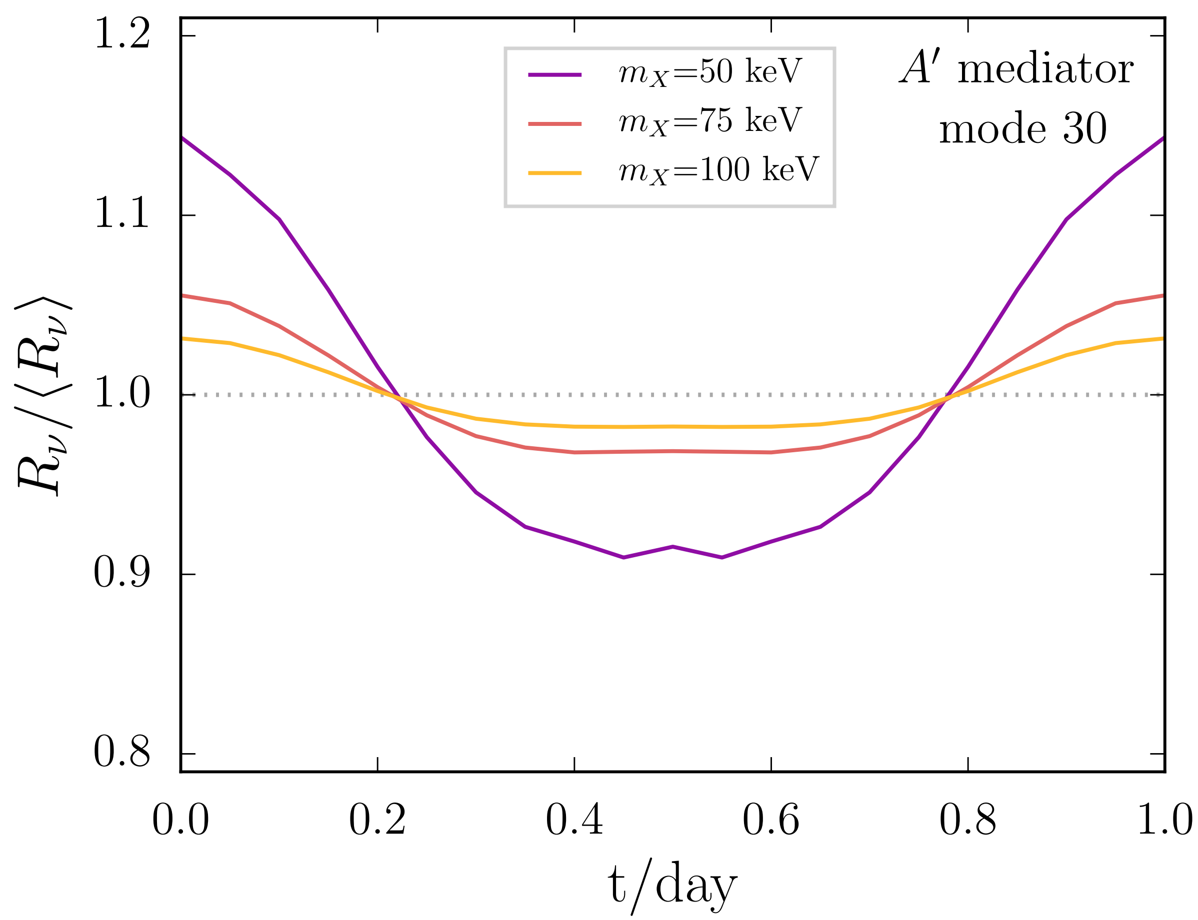
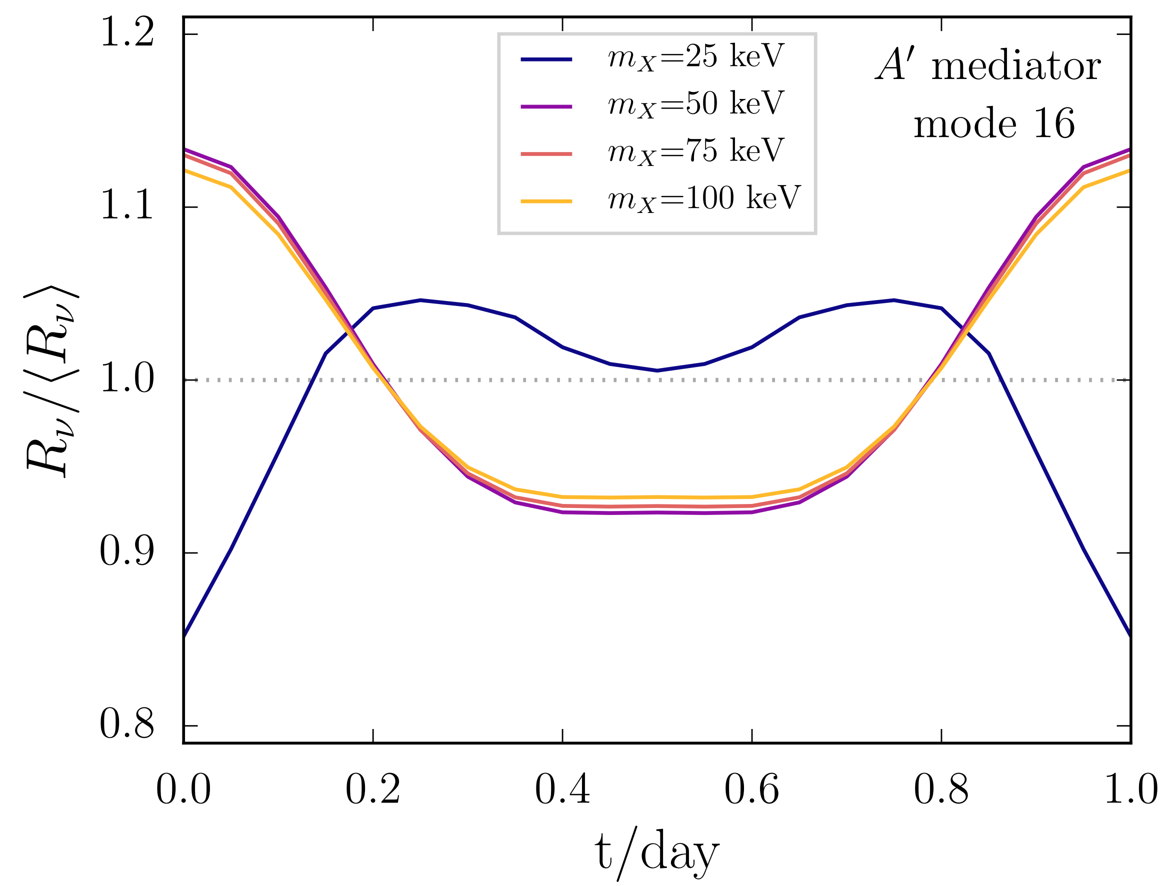
Daily modulation (dark photon mediator)







Amplitude and pattern depends on DM mass

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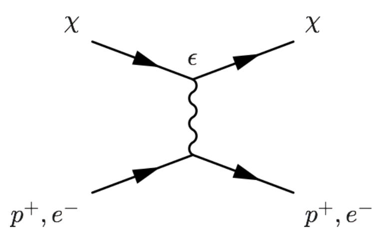

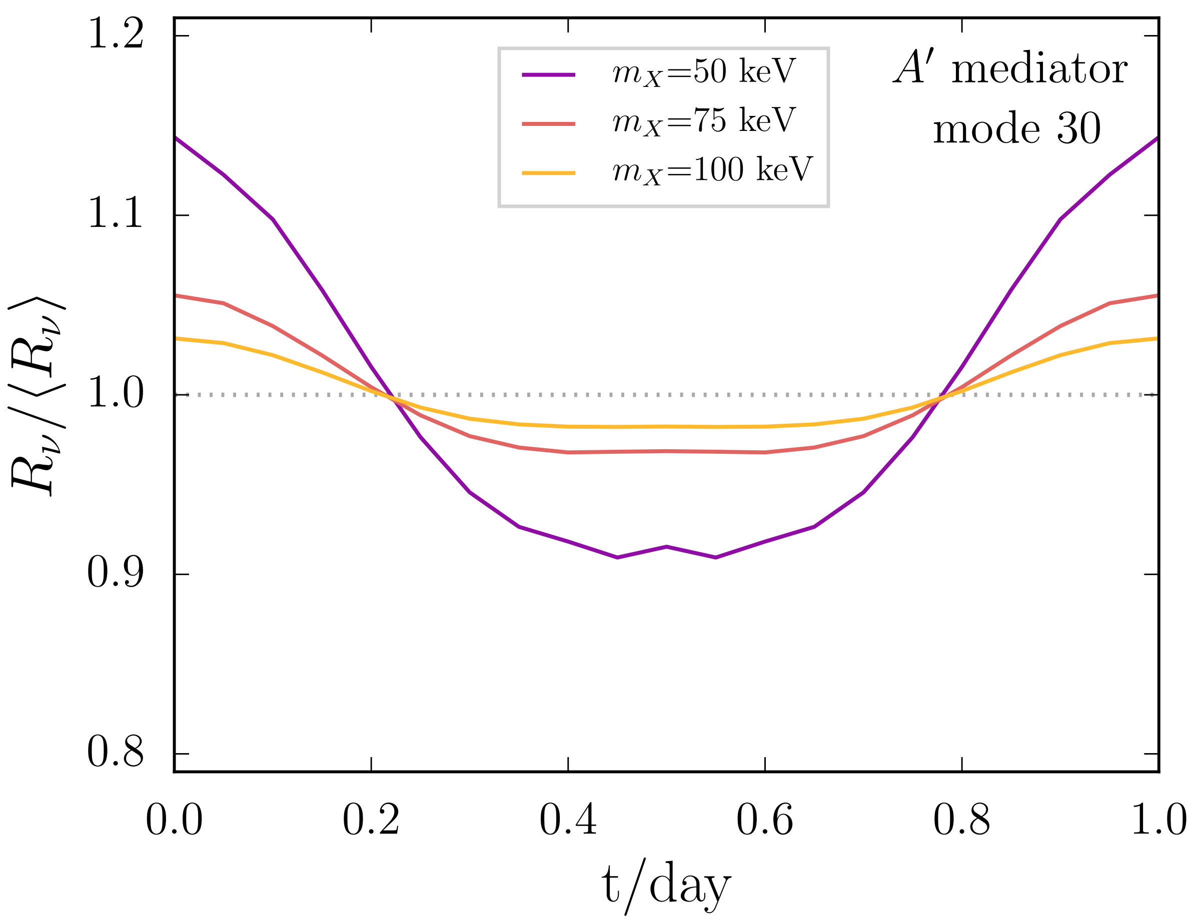

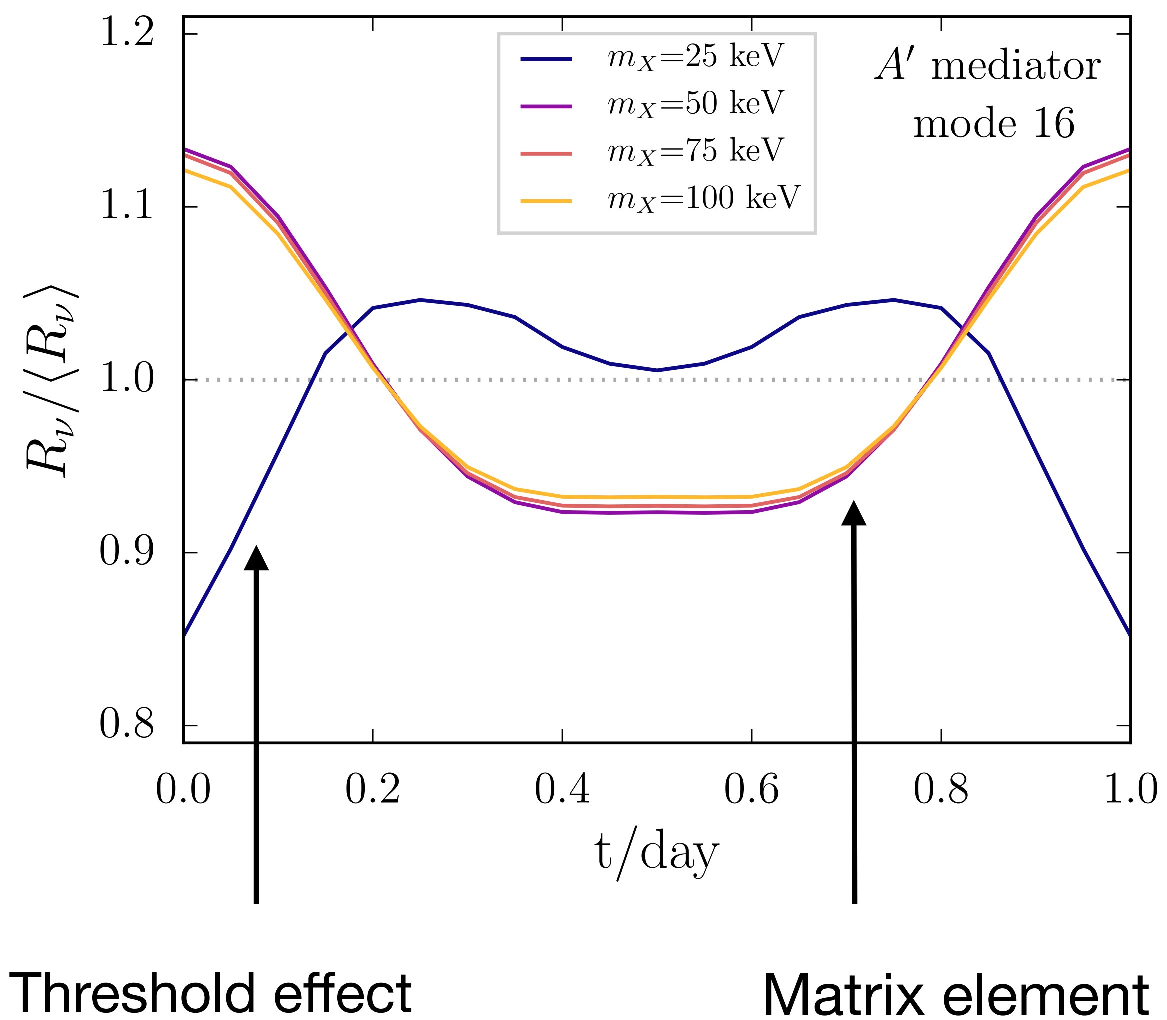

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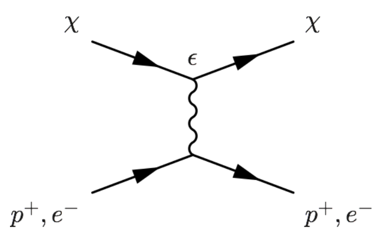

