Effective field theory near and far from equilibrium

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with
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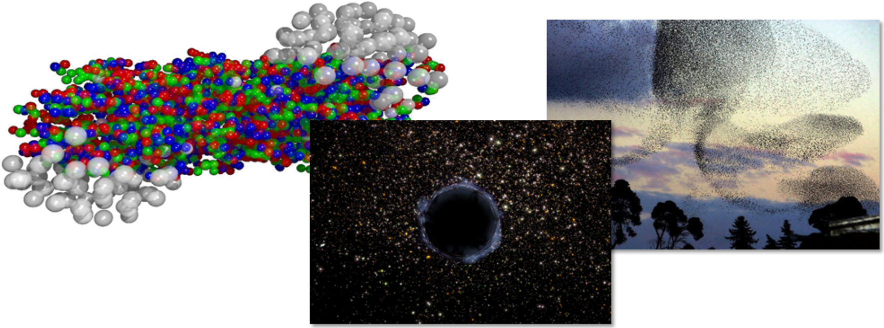
Andrey Gromov and Shinsei Ryu [1908.03217]



Non-equilibrium physics displays a huge variety of phenomena in nature. These range from heavy ion collisions to black holes dynamics, from driven systems to non-equilibrium steady states, etc.

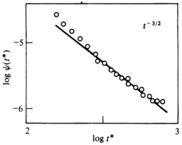
Many open questions: thermalization, information paradox, turbulence, ...

What is a suitable framework which captures systematically these phenomena?

Goal: encode low-energy description of non-equilibrium systems into effective field theories, independent of microscopic details.

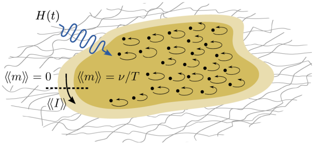
Direct applications include:

- Systematic computation of hydrodynamic fluctuations. E.g. long-time tails, renormalization of transport. Current methods (e.g. stochastic hydro) are not systematic.



[Boon, "Molecular hydrodynamics," '91]

- Topological response of periodically driven systems, which are inherently far from equilibrium.



[Nathan et al., '16]

Outline

1. Near equilibrium: infrared instability of chiral diffusion
2. Far from equilibrium: Floquet topological response
3. Conclusions

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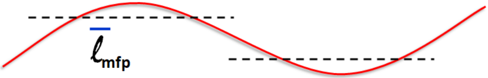
2. Far from equilibrium: Floquet topological response
3. Conclusions

Chiral diffusion in 1+1

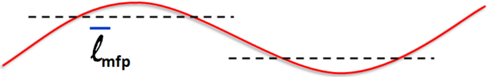
Quantum systems in local thermal equilibrium → hydrodynamics



At sufficiently low energy, the only degrees of freedom are conserved charges. Example: $U(1)$ charge, momentum, ...

Chiral diffusion in $1+1$

Quantum systems in local thermal equilibrium $\to$ hydrodynamics



At sufficiently low energy, the only degrees of freedom are conserved charges. Example: $U(1)$ charge, momentum, ...

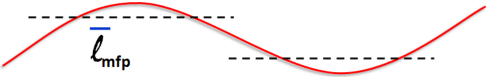
We will be interested in systems with anomalously non-conserved $U(1)$ current with chiral anomaly:

$$\partial_\mu J^\mu = c\varepsilon^{\mu\nu} F_{\mu\nu}$$

$c = \frac{\nu}{4\pi}$: anomaly coefficient, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

- Neglect energy-momentum conservation
- Local equilibrium: $\rho = e^{-\frac{1}{T}(H-\mu(t,x)Q)}$

