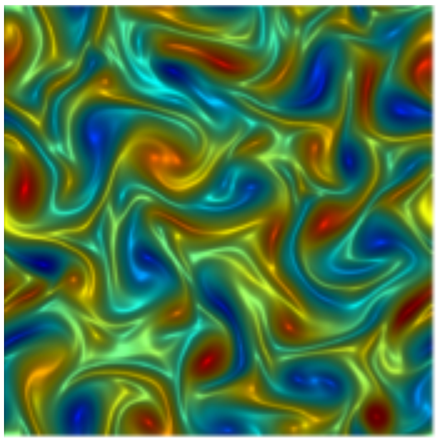
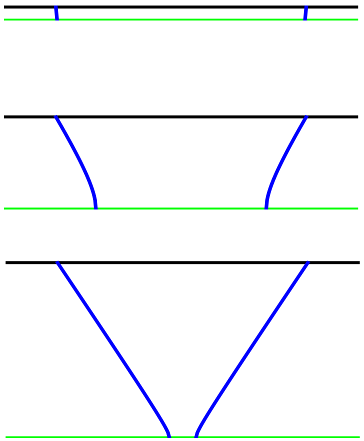
Fine probes of quantum chaos





Márk Mezei (SCGP)

In collaboration with: Agón, Bao, Casini, Choi, Cotler, Hertzberg, Liu, Mueller, Sárosi, Stanford, van der Schee, Virrueta

University of Michigan HET Brown Bag Seminars, 10/23/2019

Quantum chaotic dynamics

Phenomena associated with chaotic dynamics:

Transport

Thermalization

Butterfly effect

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Ultimate goal: understand these phenomena and their relation in quantum systems

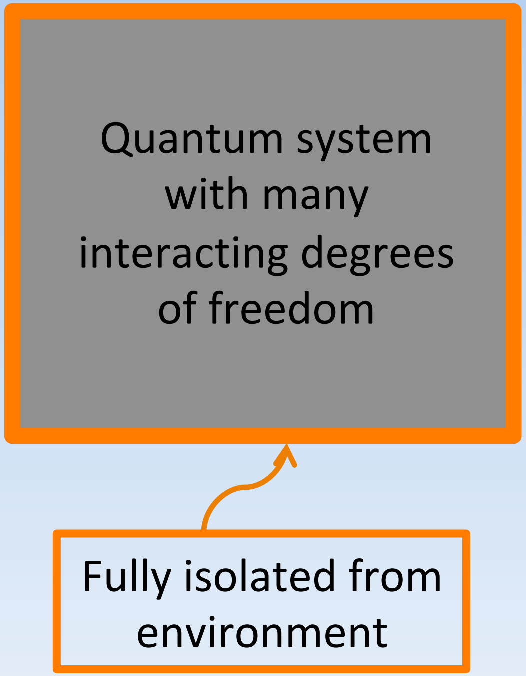
Goal of talk:

- Develop an effective field theory at long distances and at late times for each process
- Study their interplay
- Relate them to gravity through AdS/CFT

Quantum chaotic dynamics

Setup:

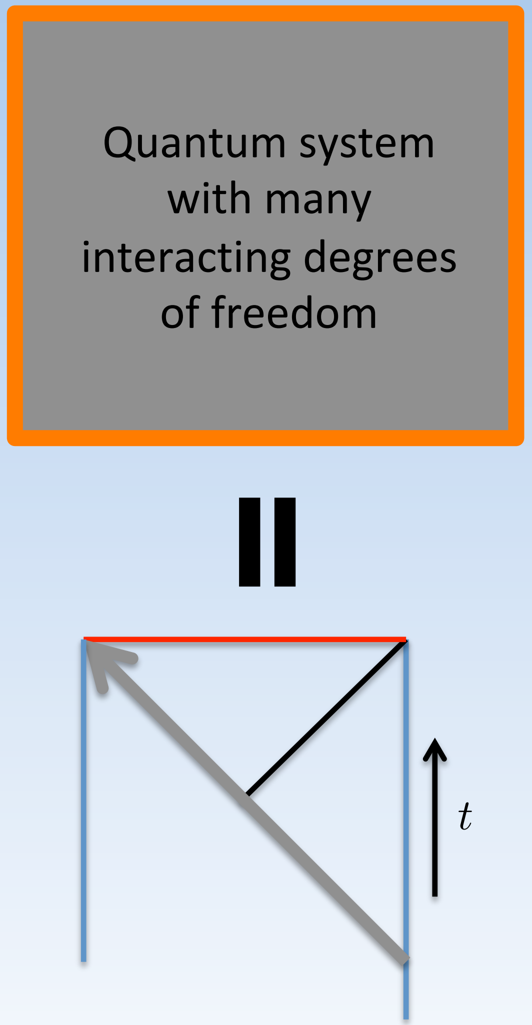
- Constituents: lattice QFT (HEP), spins/electrons (CMT), atoms (AMO), qubits (QI)
- Degrees of freedom interacting strongly through local **chaotic** Hamiltonian.
- In highly excited state, out of equilibrium at $t = 0$, in equilibrium for $t \to \infty$.
- Foundational question in statistical physics. Paradigm shift from ensembles to closed systems. Subject to intense current activity in HEP, CMT, QI, and **AMO experiments**.



