



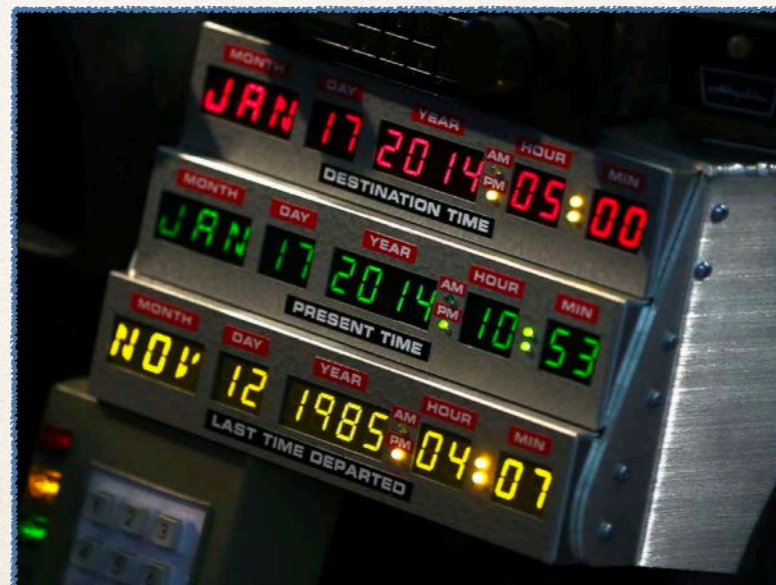
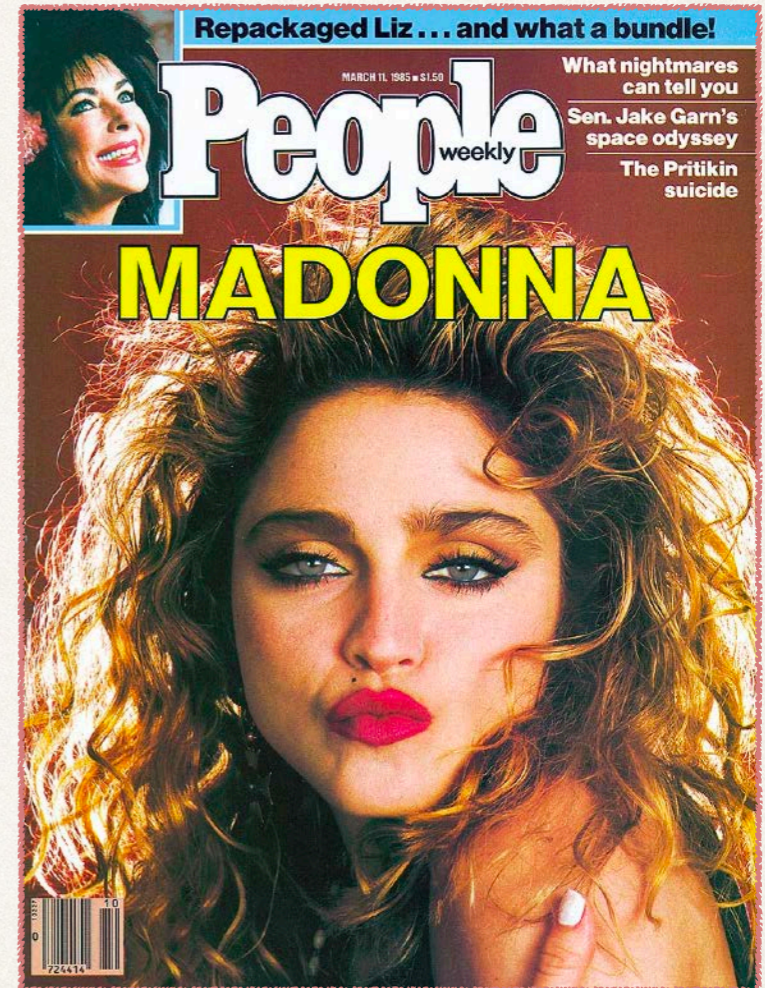
# Inconsistency of SuperFluid DM with Milky Way Observables

Oren Slone, Princeton University



arXiv: 1812.08169 and 1911.12365 - M. Lisanti, M. Moschella, N. Outmezguine and O. Slone

# Great things from the 80's



# Great things from the 80's

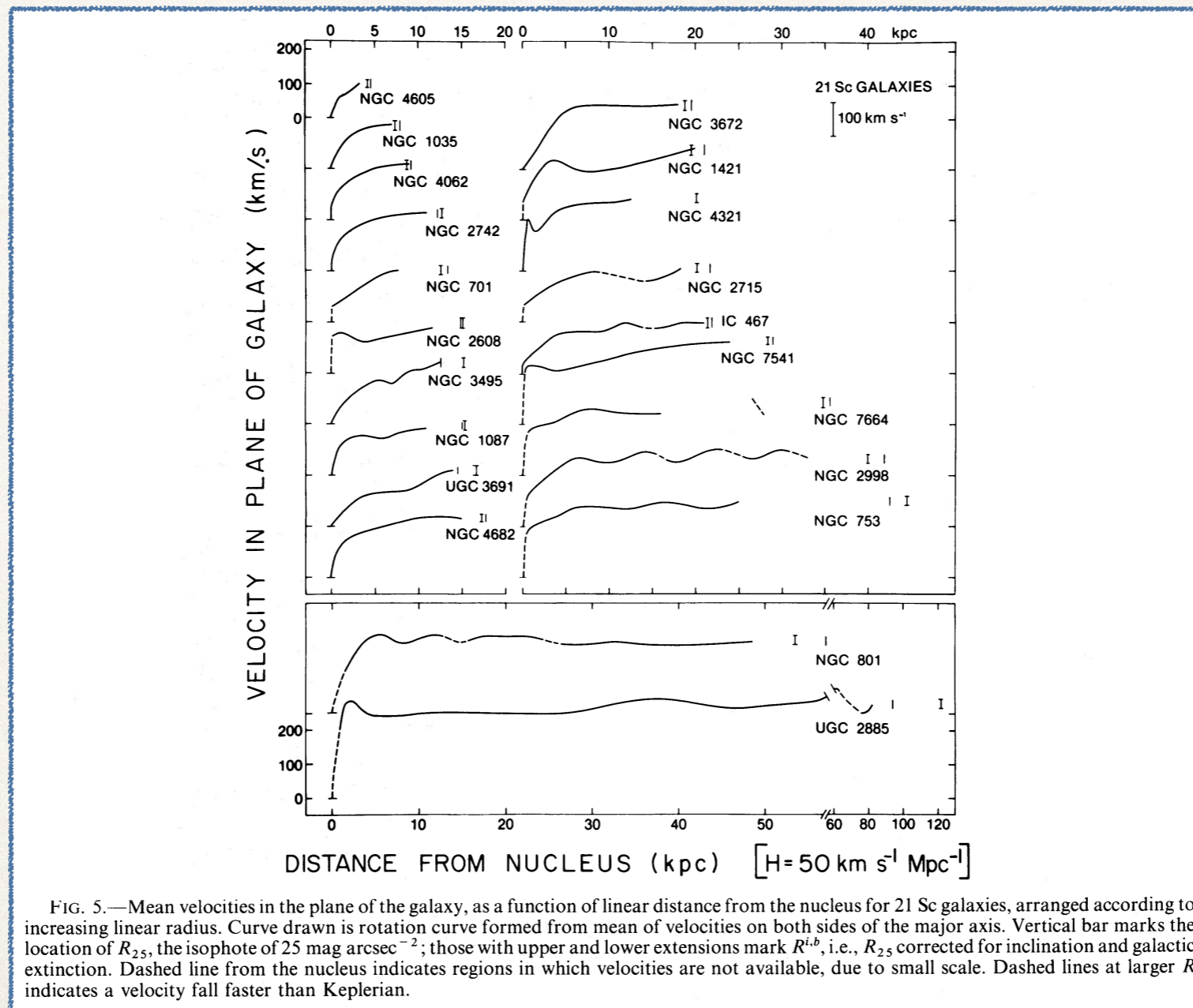


FIG. 5.—Mean velocities in the plane of the galaxy, as a function of linear distance from the nucleus for 21 Sc galaxies, arranged according to increasing linear radius. Curve drawn is rotation curve formed from mean of velocities on both sides of the major axis. Vertical bar marks the location of  $R_{25}$ , the isophote of  $25 \text{ mag arcsec}^{-2}$ ; those with upper and lower extensions mark  $R^{i,b}$ , i.e.,  $R_{25}$  corrected for inclination and galactic extinction. Dashed line from the nucleus indicates regions in which velocities are not available, due to small scale. Dashed lines at larger  $R$  indicates a velocity fall faster than Keplerian.

Vera Rubin, Ford and Thonnard, June 1980

# A Naive Solution

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$$\nabla^2 \Phi = 4\pi G\rho$$



Amazingly: Still not clear-cut on galactic scales

# Outline

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- Missing Mass and Galaxy Scale Observables
- Features of Various Classes of Solutions
- Superfluid Dark Matter
- Framework to Test Various Models using MW data
- Results and Conclusions

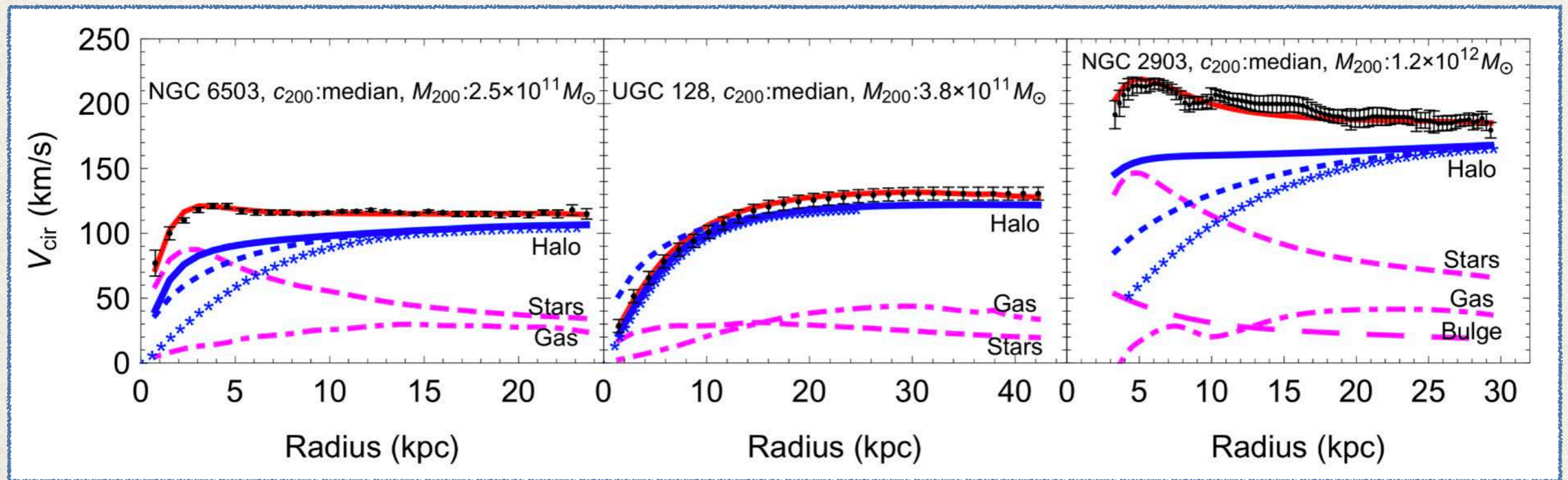
# The Missing Mass Problem on Galactic Scales, 2019

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- **Flat Rotation Curves**
- **Issues with Small Scales:**
  - Missing Satellites (maybe solved)
  - Too Big To Fail
  - Core vs Cusp
- **DM Correlates with Baryons:**

# Galaxy Scale Observables

## The Diversity Problem



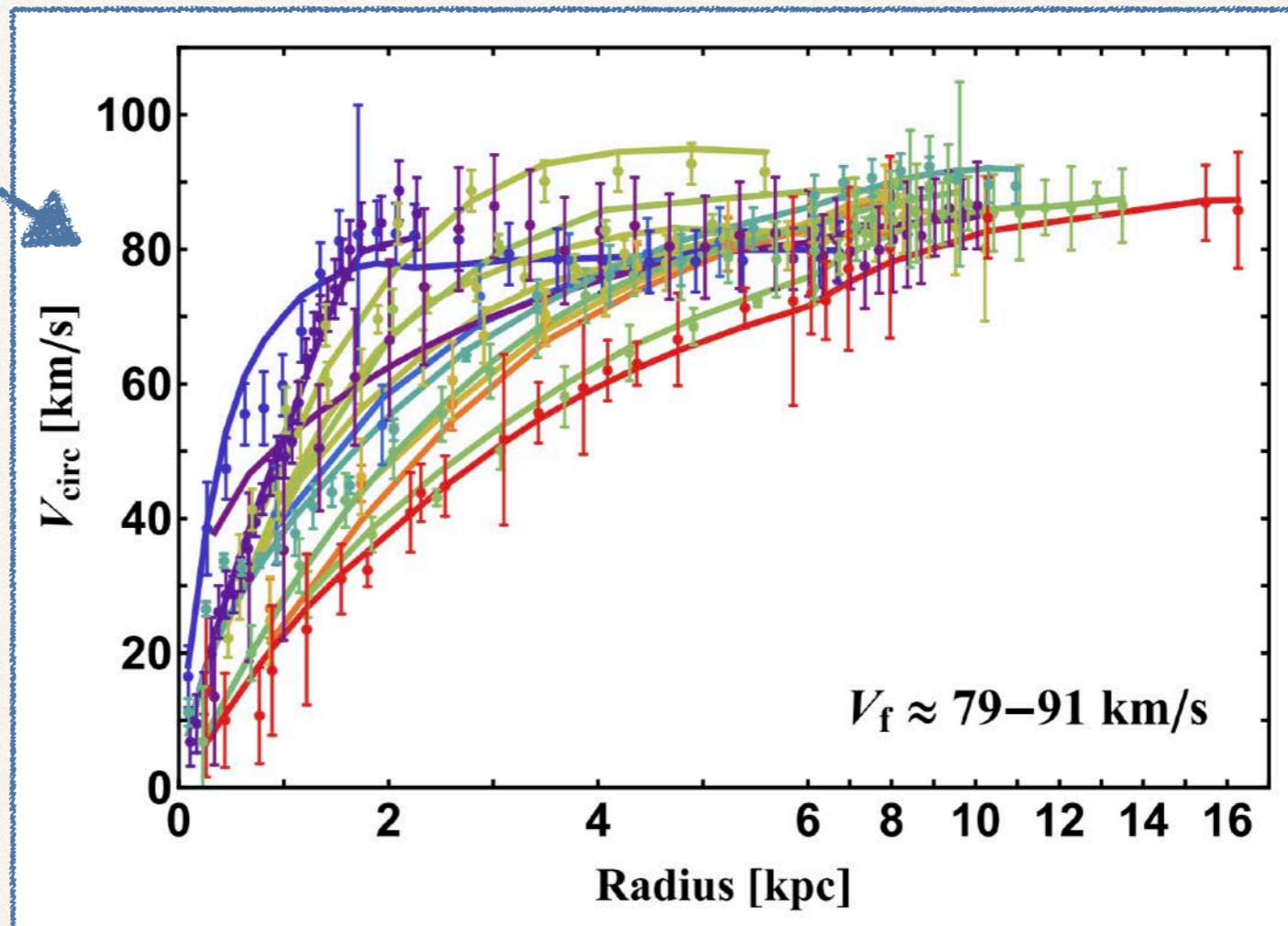
Kamada et. al., 2016

- Diversity of inner rotation curves even for galaxies with similar halo and stellar mass.
- Rotation curves correlate with baryons

# Galaxy Scale Observables

## The Diversity Problem

DM dominated galaxies!