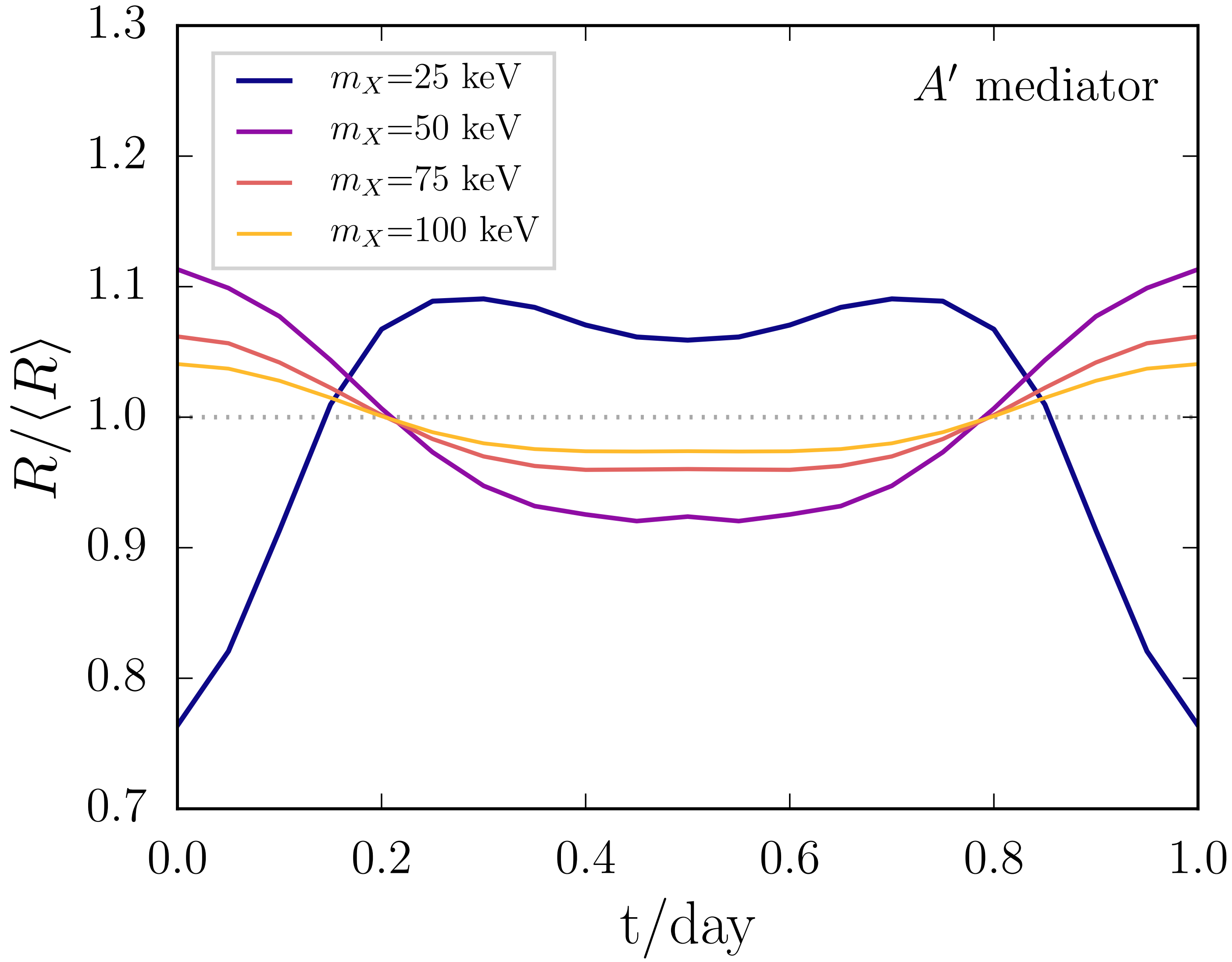

Amplitude and pattern depends on DM mass

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Dark photon absorption

Very light, bosonic dark matter can be absorbed on the target

Example: Dark photon dark matter: $\mathcal{L} \supset -\frac{\kappa}{2} F'_{\mu\nu} F^{\mu\nu}$

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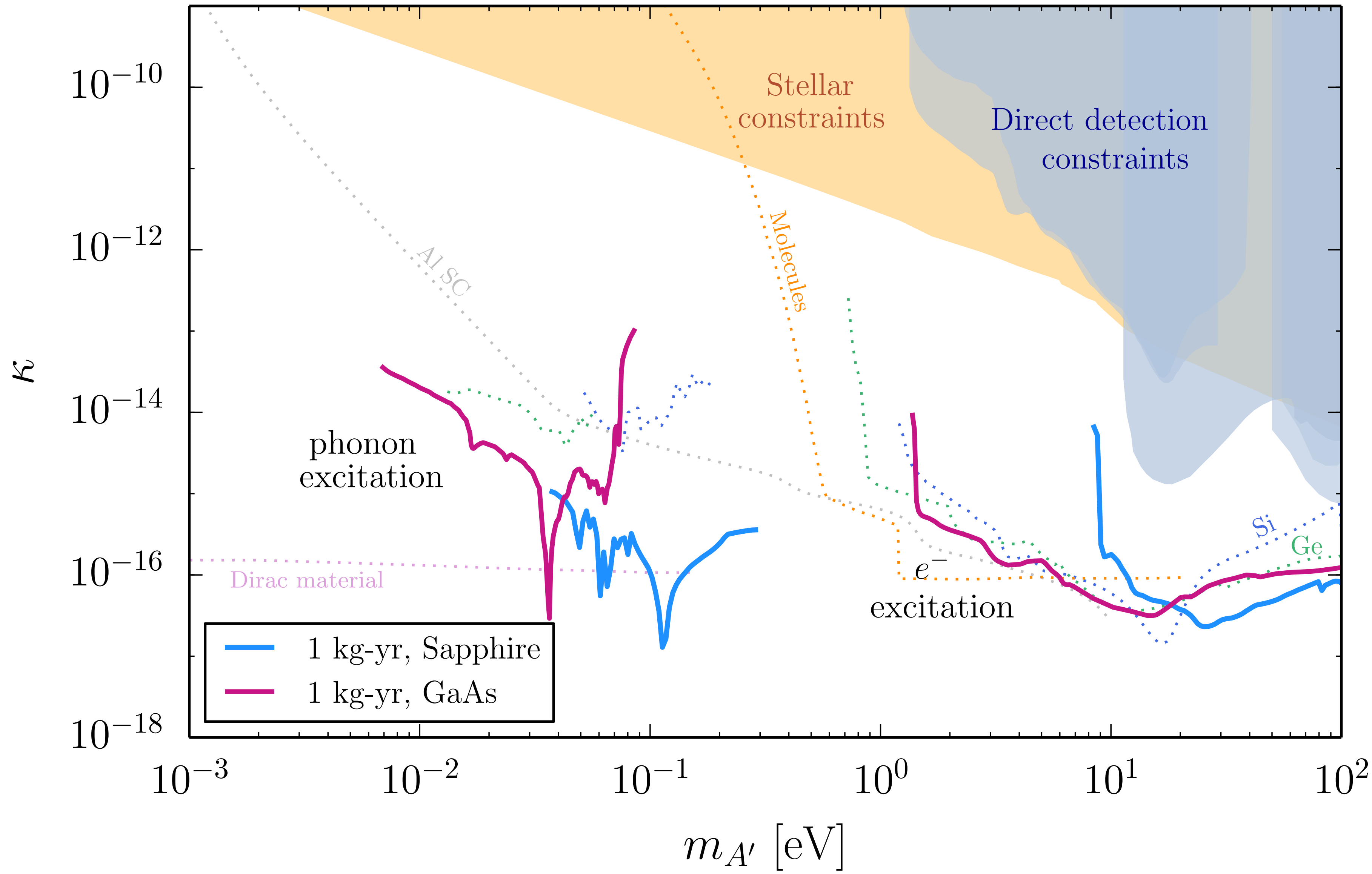

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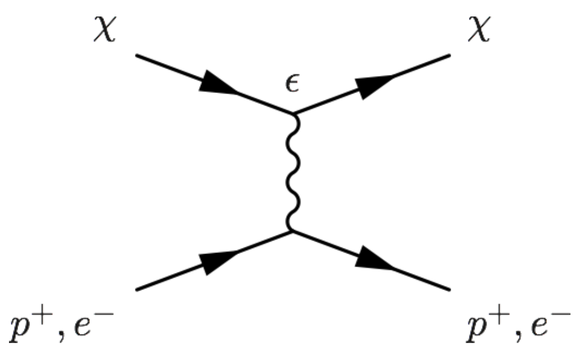
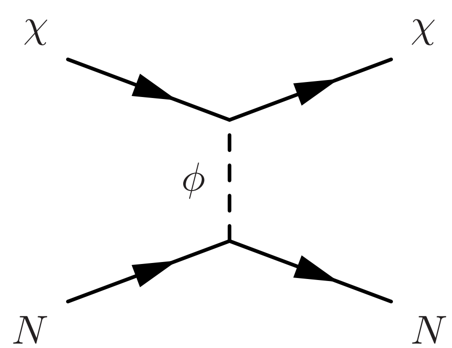
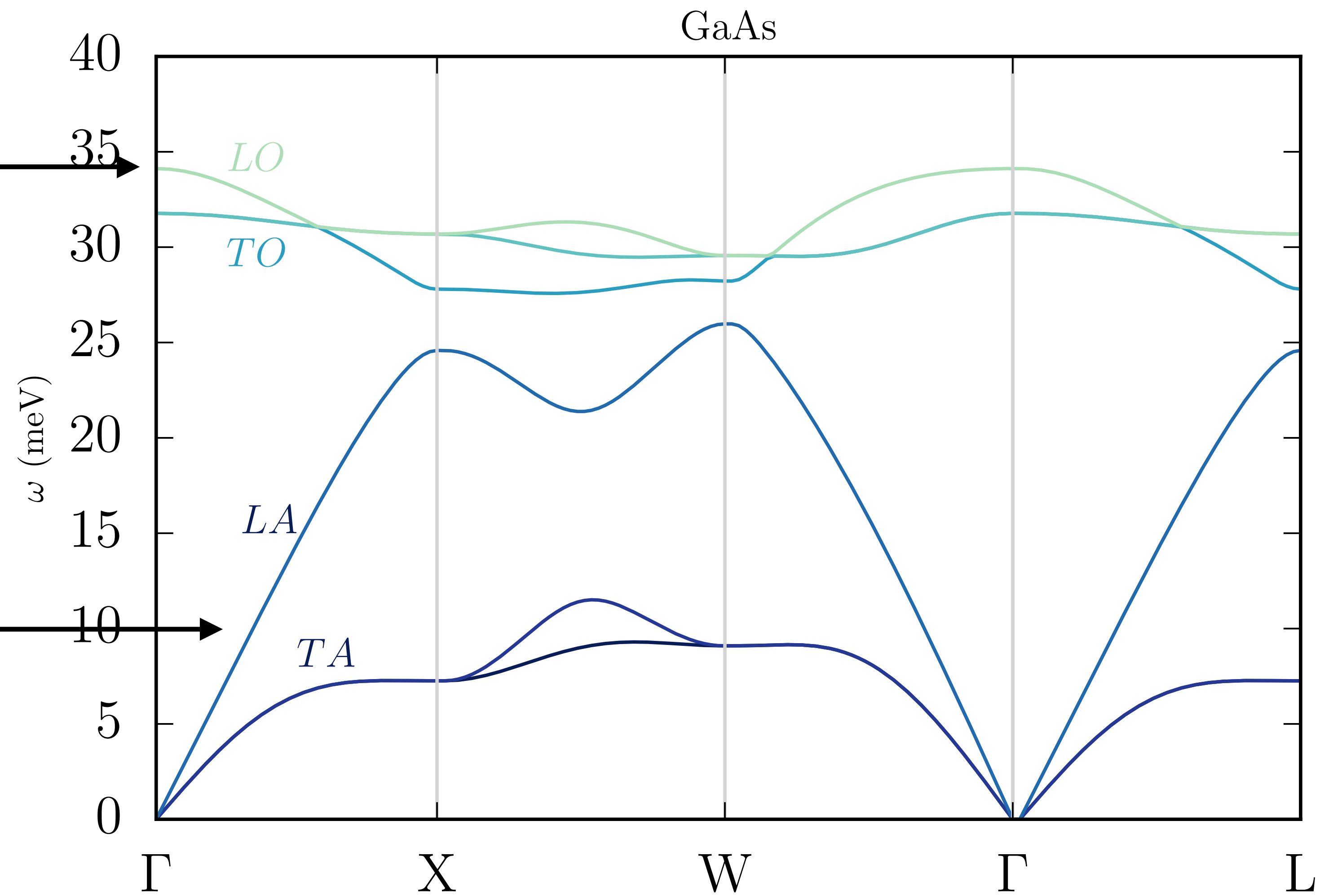
Coupling to charge