Relation to gravity

Study the setup using holographic duality:

- A QFT settling to thermal equilibrium is dual to a collapsing black hole.
- No small parameters, holography is indispensible in understanding real time quantum dynamics.
- Entanglement plays a crucial role in thermalization. Geometric prescription for computing entanglement entropy. [Ryu, Takayanagi]
- Recent breakthroughs in quantum chaos at this intersection. [Shenker, Stanford; Kitaev]



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It from Qubit:

- Holography teaches about quantum gravity through strongly interacting QFTs.
- Realization that entanglement is key to connectedness of spacetime.
- “Gravity is the hydrodynamics of entanglement”

Quantum system with many interacting degrees of freedom

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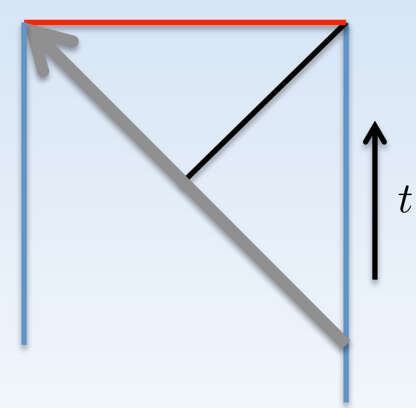
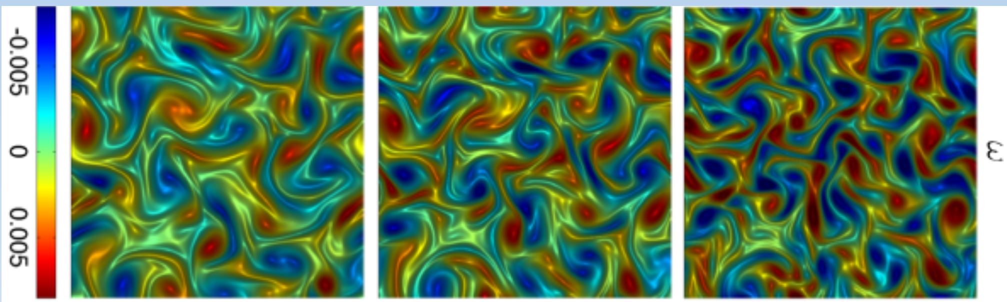


Hydrodynamics

We have an effective theory for describing conserved densities.

- Hydrodynamics applies universally for all chaotic systems. Generalized hydrodynamics for integrable systems.
- Navier-Stokes equations: $\partial_t v + (v \cdot \nabla) v - \nu \nabla^2 v = -\nabla p$



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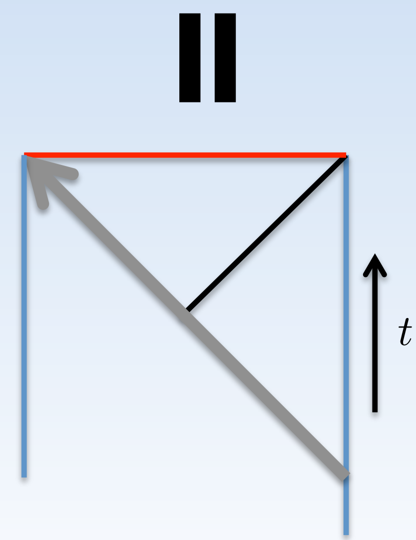

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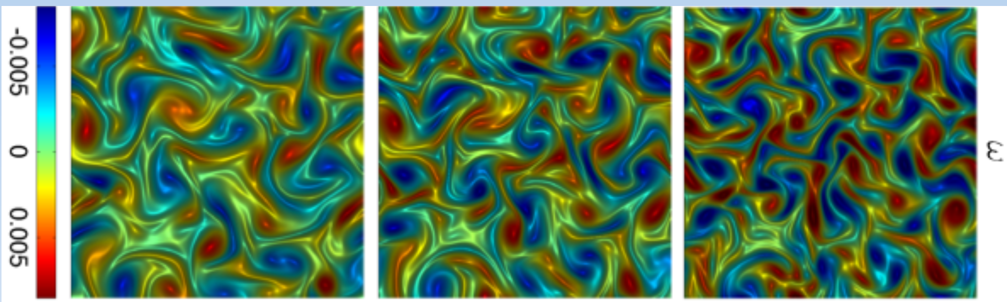

- Relativistic hydro from hep-th POV is an EFT based on systematic long distance, late time expansion. Fluid variables:

$$T_{ab} = (\rho + p) u_a u_b + p\, \eta_{ab} + \Pi_{ab}$$

For conformal fluids one transport coefficient at first order:

$$\Pi_{ab} = -2\eta\sigma_{ab} + \dots$$

- Hydrodynamics follows from the conservation of T_{ab} . Solution determines $\langle T_{ab} \rangle$ out of equilibrium.