Kamada et. al., 2016

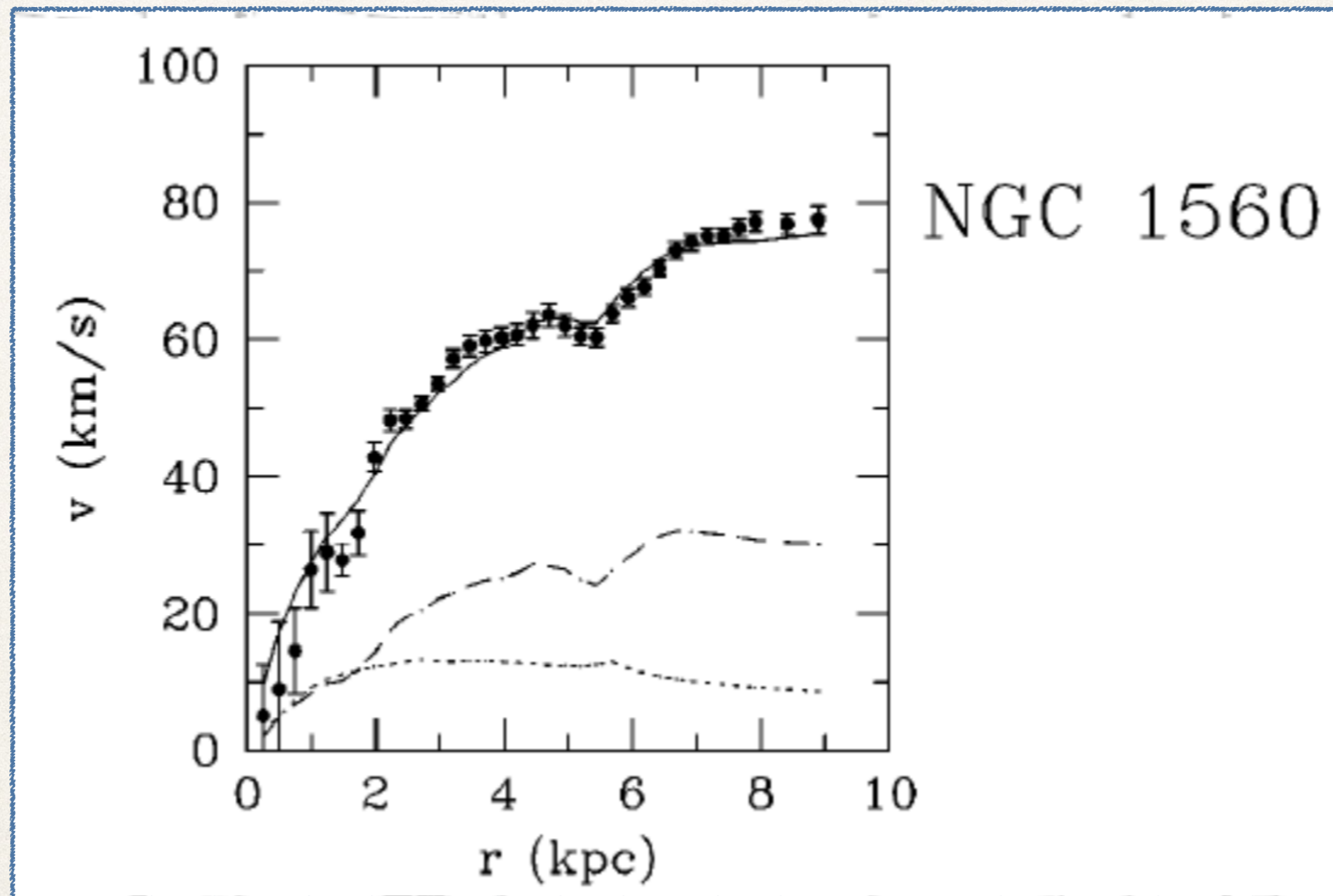
- Low surface brightness - halo is cored
- High surface brightness - halo is cusped
- Self similar if scaled to baryonic scale radius



# Galaxy Scale Observables

## Renzo's Rule

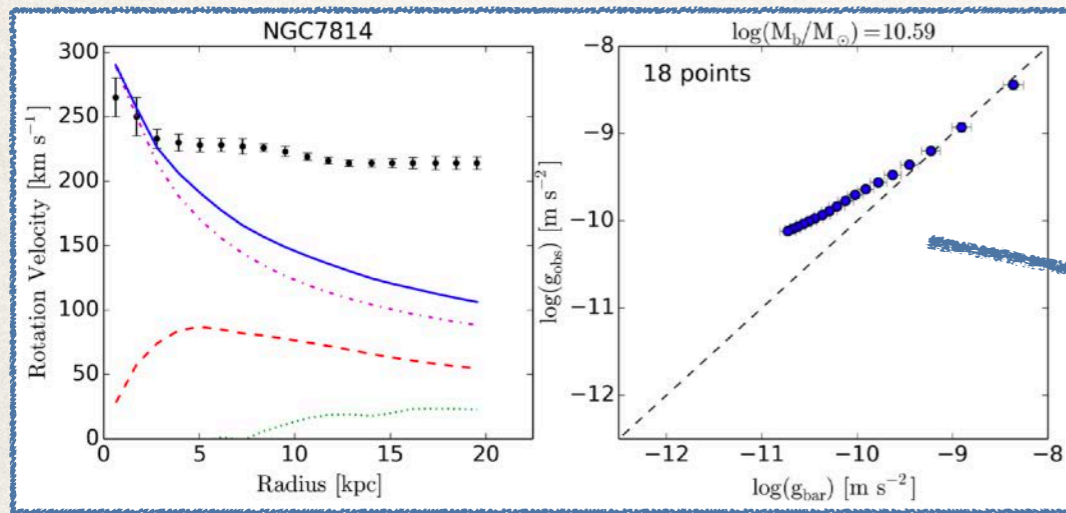
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Sancisi, 2003

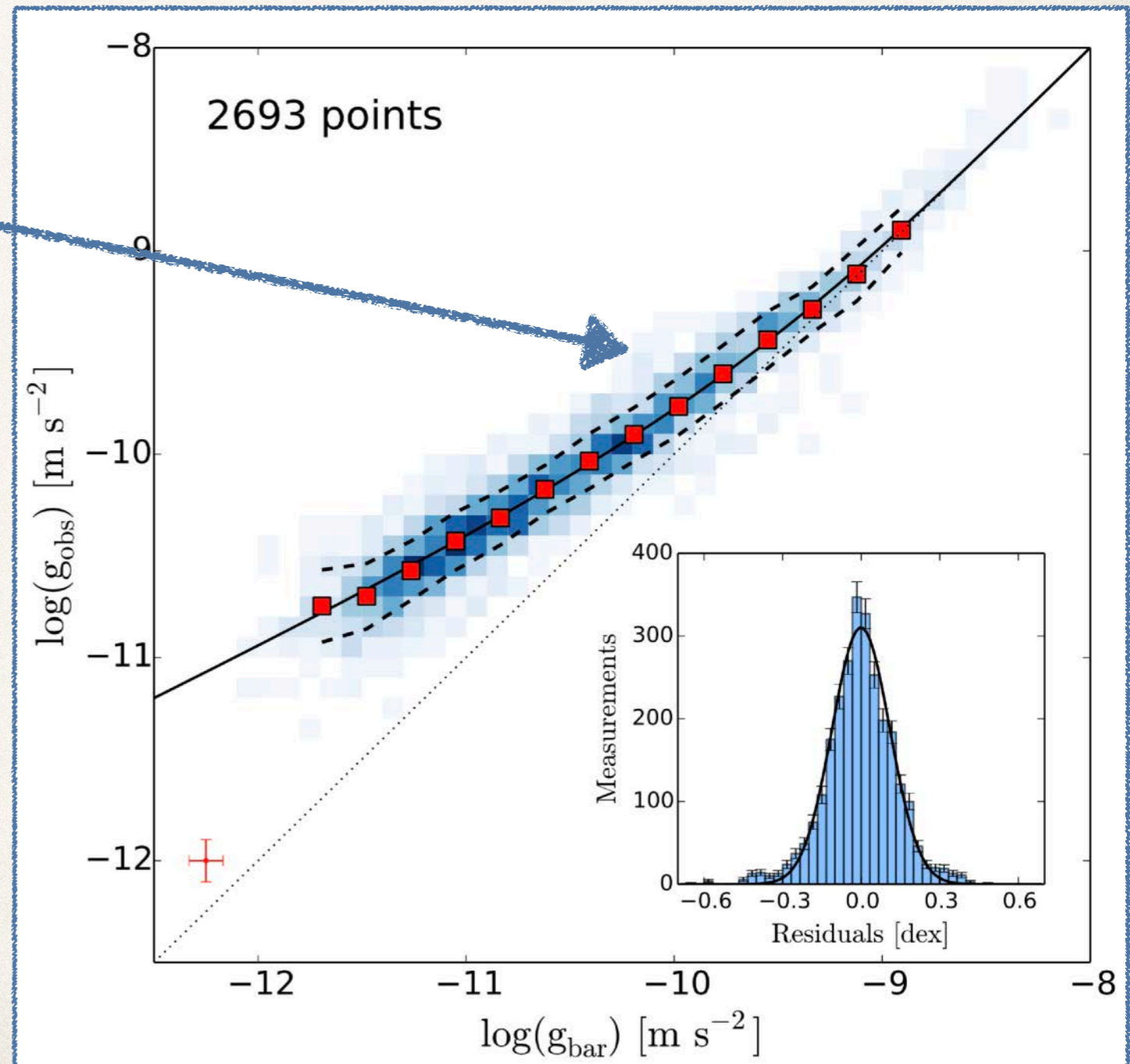
# Galaxy Scale Observables

## The Radial Acceleration Relation (RAR)



Lelli et. al, 2017

A tight correlation and an acceleration scale appear in rotation curve data from the SPARC catalog



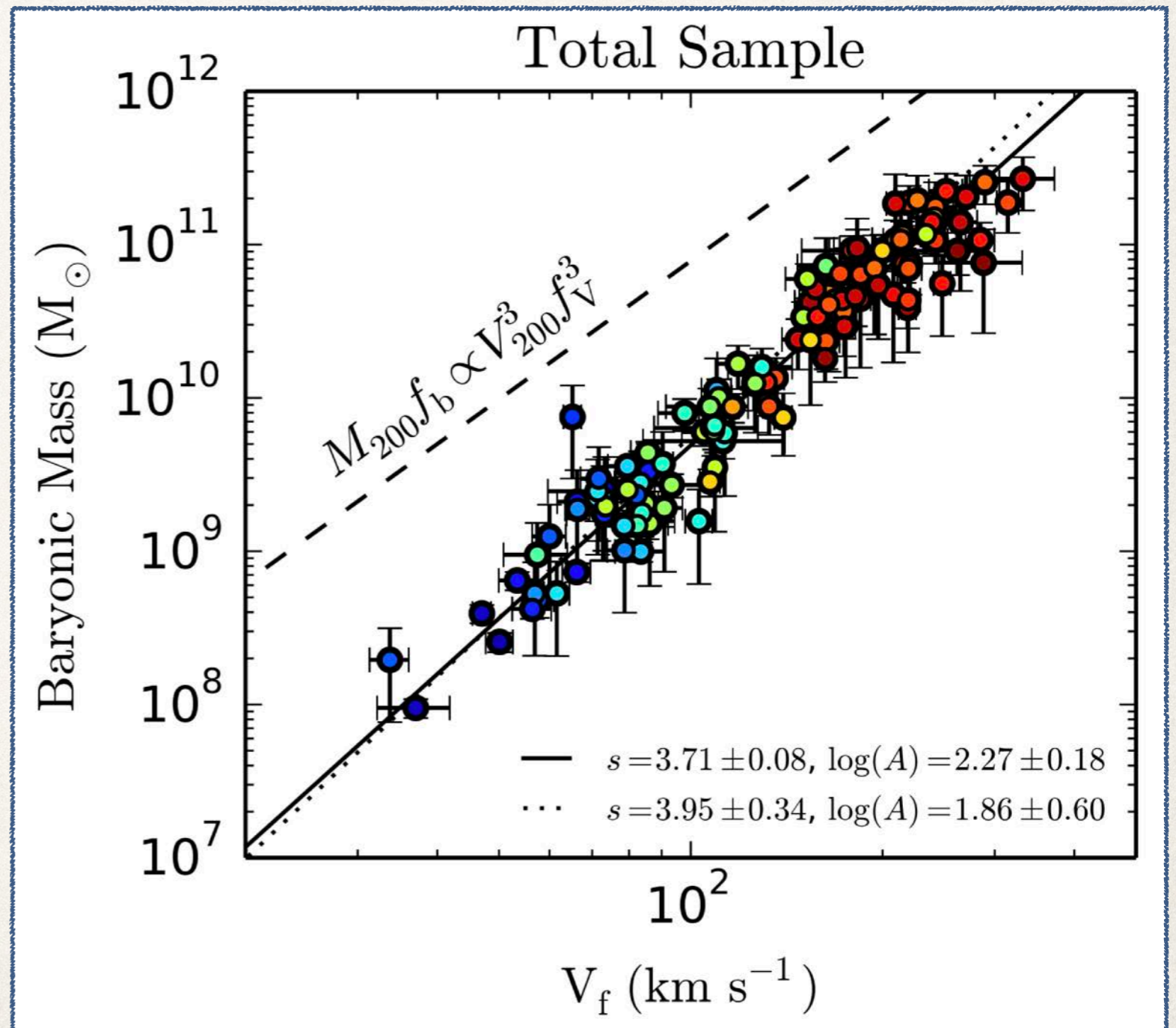
McGaugh, Lelli, 2017 10

# Galaxy Scale Observables

## The Baryonic Tully-Fisher Relation

A result of the information in the low end of the RAR

$$g_{\text{obs}} \propto \sqrt{g_{\text{bar}}} \Rightarrow \frac{V_f^2}{R} \propto \frac{\sqrt{GM_{\text{bar}}}}{R}$$




# Galaxy Scale Observables

## A Universal Scaling Relation

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A Mass Discrepancy Acceleration Relation (MDAR) appears to be a feature of galaxies:

$$a = \begin{cases} a_N & a \gg a_0 \\ \sqrt{a_0 a_N} & a \ll a_0 \end{cases}$$


An acceleration scale appears in the data

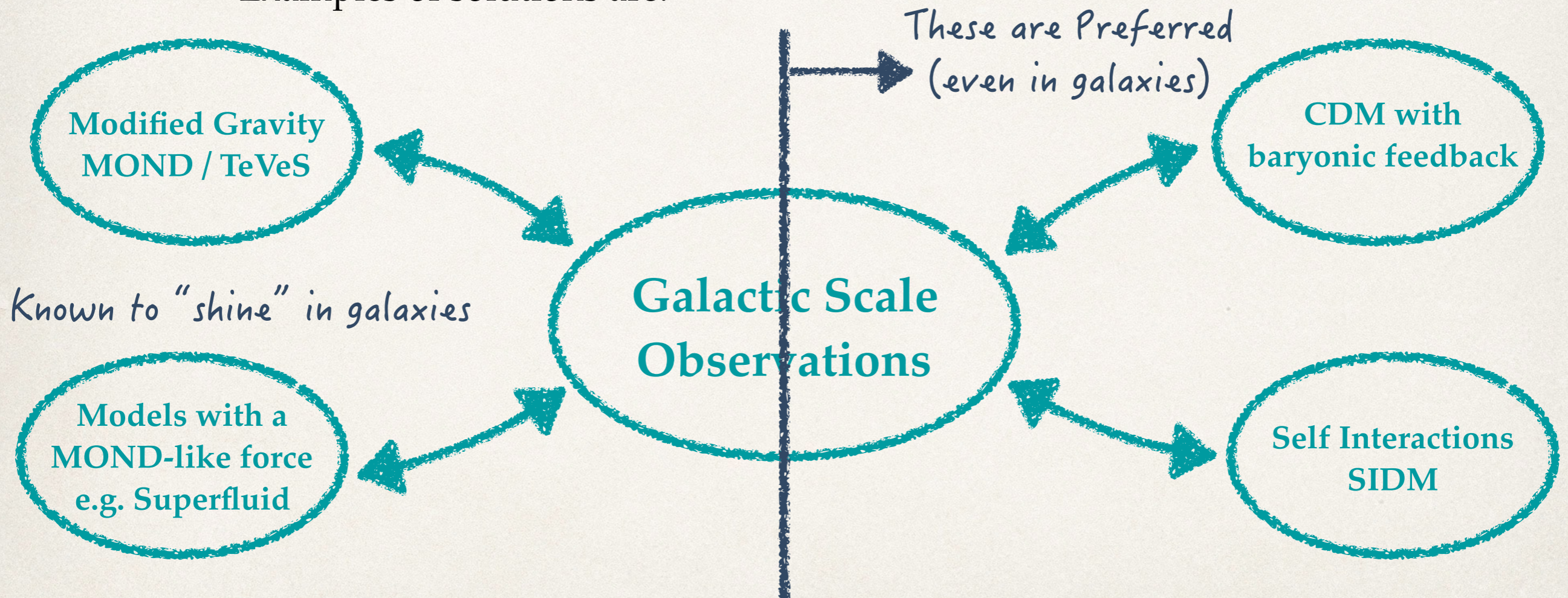
$$a_0 \sim 1.2 \times 10^{-10} \text{ m/sec}^2 \sim \frac{1}{6} H_0$$

# Galaxy Scale Observables

What models resolve these issues?

