Eigenstate Thermalization in Sachdev-Ye-Kitaev model & Schwarzian theory

by

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[basesd on 1901.xxxxx]

with Julian Sonner and Manuel Vielma

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Closed Quantum Systems

Quantum Mechanics is unitary!

$$|\psi(t)\rangle = \mathcal{U}(t, t_0)|\psi(t_0)\rangle$$

How come we observe
thermal physics?

How come we observe
black hole formation?

Plan of the talk

- ✓ Review of ETH
- ✓ Review of Sachdev-Ye-Kitaev model
- ✓ Numerical studies of ETH in SYK model
- ✓ ETH in the Schwarzian sector of the SYK model
- ✓ ETH in the Conformal sector of the SYK model
- ✓ A few thoughts on the bulk duals
- ✓ Summary and conclusion

Eigenstate Thermalization Hypothesis (ETH)

Quantum Thermalization

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Classical thermalization: **ergodicity** &$\Leftarrow$ chaos

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v.s.

Quantum thermalization: Eigenstate Thermalisation Hypothesis

$$\langle m|\mathcal{O}|n\rangle = \overline{\mathcal{O}}_{\mathrm{mc}}(\overline{E})\delta_{mn} + e^{-S(\overline{E})/2} f(\overline{E},\omega) R_{mn}$$

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micro-canonical ensemble   avg. energy of ensemble

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A generic excited state will then thermalise by **dephasing**

$$\langle\psi|\mathcal{O}|\psi\rangle = \sum_{i,j} c_i^* c_j e^{it(E_i-E_j)} \mathcal{O}_{ij} \longrightarrow \overline{O}\left(\bar{E}\right) + e^{-S}$$

expectation value of non-extensive operator

dephasing: spectral chaos

on average thermal up to exponential in S

Sachdev-Ye-Kitaev (SYK) model: a Review

SYK model: a new Guinea Pig

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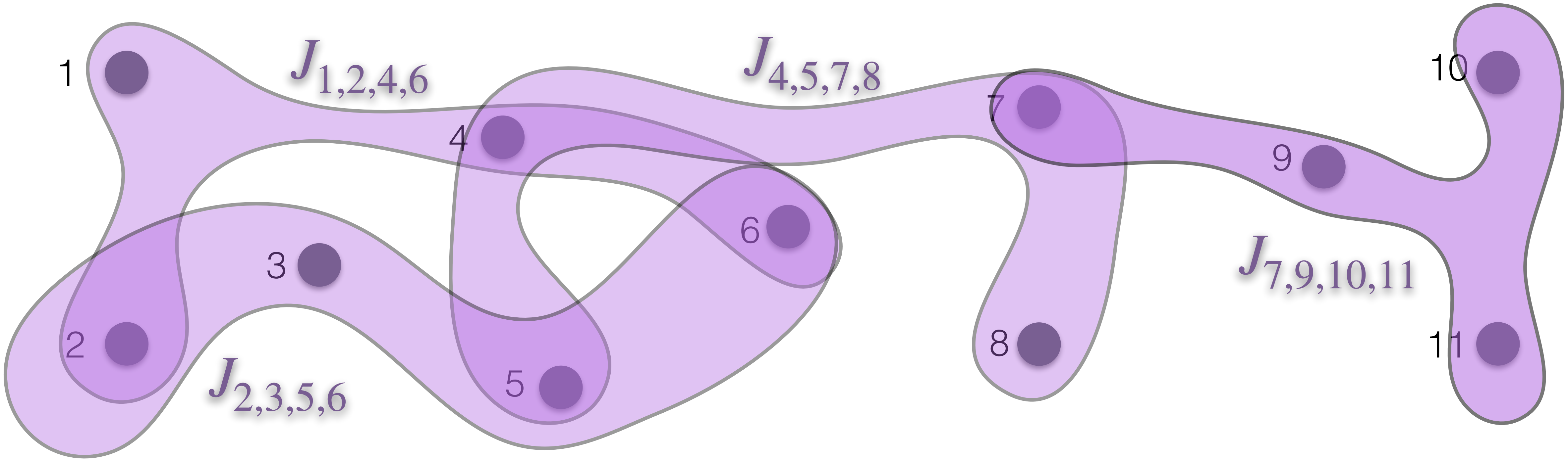

[Sachdev-Ye ’93;
Kitaev ’15]

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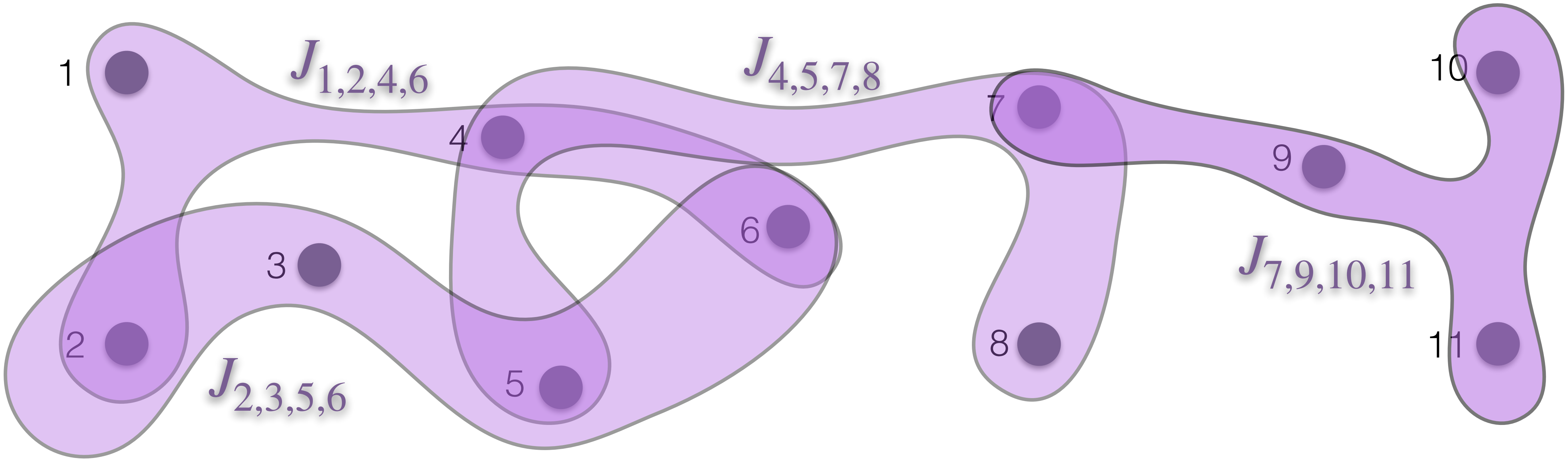

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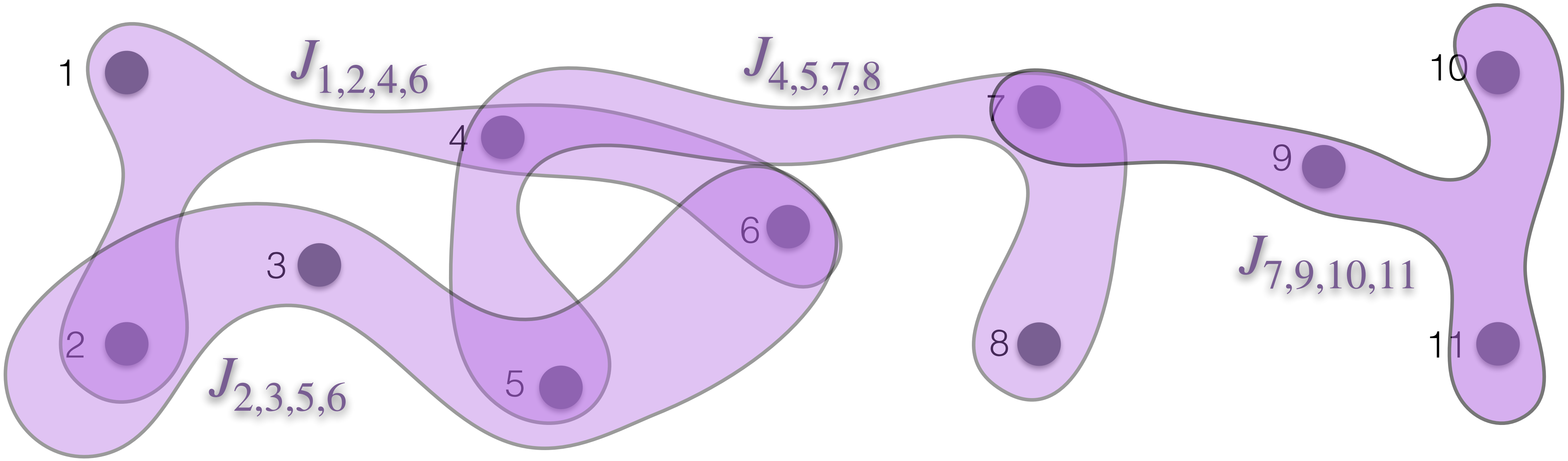

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$$H = -\sum_{1\leq i_1<i_2<i_3<i_4\leq N} J_{i_1 i_2 i_3 i_4}\, \psi_{i_1}\psi_{i_2}\psi_{i_3}\psi_{i_4}$$

where, J is chosen from a Gaussian ensemble:

$$\langle J_{ijkl}\rangle = 0 \qquad \langle J_{ijkl}^2\rangle = \frac{3!\, J^2}{N^3}$$

Solvable Limit of SYK

[Sachdev ’15; Parcollet, Georges ’00;
Kitaev ’15; Polchinski, Rosenhaus ’16; Jevicki, Suzuki, Yoon ’16; Maldacena, Stanford ’16]

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$$-\frac{S_{\rm col}}{N} = \log \mathrm{Pf}\left[\partial_\tau - \Sigma\right] + \frac{J^2}{2q}\int d\tau d\tau' \,|\, G(\tau,\tau')\,|^q - \frac{1}{2}\int d\tau d\tau' \Sigma(\tau',\tau) G(\tau,\tau')$$

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$$\partial_\tau\, G(\tau,\tau') - \left[\Sigma * G\right](\tau,\tau') = \delta(\tau-\tau')$$

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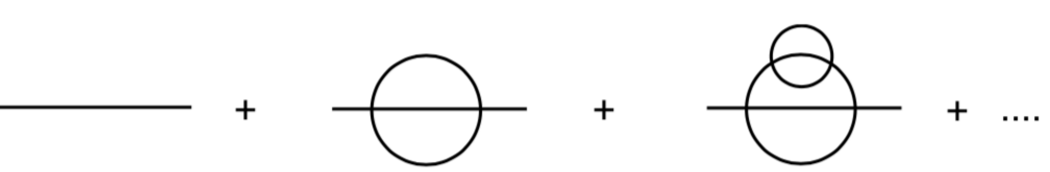
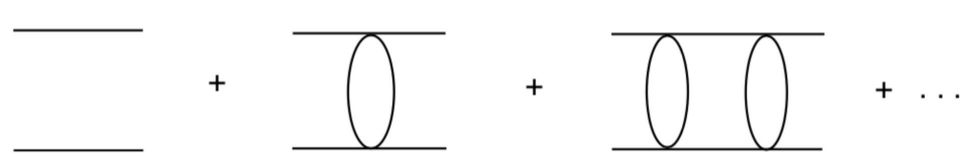


captures Melonic Physics!



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- This symmetry is explicitly broken by considering the 1/J corrections.