Chiral diffusion in 1+1



$$\rho = e^{-\frac{1}{T}(H - \mu(t,x) Q)}$$

$$\partial_\mu J^\mu = c \varepsilon^{\mu\nu} F_{\mu\nu}$$

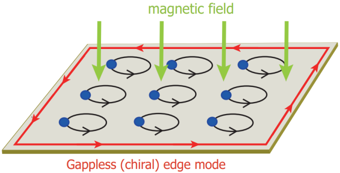
$$J^t = n(\mu) = \chi\mu + \frac{1}{2}\chi' \mu^2 + \cdots , \quad J^x = -4c\mu - \sigma \partial_x \mu$$

- $-4c\mu$ required by second law [Son,Surowka '09].
- Chiral diffusion:

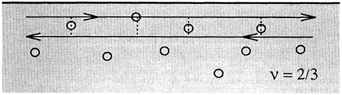
$$\chi \partial_t \mu - 4c \partial_x \mu - \sigma \partial_x^2 \mu + \frac{1}{2}\chi' \partial_t \mu^2 = 0$$

Motivations I

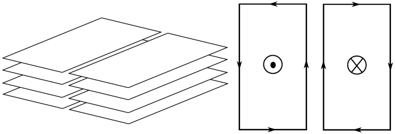
- Edge of quantum Hall systems [Kane,Fisher '95; Ma,Feldman '19]



[Hasebe, Totsuka '13]



- Surface chiral metals [Balents,Fisher '95; Sur,Lee '13]



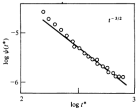
- Chiral magnetic effect [Vilenkin '80; Son,Spivak '13; Yamamoto '15]

$$\vec{J} \propto \mu \vec{B}$$

Motivations II

Hydrodynamic long time tails:

- Change qualitatively correlation functions at late time. [Alder,Wainwright '70;Kovtun,Yaffe '03;Chen-Lin,Delacretaz,Hartnoll '18]



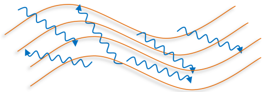
[Boon, "Molecular hydrodynamics," '91]

- Momentum conservation causes more violent effects leading to anomalous scaling [Forster,Nelson,Stephen '74] . E.g. $d = 2$:

$$\eta \sim \left(\log \frac{1}{\omega}\right)^{\frac{1}{2}}$$

  Breakdown of hydrodynamics! [Schepper,Beyeren '74]

- Result of the interplay between thermal fluctuations and interactions of collective modes



I will show:

- Chiral diffusion breaks down in the IR
- It persists even without momentum conservation!
- It furnishes a novel mechanism to flow to a non-trivial IR fixed point.

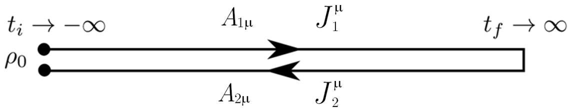
EFT of chiral diffusion

- Consider a quantum system in a thermal state $\rho_0 = e^{-\beta H}/\mathrm{Tr}(e^{-\beta H})$ with

$$\partial_\mu J^\mu = c\varepsilon^{\mu\nu} F_{\mu\nu}$$

- Background sources: $A_{1\mu}, A_{2\mu}$

$$e^{iW[A_1,A_2]} = \mathrm{Tr}\left[U(A_1)\rho_0 U^\dagger(A_2)\right] = \int_{\rho_0} D\psi_1 D\psi_2 e^{iS[\psi_1,A_1]-iS[\psi_2,A_2]}$$



- Anomalous conservation of J_1^μ and J_2^μ implies the Ward identity

$$W[A_{1\mu}+\partial_\mu\lambda_1, A_{2\mu}+\partial_\mu\lambda_2] = W[A_{1\mu}, A_{2\mu}] + c\int \lambda_1 F_{1\mu\nu} - c\int \lambda_2 F_{2\mu\nu}$$

EFT of chiral diffusion

$$W[A_{1\mu}+\partial_\mu\lambda_1, A_{2\mu}+\partial_\mu\lambda_2] = W[A_{1\mu}, A_{2\mu}] + c\int \lambda_1 F_{1\mu\nu} - c\int \lambda_2 F_{2\mu\nu}$$