- Galaxies provide clues that DM correlates with baryons.
- Examples of solutions are:

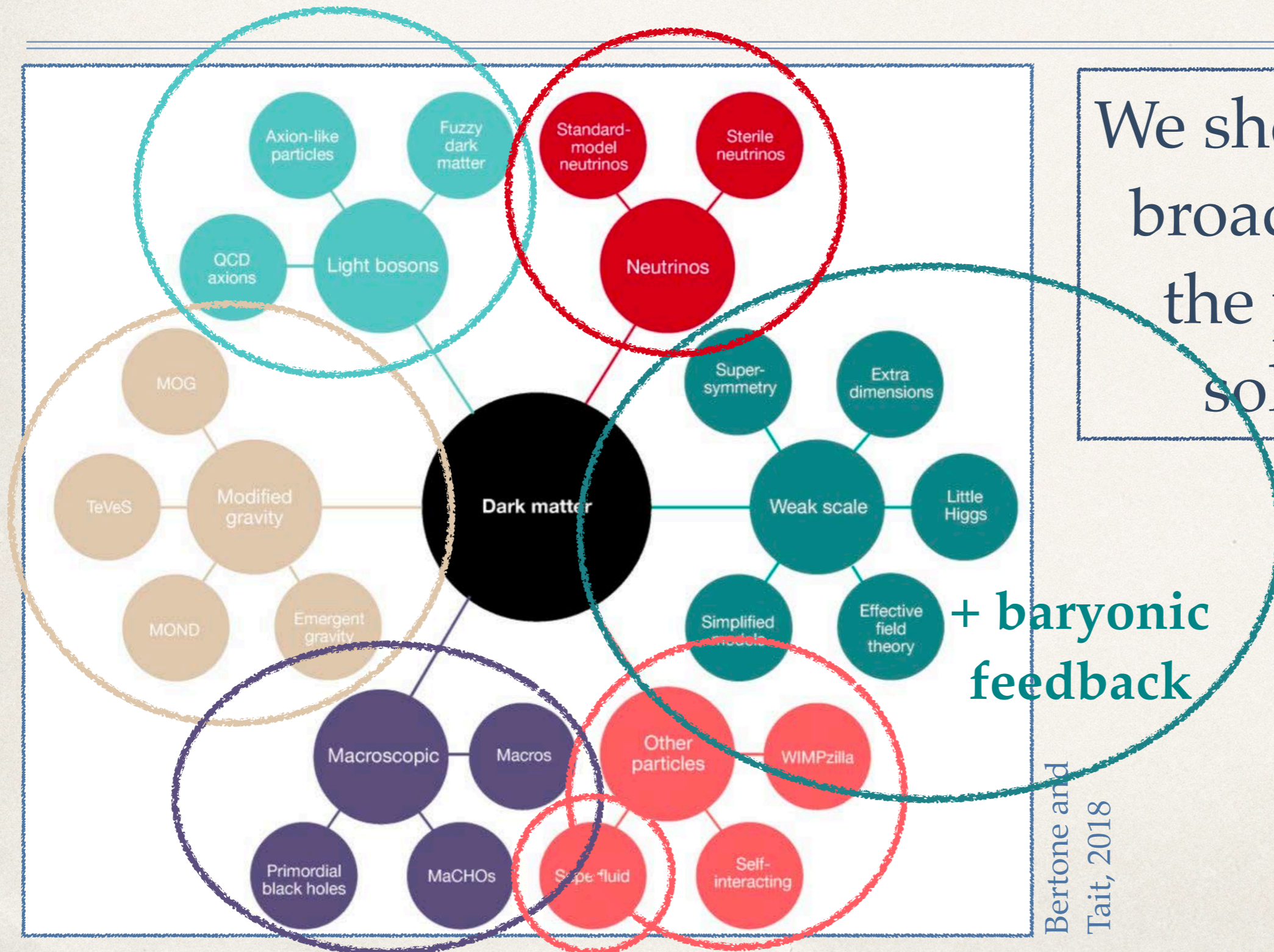
## SUMMARY OF THIS TALK



Or maybe DM mimics MOND on galactic scales?

# Solutions?

We should think broadly about the possible solutions



+ baryonic feedback

Bertone and Tait, 2018

# Fitting the MDAR with a Fundamental Force

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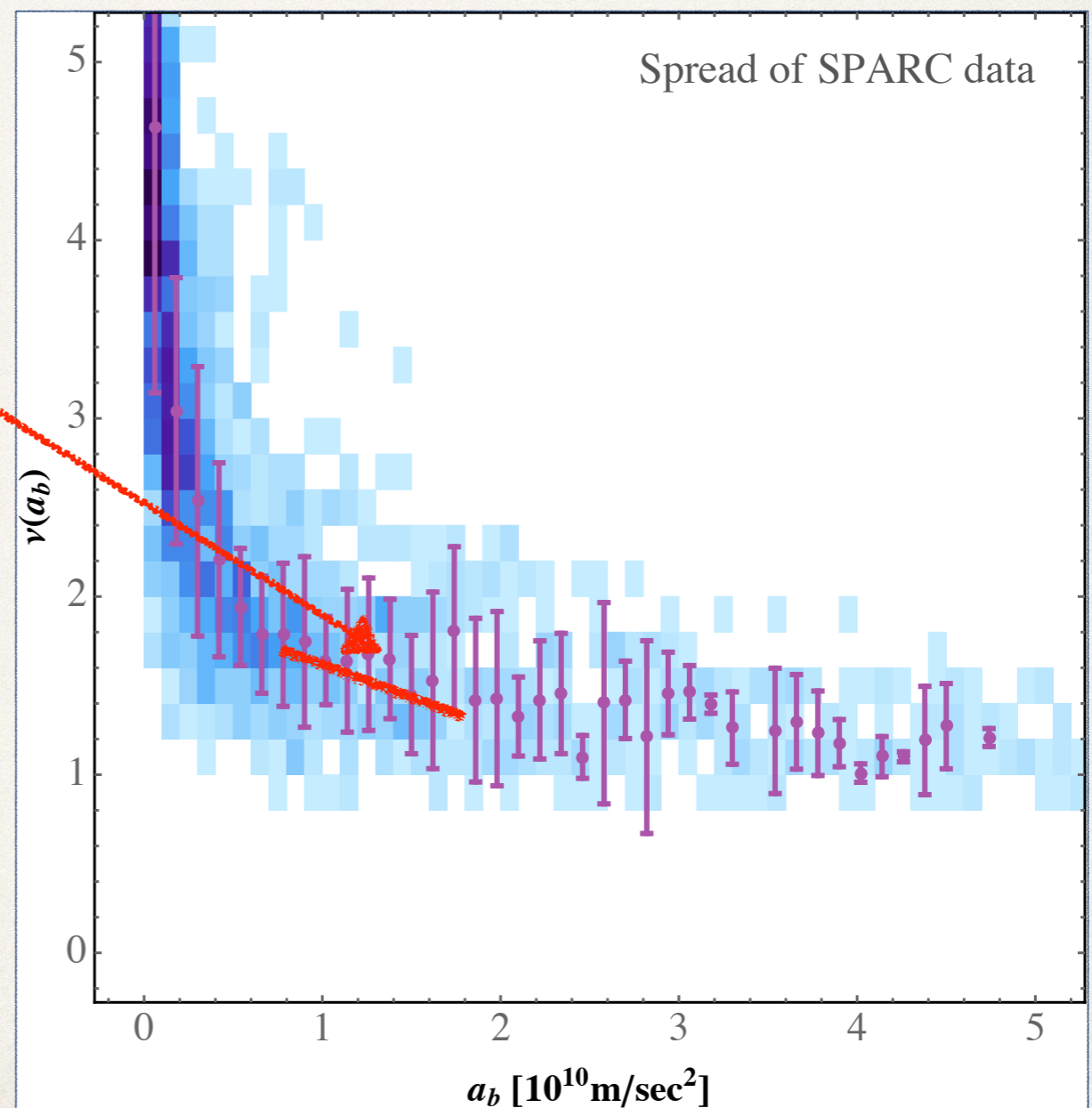
- Produce flat rotation curves:  $\Phi \propto \log r \rightarrow a \propto \frac{1}{r} \rightarrow v_c \propto \text{const}$
- Different models do this in various ways
- They typically reduce to:  $a = \nu \left( \frac{a_N}{a_0} \right) a_N$
- With an interpolation function with asymptotes:  $\nu(x_N) = \begin{cases} x_N^{-1/2} & x_N \ll 1 \\ 1 & x_N \gg 1 \end{cases}$
- This reproduces the MDAR:  $a = \begin{cases} a_N & a \gg a_0 \\ \sqrt{a_0 a_N} & a \ll a_0 \end{cases}$

# Fitting the MDAR with a Fundamental Force

For example:  
 Solar acceleration happens to live here  
 $\hat{\nu}_\alpha(x_N) = \left(1 - e^{-x_N^{\alpha/2}}\right)^{-\frac{1}{\alpha}}$   
 McCaugh, et. al. 2016

Local measurements are sensitive only to small deviation in acceleration

$$\mathbf{a} = \nu \left( \frac{a_N}{a_0} \right) \mathbf{a}_N \rightarrow \mathbf{a} = (\nu_0 + \nu_1 a_N) \mathbf{a}_N$$





# What can we do?

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## 1. Ask a model independent question:

- Can local MW measurements fit a generic model that predicts the MDAR with a fundamental force?

Anything that mimics  
MOND

## 2. Test a specific realization:

- e.g. A specific interpolation function
- e.g. Superfluid dark matter

(Test these models where they're supposed to shine!)

# Superfluid DM

Justin Khoury, Lasha Berezhiani

- Consider a light scalar DM particle with mass  $m$ .
- Require condensation to a state where the relevant DOF are phonons:

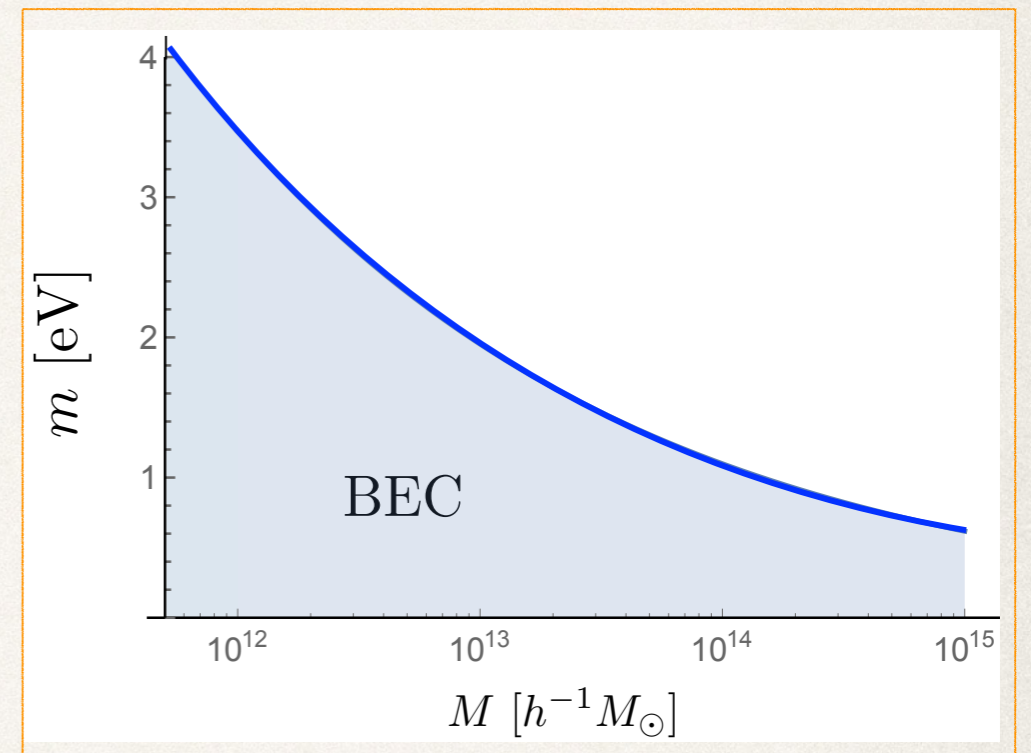
- An overlapping de Broglie wavelength:

$$\frac{1}{mv} \geq \left( \frac{m}{\rho_{\text{vir}}} \right)^{1/3} \Rightarrow m \lesssim 2\text{eV}$$

- With a critical temperature:

$$T_c \approx \frac{1}{3}mv^2 \approx \text{few} \left( \frac{\text{eV}}{m} \right)^{5/3} \text{ mK}$$

(~ known values in cold atom systems)



Berezhiani, Khoury, 2015

# Superfluid DM

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$$T \approx mv_{\text{vir}}$$

Galaxies



$$T_{\text{gal}} \approx 0.1\text{mK}$$

**Super Fluid Phase**

**MOND-Like Emergent Force**

Galaxy Clusters



$$T_{\text{cluster}} \approx 10\text{mK}$$

**Cold DM**

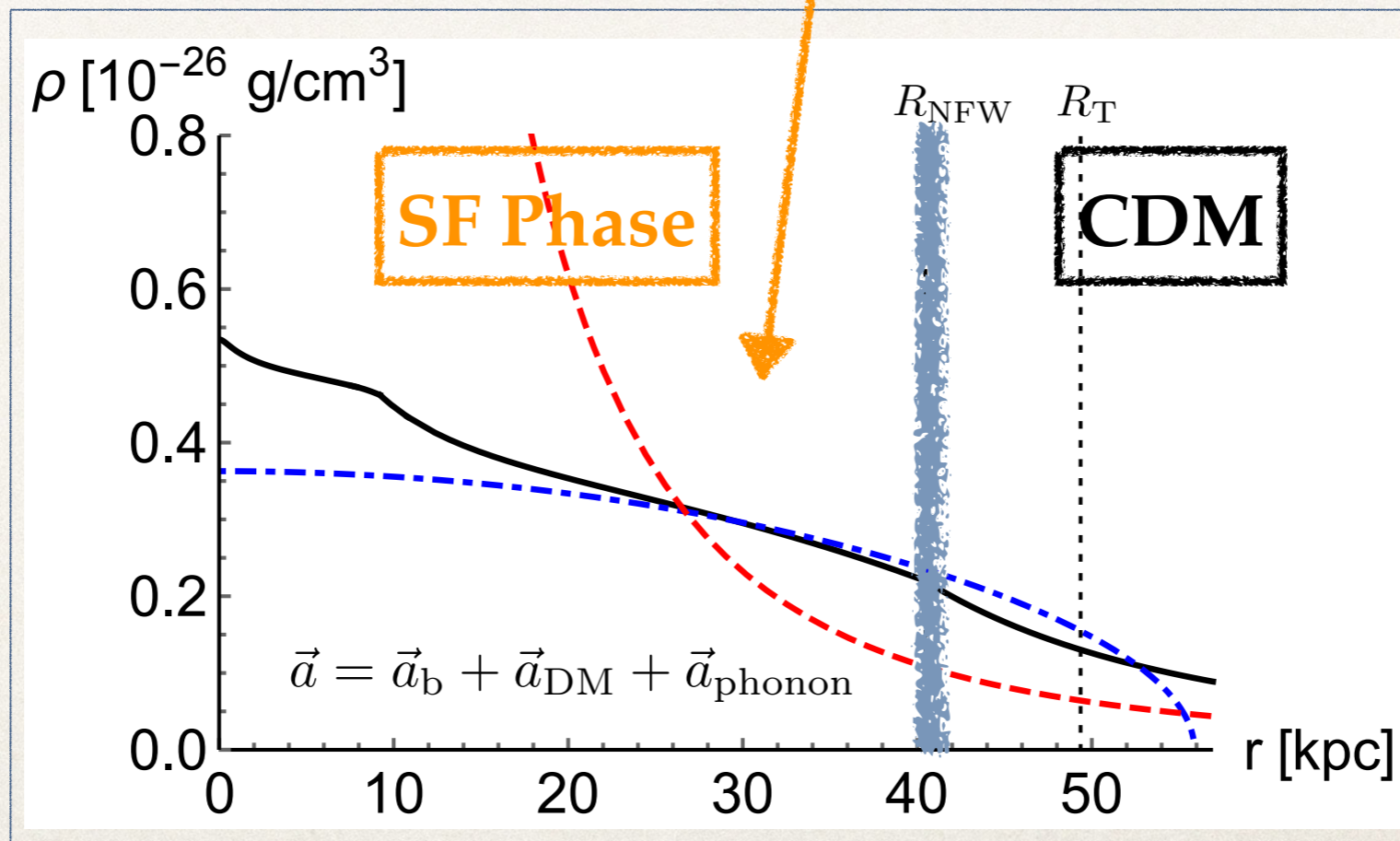
**Standard DM Dynamics**

# Superfluid DM

$$\mathcal{L}_{\text{DM}, T=0} = \frac{2\Lambda(2m)^{3/2}}{3} X \sqrt{|X|} - \alpha \frac{\Lambda}{M_{\text{Pl}}} \phi \rho_b$$