Schwarzian in SYK

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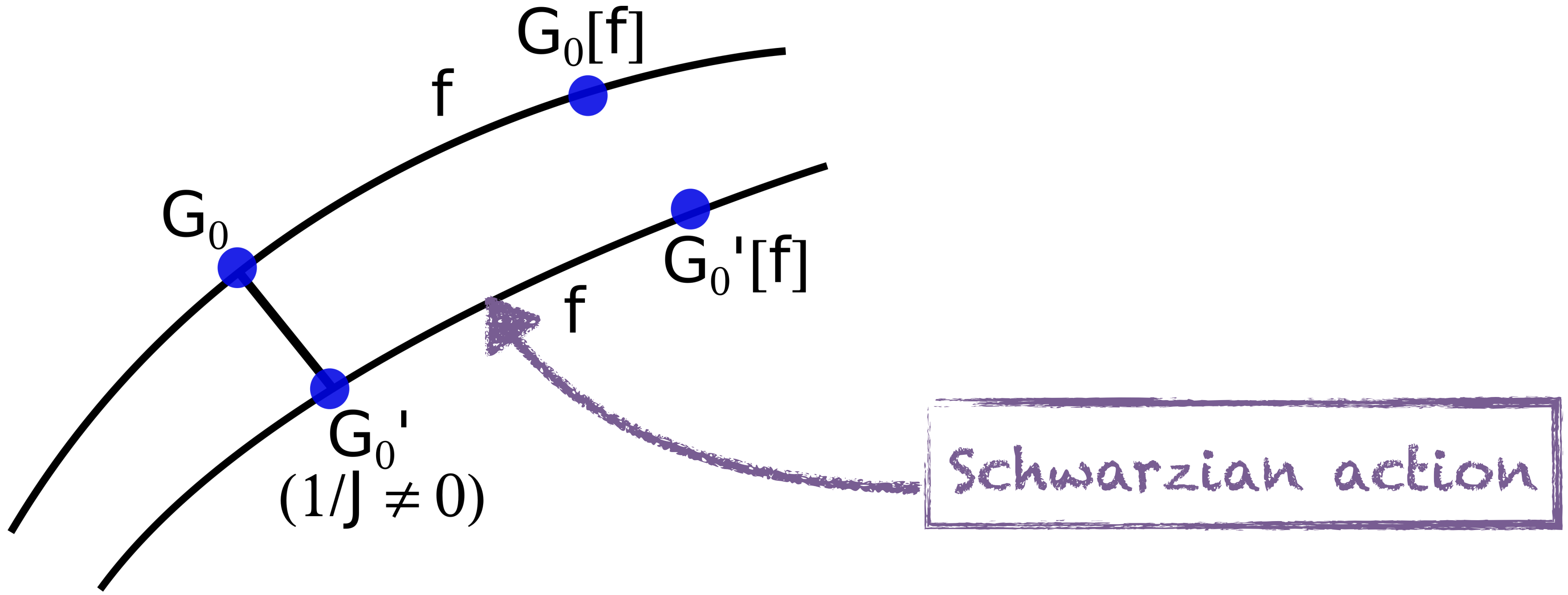

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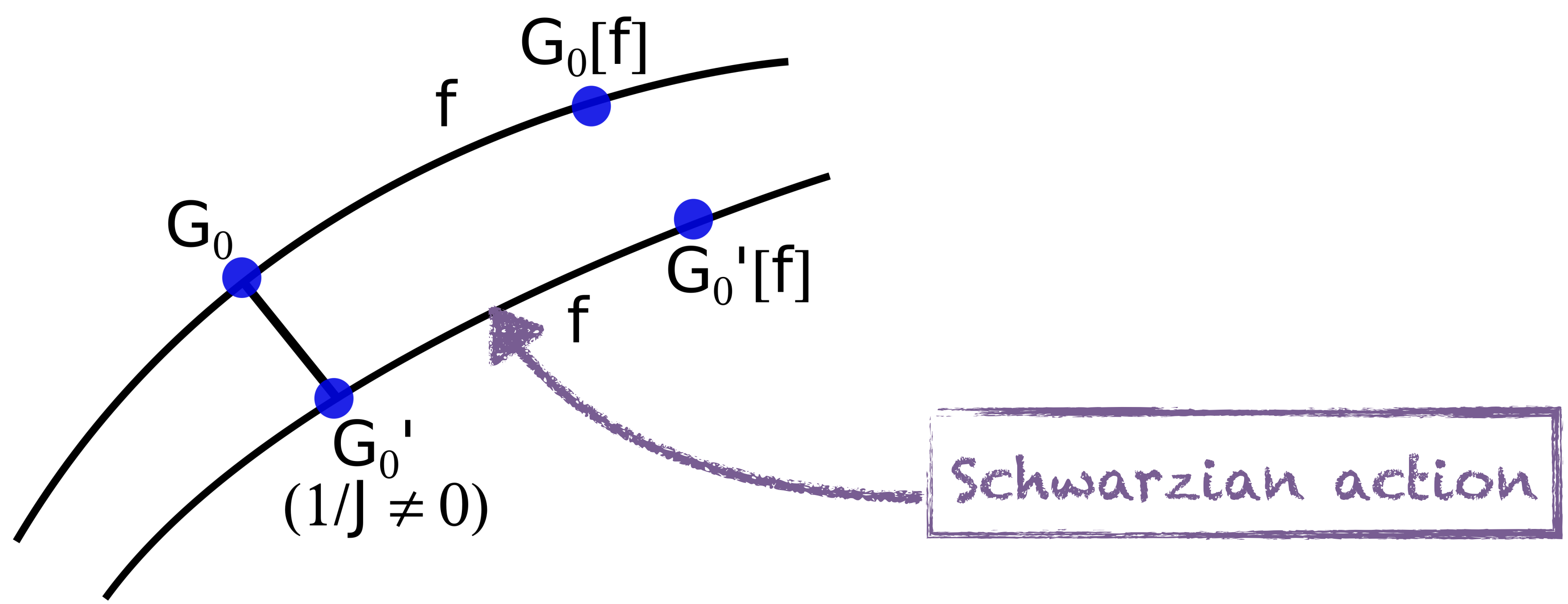

- Effective action on the ‘reparametrization modes’

$$\int \frac{f(\tau)}{\mathbb{SL}(2,\mathbb{R})} \exp\left[-\frac{1}{g^2}\int d\tau \, \{f(\tau),\tau\}\right]$$

where, $\{f(\tau),\tau\} = \frac{f'''(\tau)}{f'(\tau)} - \frac{3}{2}\left(\frac{f''(\tau)}{f'(\tau)}\right)^2$ $\qquad g^2 \sim \frac{\beta J}{N}$

Spectrum

[Kitaev ’15; Polchinski, Rosenhaus ’15; Jevicki, Suzuki, Yoon ’16; Maldacena, Stanford ’16; Gross, Rosenhaus ’17]

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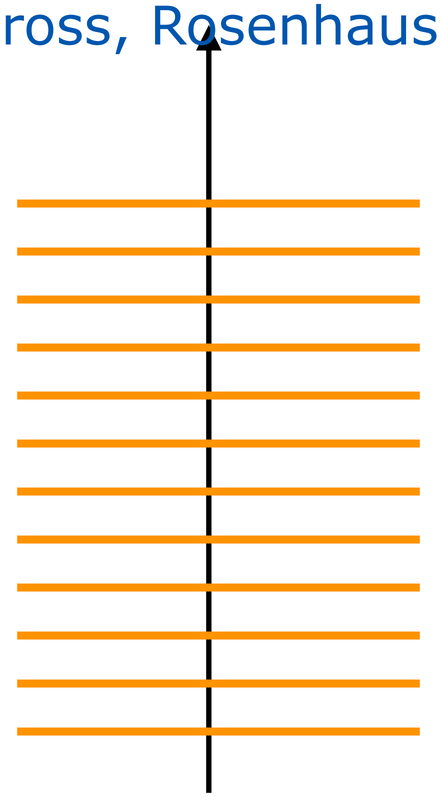

discrete tower of states

$$\mathcal{O}_n \sim \psi_i \, \partial^{2n+1} \psi_i$$

$$h_n = 2n + 1 + \epsilon_n$$

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continuum: Schwarzian

$$S = -\frac{1}{g^2}\int d\tau \, \{f(\tau), \tau\}$$

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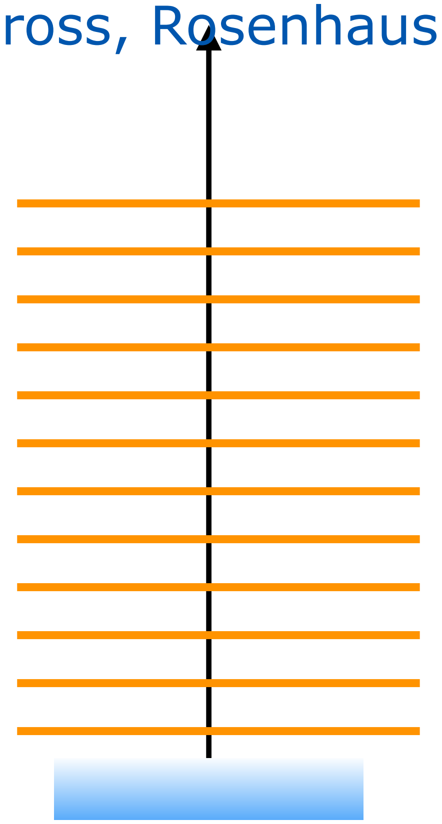

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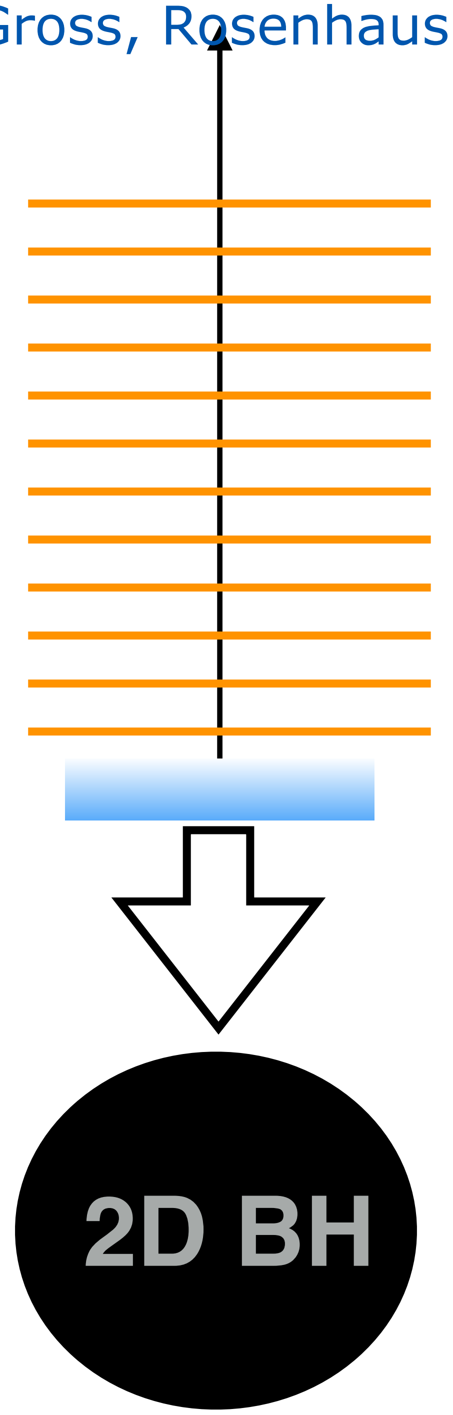

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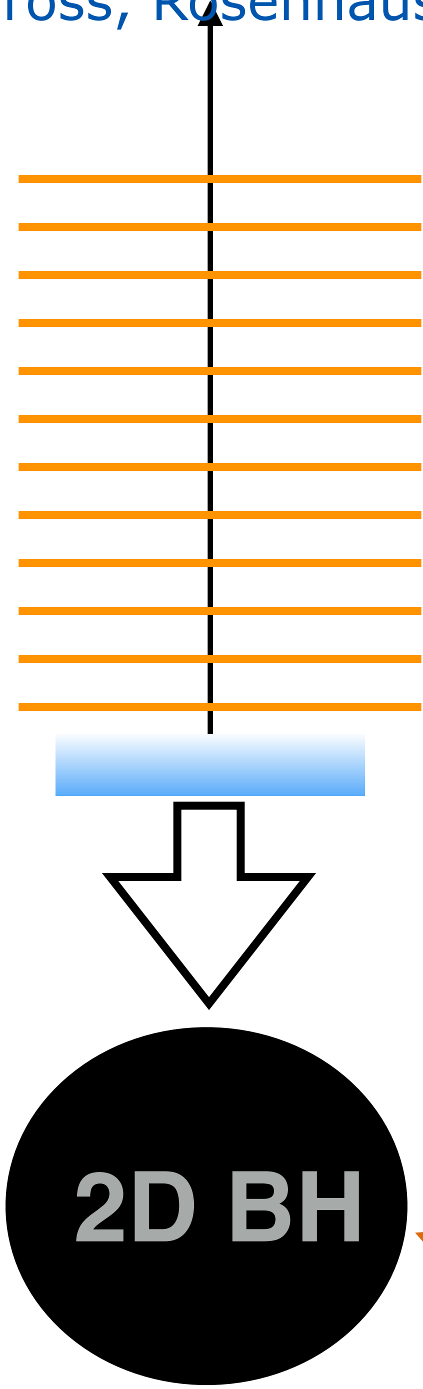

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Limit of conformal six-pt functions OPE coeffs.