Quantum system with many interacting degrees of freedom

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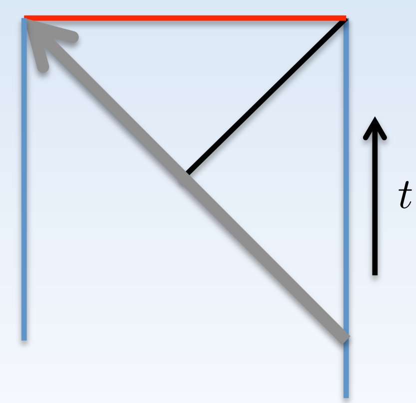

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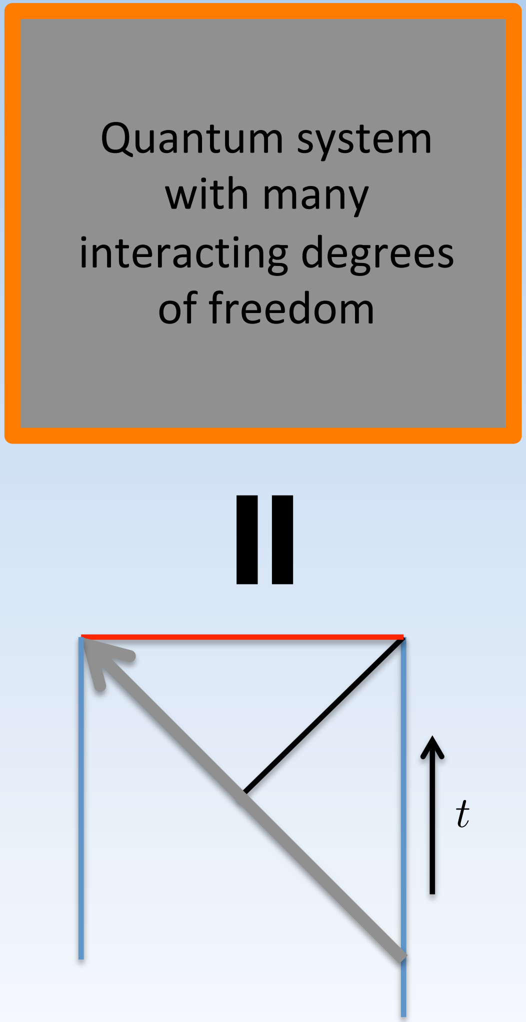

Hydrodynamics

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- Alternative history: String theorists discover hydrodynamics by studying AdS black holes.
- Interested in more data than $\langle T_{ab} \rangle$: entanglement entropy, butterfly effect, etc.
- I want to follow the “alternative history” path to discover a hydrodynamic effective theory of entanglement dynamics and operator growth.
- Hydrodynamics applies universally for all chaotic systems, and there is evidence for the universality of the other effective theories.

Quantum system with many interacting degrees of freedom

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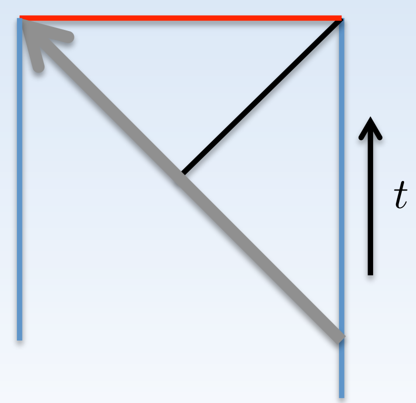

Outline

Transport
- Hydro as an EFT
- Holography for real time dynamics

Thermalization
- Entanglement entropy as a probe
- Membrane theory is the EFT

Butterfly effect
- Butterfly effect, operator growth and OTOC
- Refinement of the chaos bound
- Towards an EFT

Summary and open questions

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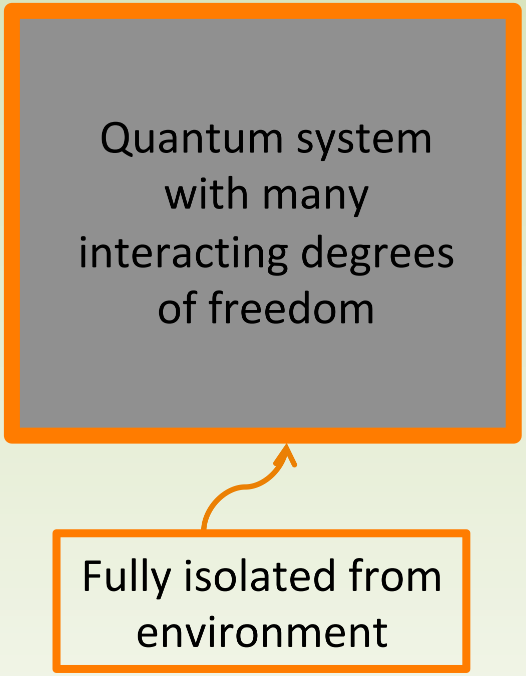
Summary and open questions

Quantum thermalization and subsystems

Quantum thermalization

- Pure state with nonzero energy density: $|\psi(0)\rangle$
  Unitary time evolution: $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$
- $\rho(t) \equiv |\psi(t)\rangle\langle\psi(t)| \not\to \dfrac{e^{-\beta H}}{Z}$ cannot mean thermalization.

  $\rho(t)$ encodes all the information in $|\psi(0)\rangle$, but at late times in a very nonlocal way.