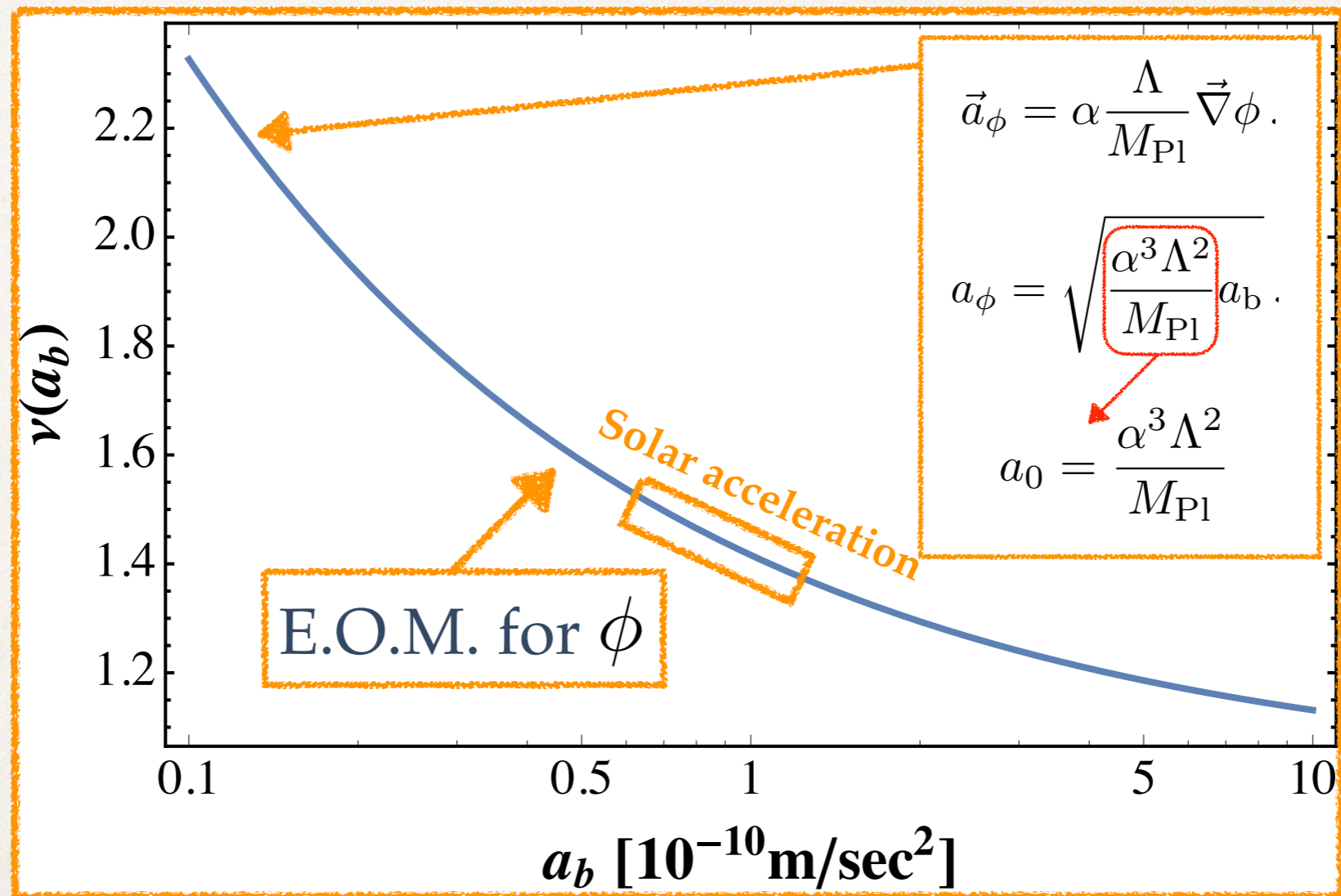
$$X = -m\Phi - (\vec{\nabla}\phi)^2/2m.$$

$$\rho_{\text{SF}} = \frac{\partial \mathcal{L}}{\partial \Phi}$$



# Superfluid DM

## The SF Interpolation Function:

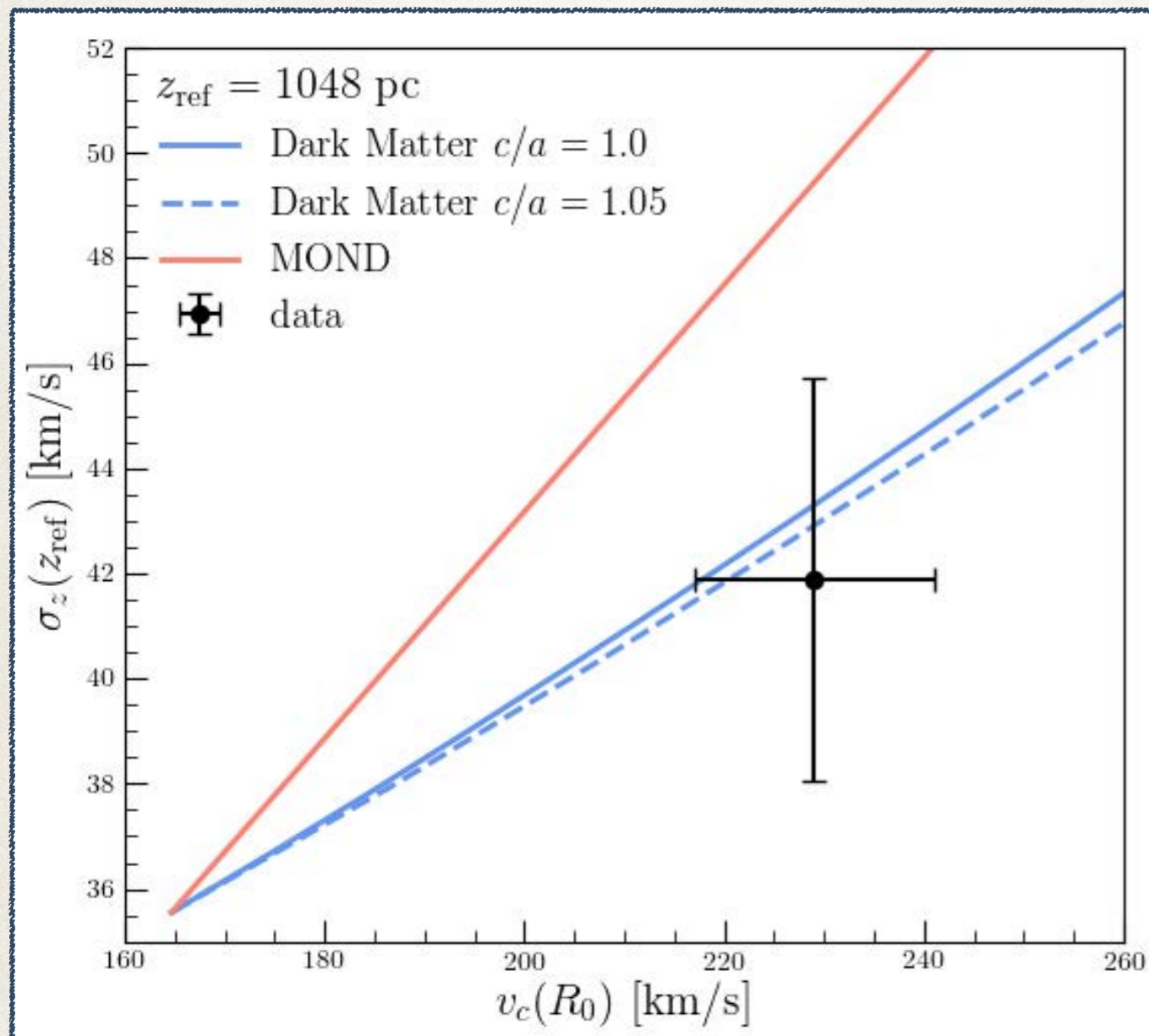


# Constraining These Models

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# Local MW Observations Provide Differentiating Power

Compare accelerations in the R and z directions:



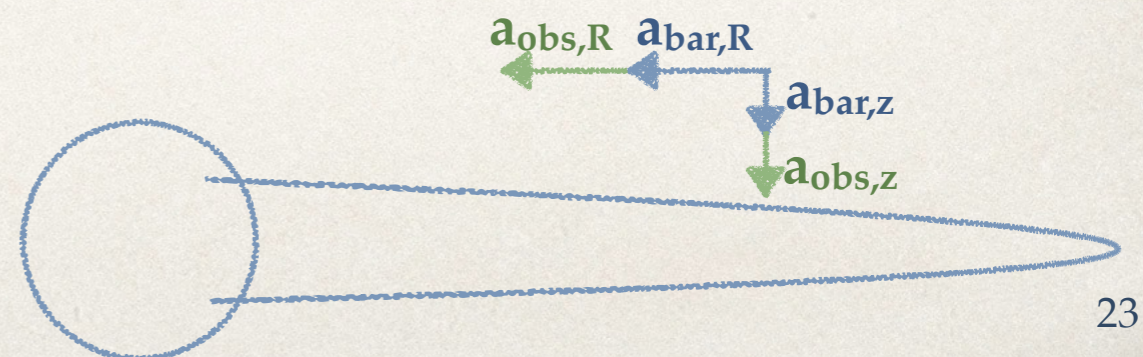
Lisanti, Moschella, Outmezguine, O.S., 2018

- Data requires amplification in  $a_R$  but essentially none in  $a_z$ .
- A spherical DM halo does precisely this:

$$\mathbf{a}_{\text{DM}} \approx -G \frac{M(R_0)}{R_0^2} \left( 1, \frac{z}{R_0} \right)$$

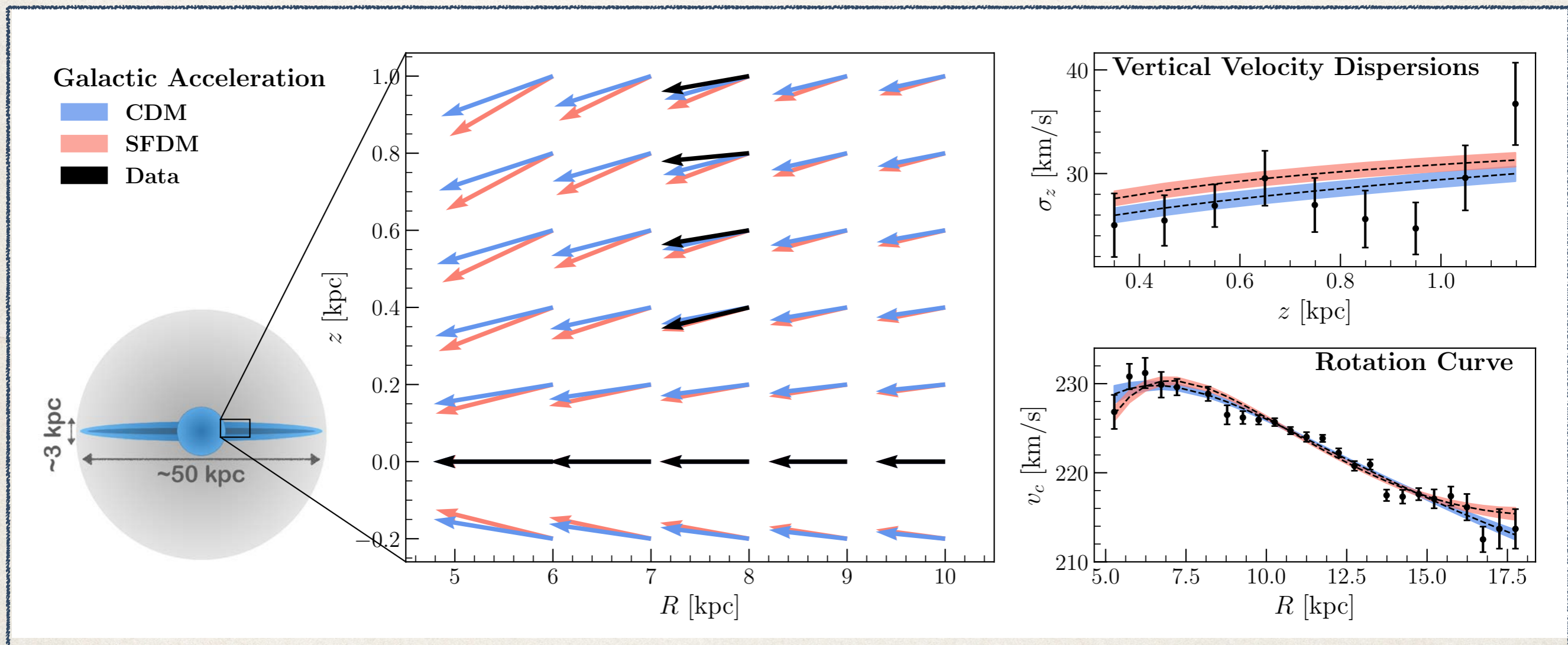
- A slightly prolate halo is slightly better.
- A MOND-like force amplifies  $a_R$  too little or  $a_z$  too much:

$$\frac{a_z}{a_R} = \frac{a_{z,N}}{a_{R,N}} \Big|_{\text{disk}}$$



# Local MW Observations Provide Differentiating Power

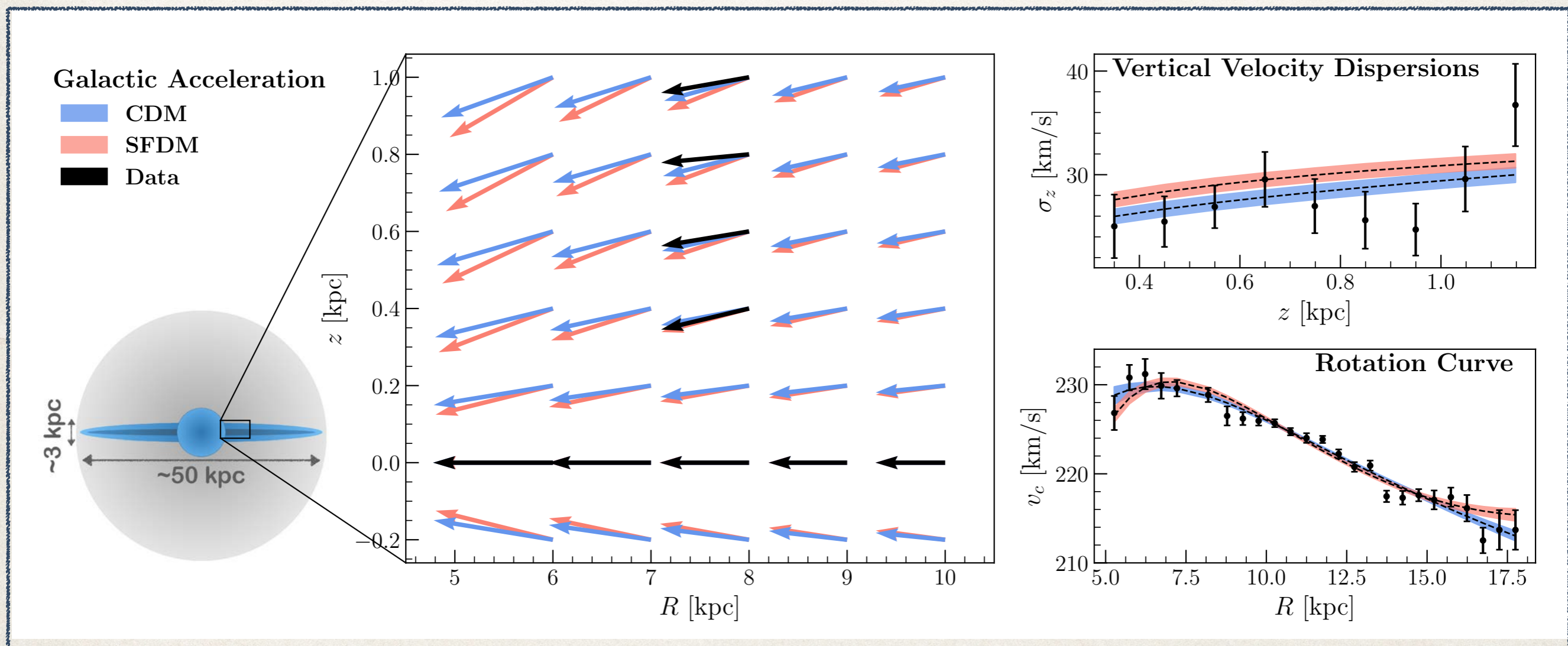
Superfluid Dark Matter is even more predictive:





# Local MW Observations Provide Differentiating Power

A new criterion for any theory which attempts to reproduce the MDAR



# Local MW Observations Provide Differentiating Power

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- In principle: measure  $\mathbf{a}$  and  $\mathbf{a}_N$  and you're done!
- However measurements are imperfect:
  - Baryonic profile is not perfectly measured.
  - Accelerations are not directly measured.  
Velocities and velocity dispersions are.
- Therefore: Adopt a **Bayesian Approach**

# Local MW Observations Provide Differentiating Power

## Bayesian Approach

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- Given a model:  $M = \text{CDM vs SFDM} / \text{a generic MOND-like force}$
- With parameters:  $\theta_{\mathcal{M}}$
- Construct a likelihood function:  $\mathcal{L}(\theta_{\mathcal{M}}) \propto \exp \left[ -\frac{1}{2} \sum_{j=1}^N \left( \frac{X_{j,\text{obs}} - X_j(\theta_{\mathcal{M}})}{\delta X_{j,\text{obs}}} \right)^2 \right]$
- $\mathbf{X}_{\text{obs}}$  : a set of measured values imposed as constraints
- $\mathbf{X}(\theta_{\mathcal{M}})$  : the corresponding model predictions
- Impose reasonable priors on  $\theta_{\mathcal{M}}$  and recover posterior distributions

# Analysis Procedure:

## Testing CDM vs SFDM / MOND-like

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# Analysis Procedure

## Milky Way Model

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SFDM/MOND-like  
For MOND use a  
Taylor expansion of  
the interpolation func

Dark Matter  
A generalized NFW  
profile

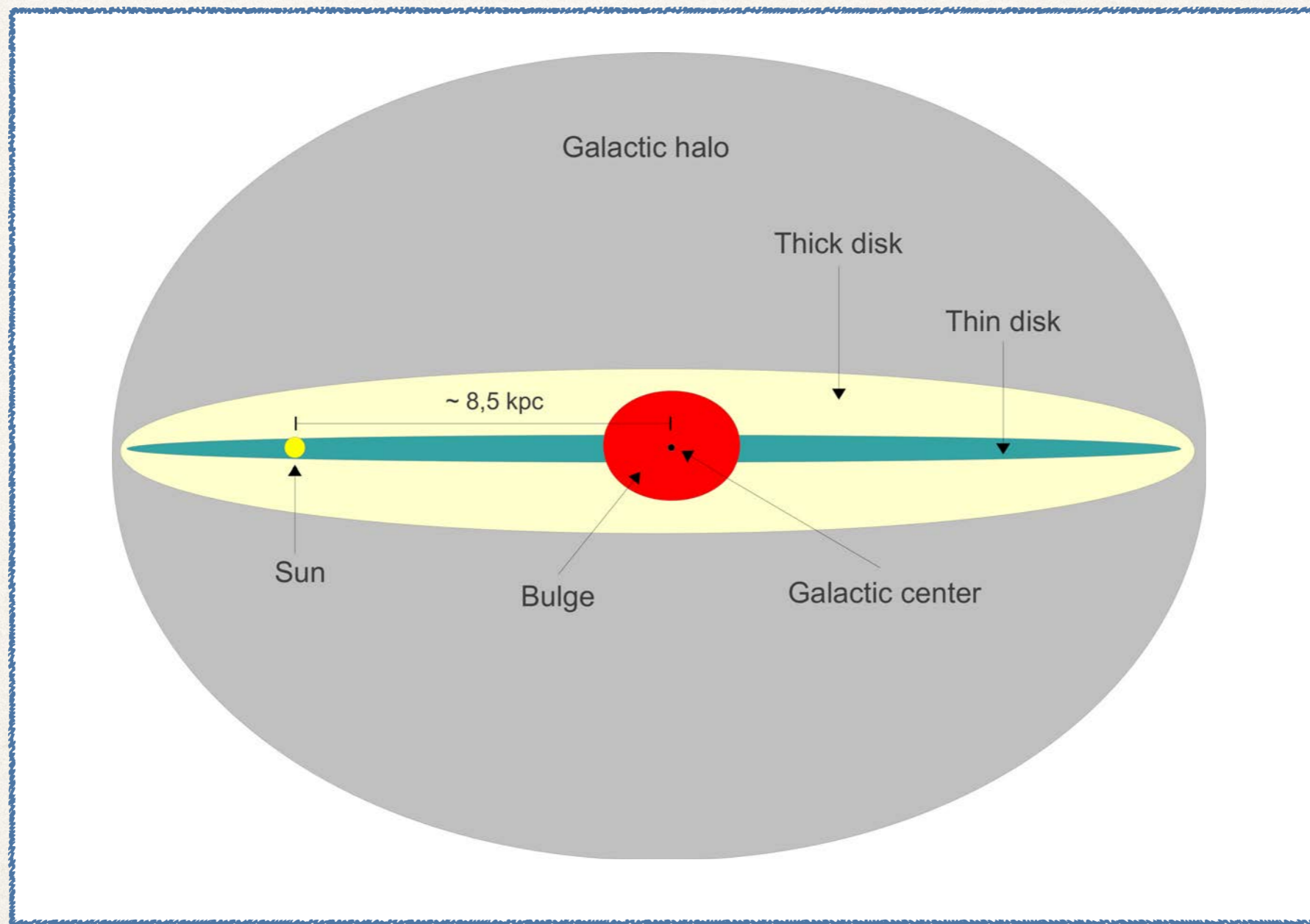
Model baryonic profile:

- Double exponential stellar disk
- Double exponential gas disk
- Hernquist stellar bulge

Perform a Markov Chain Monte Carlo analysis  
and fit parameters to MW measurements

# Analysis Procedure

## Baryonic Density Profiles



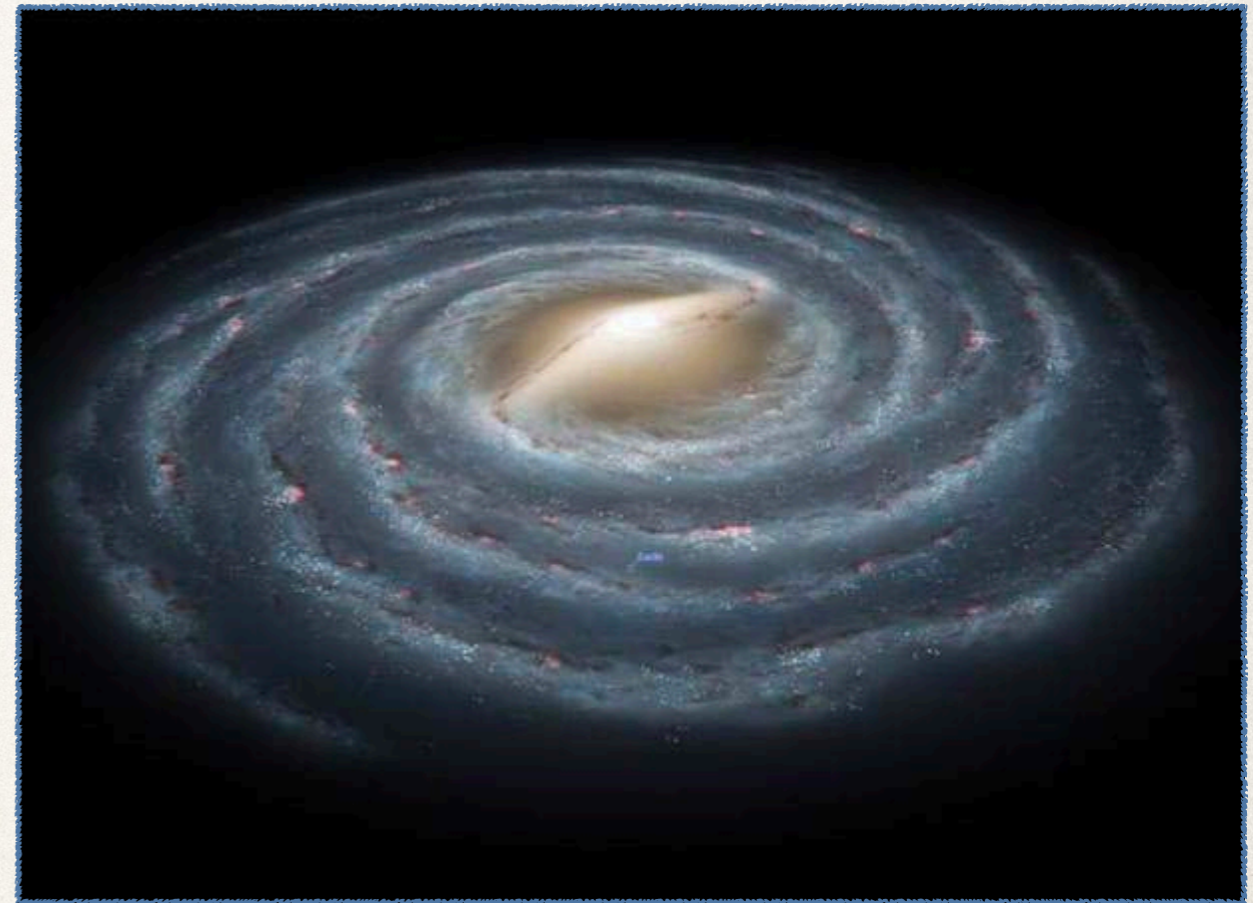
$$\rho_B = \rho_{*,\text{bulge}} + \rho_{*,\text{disk}} + \rho_{g,\text{disk}}$$

# Analysis Procedure

## Milky Way Observables

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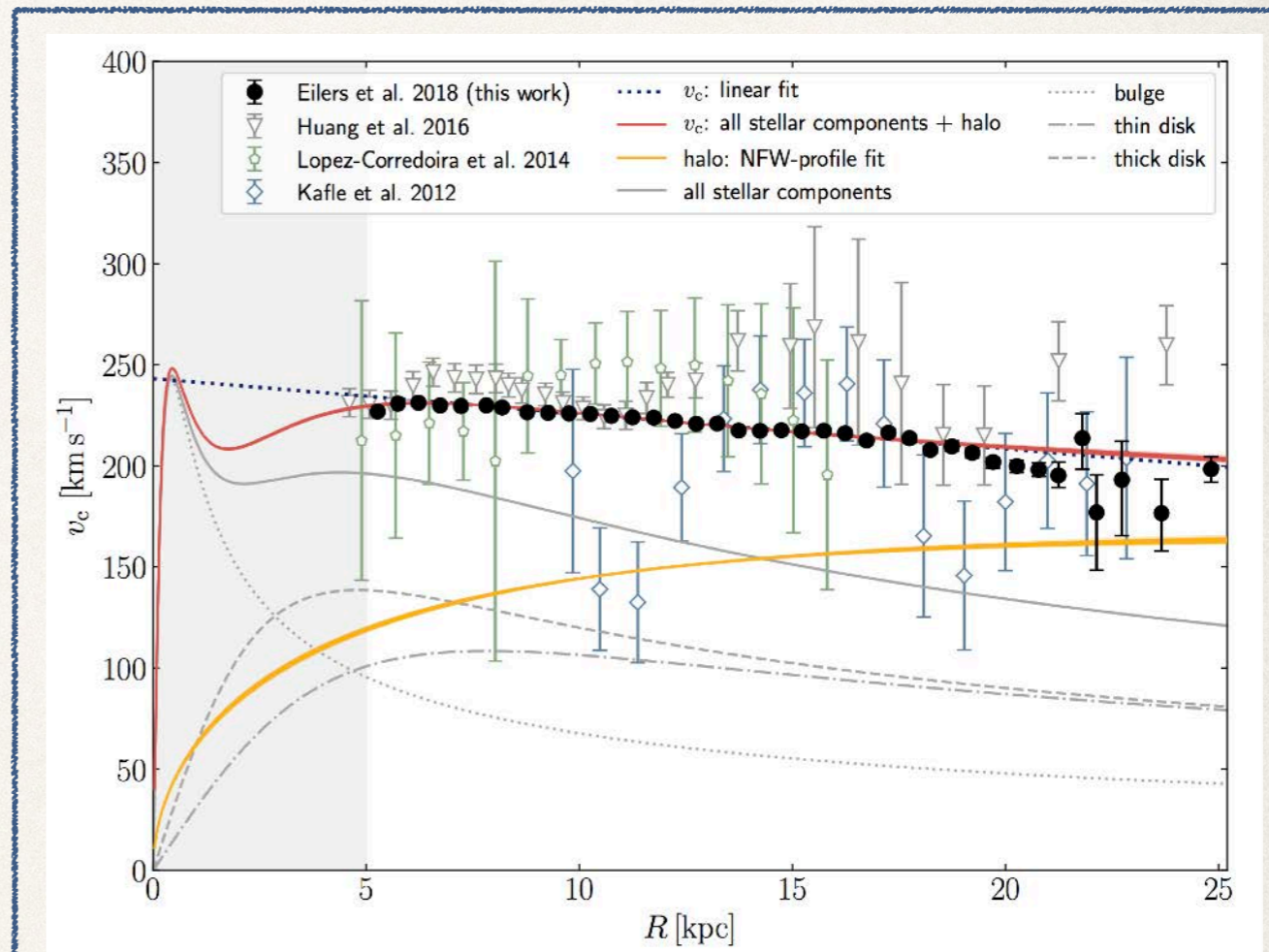
- Local stellar surface density
- Local gas surface density
- Disk scale radii (stars and gas)
- Disk scale heights (stars and gas)
- Bulge mass
- Rotation curve
- Vertical acceleration



# Analysis Procedure

## Milky Way Observables

- Local stellar surface density
- Local gas surface density
- Disk scale radii (stars and gas)
- Disk scale heights (stars and gas)
- Bulge mass
- Rotation curve**
- Vertical acceleration



Eilers et. al., 2018

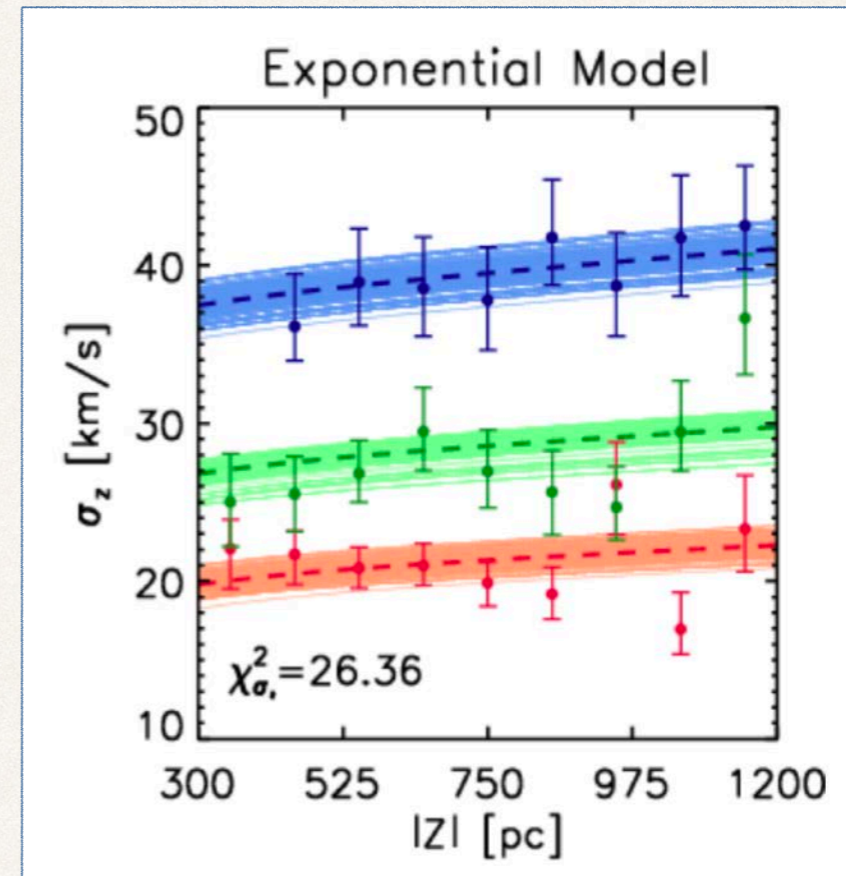
$$v_c(R) = \sqrt{R \cdot a(R)} \Big|_{z=0}$$