- W is non-local due to long-living modes associated to $\partial_\mu J_1^\mu = 0$ and $\partial_\mu J_2^\mu = 0$.
- "Unintegrate" long-living modes [Crossley, PG, Liu '15; PG, Liu '18]

$$e^{iW[A_1,A_2]} = \int D\varphi_1 D\varphi_2\, e^{iS_{\text{hydro}}[A_1,\varphi_1;A_2,\varphi_2]}$$

$$\varphi_1, \varphi_2 : \text{ long living modes}$$

- S_{hydro} local, satisfies several symmetries. Precisely recovers diffusion in the saddle-point limit.

*see also [Haehl,Loganayagam,Rangamani '15; Jensen, Pinzani-Fokeeva, Yarom '17;...]

IR instability

Action for chiral diffusion:

$$S = \int d^2x \left(-\left(\chi \partial_t \mu - 4c\partial_x \mu - \sigma \partial_x^2 \mu + \frac{1}{2}\chi' \partial_t(\mu^2) \right) \varphi_a + iT\sigma (\partial_x \varphi_a)^2 \right)$$

where $\mu = \partial_t \varphi_r$ is the chemical potential, and

$$\begin{aligned} \varphi_r &= \frac{1}{2}(\varphi_1 + \varphi_2) \quad \text{classical variable} \\ \varphi_a &= \varphi_1 - \varphi_2, \quad \text{noise variable} \end{aligned}$$

At tree-level, this action recovers:

$$\partial_\mu J^\mu = \chi \partial_t \mu - 4c\partial_x \mu - \sigma \partial_x^2 \mu + \frac{1}{2}\chi' \partial_t(\mu^2) = 0$$

IR INSTABILITY

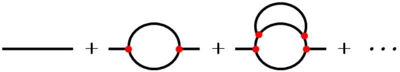
$$\partial_\mu J^\mu = \chi \partial_t \mu - 4c\partial_x \mu - \sigma \partial_x^2 \mu + \frac{1}{2}\chi' \partial_t \mu^2 = 0$$

It is convenient to change coordinates to a frame co-moving with the chiral front: $x \to x + \frac{4a}{\chi} t$. Upon rescaling various quantities:

$$\partial_t \mu - \partial_x^2 \mu + \lambda \partial_x(\mu^2) = 0$$

Scaling $\partial_t \sim \partial_x^2$, the interaction λ is relevant! This has dramatic consequences:

$$\langle J^i(\omega) J^i(-\omega)\rangle_{\text{ret}} \sim \sigma i\omega + \lambda^2 (i\omega)^{-\frac{1}{2}} + \lambda^4 (i\omega)^{-1} + \cdots$$



Correlation function grows with time!

Fate in the IR

What is the fate of chiral diffusion in the IR?

To get a sense, consider higher-dimensional generalization:

$$J^x = -4c\mu - \sigma\partial_x\mu, \qquad J^\perp = -\sigma_\perp\nabla_\perp\mu$$

- $(2+1)-d$: surface chiral metals
- $(3+1)-d$: chiral magnetic effect with large background magnetic field.

Upon rescaling various quantities:

$$\partial_t\mu - \partial_x^2\mu + \lambda\partial_x(\mu^2) - \sigma_\perp\partial_\perp^2\mu = 0$$

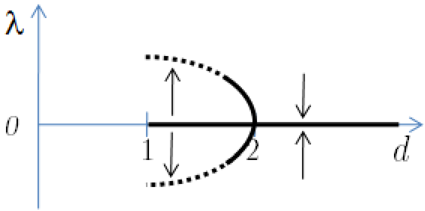
Rescaled coupling λ is marginal in $2+1$ and irrelevant in $3+1$.