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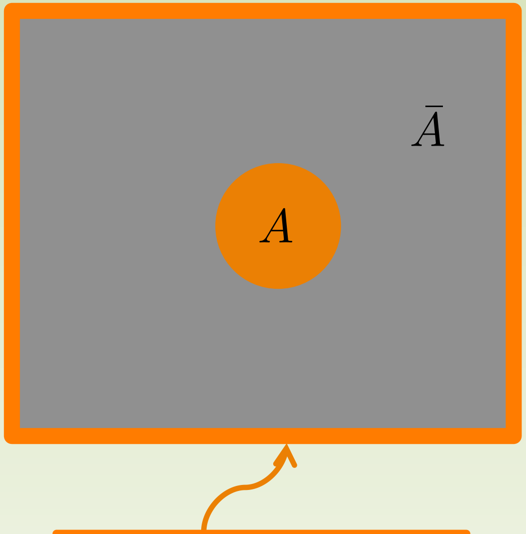
- Consider subsystems:
  In a local system: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$
  Reduced density matrix: $\rho_A = \mathrm{Tr}_{\bar{A}} \, |\psi\rangle \langle\psi|$



Fully isolated from environment

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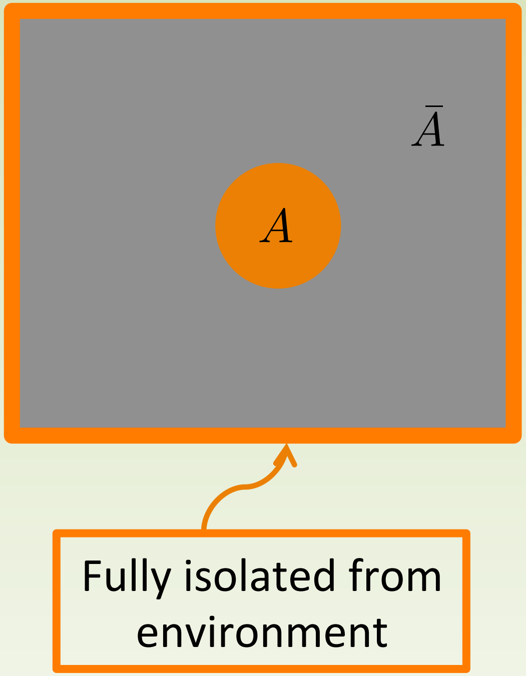
- Thermalization: $\rho_A(t) \;\to\; \rho_A^{(\mathrm{eq})}(\beta) = \mathrm{Tr}_{\bar{A}} \dfrac{e^{-\beta H}}{Z}$

  For $t \to \infty$, in the thermodynamic limit $\bar{A} \to \infty$, with β determined by the energy density. **Entanglement is crucial in making this possible.**



Entanglement entropy

Entanglement entropy is a good diagnostic of thermalization, we **focus on this quantity**.

- Entanglement entropy:

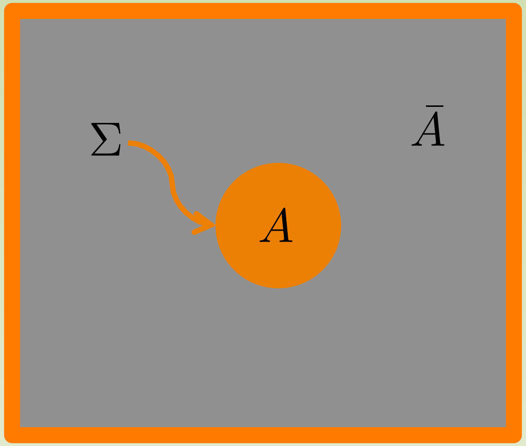
  $$S_A \equiv -\mathrm{Tr}_A\, \rho_A \log \rho_A$$

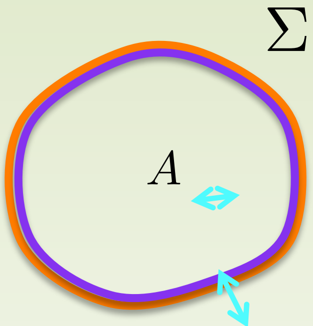
- In ground states of local Hamiltonians the entropy scales with the area:

  $$S_A = \# \frac{\mathrm{area}(\Sigma)}{\delta^{d-2}} + \dots$$

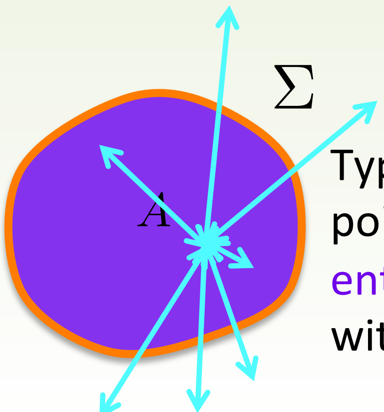
- A generic state in the Hilbert space shows volume scaling.

  $$S_A = s_{\mathrm{th}}\, \mathrm{vol}(A) + \dots$$





Typical point inside unentangled with outside



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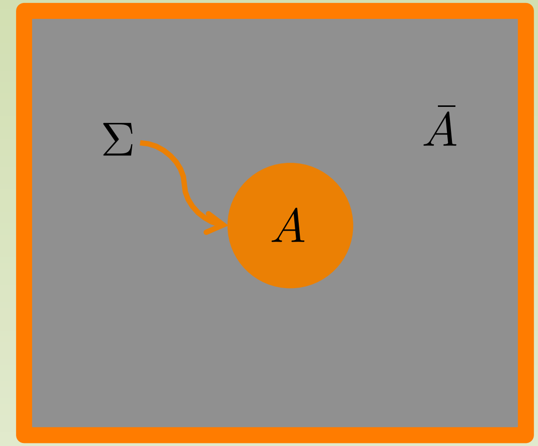