# Analysis Procedure

## Milky Way Observables

- Local stellar surface density
- Local gas surface density
- Disk scale radii (stars and gas)
- Disk scale heights (stars and gas)
- Bulge mass
- Rotation curve
- **Vertical acceleration**  
Inferred from 9000 K-dwarfs in the SEGUE sub-survey of the SDSS



Zhang et. al., 2013

$$\sigma_{i,z}(z)^2 = \frac{n_i(0) \sigma_{i,z}(0)^2}{n_i(z)} + \frac{1}{n_i(z)} \int_0^z n_i(z') a_z(z') dz'$$

# Analysis Procedure

## Ensure Self Consistency

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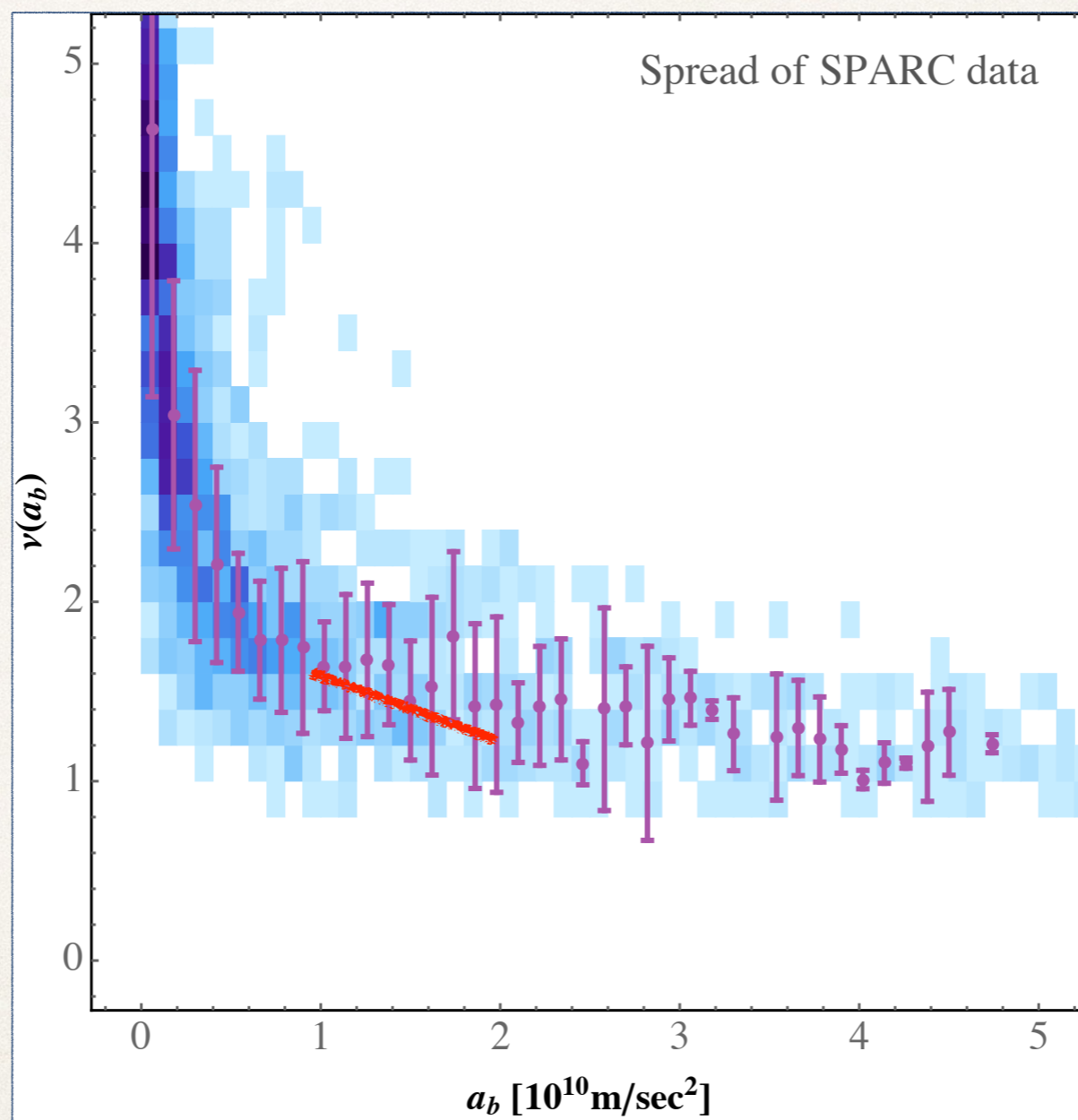
- Only use measurements from locations where non-linear effects are negligible
- Only use measurements which were not inferred dynamically under the assumption of DM

# RESULTS

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# Results for any MOND-like Model

FIT ONLY LOCAL  
ROTATION  
CURVE

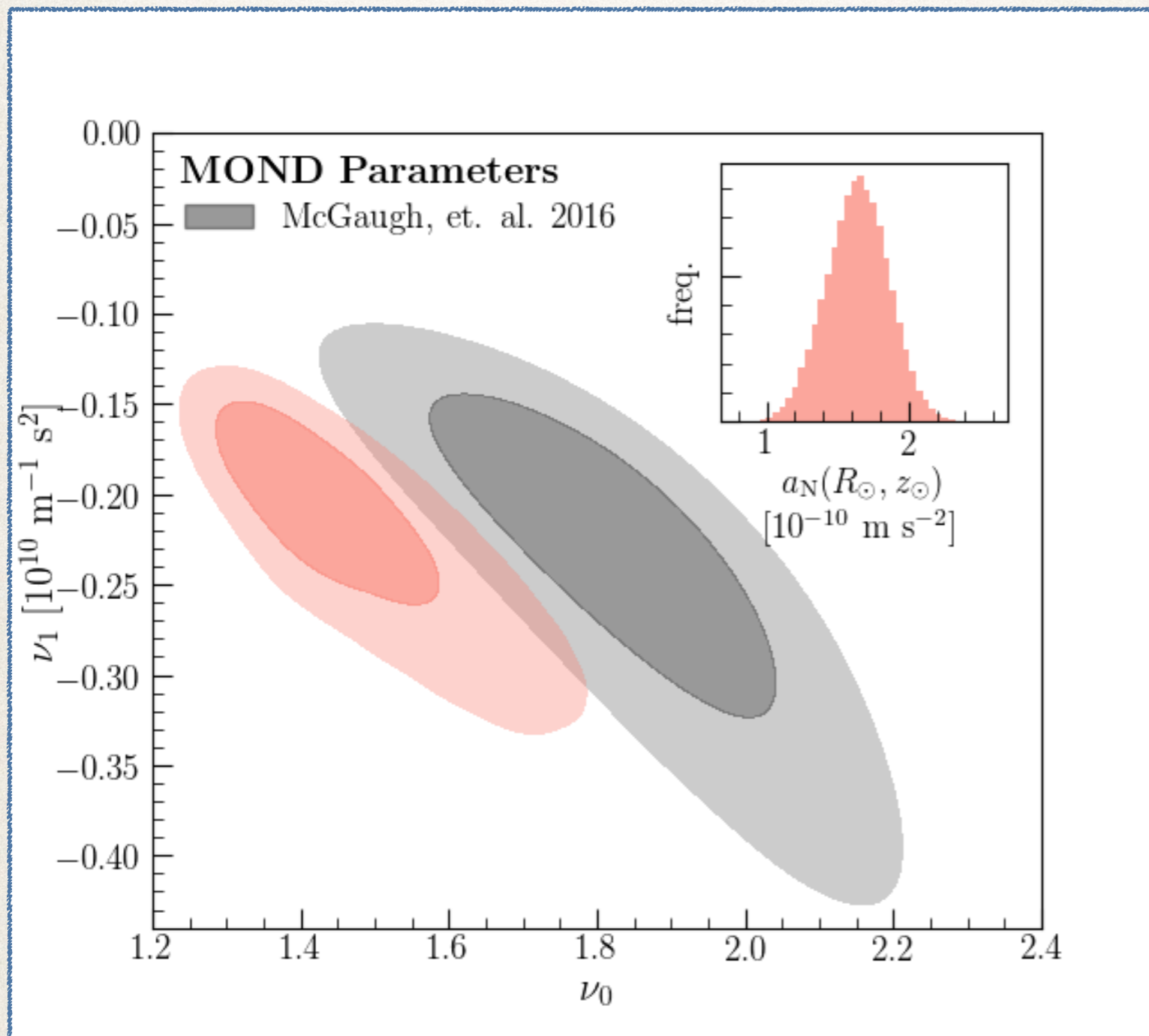


Lisanti, Moschella, Outmezguine, OS  
(PRELIMINARY)

$$\mathbf{a} = \nu \left( \frac{a_N}{a_0} \right) \mathbf{a}_N \rightarrow \mathbf{a} = (\nu_0 + \nu_1 a_N) \mathbf{a}_N$$

# Results of MCMC Scans

## Interpolation Function Parameters



Interpolation function  
 fitted to RAR:

$$\nu(a_N/a_0) = \frac{1}{1 - e^{-\sqrt{a_N/a_0}}}$$

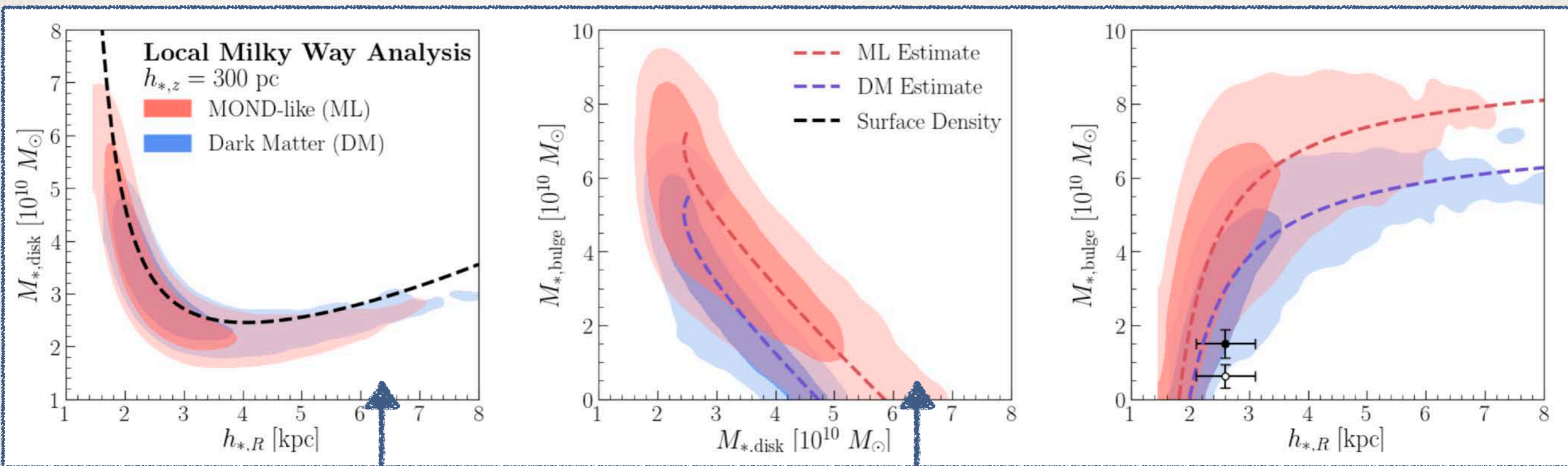
with

$$a_0 = 1.20 \pm 0.24 \times 10^{-10} \text{ m s}^{-2}$$

Excluded at 95%  
 confidence

# Results of MCMC Scans

## Tension with MW Observations



1812.08169 - Lisanti, Moschella, Outmezzguine, O.S.

Driven by stellar surface density constraint

$$M_{*,\text{disk}} = \frac{2\pi h_{*,R}^2 \Sigma_{*,\text{obs}}^{z_{\text{max}}} \exp(R_{\odot}/h_{*,R})}{1 - \exp(-z_{\text{max}}/h_{*,z})}$$

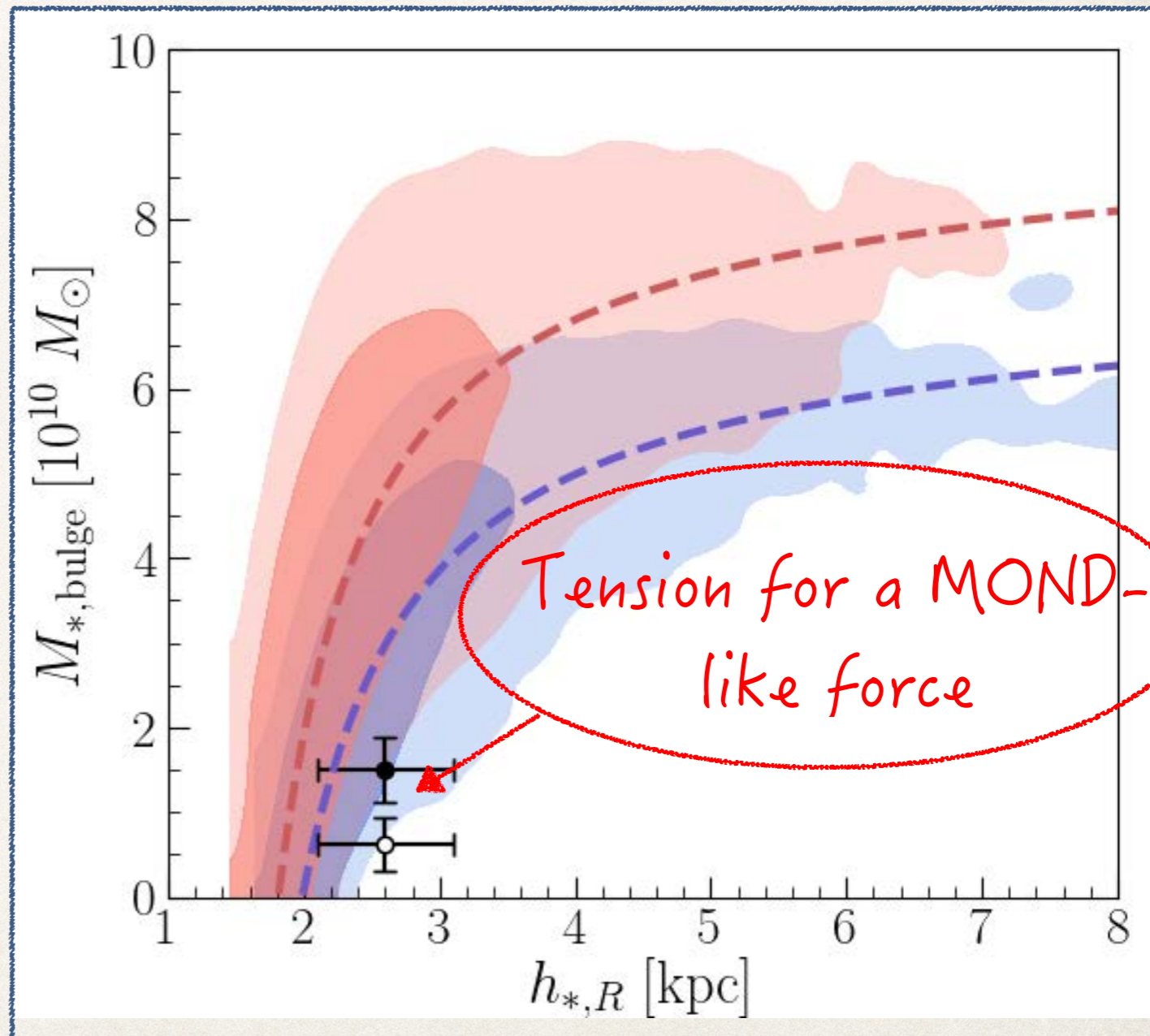
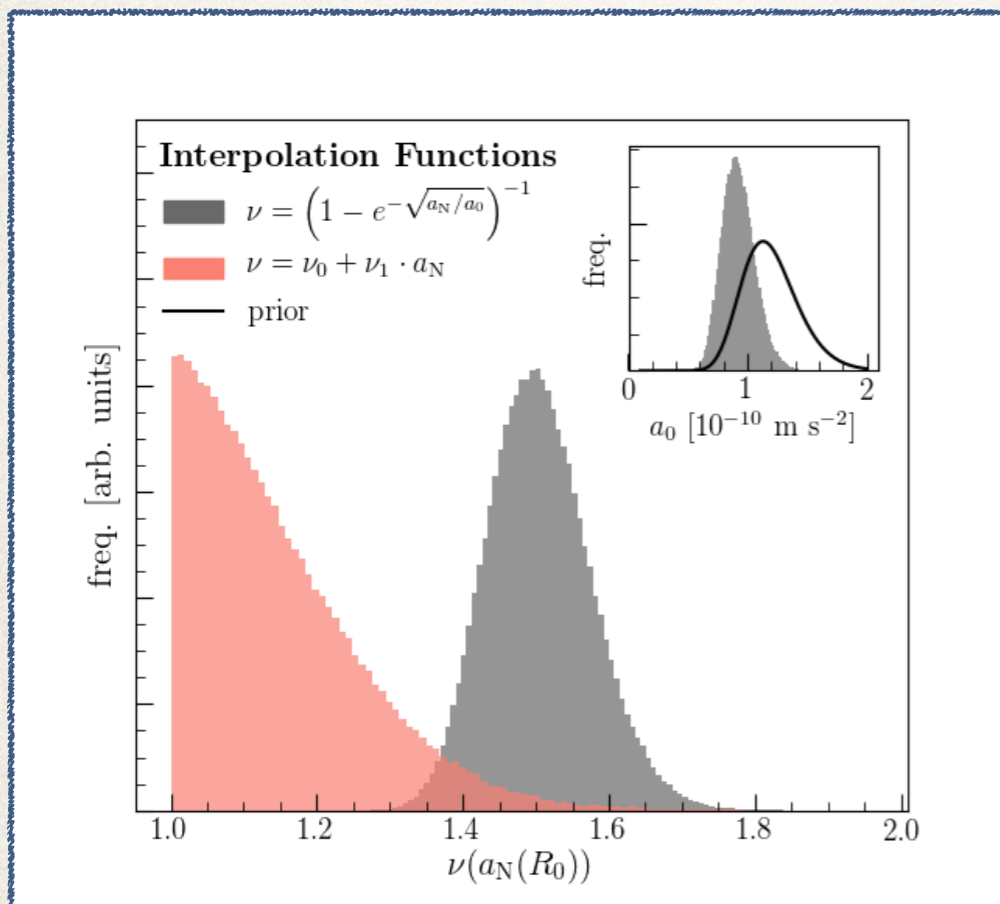
Driven by local value of rotation curve constraint