Fate in the IR

Integrate out momentum shell $e^{-l}\Lambda < |k| < \Lambda$:

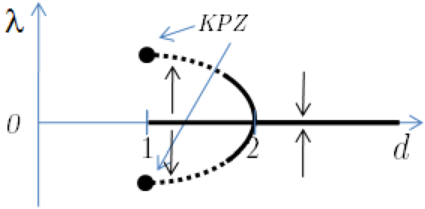
$$\frac{\partial\lambda}{\partial l} = \frac{1}{2}\varepsilon\lambda - \frac{\lambda^3}{2\pi}, \qquad \varepsilon = 2 - d$$

The theory is marginally irrelevant in $d = 2$ and has a non-trivial fixed point at $\varepsilon = 2 - d > 0$!



Fate in the IR

In $1+1$ dimensions, the theory is equivalent to KPZ (Kardar-Parisi-Zhang) universality class.



- Diffusive fluctuations around the chiral front at $x + \frac{4c}{\chi} t$ are in the KPZ universality class.
- Chiral diffusion flows to $\omega = k + k^z$, $z = \frac{3}{2}$, leading to the exact scaling

$$\sigma(\omega) = \langle J^i(\omega) J^i(-\omega) \rangle_{\mathrm{sym}} \sim \frac{T^{\frac{2}{3}} (c\chi')^{\frac{4}{3}}}{\chi} \frac{1}{\omega^{1/3}}$$

Remarks

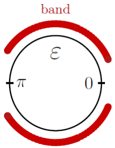
1. Infrared instability of chiral diffusion
2. Persists without momentum conservation
3. Relevant to edge physics

Non-equilibrium topology and Floquet systems

Floquet systems have time-dependent periodic Hamiltonian

$$H(t+T) = H(t), \quad U(t) = \mathcal{T} e^{-i\int_0^t H(s)ds}$$

- There is no strict notion of energy.
- Can define quasi-energies $\varepsilon_n \sim \varepsilon_n + \frac{2\pi}{T}$. Energy analog of Bloch theory.

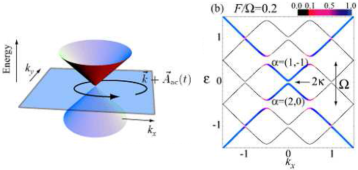


[Fruchart, '15]

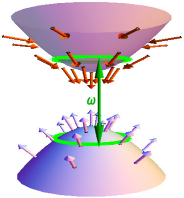
- Numerous recent theoretical works
Reviews: [Harper,Roy,Rudner,Sondhi '19; Rudner,Lindner '19]

Dynamical generation of topology

- Circularly polarized light opens a gap [Oka,Aoki '09]



- Time periodic magnetic field [Lindner,Refael,Galitski '11]



Experiments

[Wang,Steinberg,Jarillo-Herrero,Gedik '13]

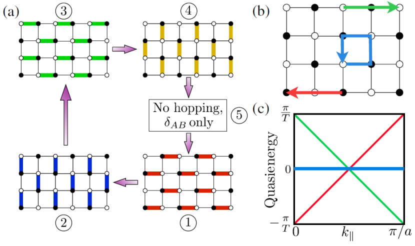
[Rechtsman,Zeuner,Plotnik,Lumer,Nolte,Segev,Szameit '13]

[Jotzu,Messer,Desbuquois,Lebrat,Uehlinger,Greif,Esslinger '14]

A canonical model: chiral Floquet drive

[Rudner, Lindner, Berg, Levin '12]

$$H(t) = -J \sum_{r \in A} (c^\dagger_{r+d(t)} c_r + c^\dagger_r c_{r+d(t)}), \qquad d(t) = \uparrow \to \downarrow \leftarrow$$



[Rudner et al., '12]

- Edge states [Fidkowski, Po, Potter, Vishwanath '16; Roy, Harper '16; Po et al. '16; von Keyserlingk, Sondhi '16]
- Quantized topological invariants [Rudner et al. '12; Iadecola, Hsieh '17]
- Quantized response: magnetization [Nathan et al. '16; Nathan et al. '19]

Recall: Static topological phases

For time-independent Hamiltonians, $H(t) = H_0$, a successful approach to many-body topological systems is that of topological field theory.