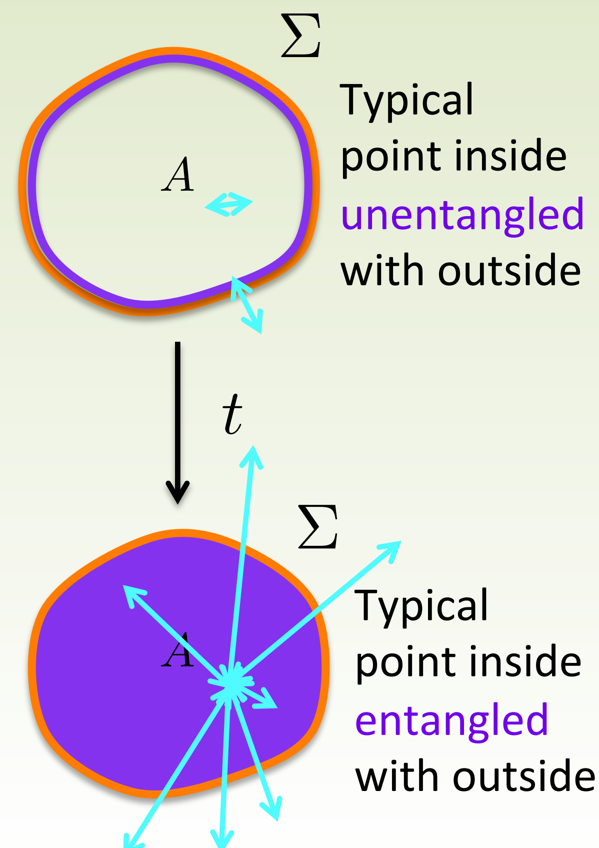

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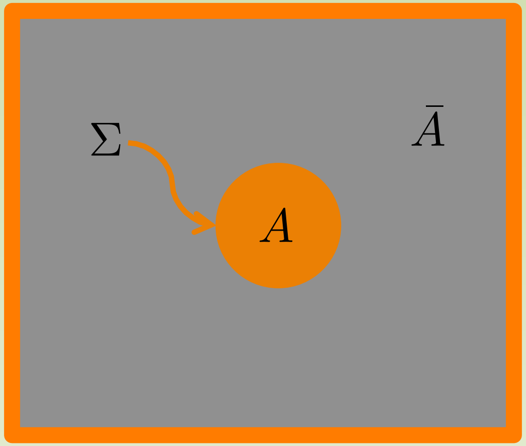
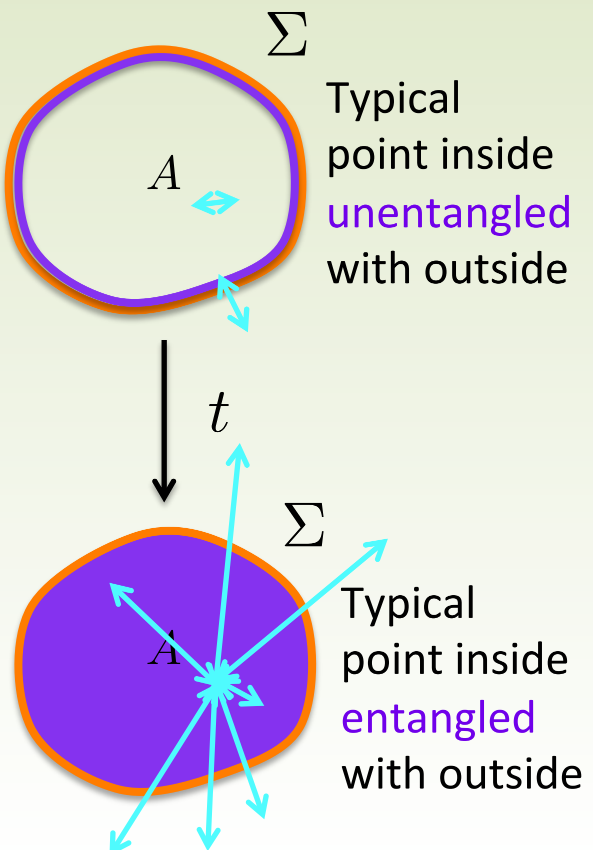
- Instead of following an operator (matrix), we follow a number.

$$\rho_A(t) \;\to\; \rho_A^{(\mathrm{eq})}(\beta) = \mathrm{Tr}_{\bar{A}}\, \frac{e^{-\beta H}}{Z}$$

$$S_A(t) \;\to\; S_A^{(\mathrm{eq})}(\beta) = s_{\mathrm{th}}(\beta)\, \mathrm{vol}(A)$$

Captures the essence of thermalization.