$$v_c(R) = \sqrt{R \cdot a(R)} \Big|_{z=0}$$

# Results of MCMC Scans

## Stellar Scale Radius $\nu$ s Stellar Bulge Mass

Driven by local surface density and rotation curve



# Results of MCMC Scans

## Bulge Mass is Poorly Constrained

Reference	$M_{\star}^{\text{B}} \pm 1\sigma$ ( $10^{10} M_{\odot}$ )	$R_0$ assumed (kpc)	Constraint type	$\beta^{\text{a}}$	$M_{\star}^{\text{B}} \pm 1\sigma(R_0 = 8.33\text{kpc})$ ( $10^{10} M_{\odot}$ )
<a href="#">Kent (1992)</a>	$1.69 \pm 0.85$	8.0	Dynamical	1	$1.76 \pm 0.88$
<a href="#">Dwek et al. (1995)</a>	$2.11 \pm 0.81$	8.5	Photometric	2	$2.02 \pm 0.78$
<a href="#">Han &amp; Gould (1995)</a>	$1.69 \pm 0.85$	8.0	Dynamical	1	$1.76 \pm 0.88$
<a href="#">Blum (1995)</a>	$2.63 \pm 1.32$	8.0	Dynamical	1	$2.74 \pm 1.37$
<a href="#">Zhao (1996)</a>	$2.07 \pm 1.03$	8.0	Dynamical	1	$2.15 \pm 1.08$
<a href="#">Bissantz et al. (1997)</a>	$0.81 \pm 0.22$	8.0	Microlensing	0	$0.81 \pm 0.22$
<a href="#">Freudenreich (1998)<sup>b</sup></a>	$0.48 \pm 0.65$	...	Photometric	...	$0.48 \pm 0.65$
<a href="#">Dehnen &amp; Binney (1998)</a>	$0.61 \pm 0.38$	8.0	Dynamical	1/2	$0.62 \pm 0.38$
<a href="#">Sevenster et al. (1999)</a>	$1.60 \pm 0.80$	8.0	Dynamical	1	$1.66 \pm 0.83$
<a href="#">Klypin et al. (2002)</a>	$0.94 \pm 0.29$	8.0	Dynamical	1	$0.98 \pm 0.31$
<a href="#">Bissantz &amp; Gerhard (2002)<sup>c</sup></a>	$0.84 \pm 0.09$	8.0	Dynamical	1	$0.87 \pm 0.09$
<a href="#">Han &amp; Gould (2003)</a>	$1.20 \pm 0.60$	8.0	Microlensing	0	$1.20 \pm 0.60$
<a href="#">Picaud &amp; Robin (2004)</a>	$0.54 \pm 1.11$	8.5	Photometric	0	$0.54 \pm 1.11$
<a href="#">Hamadache et al. (2006)</a>	$0.62 \pm 0.31$	None	Microlensing	0	$0.62 \pm 0.31$
<a href="#">Wyse (2006)</a>	$1.00 \pm 0.50$	None	Historical review	0	$1.00 \pm 0.50$
<a href="#">López-Corredoira et al. (2007)</a>	$0.60 \pm 0.30$	8.0	Photometric	2	$0.65 \pm 0.33$
<a href="#">Calchi Novati et al. (2008)</a>	$1.50 \pm 0.38$	8.0	Microlensing	0	$1.50 \pm 0.38$
<a href="#">Widrow et al. (2008)</a>	$0.90 \pm 0.11$	7.94	Dynamical	1	$0.95 \pm 0.12$

Bland-Hawthorn, Gerhard (2016), Licquia, Newman (2015)

Conservative Range:  $0 < M_{\star,\text{bulge}} < 2 \times 10^{10} M_{\odot}$

Reference Value:  $M_{\star,\text{bulge}} = 1.50 \pm 0.38 \times 10^{10} M_{\odot}$



# Results of MCMC Scans

## Comparison between the Theories

Naming Convention	Functional Form	Prior for Scan	$\Delta$ BIC
Taylor Expansion	$\nu(a_N) = \nu_0 + \nu_1 a_N$	$\nu(a_N) > 1$ or 1.3	4.1 or 7.5
RAR [7]	$\nu(a_N) = \left(1 - e^{-\sqrt{a_N/a_0}}\right)^{-1}$	$a_0 = \text{LOGNORMAL}(1.20, 0.24^2)$	10.4
Simple [27, 52]	$\nu(a_N) = \frac{1}{2} \left(1 + \sqrt{1 + \frac{4}{a_N/a_0}}\right)$	$a_0 = \text{LOGNORMAL}(1.2, 0.4^2)$	9.6
Standard [27, 52]	$\nu(a_N) = \sqrt{\frac{1}{2} \left(1 + \sqrt{1 + \left(\frac{2}{a_N/a_0}\right)^2}\right)}$	$a_0 = \text{LOGNORMAL}(1.2, 0.4^2)$	4.8

Bayesian Information Criterion:  
(a proxy for the Bayes Evidence)

$$\text{B.I.C.} = k \log n - 2 \log \hat{\mathcal{L}}$$

$k$ : number of model parameters

$n$ : number of data points

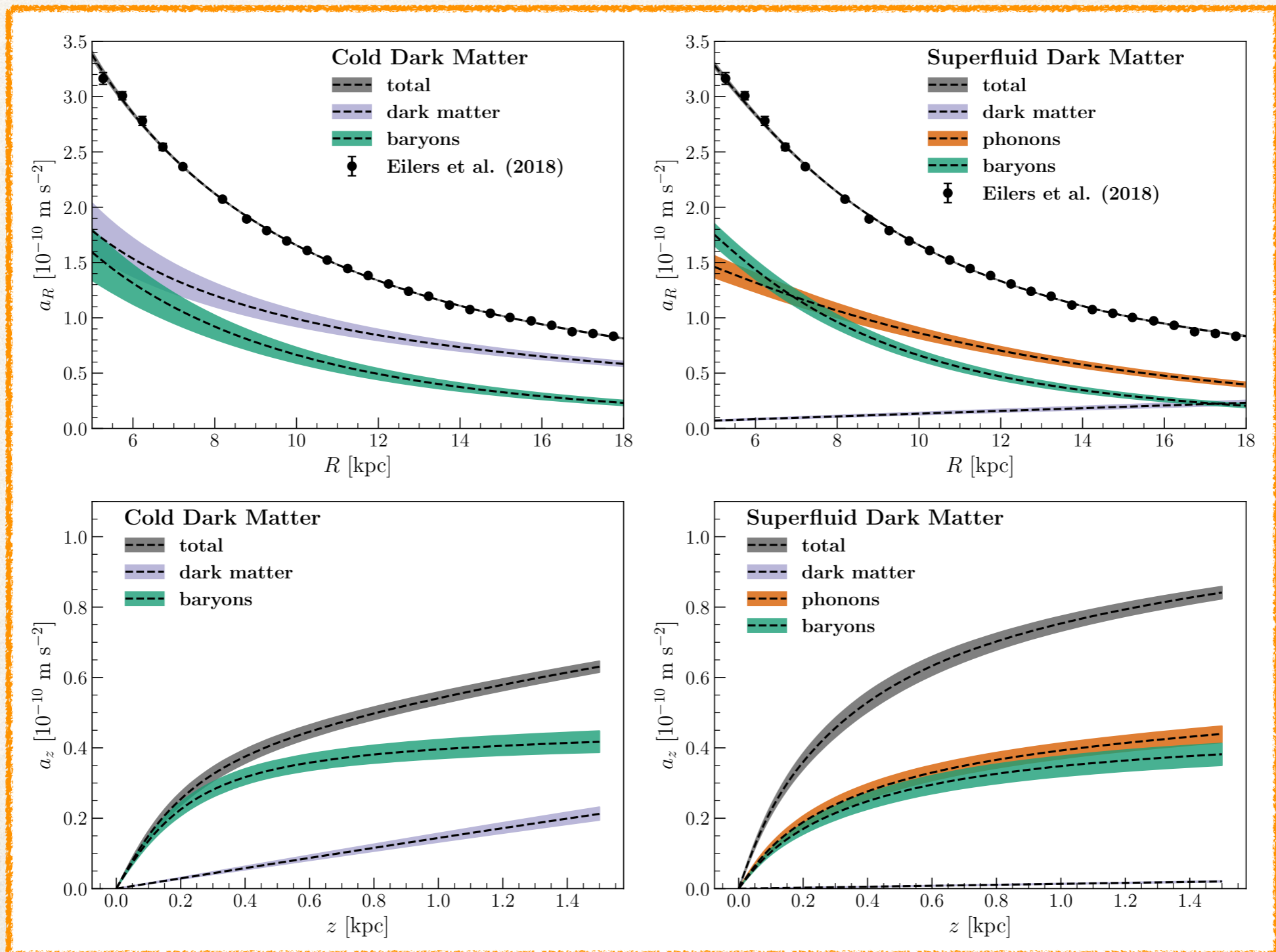
$\hat{\mathcal{L}}$ : maximum likelihood

# Results for Superfluid DM

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# Results for SuperFluid DM

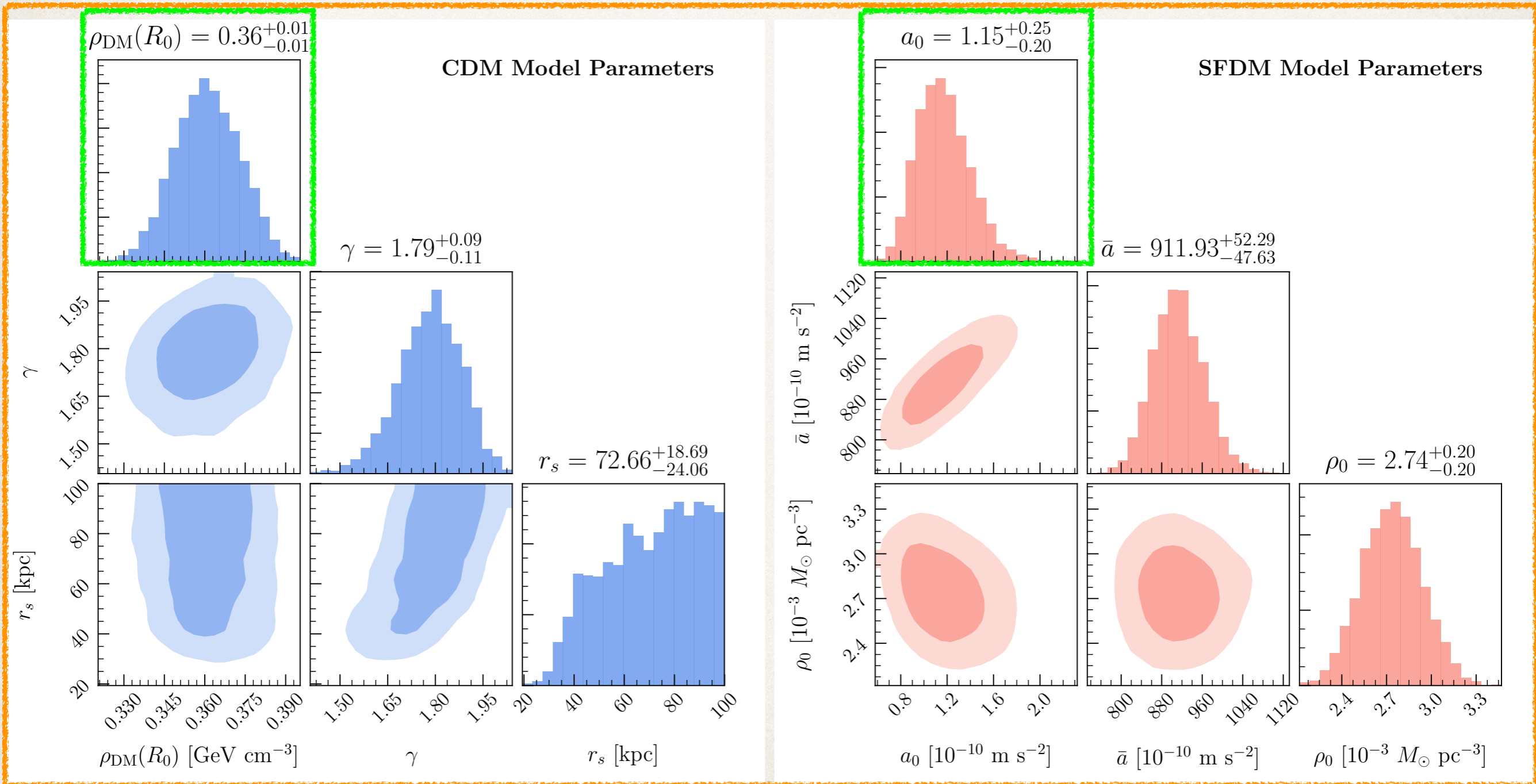
## Full Rotation Curve and Vertical Accelerations