ETH

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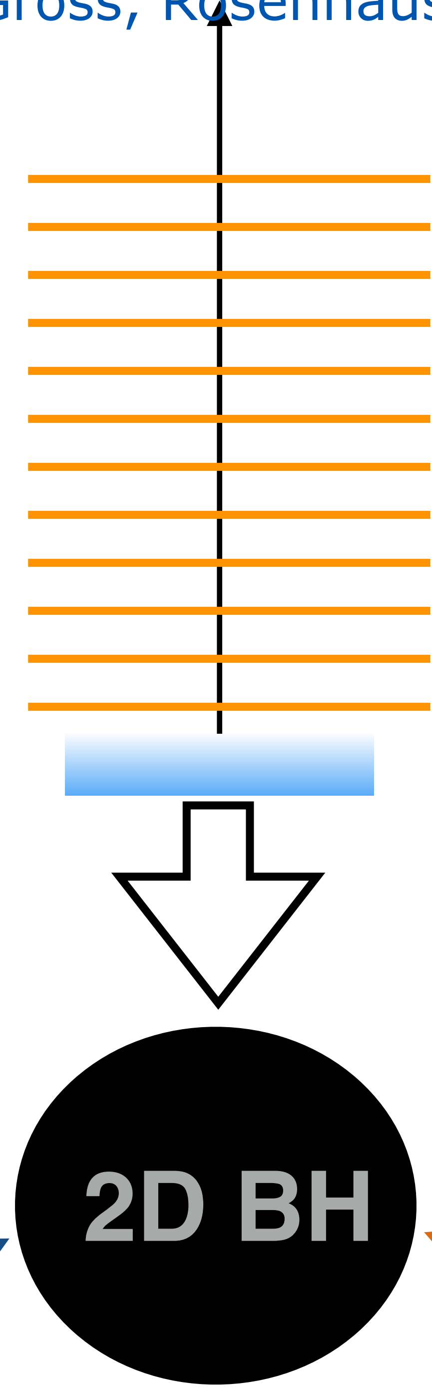

Ward identities for Schwarzian
~ monodromy method

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weak ETH

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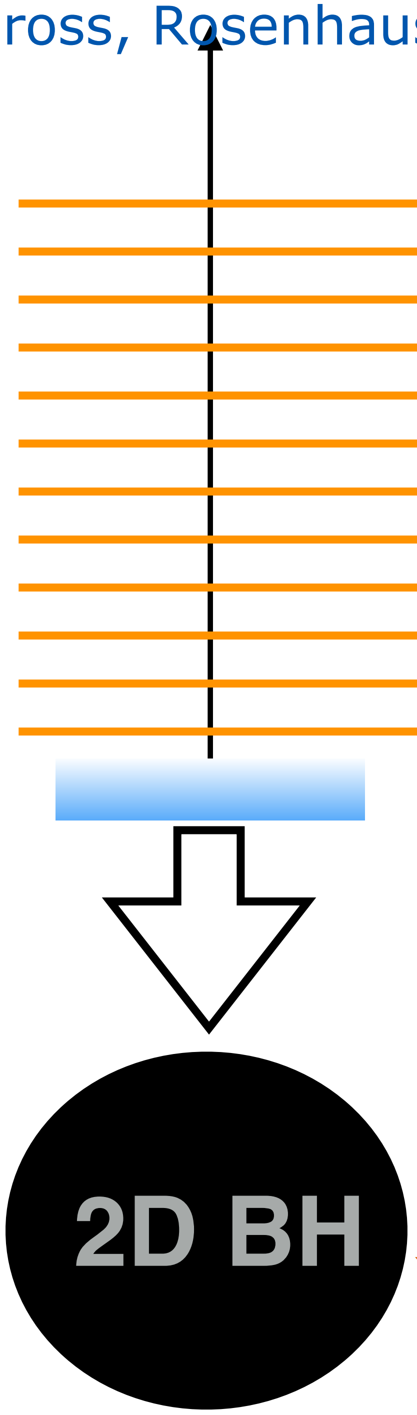

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ETH

[PN, Julian Sonner & Manuel Vielma]

ETH in the SYK model: A numerical study

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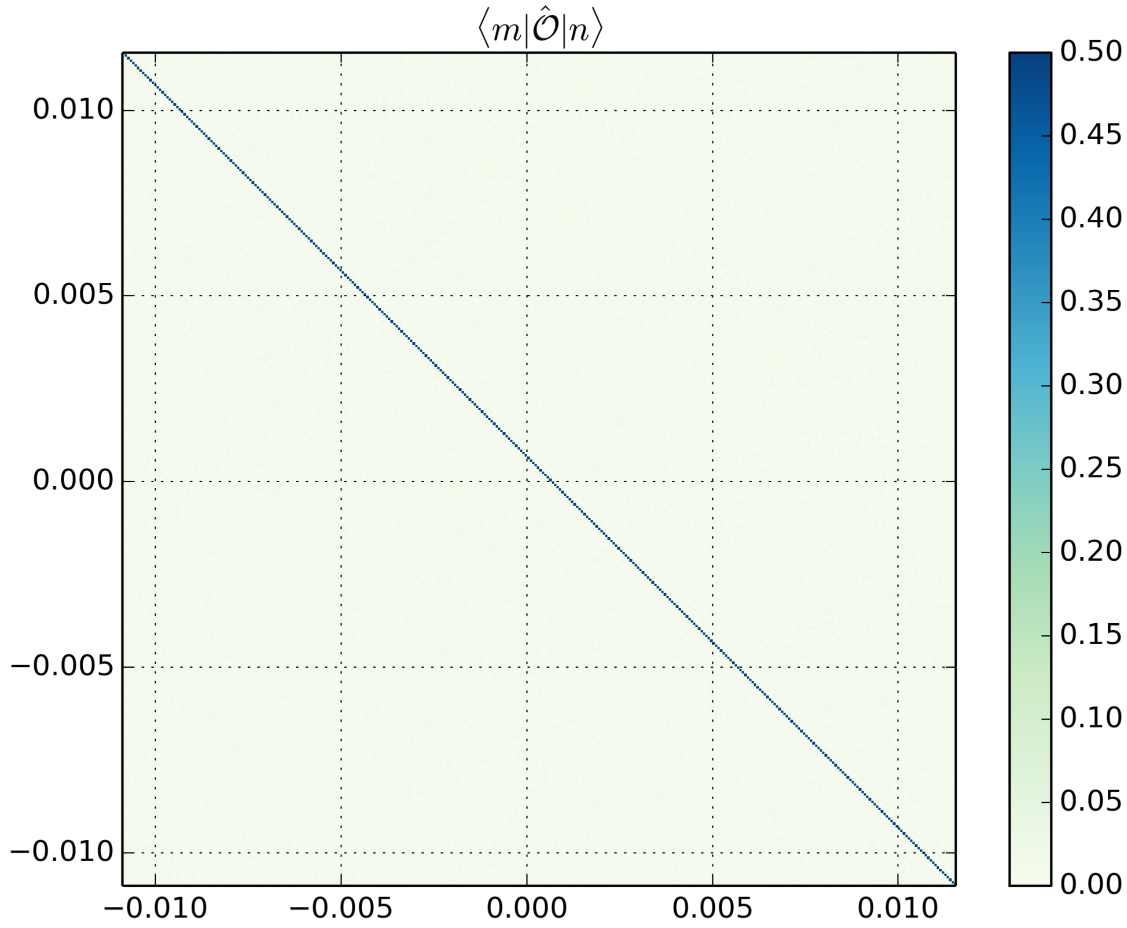

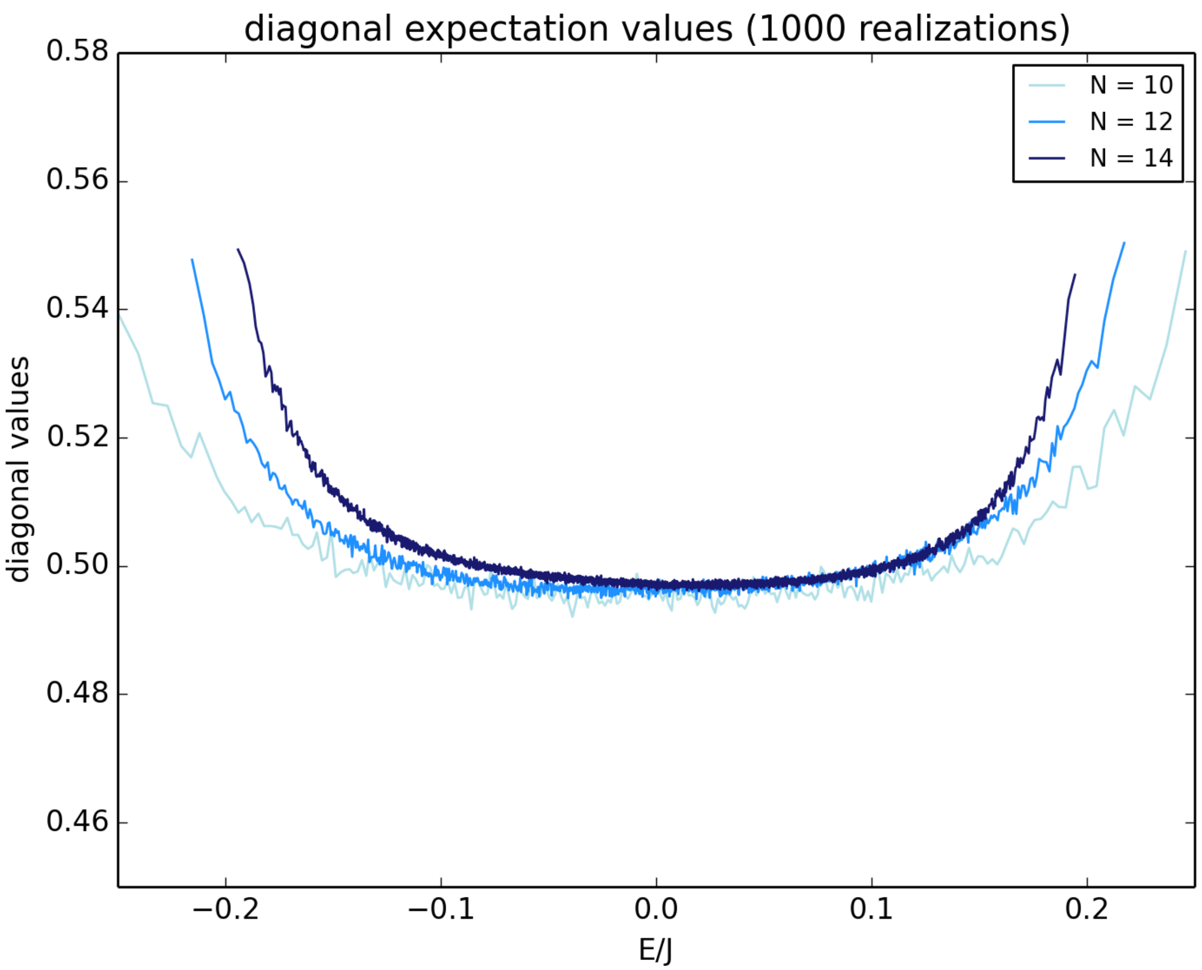

RMT or Not?