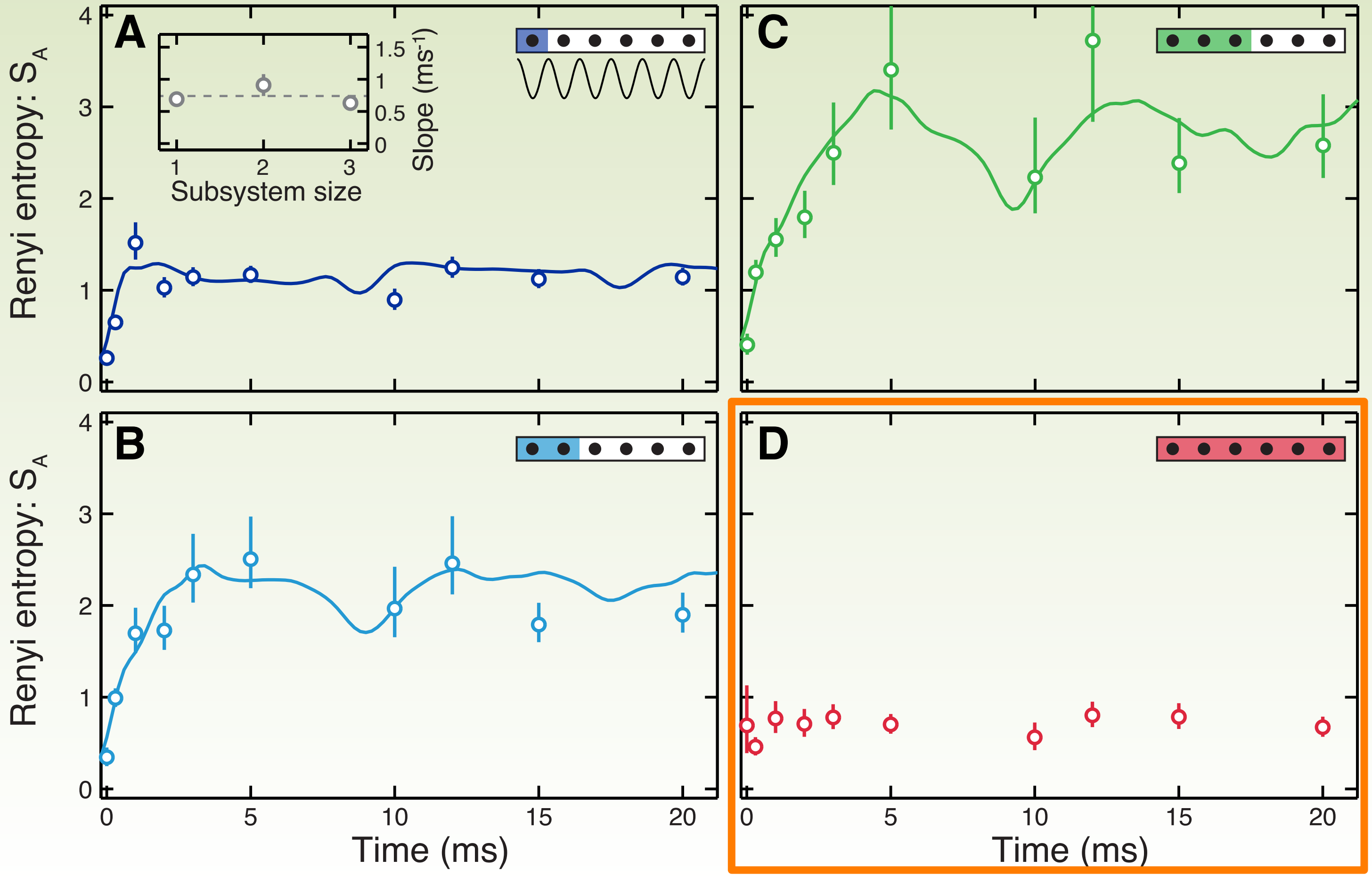




Entanglement entropy in experiment

In cold atom experiments we can realize quenches and measure the entanglement of subsystems. [Kaufman et al.]

- The state of the total system is pure.



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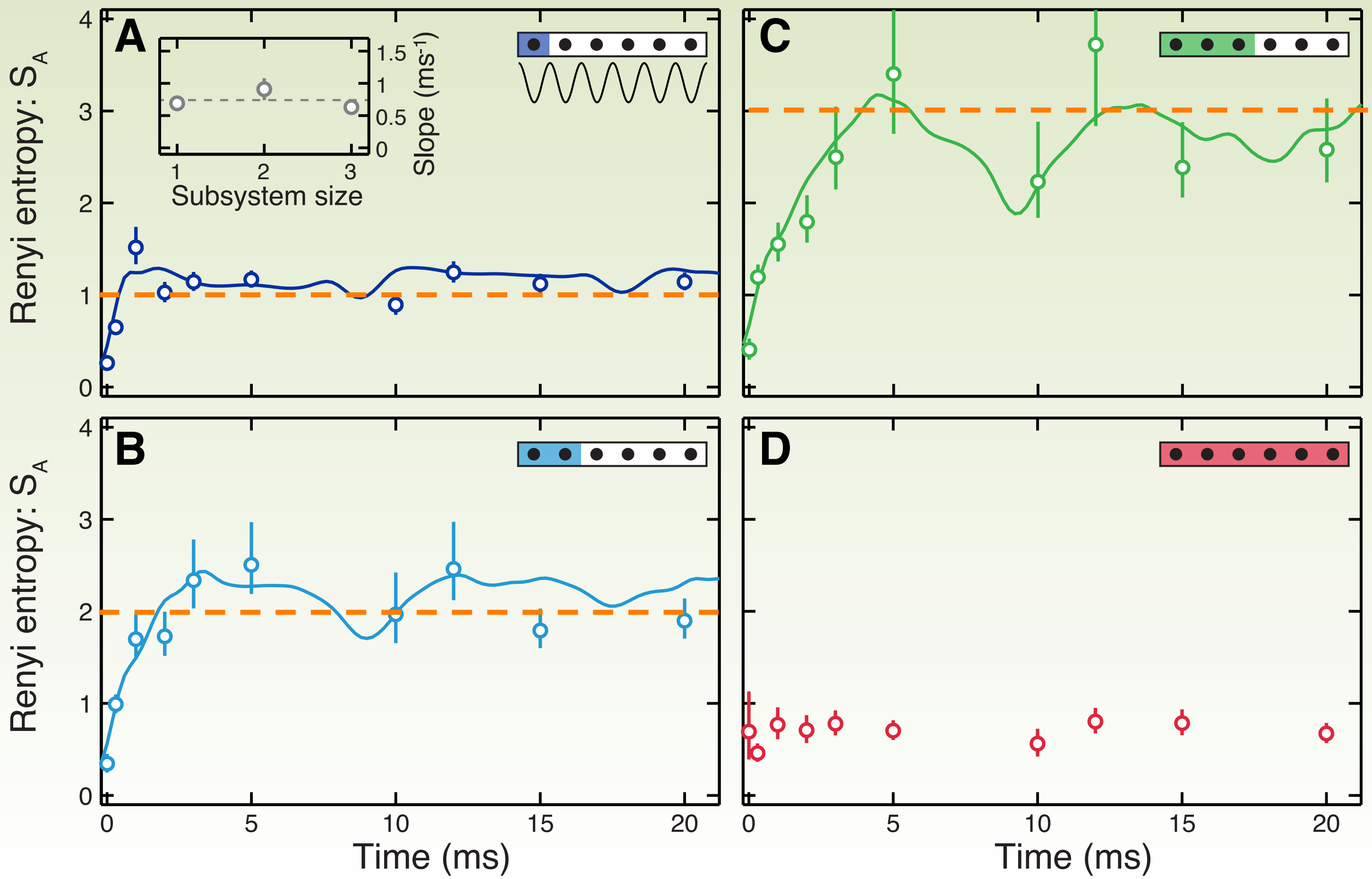