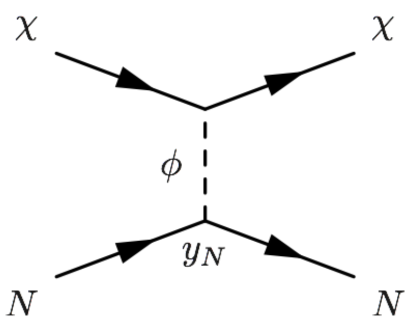


Coupling to mass

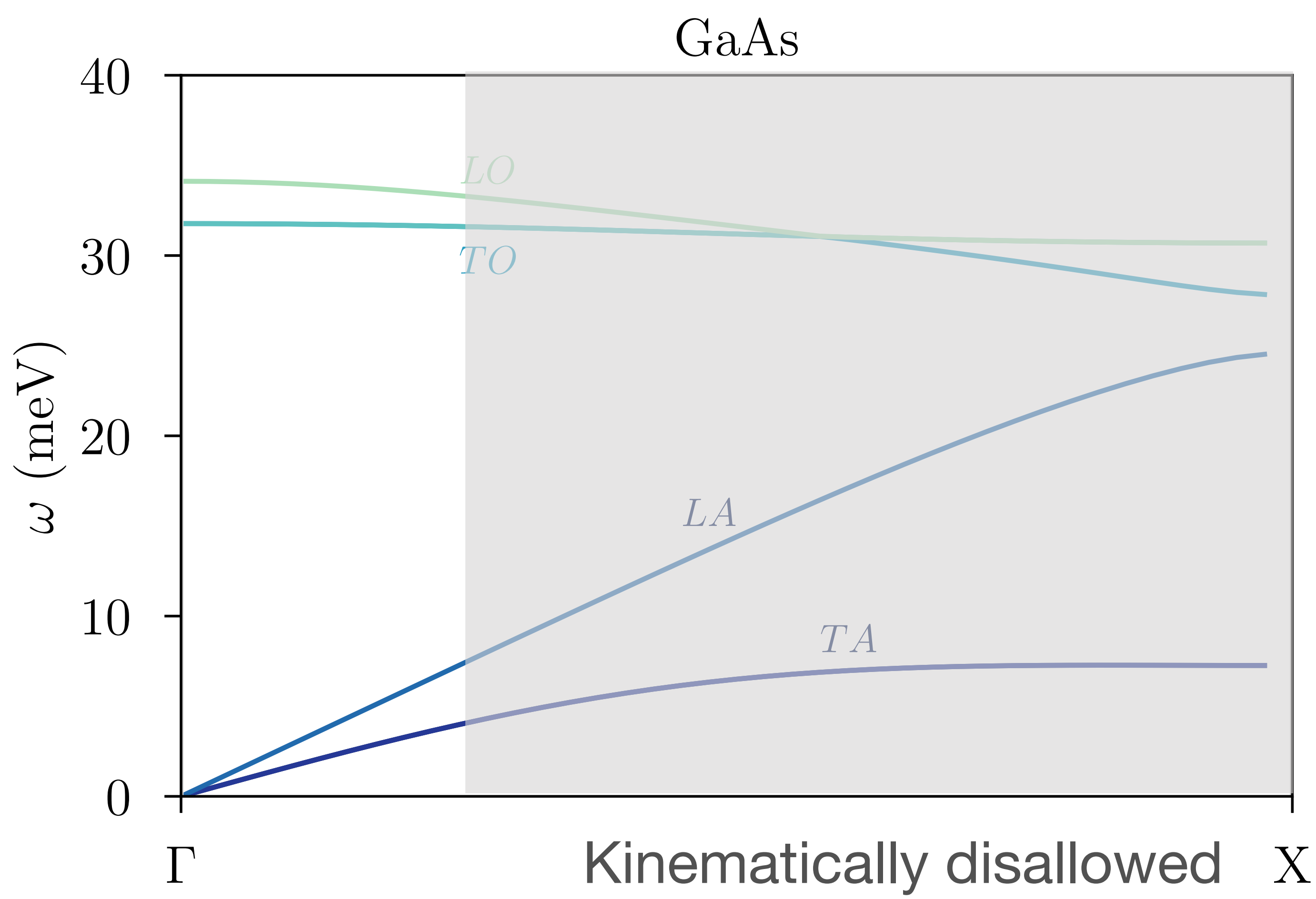




Kinematics



Which modes to use?



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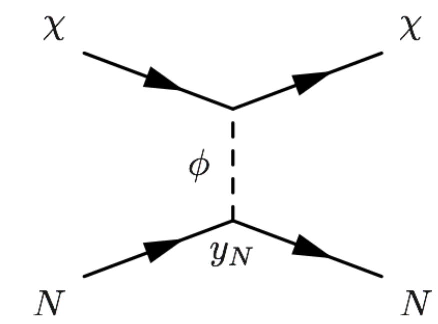
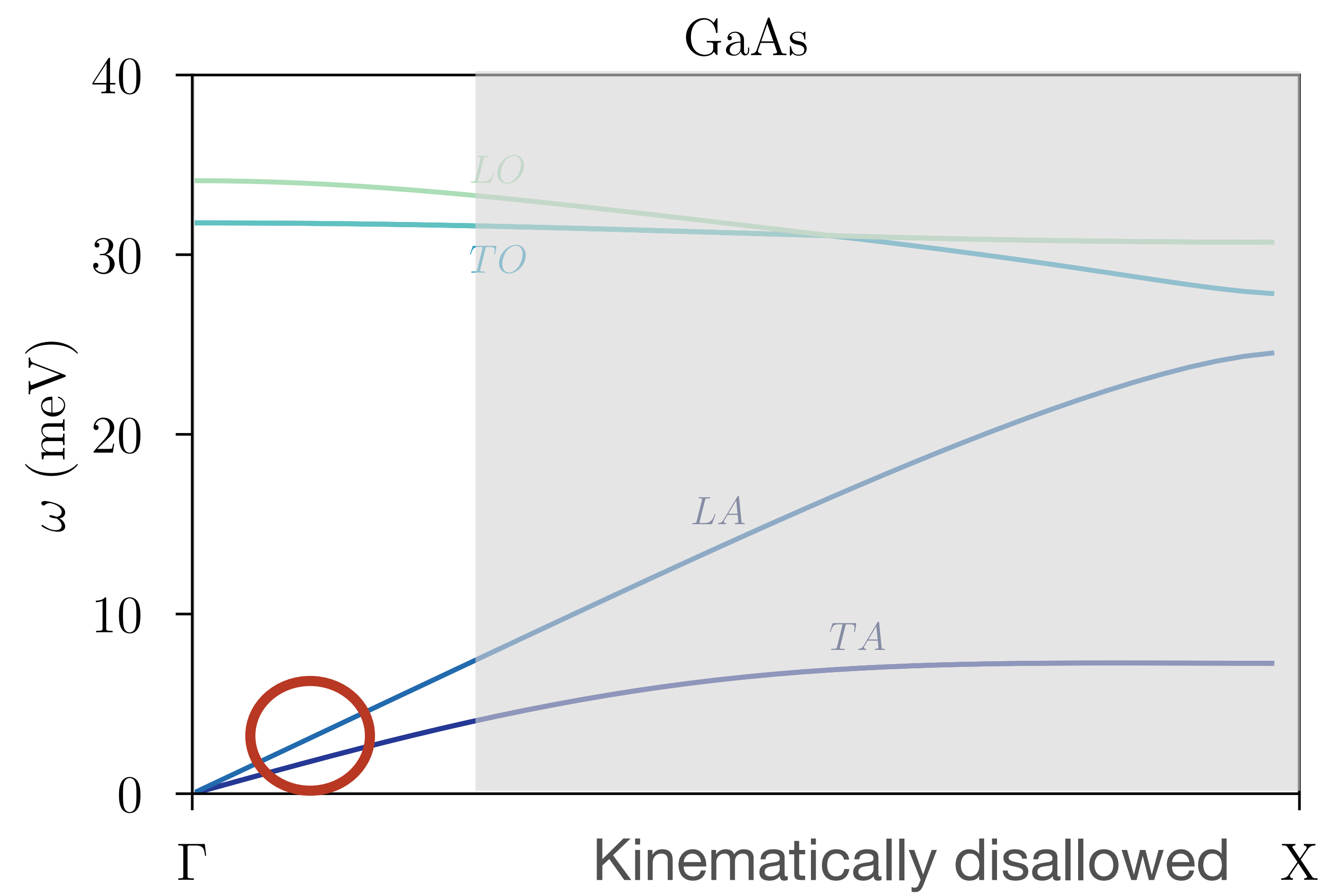


Which modes to use?

1. Single acoustic:

   Experimentally extremely challenging



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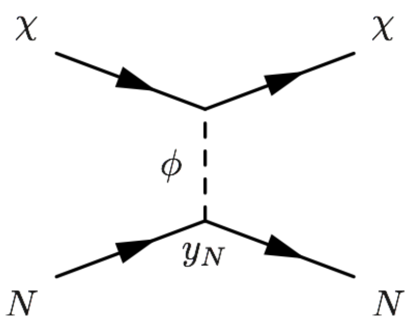

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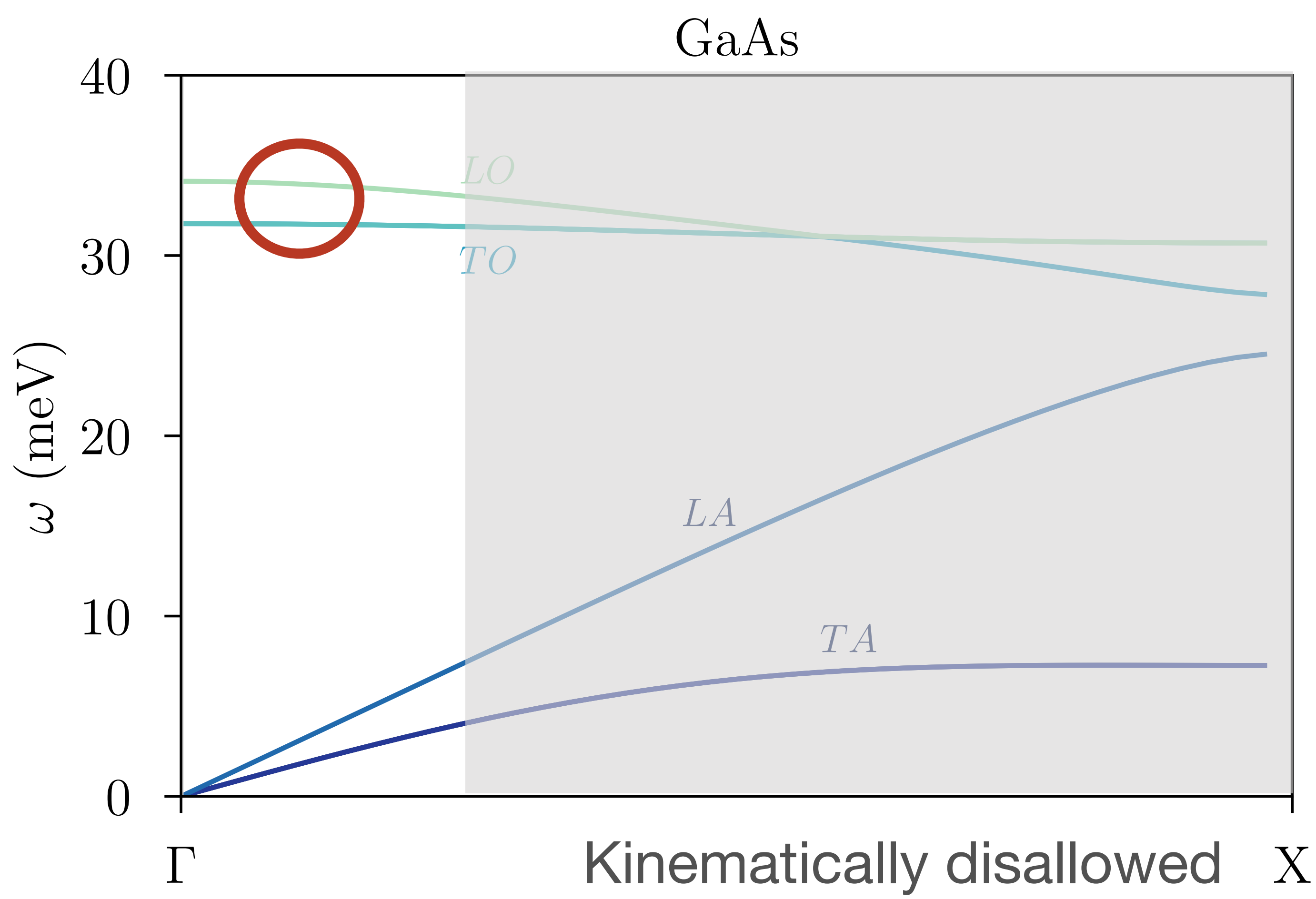
   Experimentally extremely challenging

2. Single optical:

   Strong destructive interference



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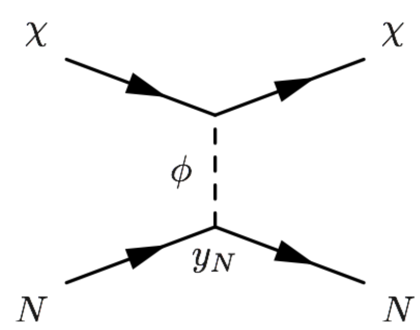


Which modes to use?