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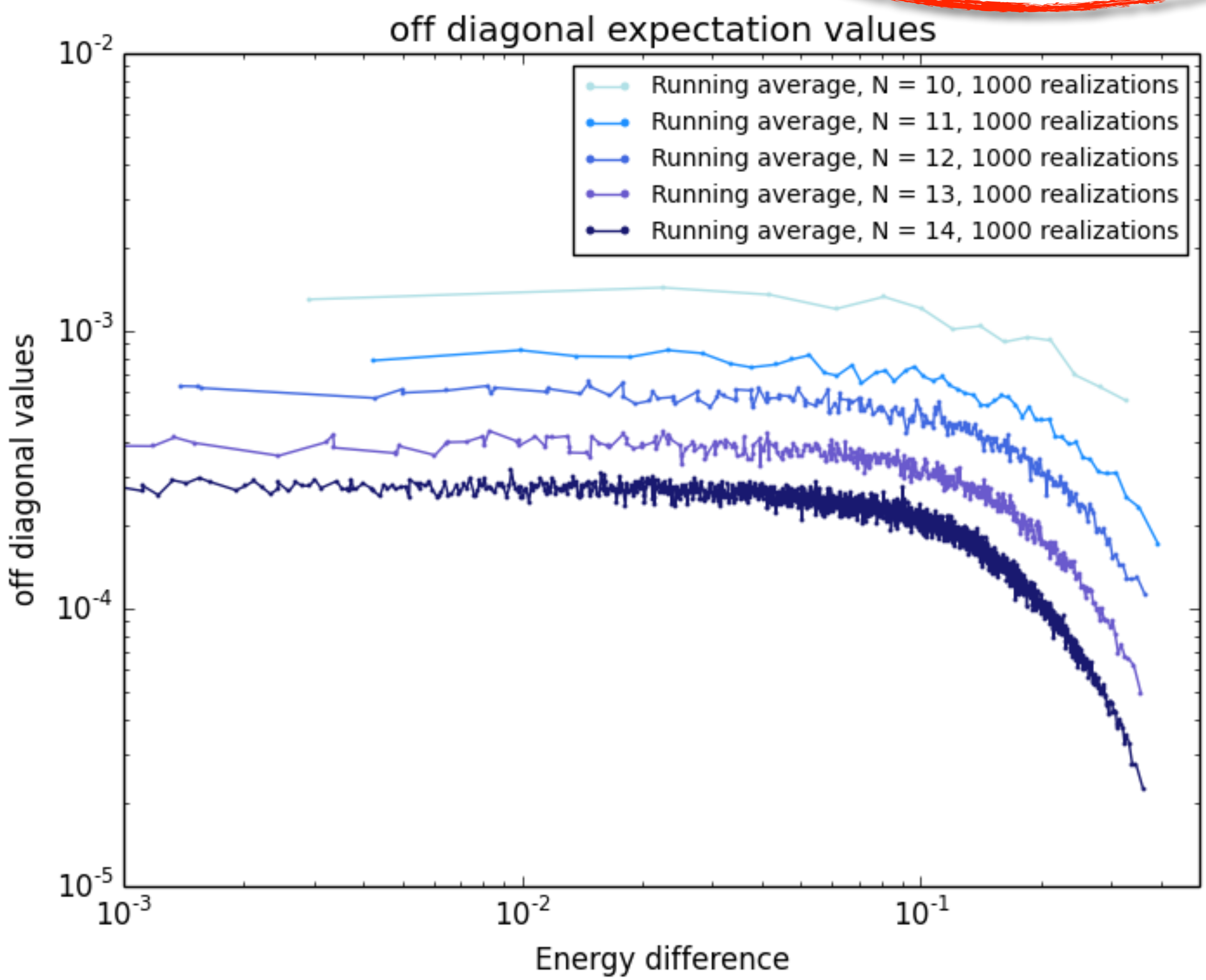

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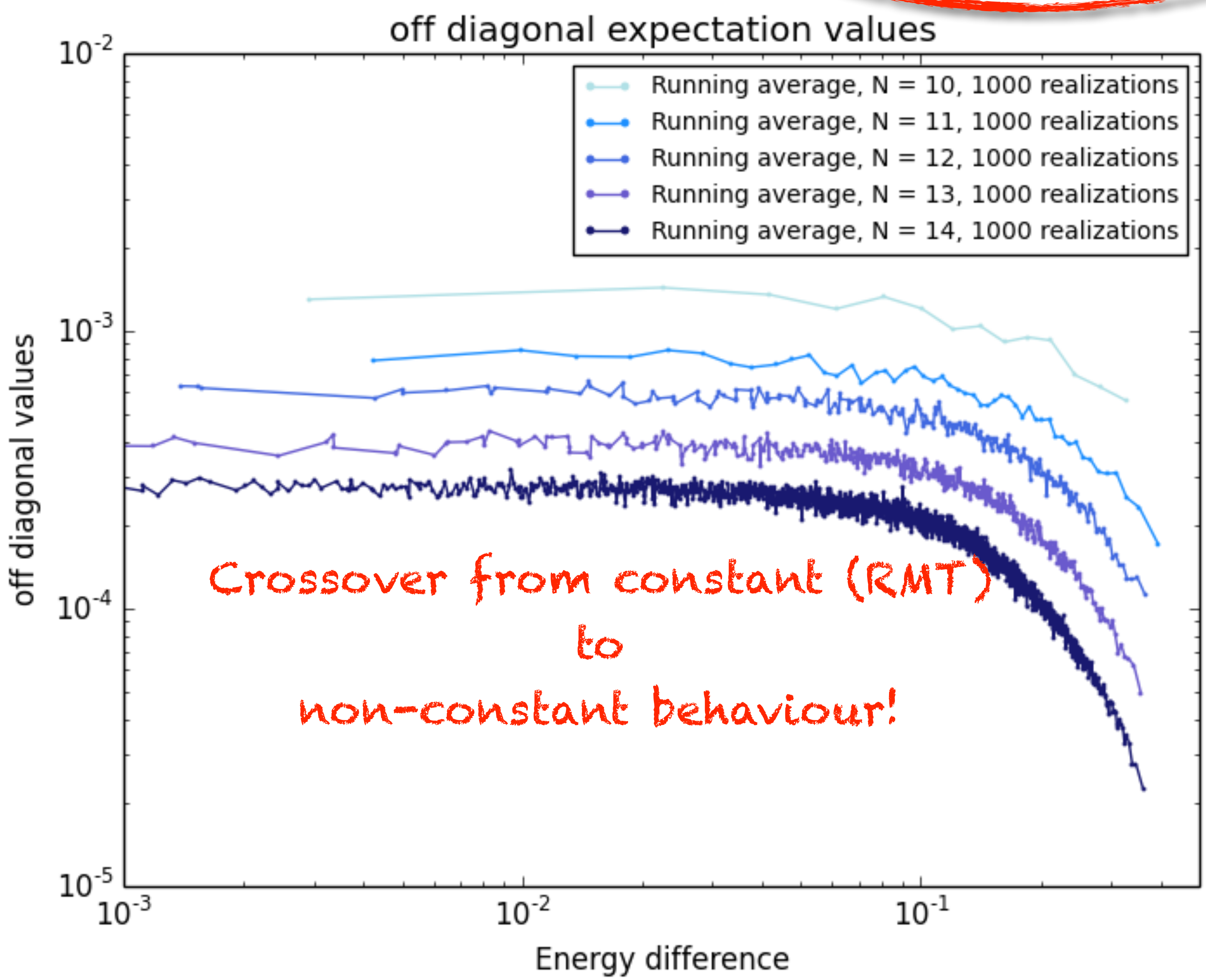