- Detect topological phases by coupling the system to background gauge fields.
- Example: integrate out fermions in $(2+1)$-dimensions

$$Z[A] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S[\psi,\bar{\psi},A]} = e^{-S_{\text{eff}}[A]}$$

- Response action S_{eff} is local, imaginary, and topological

$$S_{\text{eff}}[A] = i\frac{\nu}{4\pi}\int d^3x \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho, \qquad \nu = \text{integer}$$

- Powerful to diagnose and predict new topological phases.
- Works only when notion of ground state and gap are well-defined.

Aim: Reproduce the success of time-independent approach to Floquet systems.

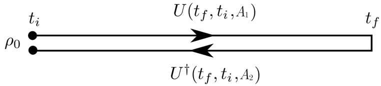
- Effective field theory approach.
- Diagnostic tool of topological order.

General setup – Schwinger-Keldysh approach

For systems out of equilibrium, the natural starting point is the Schwinger-Keldysh trace

$$e^{iW[A_1,A_2]} = \mathrm{Tr}\left[U(t_i,t_f;A_1)\rho_0 U^\dagger(t_i,t_f;A_2)\right]$$

- Analog of $Z[A_\mu]$ for time-independent Hamiltonians.
- ρ_0 is an initial state
- $U(t_i,t_f;A)$: unitary coupled to an external gauge field
  - Two unitaries for forward and backward evolutions
  - Two gauge fields A_1, A_2 for forward and backward evolutions



General setup

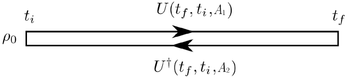
$$e^{iW[A_1,A_2]} = \mathrm{Tr}\left[U(t_i,t_f;A_1)\rho_0 U^\dagger(t_i,t_f;A_2)\right]$$

SK for Floquet topological systems:

- Initial state: Infinite temperature Gibbs ensemble

$$\rho_0 = \frac{e^{\alpha Q}}{\mathrm{Tr} e^{\alpha Q}}, \qquad \mathcal{Q} = \sum_r (n_r - \tfrac{1}{2})$$

- Real time contour: Integer multiple of Floquet period T



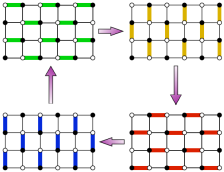
$$t_f - t_i = \kappa T, \quad \kappa = \text{integer}$$

- Background: Static background

$$A_0 = 0, \qquad \vec{A}(t,\vec{r}) = \vec{A}(\vec{r})$$

up to gauge fixing.
Note: gauge invariance under $A_{1,2} \to A_{1,2} + \partial\lambda_{1,2}$ with $\lambda_1 = \lambda_2$ at $t = t_i, t_f$.

Topological response: chiral Floquet drive



On a closed manifold:

$$e^{iW[A_1,A_2]} = \frac{1}{2\cosh(\frac{\alpha}{2})^N} \prod_r \left[e^{-\frac{\alpha}{2}} + e^{\frac{\alpha}{2}} e^{i\int \frac{dt}{T}(B_{1r}-B_{2r})} \right]$$

where

- $\int \frac{dt}{T} =$integer
- N total number of lattice sites
- $B_r = A_x(r) + A_y(r+b_1) - A_x(r+b_1+b_2) - A_y(r+b_2)$ flux collected by a particle starting at r

Topological response: chiral Floquet drive

For slowly varying background, leads to a spatial theta term:

$$e^{iW[A_1,A_2]} = e^{i\frac{\Theta(\alpha)}{2\pi}\int\frac{dt}{T}\int d^2r[B_1(r)-B_2(r)]}, \qquad B(r) = d\vec{A}(r)$$

where

$$\Theta(\alpha) = \theta + f(\alpha), \quad \theta = \Theta(\alpha = 0), \quad f(\alpha) = -f(-\alpha)$$

- θ is quantized due to flux quantization and a charge-conjugation symmetry.
- From explicit evaluation:

$$\theta = \pi, \qquad f(\alpha) = -\pi \tanh\frac{\alpha}{2}$$

- Independent of metric of the spatial manifold $\Rightarrow$ topological term.