1. Single acoustic:

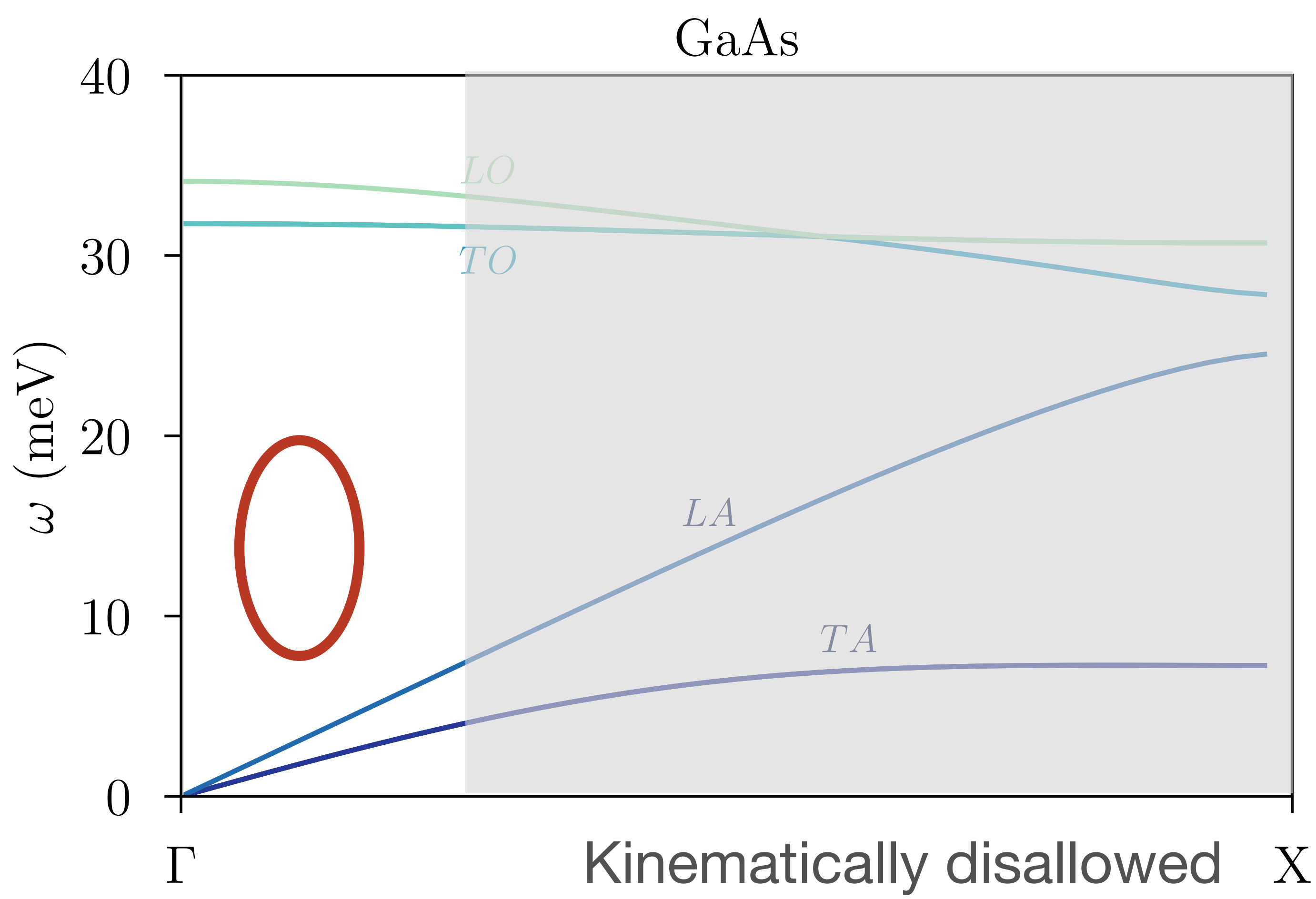
   Experimentally extremely challenging

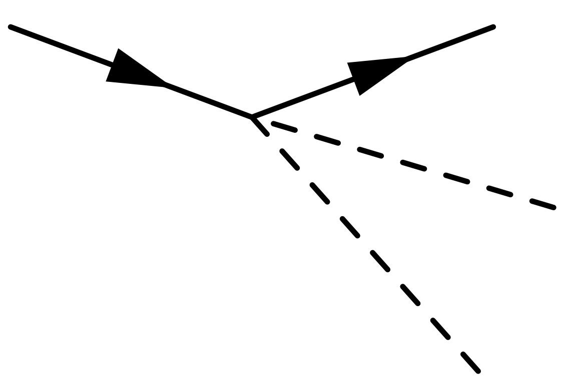
2. Single optical:

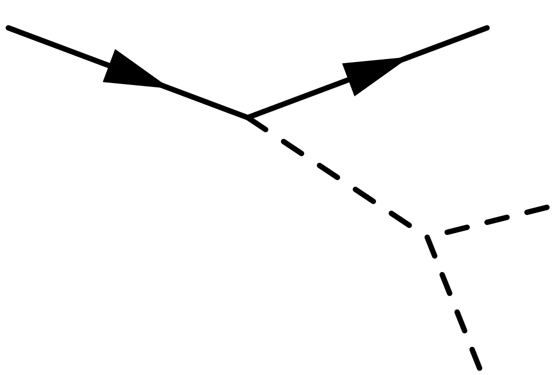
   Strong destructive interference

3. Double acoustic:

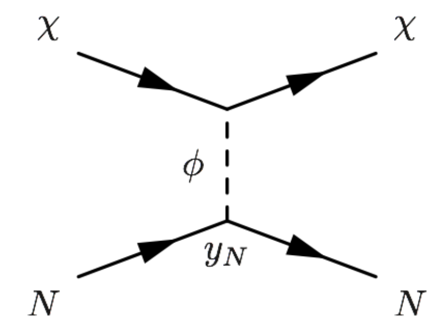
   Next-to-leading order







Scattering formalism



Scattering potential:

$$V(\mathbf{r}) = \frac{2\pi b_X}{m_X} \sum_{\ell,j} A_j \delta(\mathbf{r}_{\ell,j} - \mathbf{r})$$

Phonon form factor:

$$|F_\nu(\mathbf{q})|^2 = \left| \sum_j \frac{A_j}{\sqrt{m_j}} e^{-W_j(\mathbf{q})} \mathbf{q} \cdot \mathbf{e}_{\nu,j,\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{r}_j} \right|^2$$

SK, T. Lin, M. Pyle, K. Zurek: 1712.06598
S. Griffin, SK, T. Lin, M. Pyle, K. Zurek: 1807.10291
P. Cox, T. Melia, S. Rajendran: 1905.05575

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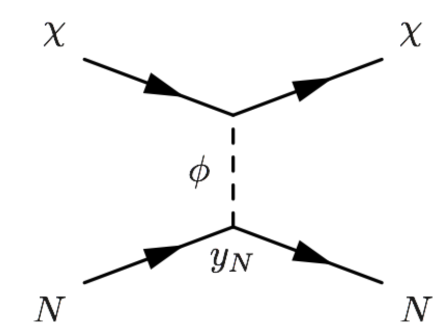

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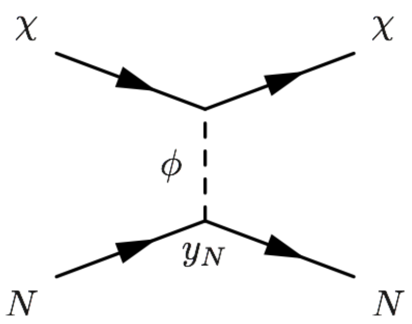
Daily modulation

Phonon form factor:

$$|F_\nu(\mathbf{q})|^2 = \left| \sum_j \frac{A_j}{\sqrt{m_j}} e^{-W_j(\mathbf{q})} \mathbf{q} \cdot \mathbf{e}_{\nu,j,\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{r}_j} \right|^2$$

SK, T. Lin, M. Pyle, K. Zurek: 1712.06598
S. Griffin, SK, T. Lin, M. Pyle, K. Zurek: 1807.10291
P. Cox, T. Melia, S. Rajendran: 1905.05575

Scattering formalism



Scattering potential:

$$V(\mathbf{r}) = \frac{2\pi b_X}{m_X} \sum_{\ell,j} A_j \delta(\mathbf{r}_{\ell,j} - \mathbf{r})$$

Phonon form factor:

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Acoustic

$$|F_\nu^{(ac)}(\mathbf{q})|^2 \approx \frac{q^2}{m_p} \left| \sum_j \sqrt{A_j} e^{-i\mathbf{q}\cdot\mathbf{r}_j} \right|^2$$

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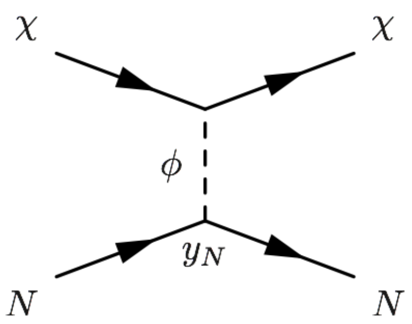

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Phonon form factor:

Daily modulation

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Acoustic

Optical

$$|F_\nu^{(ac)}(\mathbf{q})|^2 \approx \frac{q^2}{m_p} \left| \sum_j \sqrt{A_j} e^{-i\mathbf{q}\cdot\mathbf{r}_j} \right|^2$$

$$|F_\nu^{(opt)}(\mathbf{q})|^2 \approx \frac{q^2}{m_p} \times \mathcal{O}\left(q^2 a^2\right)$$

Lattice constant

Destructive interference kills leading order piece for optical phonons

SK, T. Lin, M. Pyle, K. Zurek: 1712.06598
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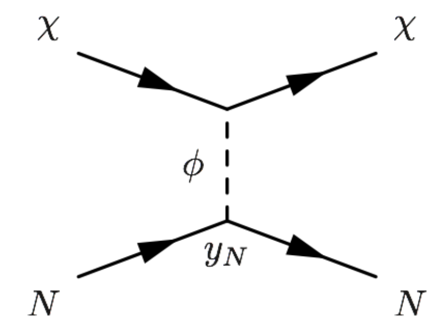
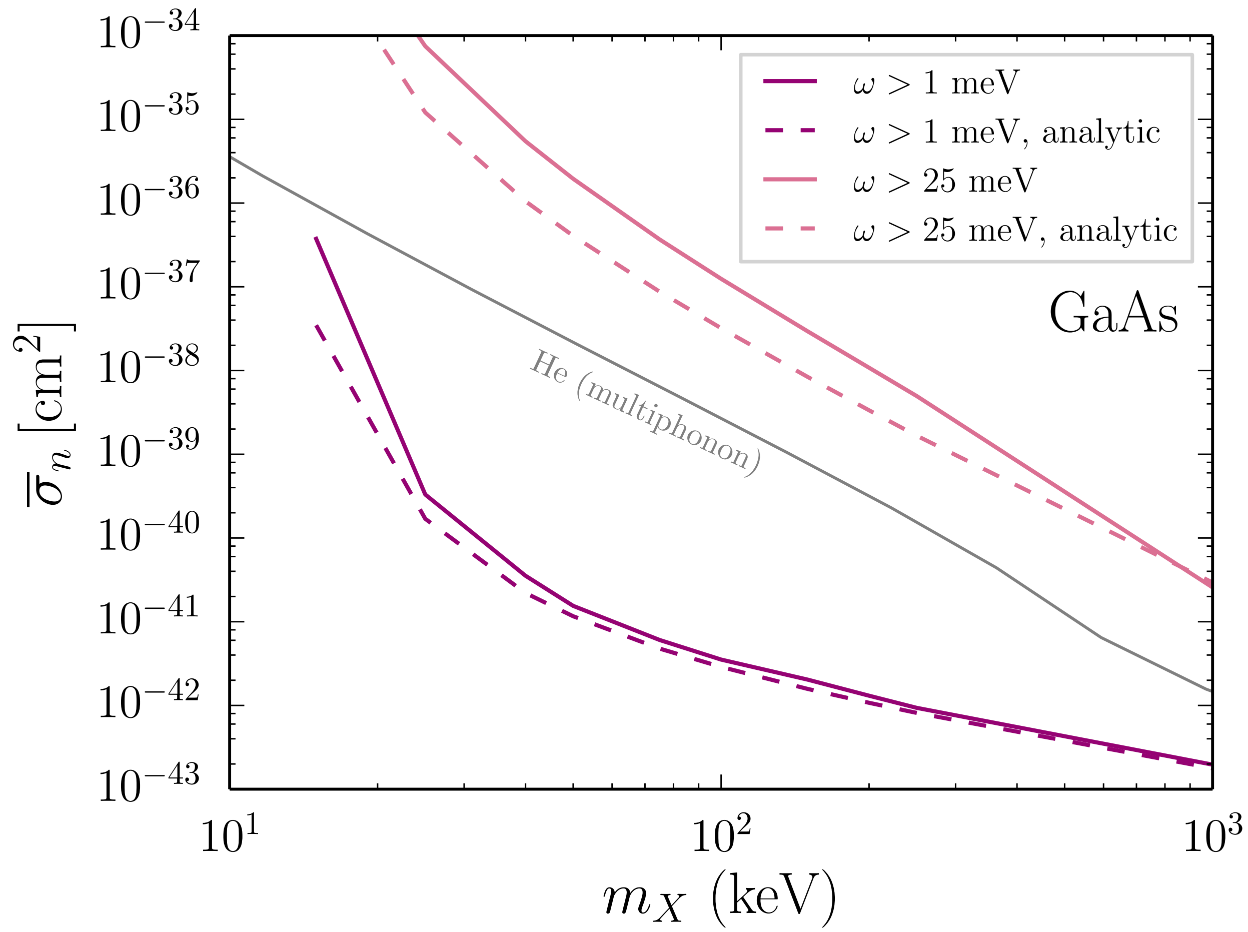
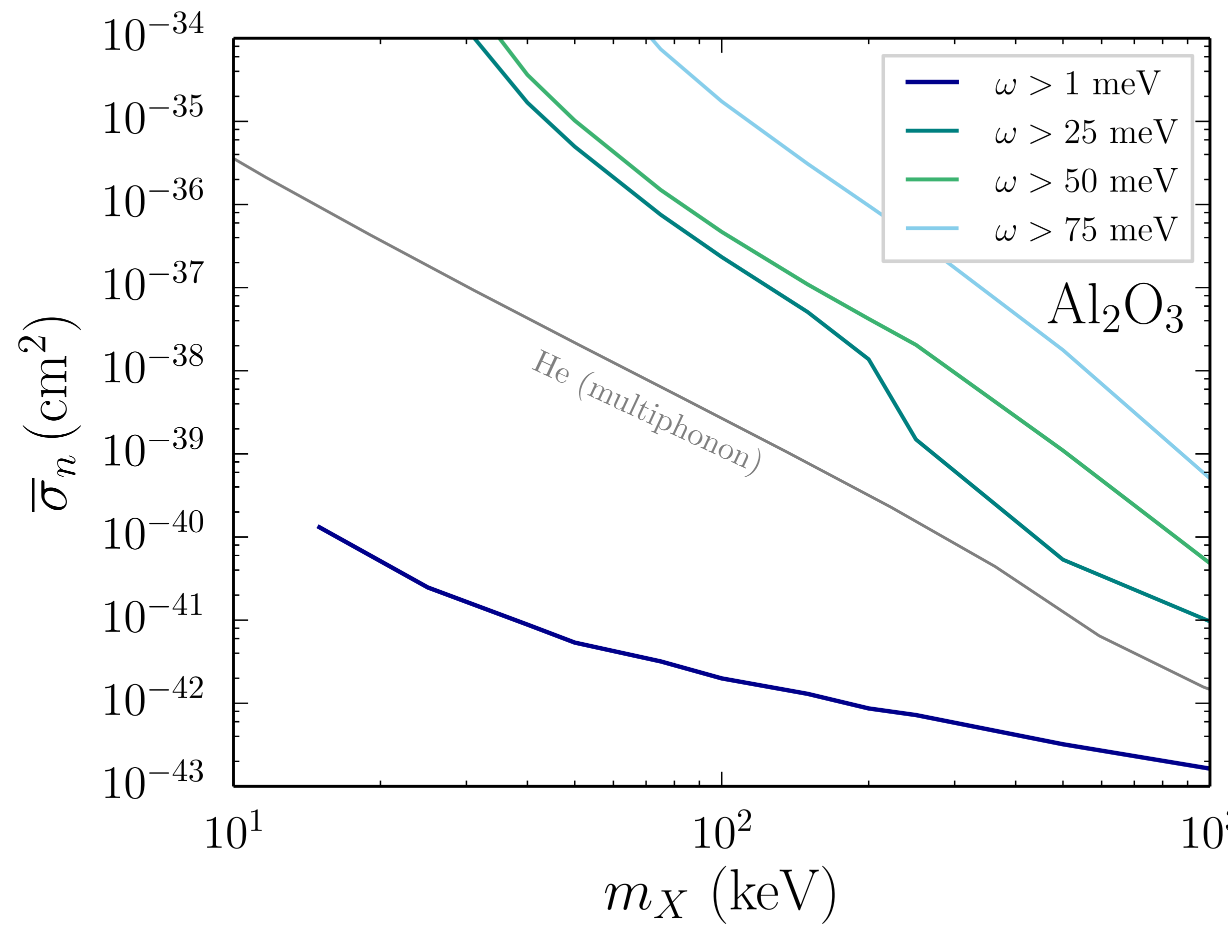
Rate for scalar mediator