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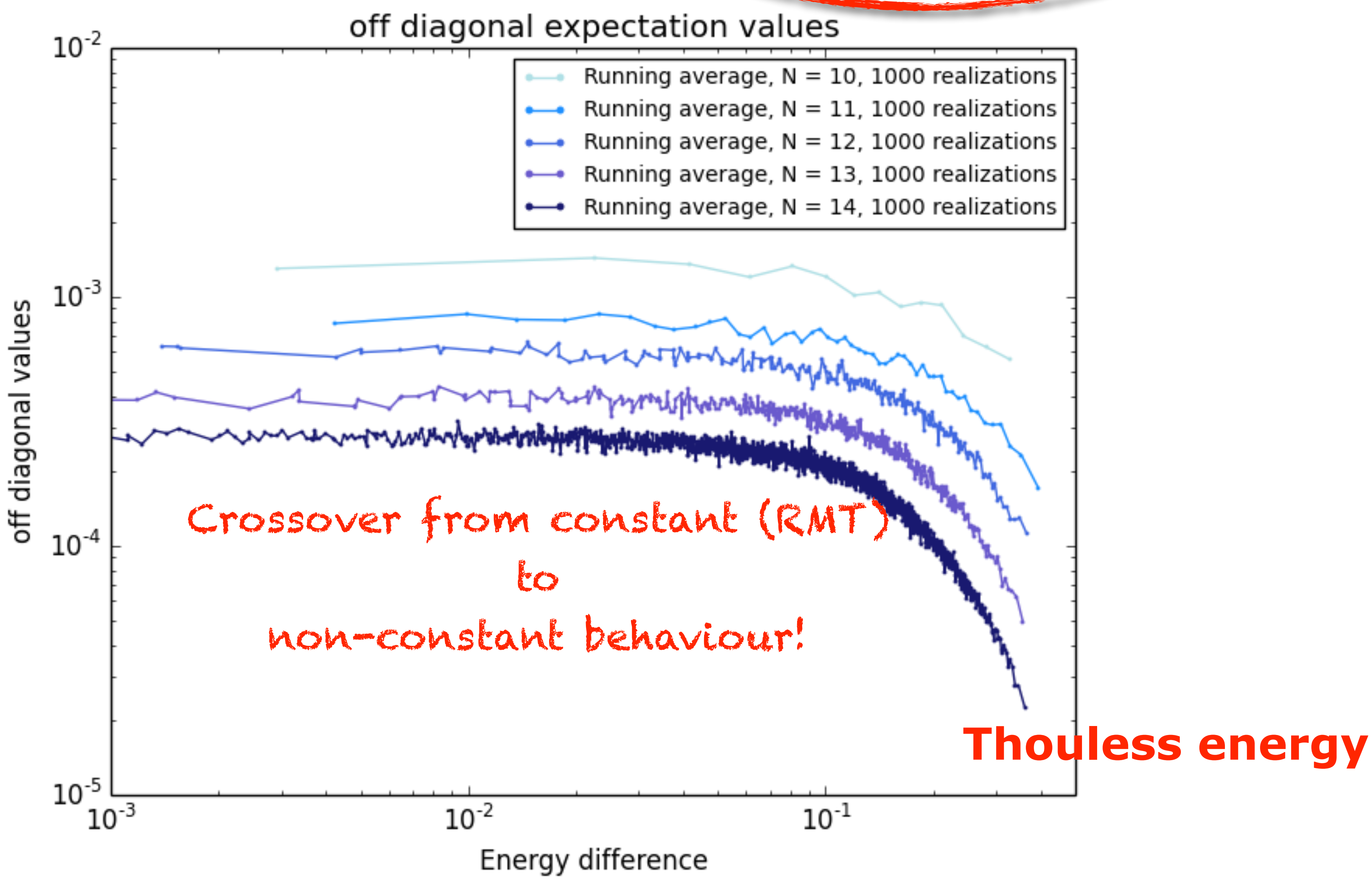
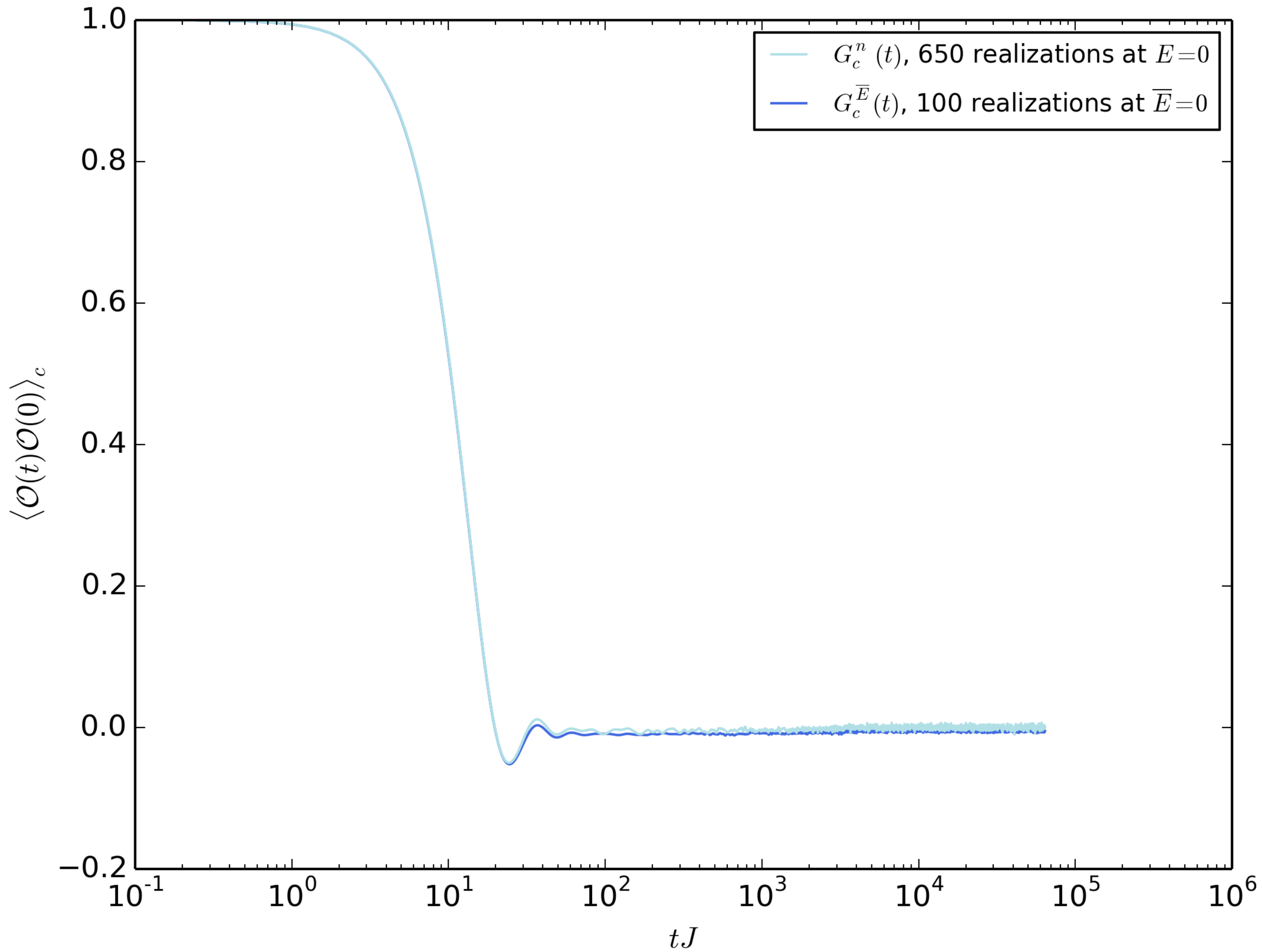
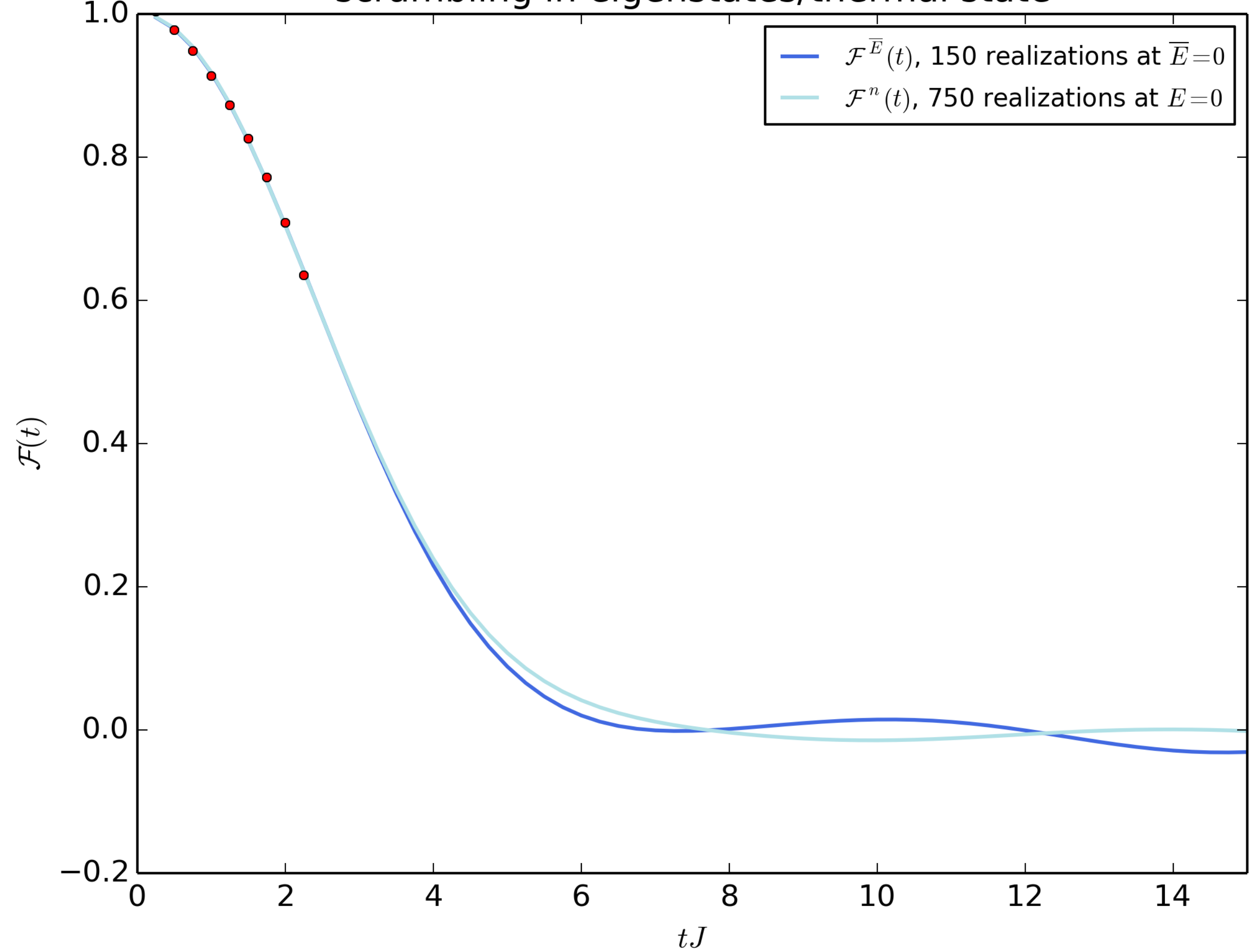


More evidence of ETH

[Sonner, Vielma '17]

Compare OTOC in eigenstates to thermal result





Become essentially indistinguishable as system size increases

Conjecture: $\exists\ \lambda_L^{\mathrm{ETH}} = \dfrac{2\pi}{\beta(E)}$

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- ? Can we do a dual bulk computation to understand black hole formation in 2 dimensions?

ETH in the Schwarzian Limit

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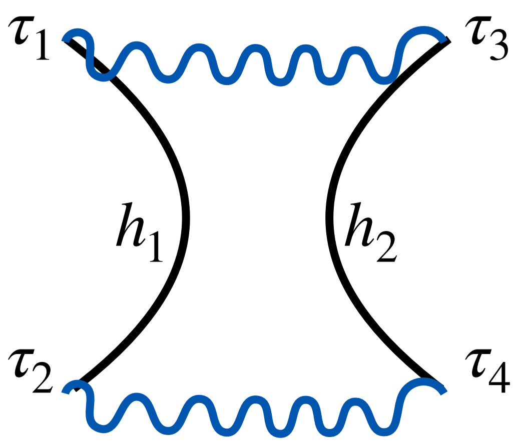

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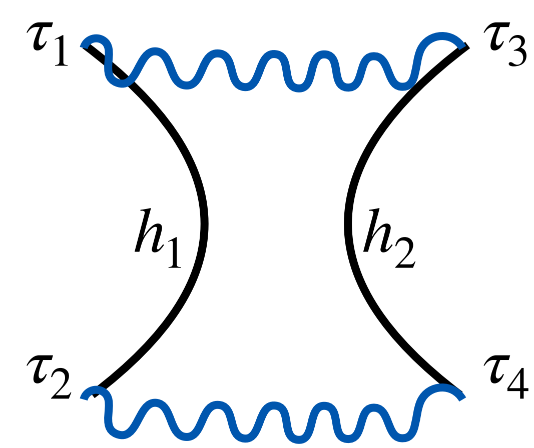

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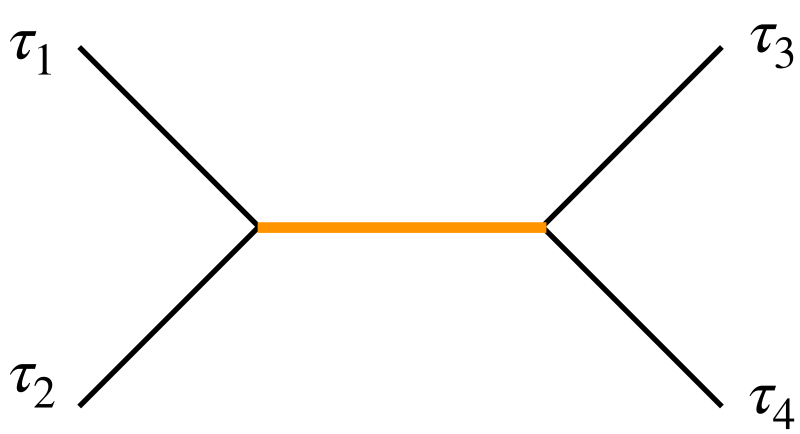
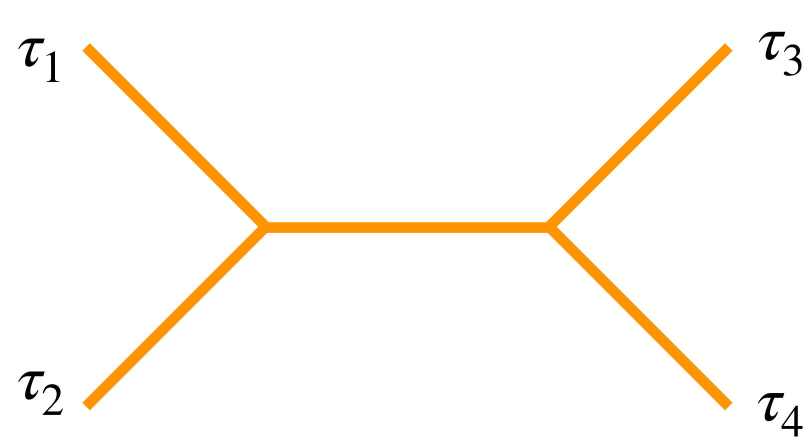
  ✦ Exchange of excited modes (conformal limit)





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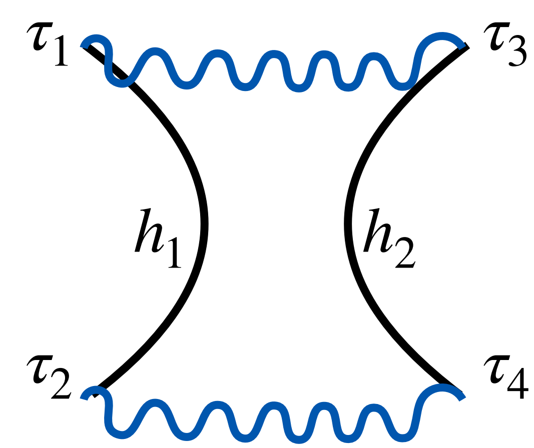