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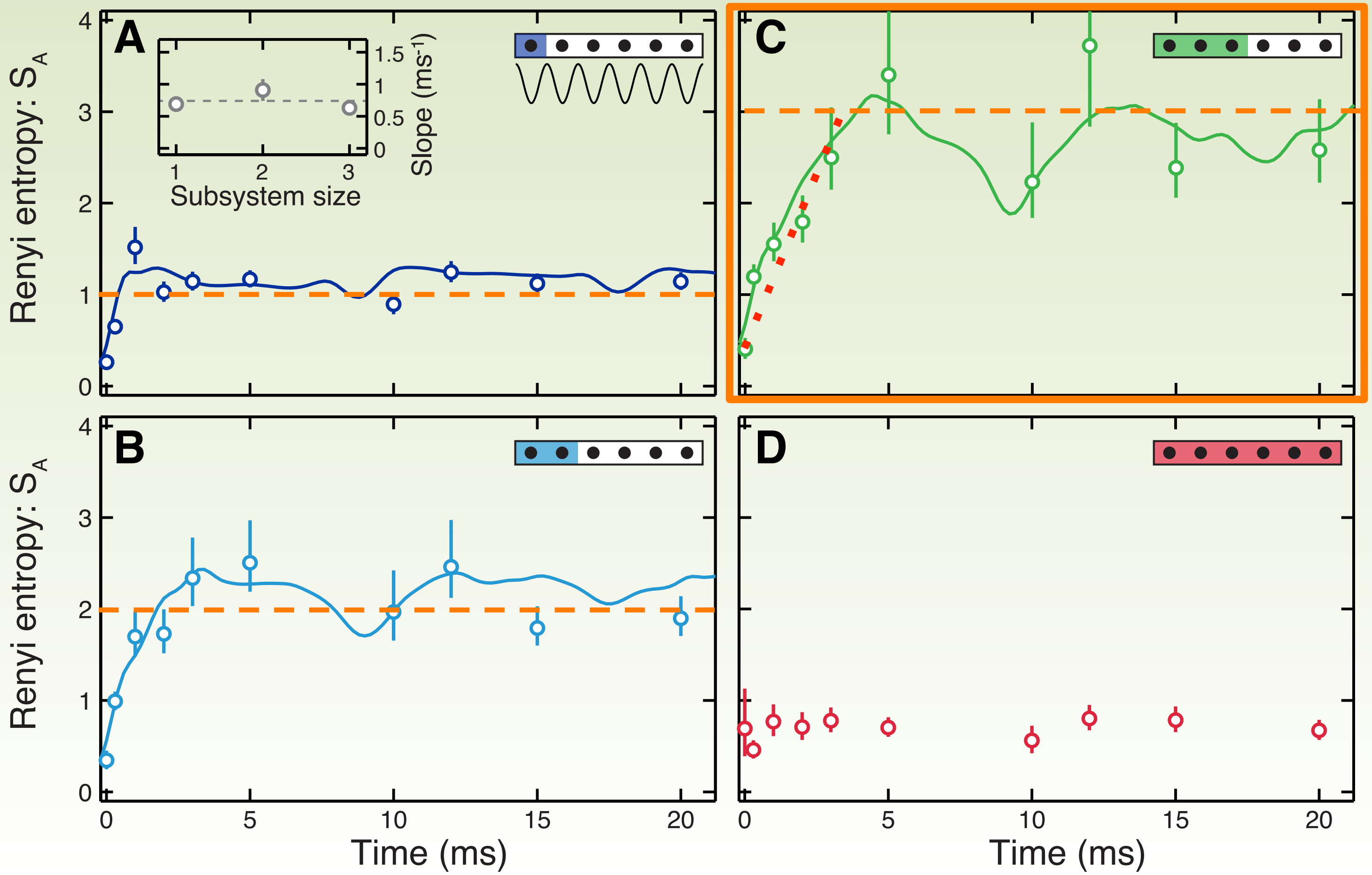
- Linear growth at early times.