Huge enhancement for the acoustic mode (but needs ultra low threshold)

SK, T. Lin, M. Pyle, K. Zurek: 1712.06598
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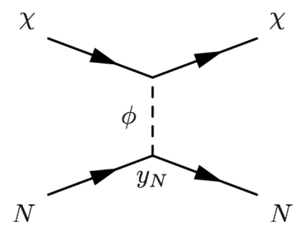
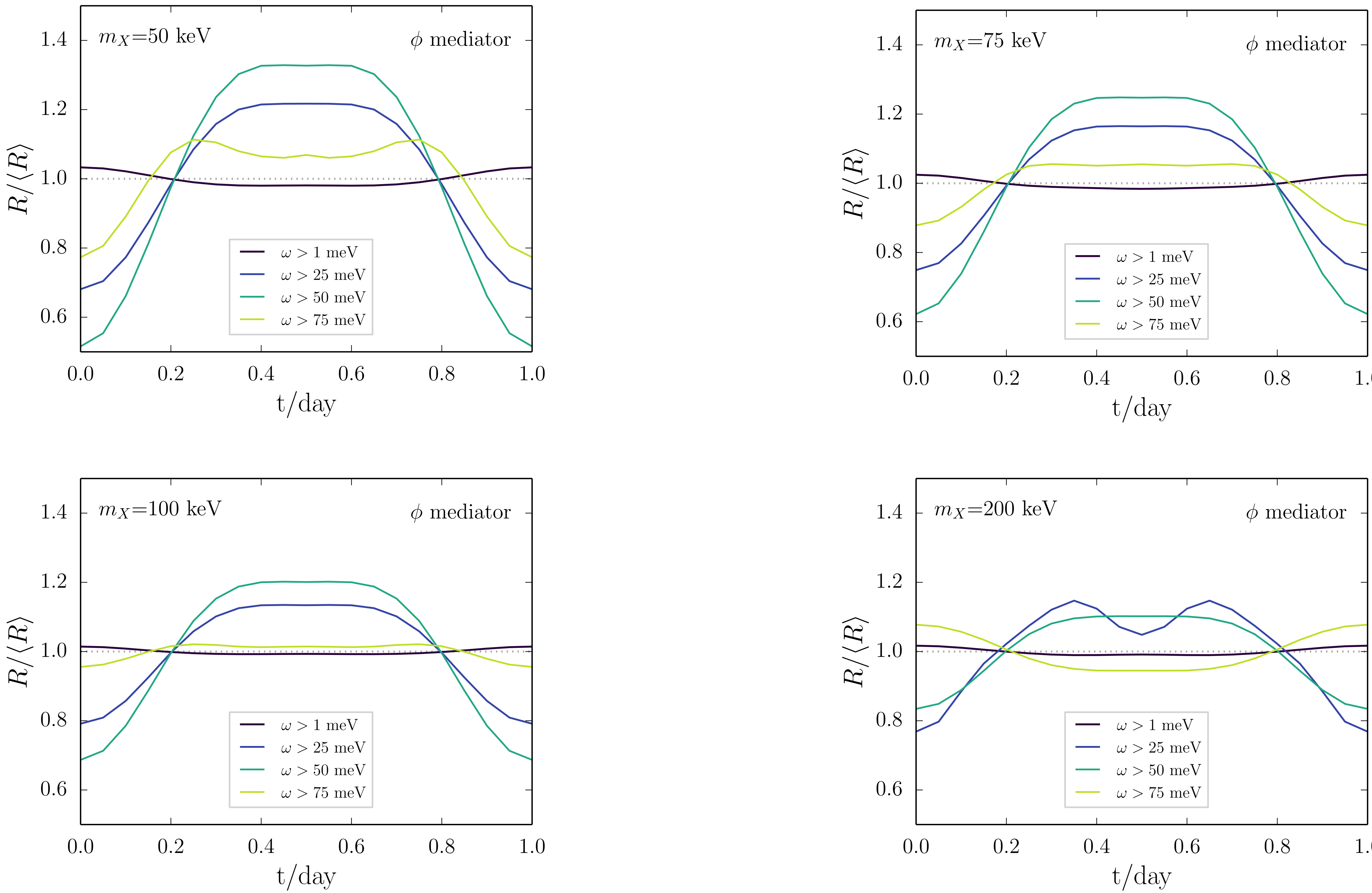
Daily modulation (Sapphire)

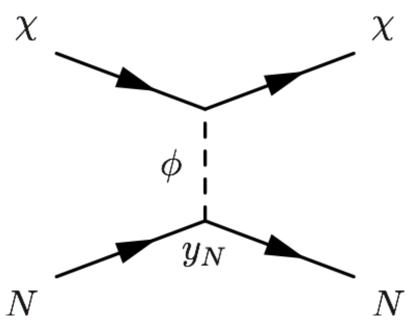




Qualitatively different from dark photon mediator!

S. Griffin, SK, T. Lin, M. Pyle, K. Zurek: 1807.10291

Kinematics



Which modes to use?

1. Single acoustic:

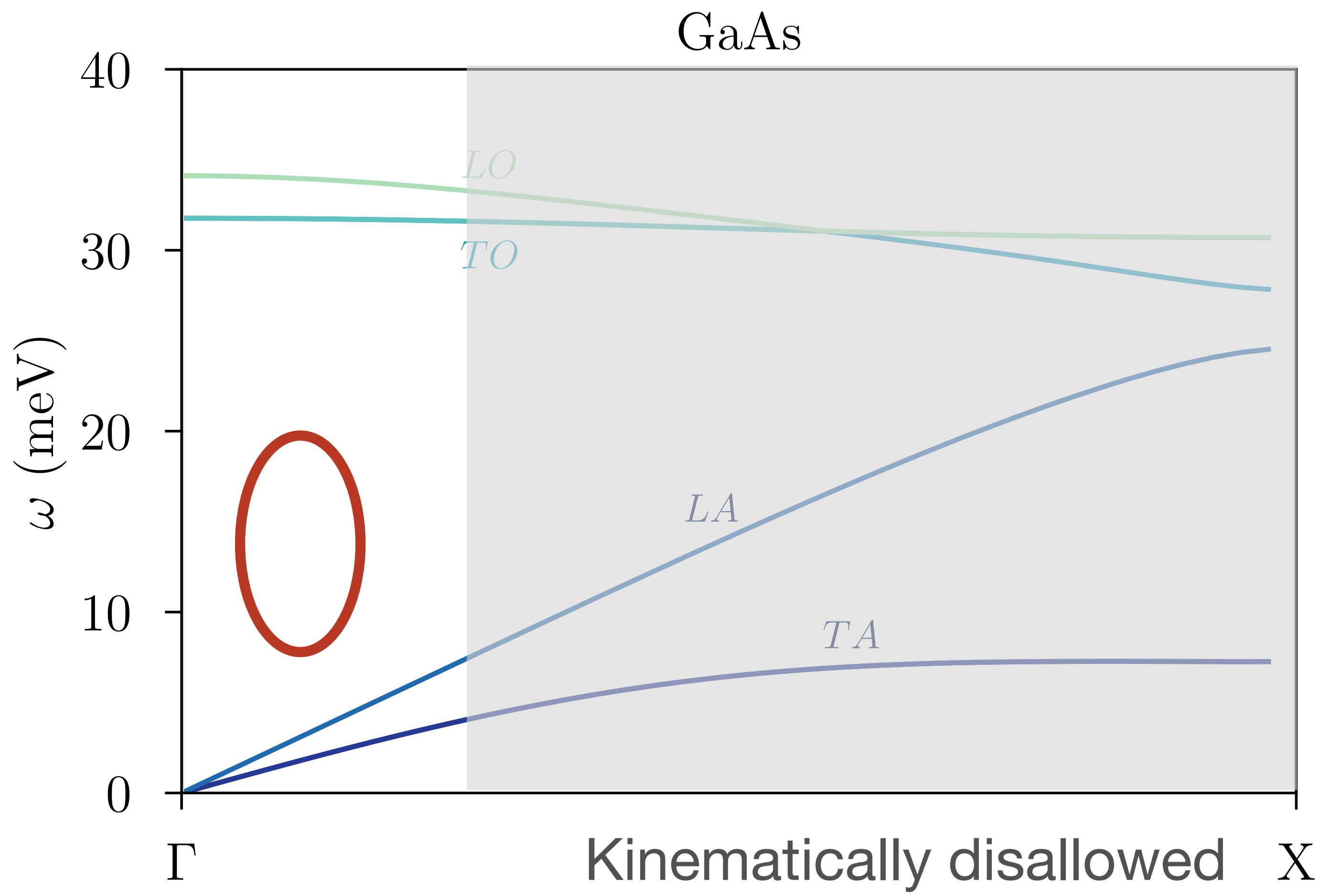
   Experimentally extremely challenging

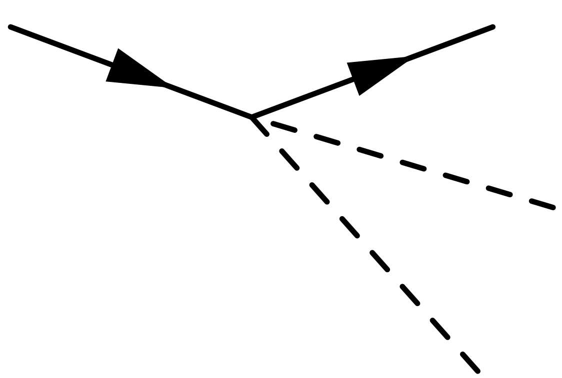
2. Single optical:

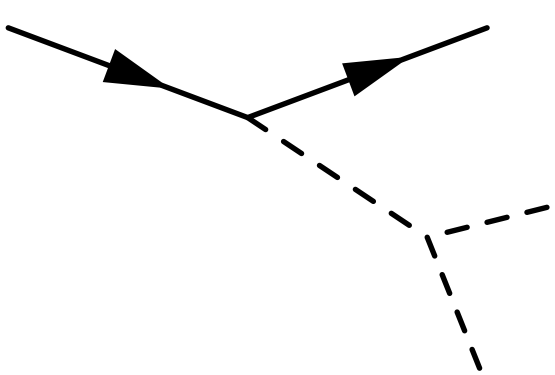
   Strong destructive interference

3. Double acoustic:

   Next-to-leading order

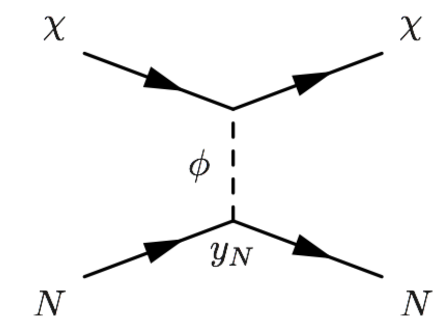






In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

Phonon perturbation theory



Scattering potential:

$$V(\mathbf{r}) = \frac{2\pi b_X}{m_X} \sum_{\ell,j} A_j \delta(\mathbf{r}_{\ell,j} - \mathbf{r}) \qquad \Rightarrow \qquad V(\mathbf{q}) = \frac{2\pi b_X}{m_X} \sum_{\ell,j} A_j e^{i\mathbf{q}\cdot\mathbf{r}_{\ell,j}}$$

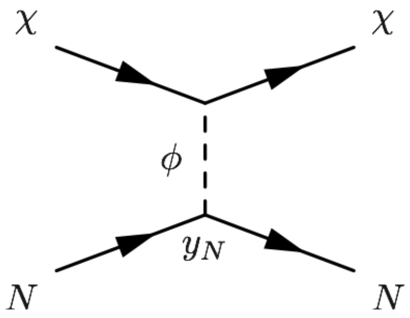
Matrix element:

$$|\mathcal{M}|^2 \sim \left| \langle \Phi_f | \sum_{\ell,l} A_j e^{i\mathbf{q}\cdot\mathbf{r}_{\ell,j}} |0\rangle \right|^2$$