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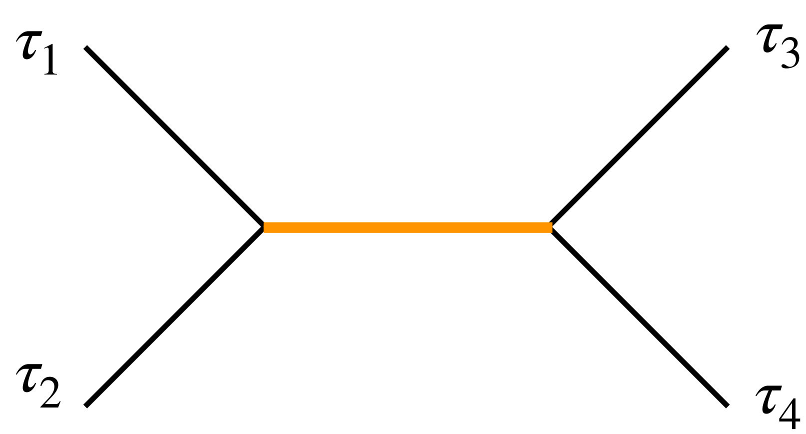

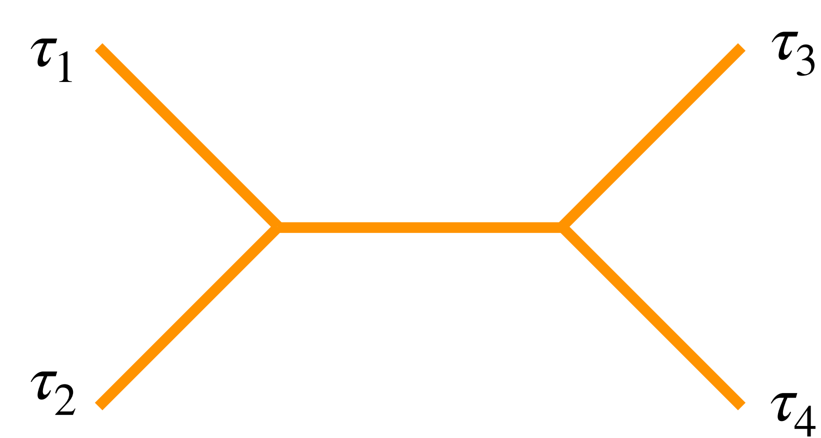

The Schwarzian theory

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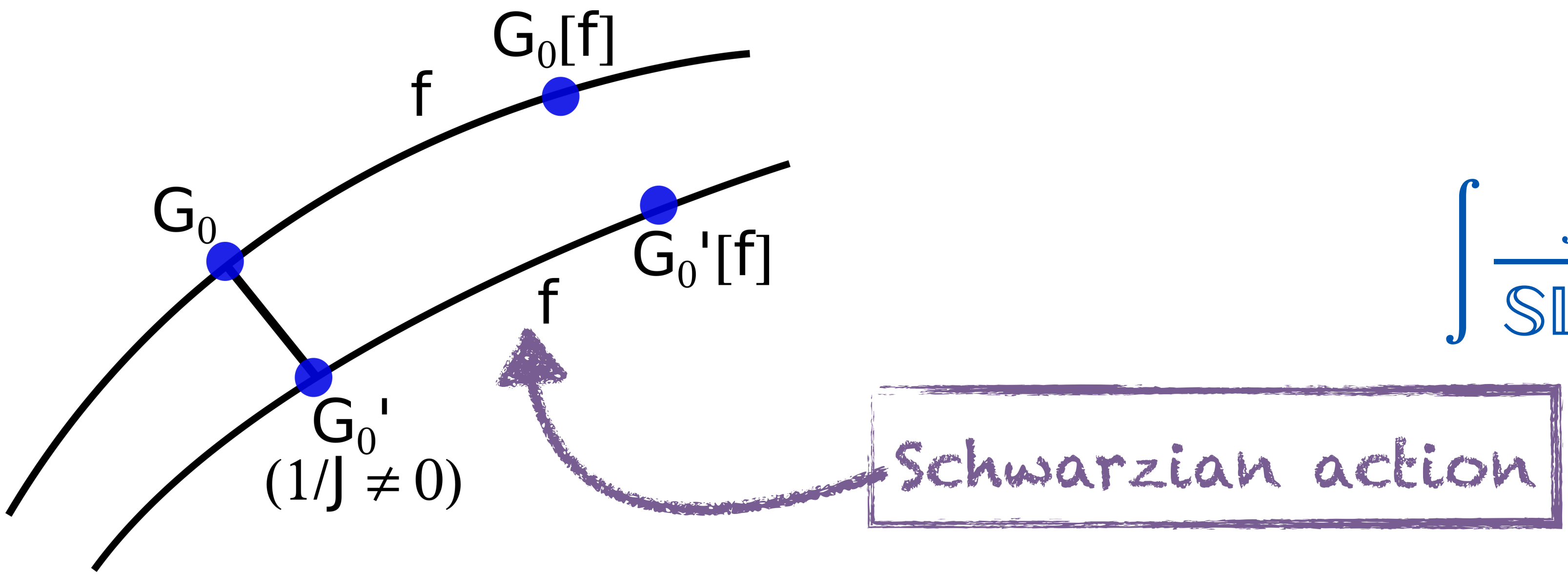

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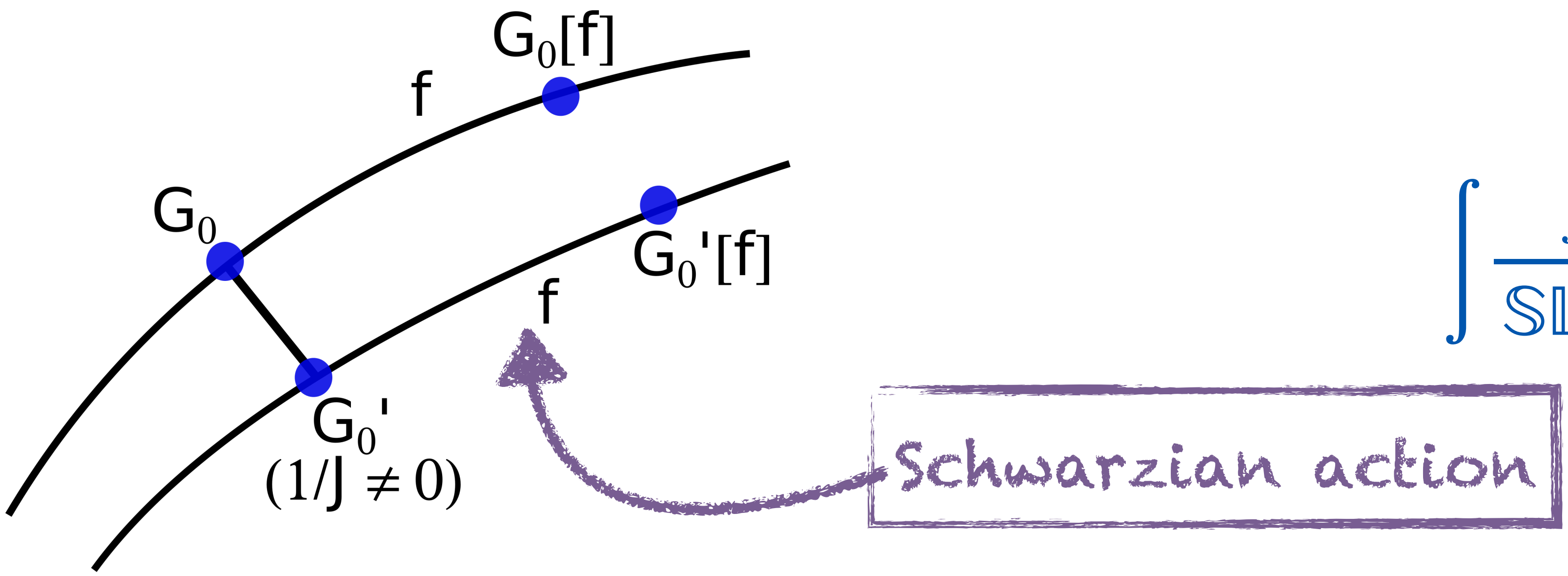

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- $f(\tau)$ are the pseudo-Goldstone modes

The leading contribution to the physical observables is due to exchange of these modes

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[PN, Sonner, Vielma '19]

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$$\sum_{i=1}^{m}\left[h_i\Big(\delta(x-x_{2i-1})+\delta(x-x_{2i})\Big)+\Big(sgn(x-x_{2i-1})-sgn(x-x_{2i})\Big)\partial_{x_{2i}}\right]\prod_{j=1}^{n}\langle\mathcal{O}_j\Big(x_{2j-1},x_{2j}\Big)\rangle$$

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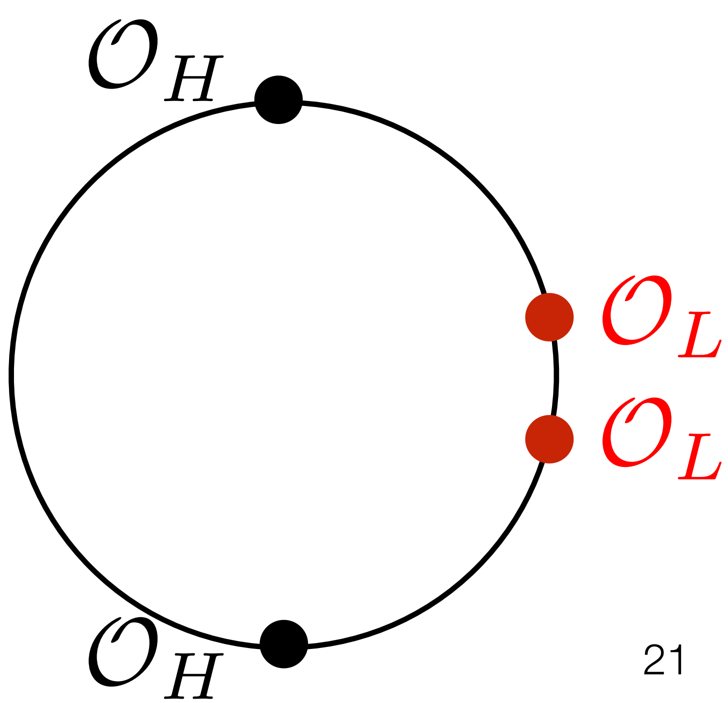

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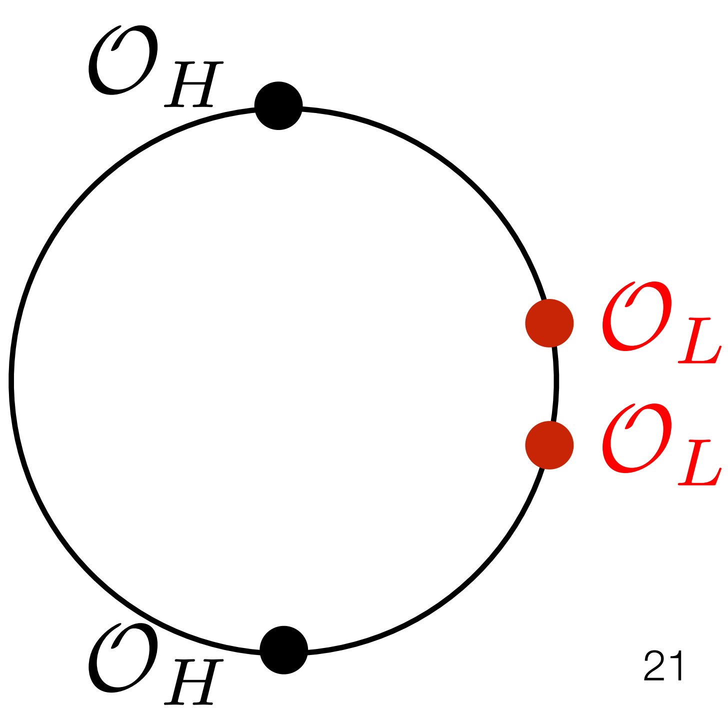

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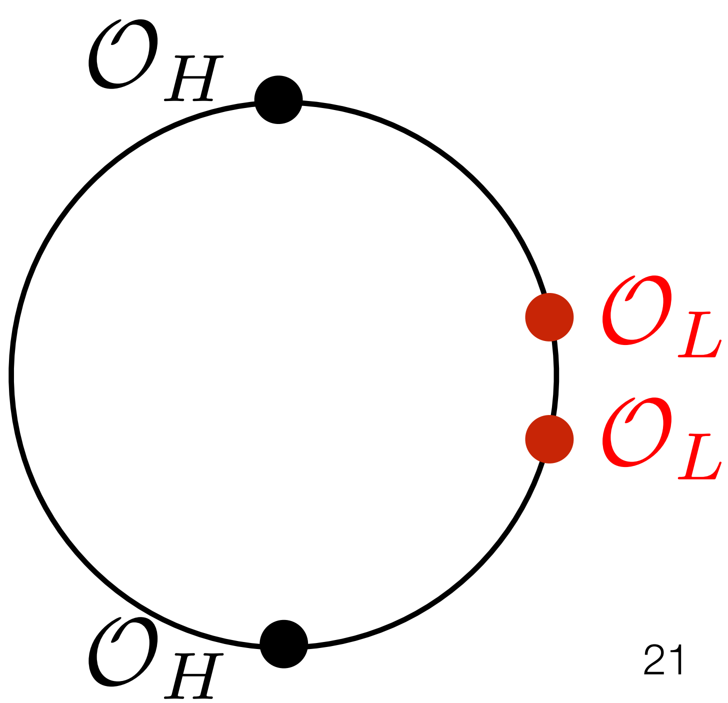

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Weak ETH!