Lisanti, Moschella, Outmezguine, O.S., 2019

# Results for SuperFluid DM

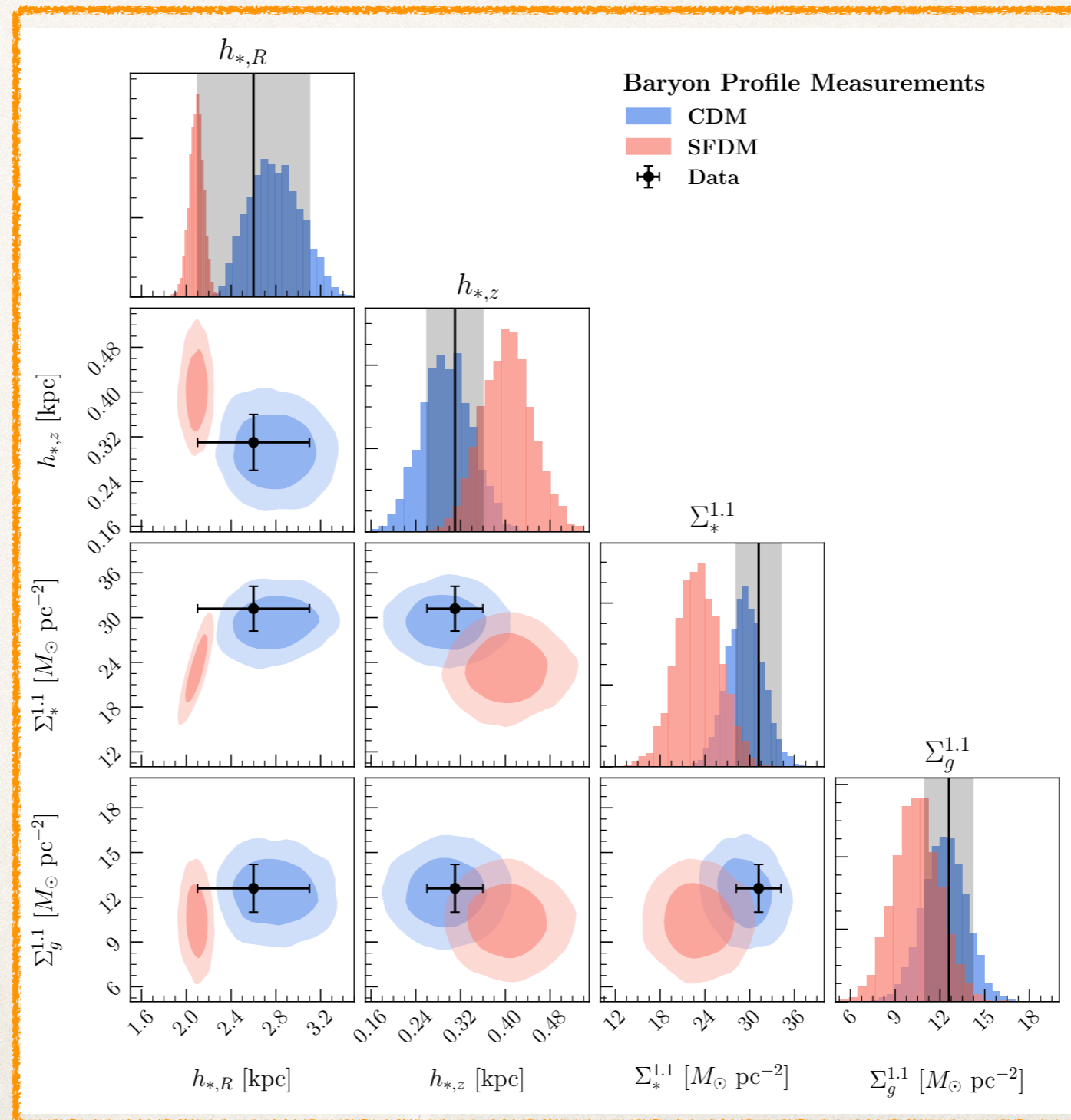
## Model Parameters



# Results for SuperFluid DM

## Baryonic Parameters

Bayes Factor:  
 $\ln\text{BF}=32$



Lisanti, Moschella, Outmezguine, O.S., 2019

# Additional Tests

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Redo analysis with:

- Only one mono-abundance population for velocity dispersions
- Various choices of priors for all parameters
- Artificially enhanced errors by factor of 2

⇒ Qualitatively same results for all cross checks

# Summary of the Results

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- ❖ Local accelerations only
- ❖ Taylor interpolation func

$$\Delta\text{BIC} \approx 4$$

**POSITIVE EVIDENCE**  
(with  $\nu \approx 1$ )

- ❖ Local accelerations only
- ❖ Specific interpolation func

$$\Delta\text{BIC} \approx 10$$

**STRONG EVIDENCE**

- ❖ All rotation curve and velocity dispersion data
- ❖ Superfluid DM

$$\ln\text{BF} \approx 30$$

**DECISIVE EVIDENCE**

# Conclusions

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- Standard lore is that “MOND-like forces work on Galactic scales”. This is not precisely true.
- Our results establish a new criterion for any DM model which attempts to reproduce the MDAR.
- SFDM is a representative example of a broad class of such theories.
- MW measurements seem to prefer CDM over these models.





A strictly MOND-like force has trouble simultaneously explaining rotation curves and velocity dispersions... so, probably something else

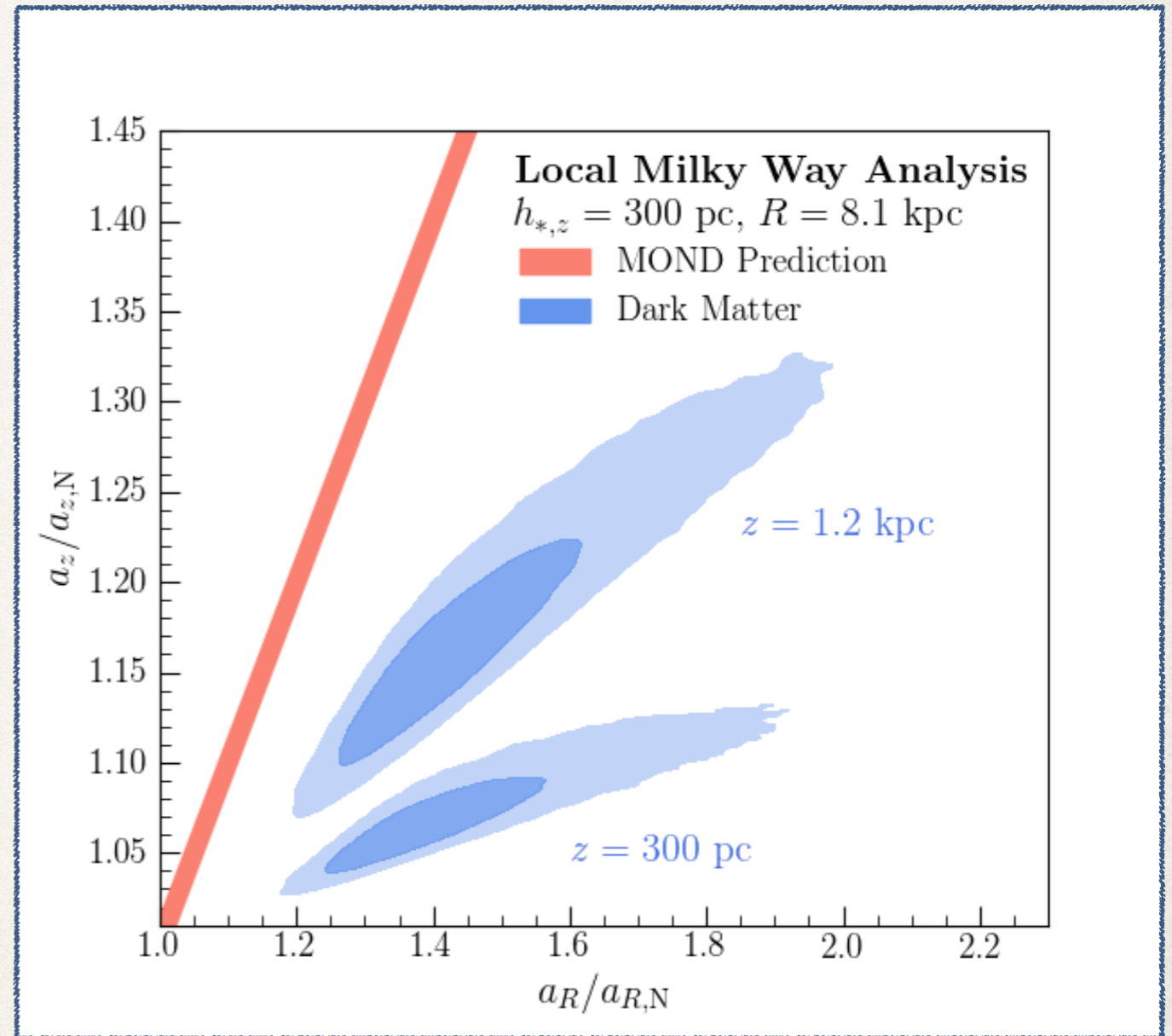
**THANK YOU**

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# Results of MCMC Scans

## Tension between models for any Scalar Enhancement

Each axis is the local enhancement of acceleration in the R/z directions  
*or*  
an independent measurement of the local value of the interpolation function



# Some general comments (and more on MOND-like forces)

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# Some Comments

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- Could be done for any model where dynamics are predicted locally by baryons

- The starting point could have been something of the form:

Example of a MONDian  
Poisson Equation

$$\nabla \left( \mu \left( \frac{|\nabla\Phi|}{a_0} \right) \nabla\Phi \right) = 4\pi G\rho \quad \rightarrow \quad \Phi \propto \log r$$

Inverse of interp. func.

- This equation is non-linear and difficult to calculate
- Is VERY model dependent
- Starting from an acceleration relation can map onto other theories

# MOND / Superfluid DM

## Non-Linear Effects

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- Non-linear effects must be accounted for!
- Potential problems include:
  - A possible non-trivial correction to the acceleration relation.
  - Small perturbations to a smooth potential can cause large effects.

# MOND / Superfluid DM

## A Divergenceless Field

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Poisson Equation:

$$\nabla (\nabla \Phi_{\text{N}}) = 4\pi G \rho$$

MONDian Poisson Equation:

$$\Phi \propto \log r$$

$$\nabla \left( \mu \left( \frac{|\nabla \Phi|}{a_0} \right) \nabla \Phi \right) = 4\pi G \rho$$

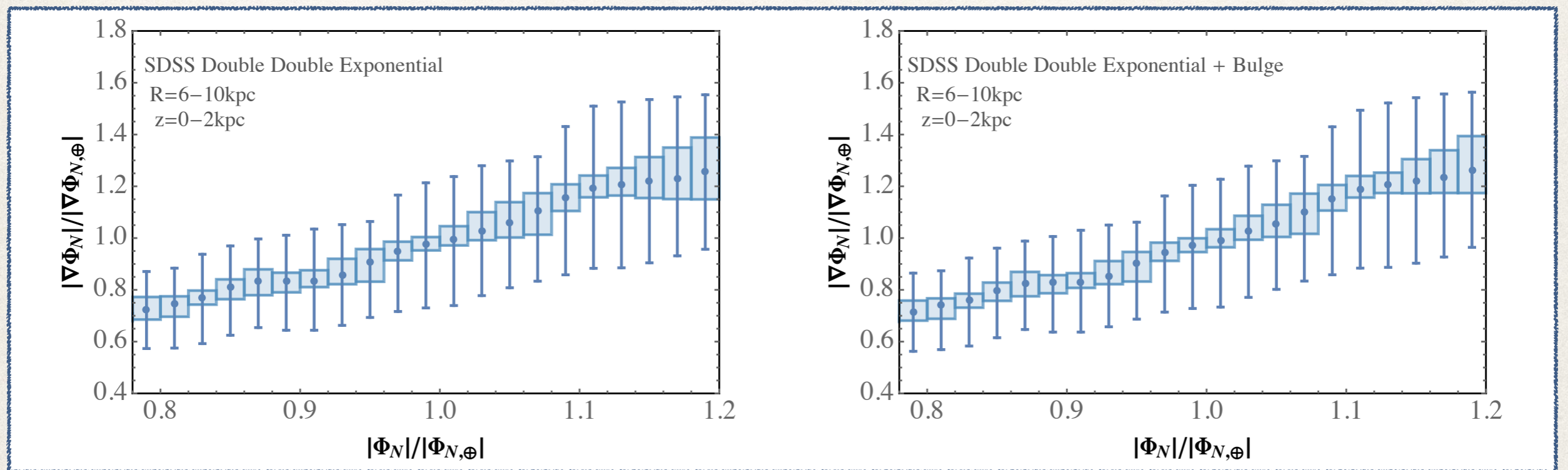
Inverse of

Acceleration Relation known  
up to a divergenceless field:

$$\mathbf{a} = \nu \left( \frac{a_{\text{N}}}{a_0} \right) \mathbf{a}_{\text{N}} + \mathbf{S}$$

# MOND

## A Divergenceless Field



Can be shown that  $S=0$  for 1D  
symmetrical potentials, or:

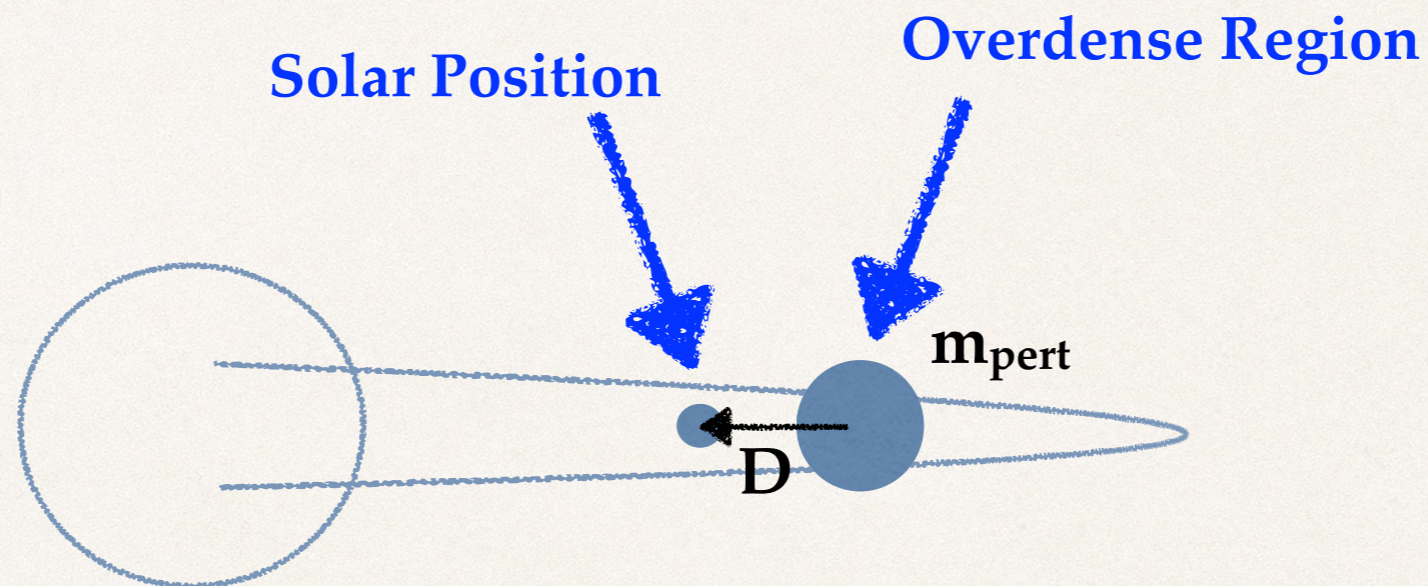
$$\nabla |\nabla \Phi_N| \times \nabla \Phi_N = 0$$

$$|\nabla \Phi_N| = f(\Phi_N)$$



# MOND / Superfluid DM

## Small Perturbations



The External Field Effect (EFE)  
is small as long as:

$$D \gg 0.1 \text{ kpc} \times \left[ \nu \left( \frac{a_{\text{N,BG}}}{a_0} \right) \cdot \frac{m_{\text{pert}}}{10^7 M_{\odot}} \cdot \frac{2 \cdot 10^{-10} \text{ m/s}^2}{a_{\text{loc}}} \right]^{1/2}$$

$$a_{\text{loc}} = \frac{v_c^2}{R_0} \approx 2 \cdot 10^{-10} \text{ m/s}^2.$$

# MOND

So for a local MW study:

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Using  $\mathbf{a} = \nu \left( \frac{a_N}{a_0} \right) \mathbf{a}_N$   
with  $\nu(x_N) \rightarrow \nu_0 + \nu_1 \cdot x_N$

- A good local approximation.
- Holds for many MOND-like theories.
- Independent of specific interpolation function.