Double expansion in the phonon self-coupling and momentum transfer

In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

Phonon perturbation theory



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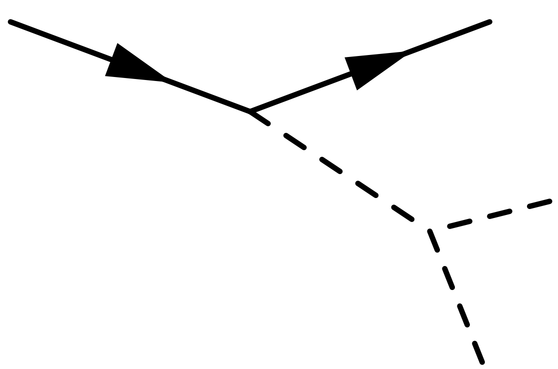
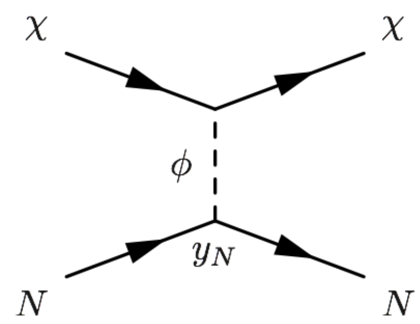


$$\mathcal{O}\left(\lambda^2 q^2\right)$$

Double expansion in the phonon self-coupling and momentum transfer

In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

Phonon perturbation theory



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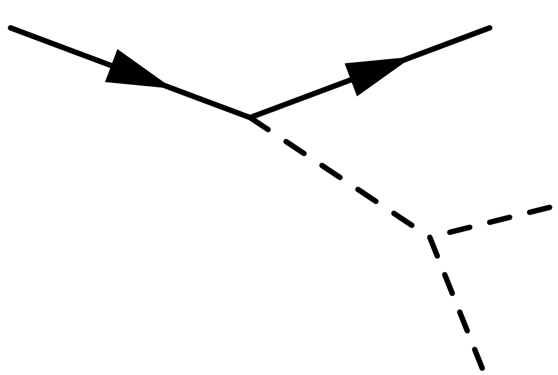
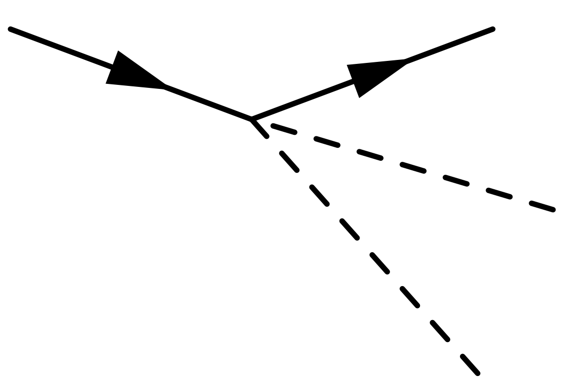


$\mathcal{O}\left(\lambda^2 q^2\right)$

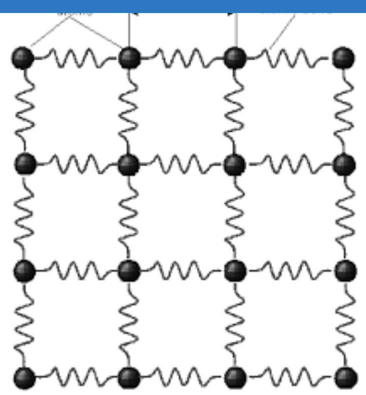


$\mathcal{O}\left(q^4\right)$

Double expansion in the phonon self-coupling and momentum transfer

In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

Phonon self-interactions



Potential:

$$\mathcal{V} = \mathcal{V}^{(0)} + \cancel{\sum_{\mathbf{l},j} \boldsymbol{\mathcal{V}}^{(1)}_{\mathbf{l},j}} \cdot \mathbf{u}_{j,\mathbf{l}} + \frac{1}{2} \sum_{\mathbf{l},\mathbf{l}',j,j'} \mathbf{u}_{j,\mathbf{l}} \cdot \boldsymbol{\mathcal{V}}^{(2)}_{\mathbf{l},j,\mathbf{l}',j'} \cdot \mathbf{u}_{j',\mathbf{l}'} + \cdots$$

In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

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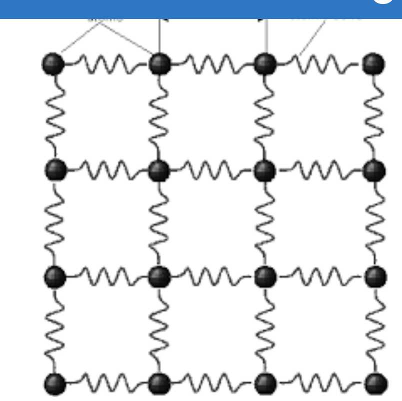

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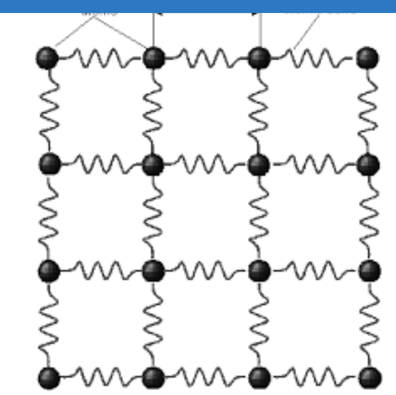
Higher order terms give effective Hamiltonian (in isotropic approximation):

$$\delta H = \int d^3\mathbf{r}\ \frac{1}{2}(\beta+\lambda)u_{ii}u_{jk}u_{jk} + (\gamma+\mu)u_{ij}u_{ki}u_{kj} + \frac{\alpha}{3!}u_{ii}u_{jj}u_{kk} + \frac{\beta}{2}u_{ii}u_{jk}u_{kj} + \frac{\gamma}{3}u_{ij}u_{jk}u_{ki}$$

With $u_{ij} \equiv \partial_i u_j$

In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

Phonon self-interactions



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With $u_{ij} \equiv \partial_i u_j$

Couplings arise from expanding the potential beyond harmonic approximation

λ, μ ⟷ Bulk modulus & Young's modulus

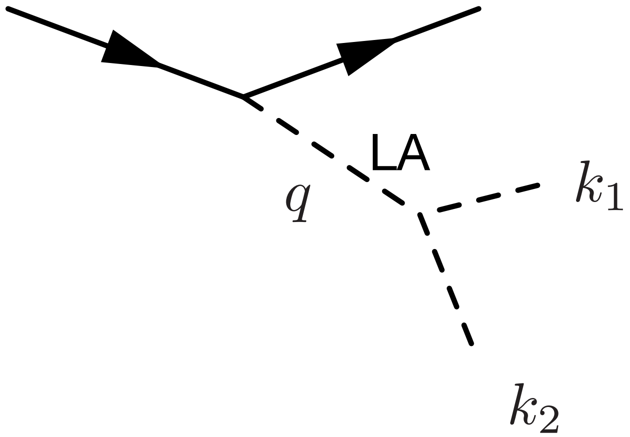
α, β, γ ⟷ Third order elastic constants

Can be measured!

In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

Self-interactions matrix element

$$
\begin{aligned}
\widetilde{\mathcal{M}} =& (\beta+\lambda)\Big[(\mathbf{q}\cdot\mathbf{e})(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{e}_1\cdot\mathbf{e}_2) + (\mathbf{k}_1\cdot\mathbf{e}_1)(\mathbf{q}\cdot\mathbf{k}_2)(\mathbf{e}\cdot\mathbf{e}_2) + (\mathbf{k}_2\cdot\mathbf{e}_2)(\mathbf{k}_1\cdot\mathbf{q})(\mathbf{e}_1\cdot\mathbf{e})\Big] \\
&+ (\gamma+\mu)\Big[(\mathbf{q}\cdot\mathbf{k}_2)\big[(\mathbf{k}_2\cdot\mathbf{e}_1)(\mathbf{e}_2\cdot\mathbf{e}) + (\mathbf{k}_2\cdot\mathbf{e})(\mathbf{e}_2\cdot\mathbf{e}_1)\big] \\
&\qquad + (\mathbf{k}_2\cdot\mathbf{k}_1)\big[(\mathbf{q}\cdot\mathbf{e}_1)(\mathbf{e}_2\cdot\mathbf{e}) + (\mathbf{q}\cdot\mathbf{e}_2)(\mathbf{e}\cdot\mathbf{e}_1)\big] \\
&\qquad + (\mathbf{q}\cdot\mathbf{k}_2)\big[(\mathbf{k}_1\cdot\mathbf{e}_2)(\mathbf{e}_1\cdot\mathbf{e}) + (\mathbf{k}_1\cdot\mathbf{e})(\mathbf{e}_1\cdot\mathbf{e}_2)\big]\Big] \\
&+ \alpha(\mathbf{q}\cdot\mathbf{e})(\mathbf{k}_1\cdot\mathbf{e}_1)(\mathbf{k}_2\cdot\mathbf{e}_2) \\
&+ \beta\Big[(\mathbf{k}_1\cdot\mathbf{e}_1)(\mathbf{q}\cdot\mathbf{e}_2)(\mathbf{k}_2\cdot\mathbf{e}) + (\mathbf{q}\cdot\mathbf{e})(\mathbf{k}_1\cdot\mathbf{e}_2)(\mathbf{k}_2\cdot\mathbf{e}_1) + (\mathbf{k}_2\cdot\mathbf{e}_2)(\mathbf{q}\cdot\mathbf{e}_1)(\mathbf{k}_1\cdot\mathbf{e})\Big] \\
&+ \gamma\Big[(\mathbf{q}\cdot\mathbf{e}_1)(\mathbf{k}_1\cdot\mathbf{e}_2)(\mathbf{k}_2\cdot\mathbf{e}) + (\mathbf{q}\cdot\mathbf{e}_1)(\mathbf{k}_1\cdot\mathbf{e})(\mathbf{k}_2\cdot\mathbf{e}_1)\Big]
\end{aligned}
$$



In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

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&\qquad + (\mathbf{q}\cdot\mathbf{k}_2)\big[(\mathbf{k}_1\cdot\mathbf{e}_2)(\mathbf{e}_1\cdot\mathbf{e}) + (\mathbf{k}_1\cdot\mathbf{e})(\mathbf{e}_1\cdot\mathbf{e}_2)\big]\Big] \\
&+ \alpha(\mathbf{q}\cdot\mathbf{e})(\mathbf{k}_1\cdot\mathbf{e}_1)(\mathbf{k}_2\cdot\mathbf{e}_2) \\
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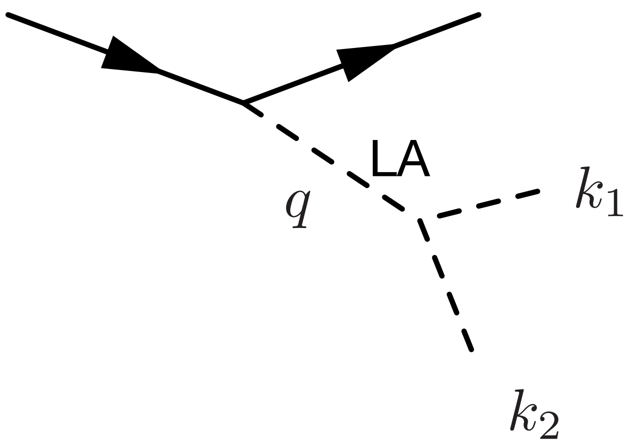
\end{aligned}
$$

4 non-vanishing channels:

- LA* → LA LA
- LA* → LA TA
- LA* → TA TA (polarized in momentum-plane)
- LA* → TA TA (polarized orthogonal to momentum-plane)

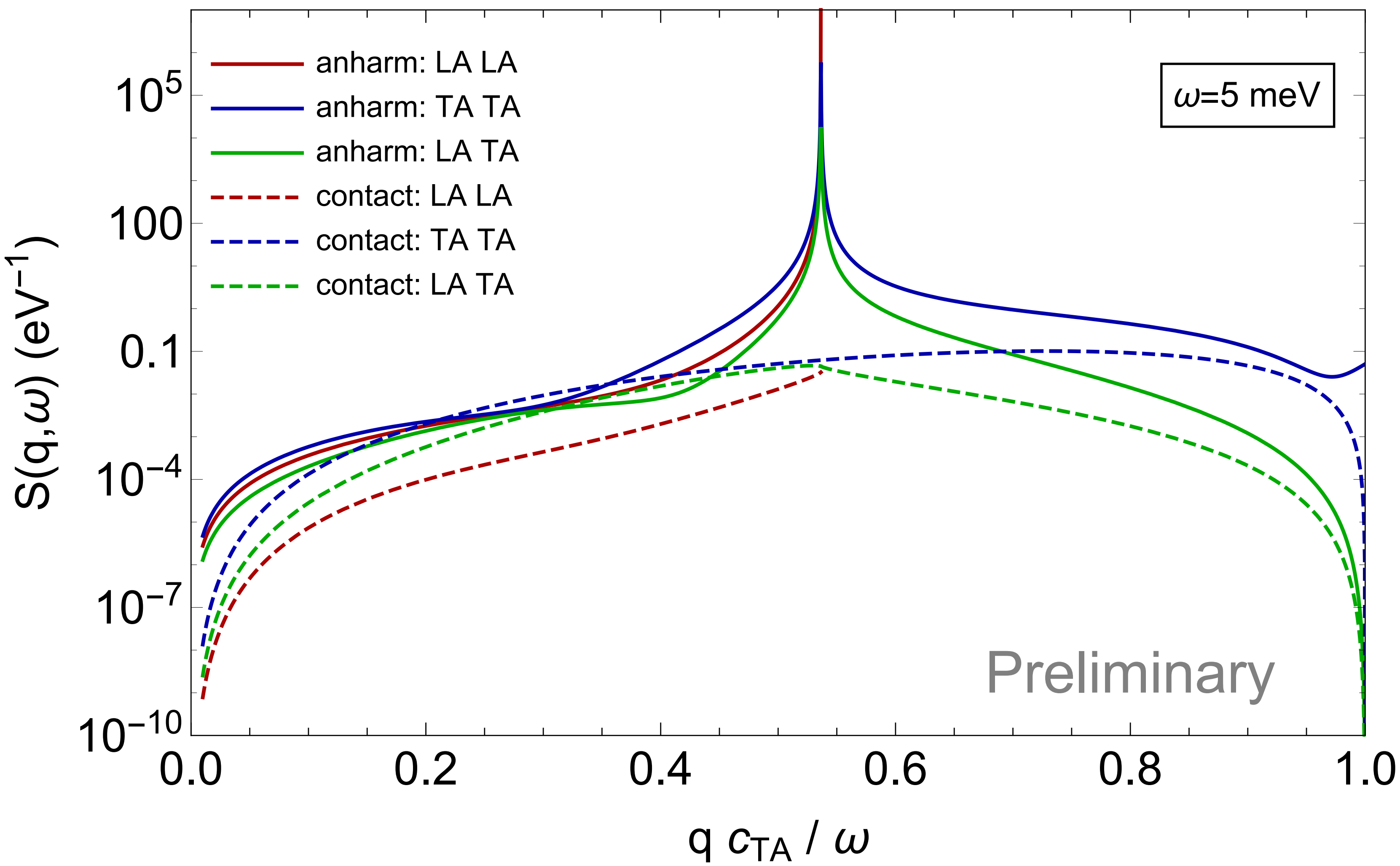


In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

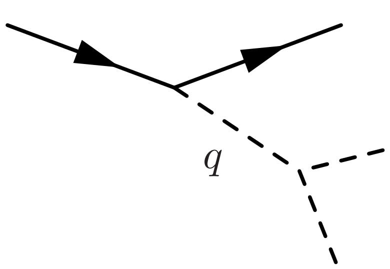
Comparing different channels

Example GaAs:

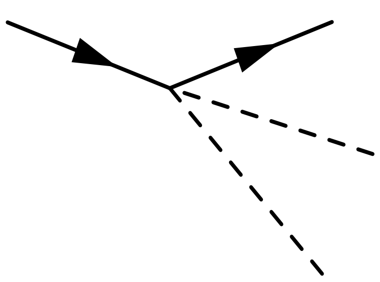
Differential scattering rate



Anharmonic:



Contact:



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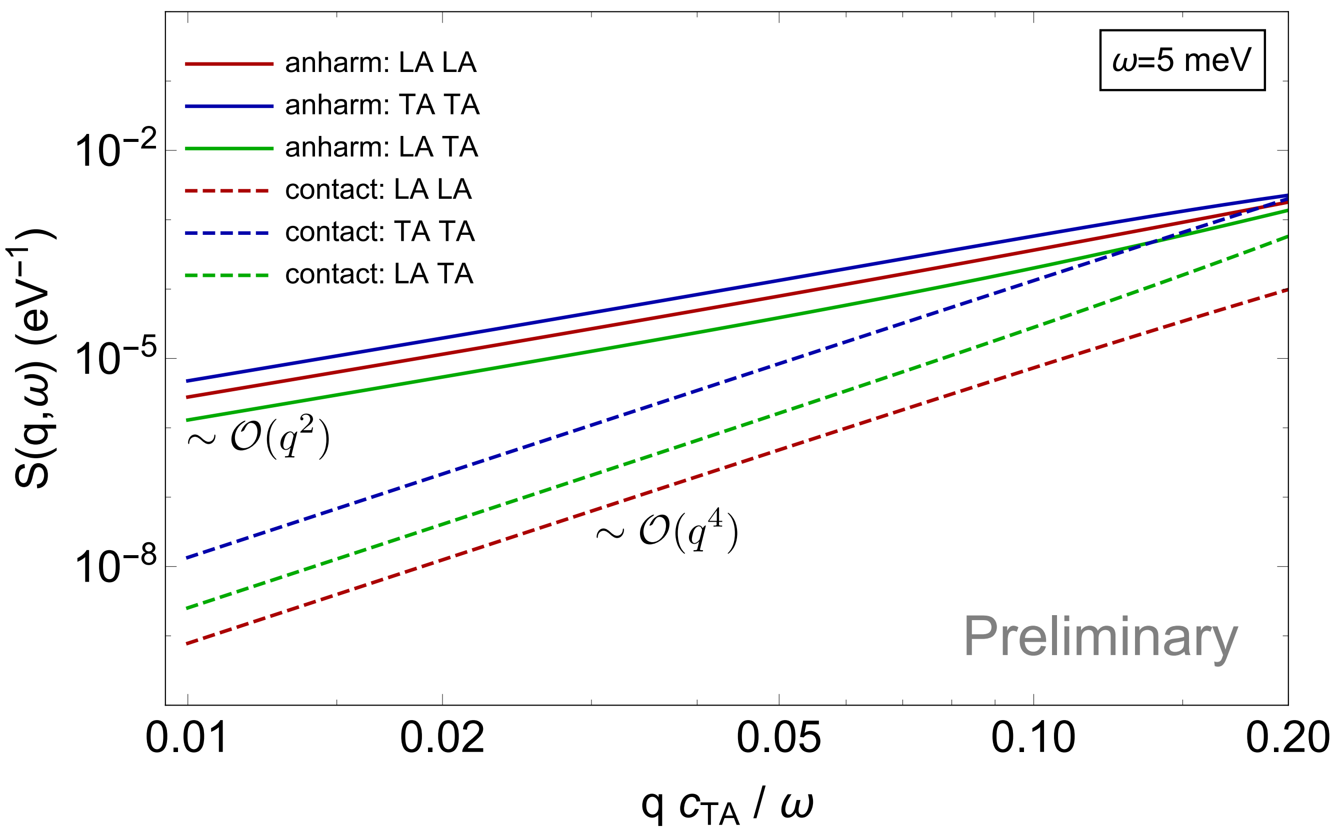

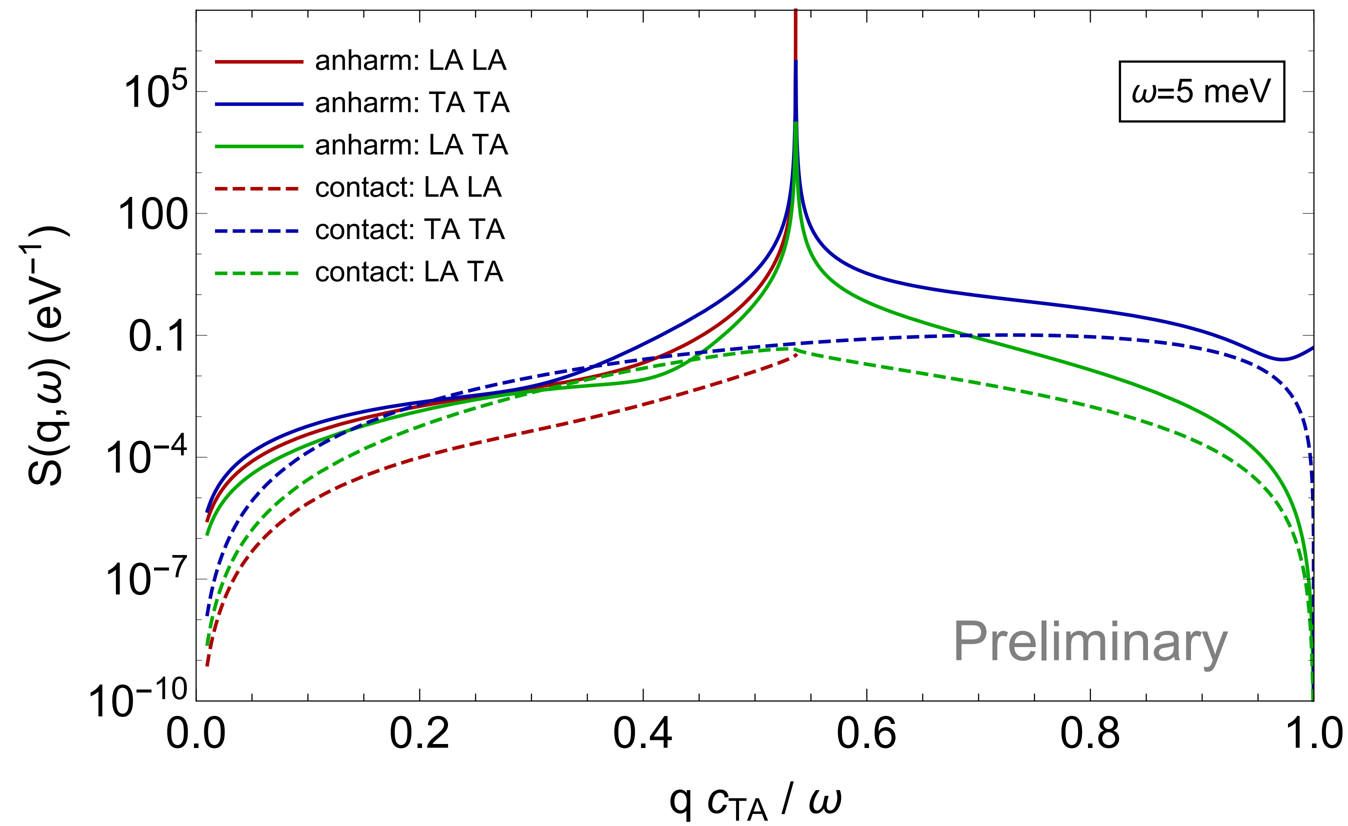

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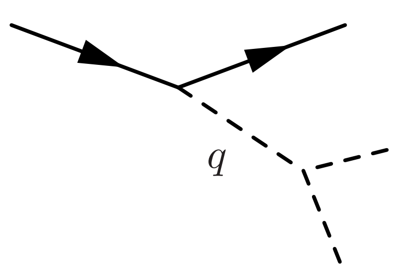

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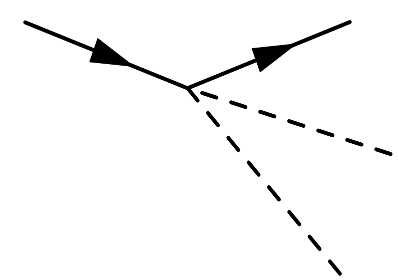
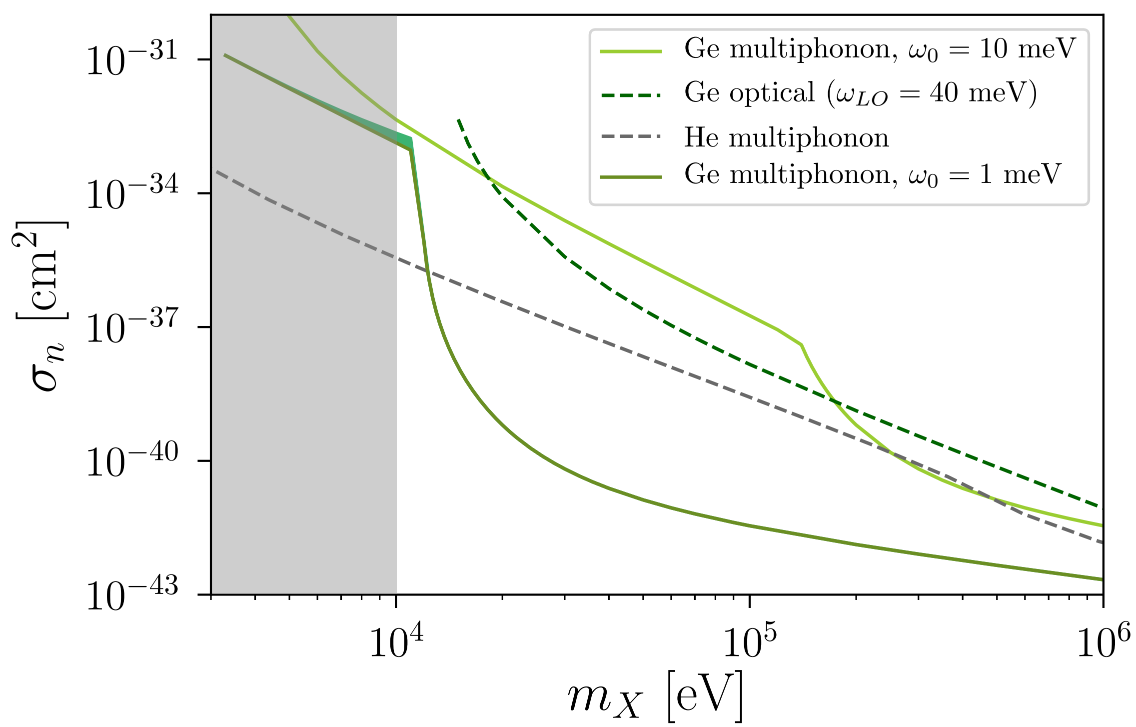
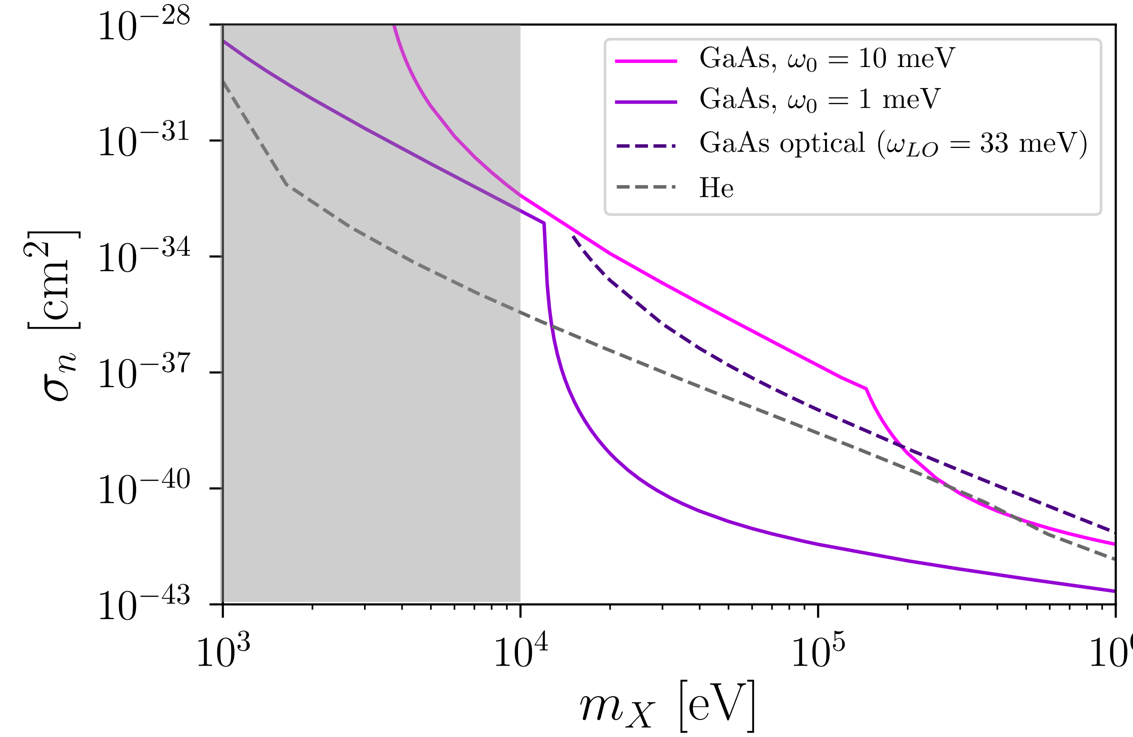