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These states *thermalize* with

$$\tilde{T}_{eff} = \frac{\sqrt{2g^2E}}{2\pi}$$

[Lam, Mertens, Turiaci, Verlinde '18]

ETH in the Conformal Limit

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- Correlation functions of the fermions as well as of the excited operators in SYK spectrum get contribution from:

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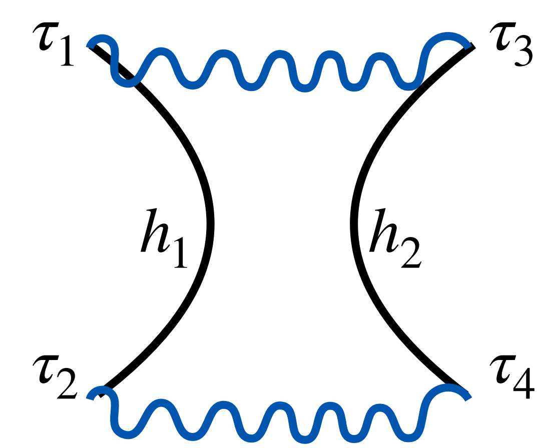

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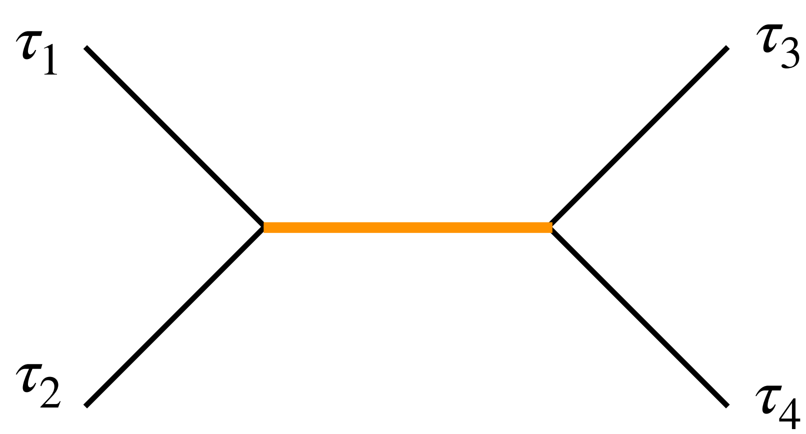

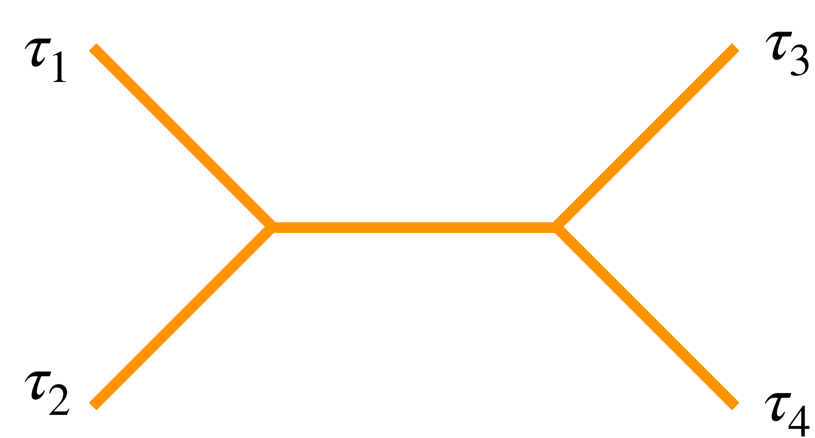

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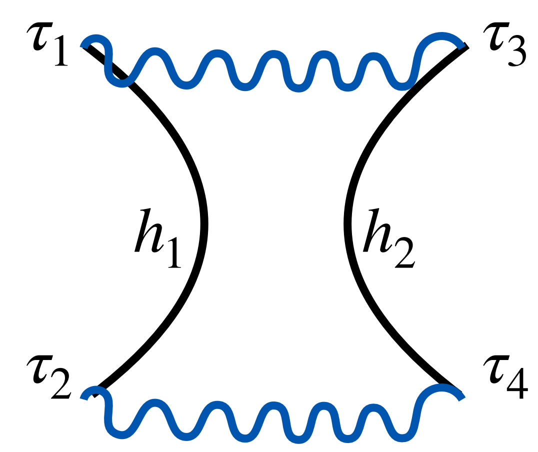

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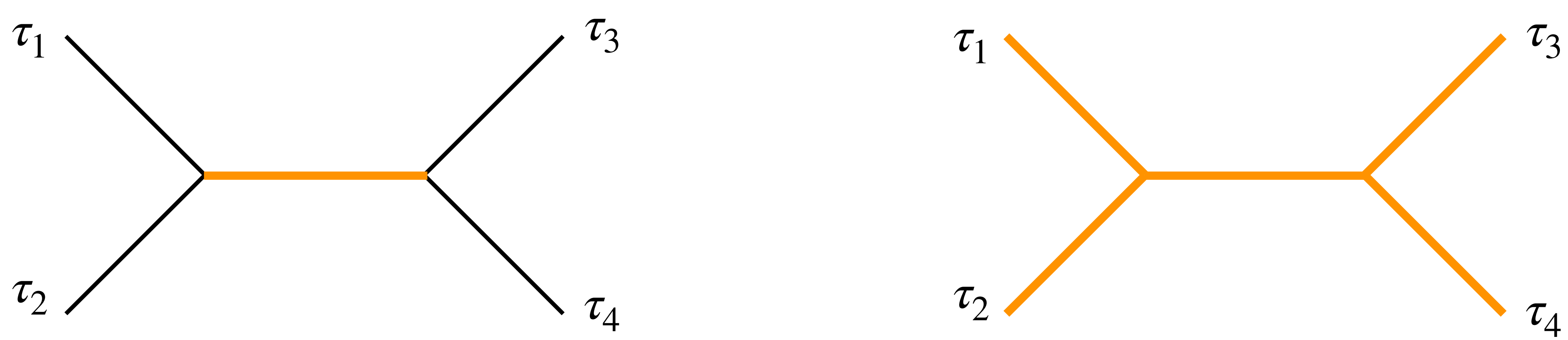

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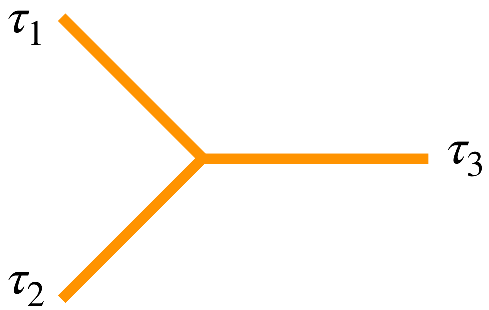

discrete tower of states

$$\mathcal{O}_n \sim \psi_i \, \partial^{2n+1} \psi_i$$

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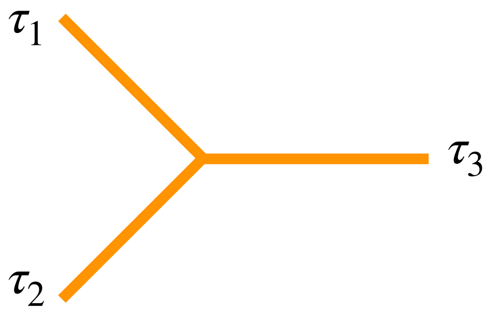

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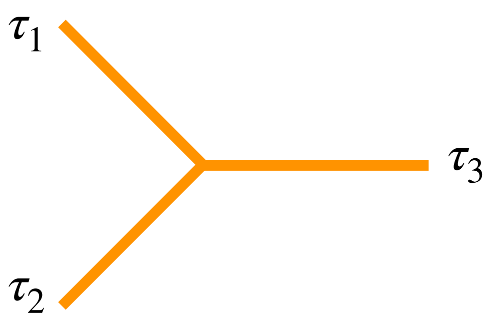

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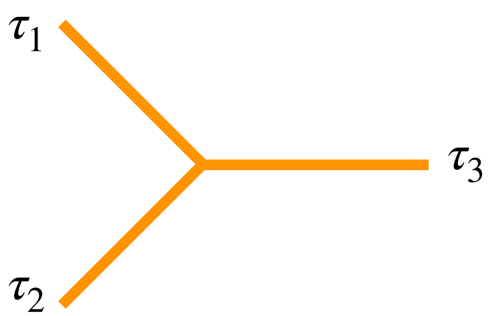

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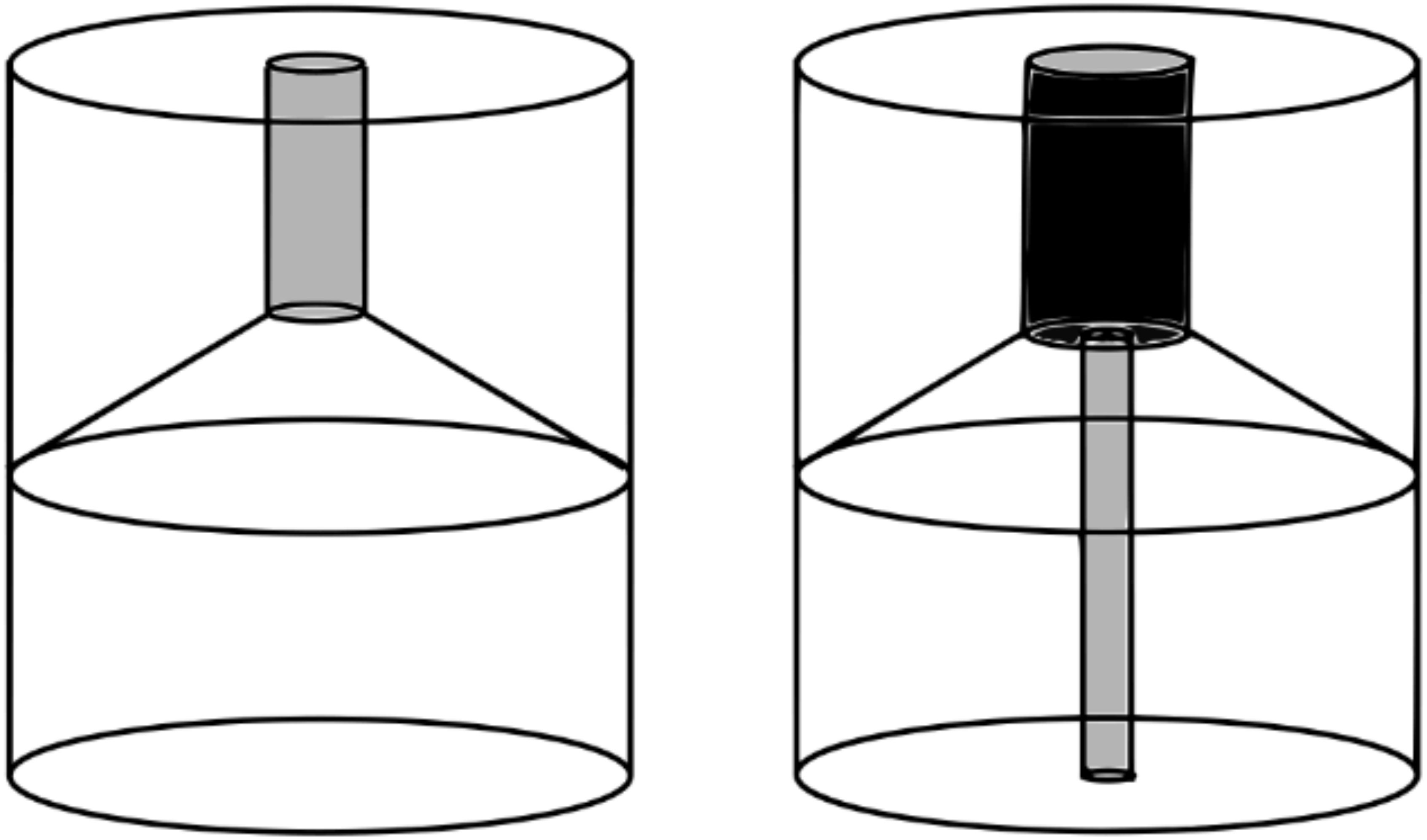
conformal sector of SYK satisfies ETH

A Bulk Story?