Topological response: chiral Floquet drive

$$e^{iW[A_1,A_2]} = e^{i\frac{\Theta(\alpha)}{2\pi}\int \frac{dt}{T}\int d^2r[B_1(r)-B_2(r)]}$$

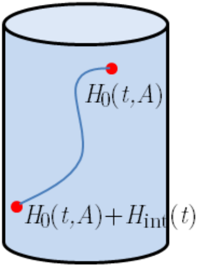
$\Theta(\alpha)$ independent of continuous deformations:

$$H(t,A) = H_0(t,A) + \lambda H_{\text{int}}(t)$$

$$H_0: \text{ chiral Floquet drive,} \quad H_{\text{int}}: \text{ many-body interaction}$$

Independent of λ as far as response remains local.

Sketch of the proof:



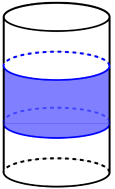
1. $\frac{\delta W}{\delta A_{1i}(r)} = -i\int dt \mathrm{Tr}[\rho_0 J^i(r,t)] \equiv -i\langle J^i(r)\rangle$
2. $\langle J^i(r)\rangle = \mathrm{Tr}[\rho\frac{\delta H_0}{\delta A_i}] = 0$
3. EFT:
   $W = \frac{1}{2\pi}\int\frac{dt}{T}\int d^2r\Theta(\alpha,r)(B_1(r)-B_2(r))$

$$\Rightarrow \quad \frac{\delta W}{\delta A_{1i}} \propto \int dt\varepsilon^{ij}\partial_j\Theta(\alpha,r) = 0$$

Topological response: chiral Floquet drive

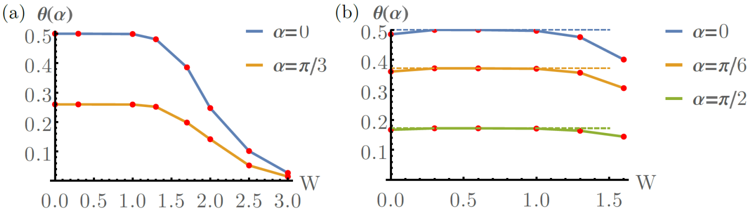
Numerical test of topological stability.

Open boundary conditions:



$$H = H_0(t, A) + \sum_r w_r c_r^\dagger c_r + V_0 \sum_r (-1)^{\eta_r} c_r^\dagger c_r$$

$w_r \in (-W, W)$ random, uniformly distributed



Plot of $\Theta(\alpha)$ as a function of disorder strength W, for various values of α when $\lambda = 0$ (a) and $\lambda = 0.1(b)$.

Remarks

1. Theta term can be related to quantized magnetization [Nathan et al. '16; Nathan et al. '19]
2. Relation to chiral unitary index [Po et al. '16]
3. Formalism provides EFT approach to topological Floquet phases (higher dimensions, geometric response, ...)

Summary

- Non-equilibrium EFT provides a very flexible tool to approach the low energy sector of a wide variety of systems
- IR instability of chiral hydrodynamics
- Topological response of driven (Floquet) systems

Future directions

1. Chiral diffusion: include energy conservation; estimate effect for realistic systems (comparison to shot noise?)
2. Floquet: geometric response; constraints on $\Theta(\alpha)$? time-ordering sensitive topological response? Non-topological properties?
3. Other directions: open systems and novel constraints