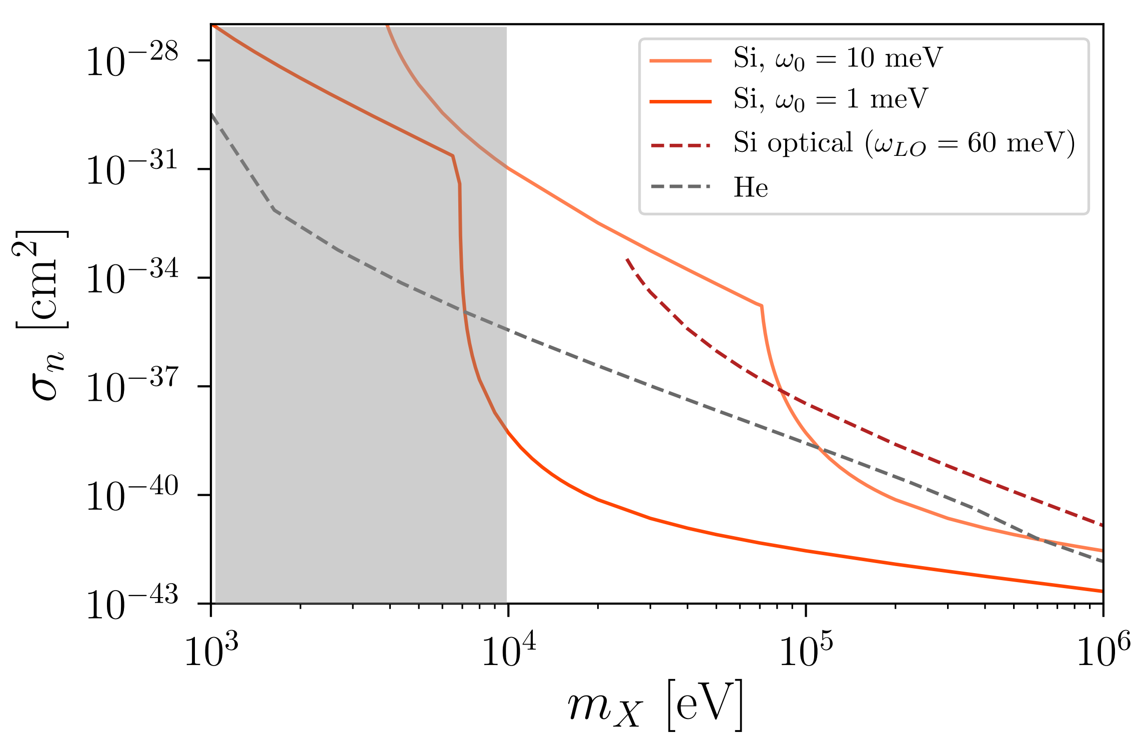
The anharmonic contribution tends to dominate

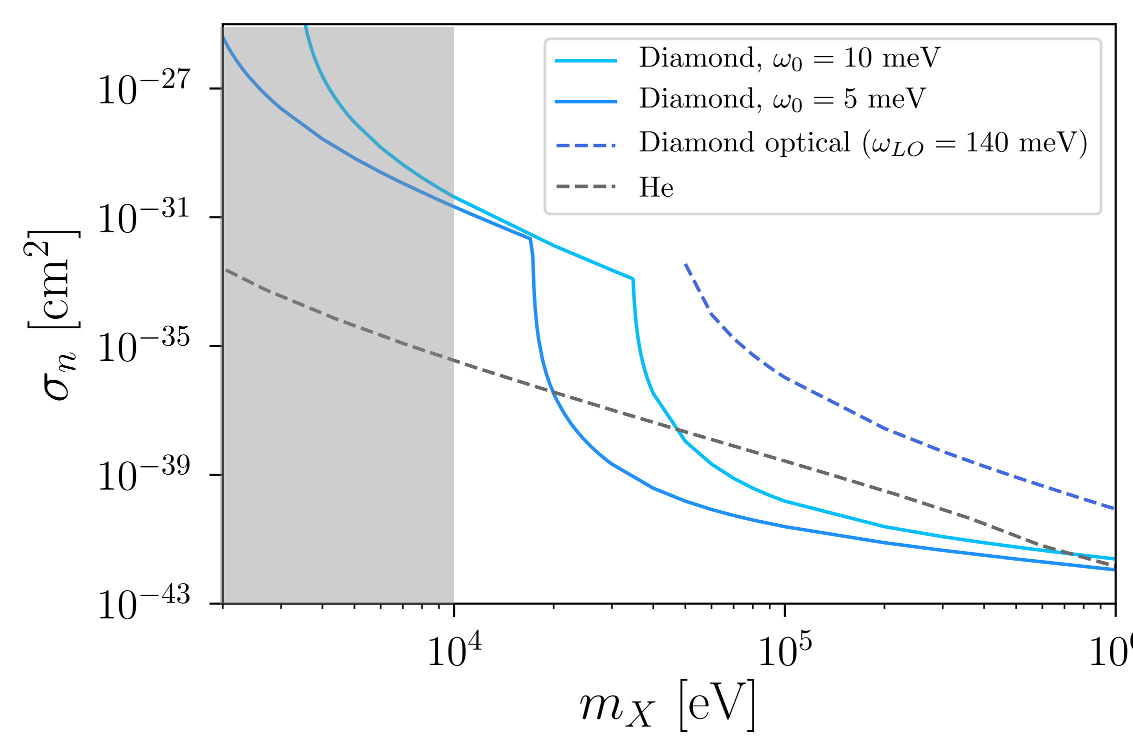
In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

Preliminary results









In progress with B. Campbell-Deem, T. Lin, P. Cox, T. Melia

Comment on superfluid helium

<u>Crystal</u>

$$\langle \mathbf{q} \,|\, \delta H \,|\, \mathbf{k}_1,\, \mathbf{k}_2 \rangle \sim (\mathbf{q}\cdot\mathbf{e})\;(\mathbf{k}_1\cdot\mathbf{e}_1)\;(\mathbf{k}_2\cdot\mathbf{e}_2) + \cdots$$
$$\sim q$$

$\longrightarrow$ Rate $\sim \mathcal{O}(q^2)$

Translations spontaneously broken, so proportional to momenta

SK, T. Lin, K. Zurek: 1611.06228
A. Caputo, A. Esposito, A. Polosa: 1907.10635

Comment on superfluid helium

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$$\sim q$$

$\longrightarrow$ Rate $\sim \mathcal{O}(q^2)$

Translations spontaneously broken, so proportional to momenta

<u>Superfluid Helium</u>

$$\langle \mathbf{q} \,|\, \delta H \,|\, \mathbf{k}_1,\, \mathbf{k}_2 \rangle \sim \lambda \mathbf{q}\cdot(\mathbf{k}_1+\mathbf{k}_2) + \lambda' \omega\, \mathbf{k}_1\cdot\mathbf{k}_2 + \lambda'' \omega\omega_1\omega_2 + \cdots$$
$$\sim q^2$$

Translations NOT spontaneously broken, but mysterious cancellation anyways…

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Comment on superfluid helium

Crystal

$$\langle \mathbf{q} \,|\, \delta H \,|\, \mathbf{k}_1,\, \mathbf{k}_2 \rangle \sim (\mathbf{q}\cdot\mathbf{e})\,(\mathbf{k}_1\cdot\mathbf{e}_1)\,(\mathbf{k}_2\cdot\mathbf{e}_2) + \cdots \longrightarrow \text{Rate} \sim \mathcal{O}(q^2)$$

$$\sim q$$

Translations spontaneously broken, so proportional to momenta

Superfluid Helium

$$\langle \mathbf{q} \,|\, \delta H \,|\, \mathbf{k}_1,\, \mathbf{k}_2 \rangle \sim \lambda \mathbf{q}\cdot(\mathbf{k}_1+\mathbf{k}_2) + \cancel{\lambda'\omega\,\mathbf{k}_1}\cdot\mathbf{k}_2 + \cancel{\lambda''\omega\omega_1}\omega_2 + \cdots \longrightarrow \text{Rate} \sim \mathcal{O}(q^4)$$

$$\sim q^2$$

Translations NOT spontaneously broken, but mysterious cancellation anyways…

SK, T. Lin, K. Zurek: 1611.06228
A. Caputo, A. Esposito, A. Polosa: 1907.10635

Comment on superfluid helium

Crystal

$$\langle \mathbf{q} \,|\, \delta H \,|\, \mathbf{k}_1,\, \mathbf{k}_2 \rangle \sim \underbrace{(\mathbf{q}\cdot\mathbf{e})}_{\sim q}\, (\mathbf{k}_1\cdot\mathbf{e}_1)\, (\mathbf{k}_2\cdot\mathbf{e}_2) + \cdots \longrightarrow \text{Rate} \sim \mathcal{O}(q^2)$$

Translations spontaneously broken, so proportional to momenta

Superfluid Helium

$$\langle \mathbf{q} \,|\, \delta H \,|\, \mathbf{k}_1,\, \mathbf{k}_2 \rangle \sim \underbrace{\lambda \mathbf{q}\cdot(\mathbf{k}_1+\mathbf{k}_2)}_{\sim q^2} + \cancel{\lambda'\omega\,\mathbf{k}_1\cdot\mathbf{k}_2} + \cancel{\lambda''\omega\omega_1\omega_2} + \cdots \longrightarrow \text{Rate} \sim \mathcal{O}(q^4)$$

Translations NOT spontaneously broken, but mysterious cancellation anyways…

Crossing symmetry then implies that the leading term in the q-expansion cancels

(The self-interaction is however much stronger in helium, overcoming this additional suppression)

SK, T. Lin, K. Zurek: 1611.06228
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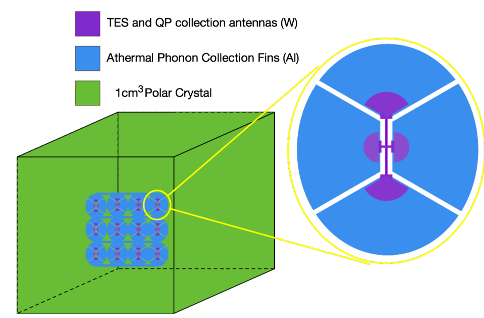
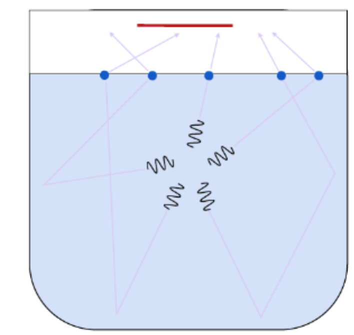
Summary



Crystals



Superfluid Helium

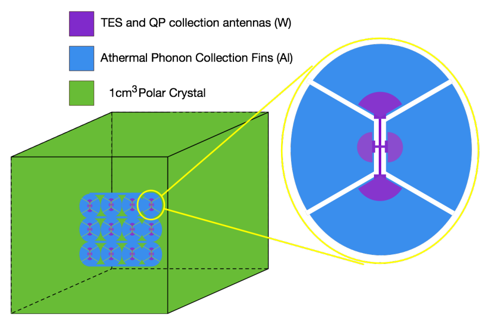
<u>Scattering</u>

		Crystals	Superfluid Helium
Coupling to charge	Single optical	Large [1,2]	X
	Single acoustic	Tiny [1,2]	Tiny [b]

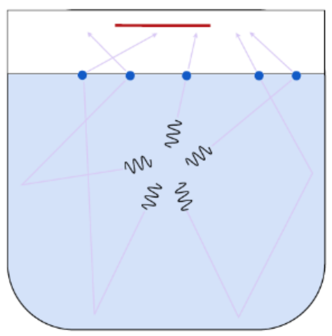
[1] **SK**, T. Lin, M. Pyle, K. Zurek: 1712.06598
[2] S. Griffin, **SK**, T. Lin, M. Pyle, K. Zurek: 1807.10291
[3] P. Cox, T. Melia, S. Rajendran: 1905.05575
[4] B. Campbell-Deem, P. Cox, T. Melia, **SK**, T. Lin: to appear

[a] K. Schutz, K. Zurek: 1604.08206
[b] **SK**, T. Lin, K. Zurek: 1611.06228
[c] F. Acanfora, A. Esposito, A. Polosa: 1902.02361
[d] A. Caputo, A. Esposito, A. Polosa: 1907.10635

Summary



Crystals



Superfluid Helium

Scattering

		Crystals	Superfluid Helium
Coupling to charge	Single optical	Large [1,2]	X
	Single acoustic	Tiny [1,2]	Tiny [b]
Coupling to mass	Single acoustic	experimentally hard [1,2]	experimentally impossible
	Single optical	small [1,2,3]	X
	multiphonon	small [4]	small [a,b,c,d]

[1] **SK**, T. Lin, M. Pyle, K. Zurek: 1712.06598
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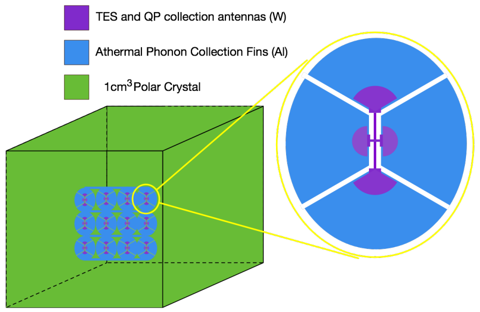
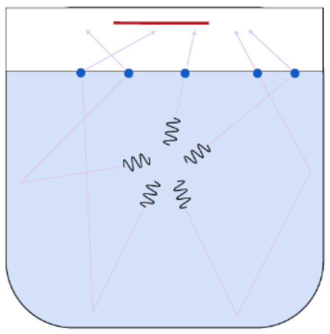


Crystals



Superfluid Helium

Scattering

		Crystals	Superfluid Helium
Coupling to charge	Single optical	Large [1,2]	X
	Single acoustic	Tiny [1,2]	Tiny [b]
Coupling to mass	Single acoustic	experimentally hard [1,2]	experimentally impossible
	Single optical	small [1,2,3]	X
	multiphonon	small [4]	small [a,b,c,d]

Absorption

	Crystals	Superfluid Helium
Dark photon	Large [2]	Tiny [b]
Scalar	Future work	Future work

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Thank you!