Dual of Schwarzian theory in 1st Limit?



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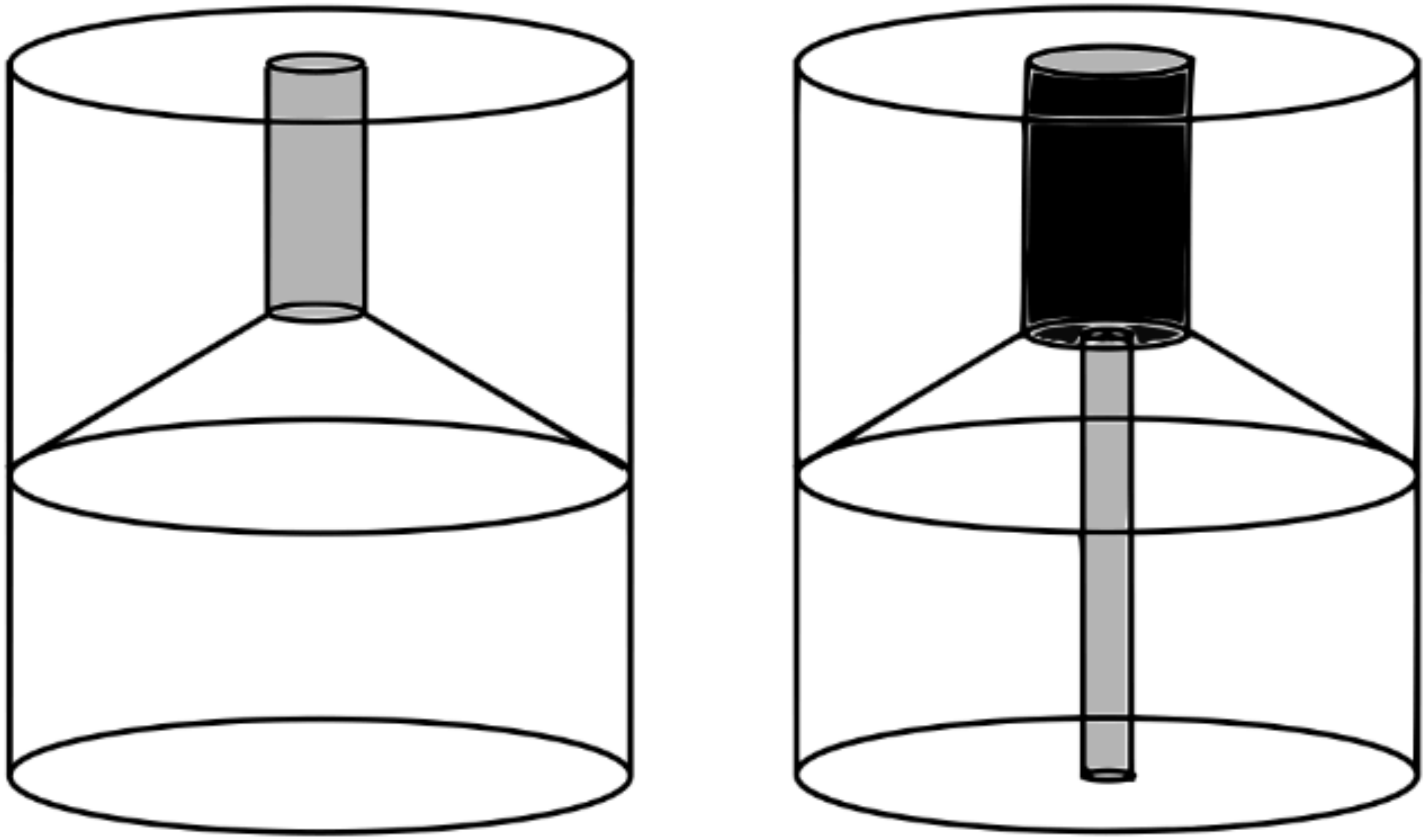
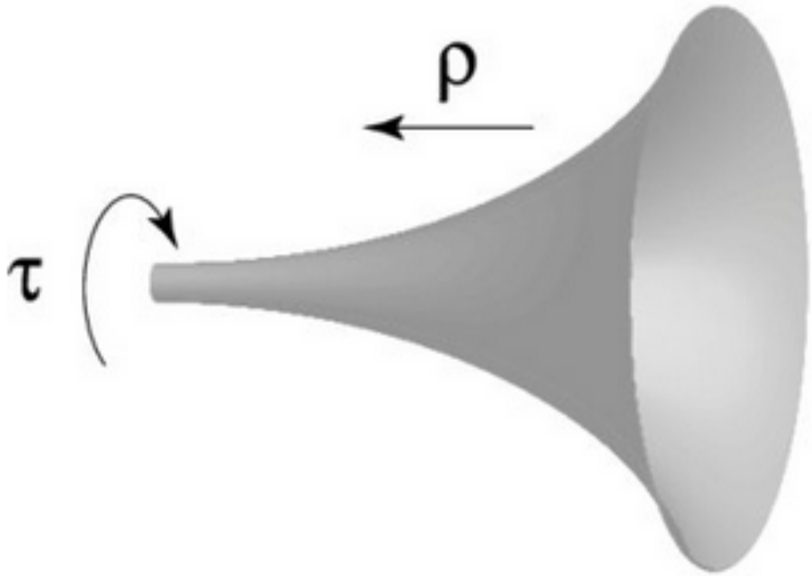


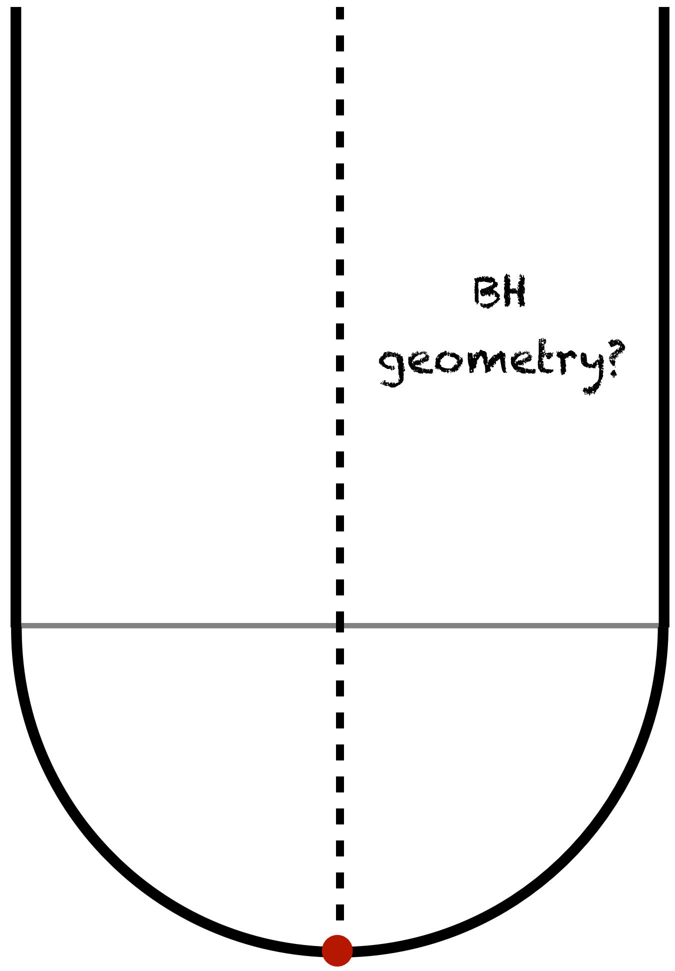
[Vos '18]

Dual of Schwarzian theory in 2^{nd} Limit?



Thermal AdS_2

Dual of SYK theory in Conformal Limit?



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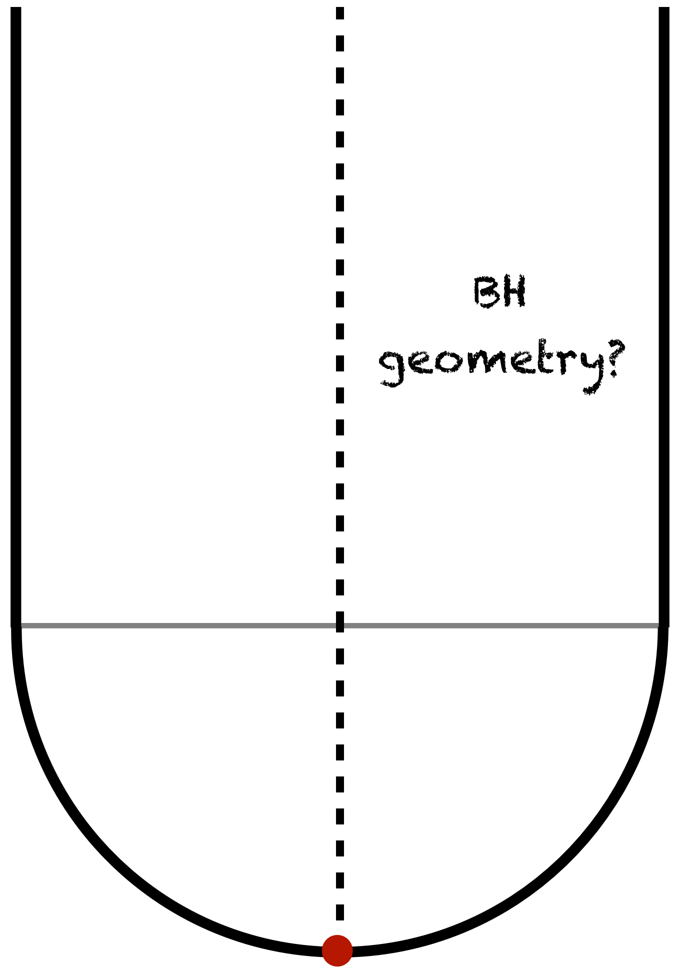

Can the ETH in conformal sector be understood as a correction?

Summary

- We showed that Eigenstate thermalization can be studied analytically in SYK model in 2 limits:

    - ✦ Schwarzian limit: where the contribution of pseudo-Goldstone modes is considered

        - First Thermodynamic limit: $N \to \infty$ **then** $\beta J \to \infty$

          Weak ETH!

        - Second Thermodynamic limit: $N, \beta J \to \infty$ **simultaneously, followed by** $g^2 \sim \dfrac{\beta J}{N} \to 0$

          No ETH!

        - ETH in boundary states of Liouville theory: ETH!

    - ✦ Conformal limit: where only the operators that are primaries of Virasoro (and their descendants) are considered

      ETH!

TO DO

- Bulk dual of the each of the above limits

- Are 'heavy' states chaotic?

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THANK YOU!