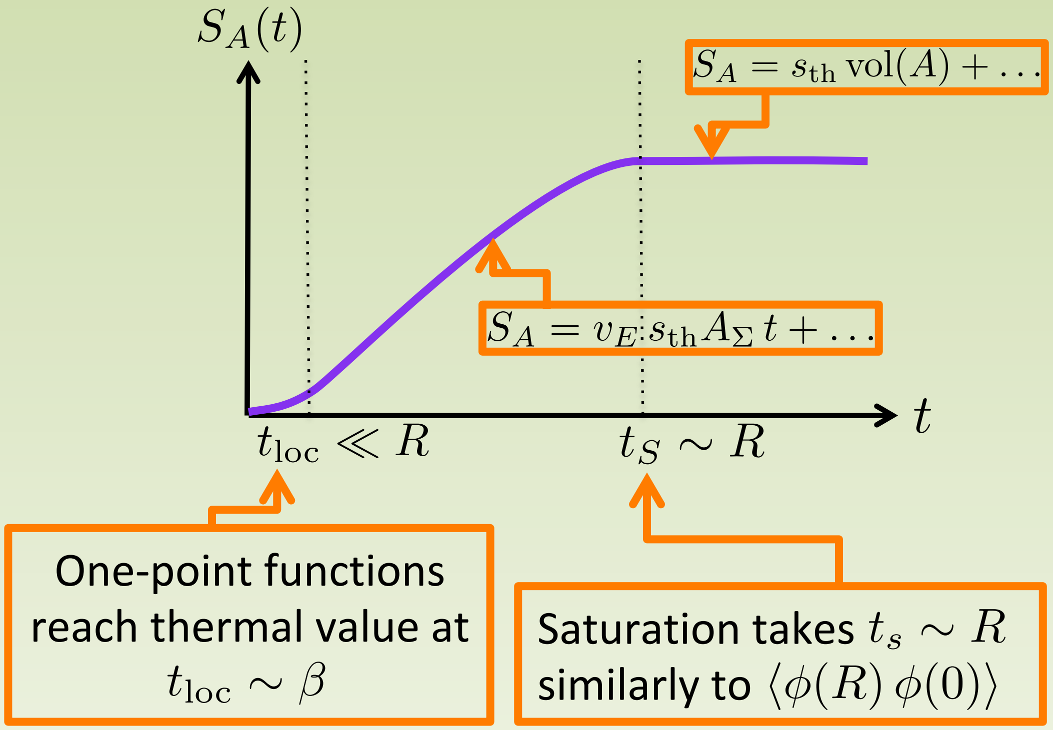
Universality classes of entropy dynamics

I propose that there are two universality classes of entropy dynamics at long distances and late times (in translationally invariant systems).

- 2d integrable models, RCFTs, d>2 free theories are described by the quasiparticle model.
- The holographic results can be reformulated in terms of a **membrane theory**, which then can be adopted to any chaotic system. Applies to holographic theories, random circuits, evidence for chaotic spin chains. [Jonay et al., MM_2]
- Is there something in between?
- Analogous to the dichotomy between generalized hydrodynamics applicable to integrable systems (giving ballistic transport) and hydrodynamics (describing diffusive transport).

Entropy in the hydrodynamic limit

- Qualitative picture of entanglement entropy at time t of a region of characteristic size R, $R, t \gg t_{\rm loc}$. [Cardy, Calabrese; Hartman, Maldacena; Liu, Suh]



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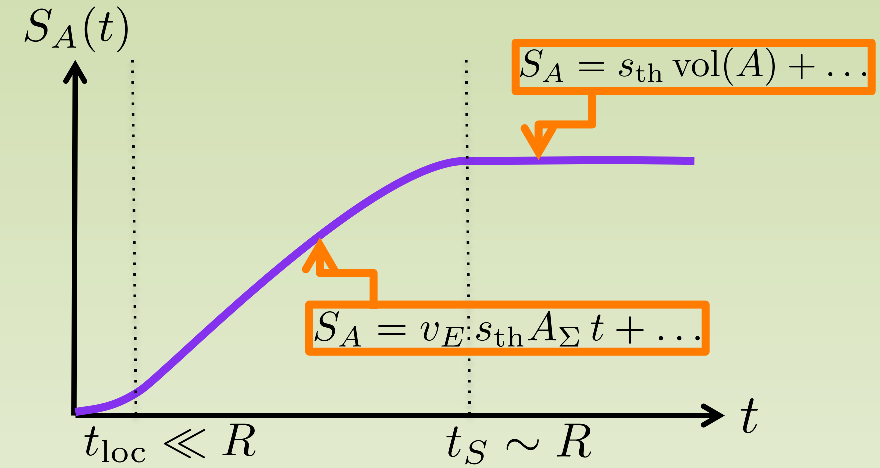
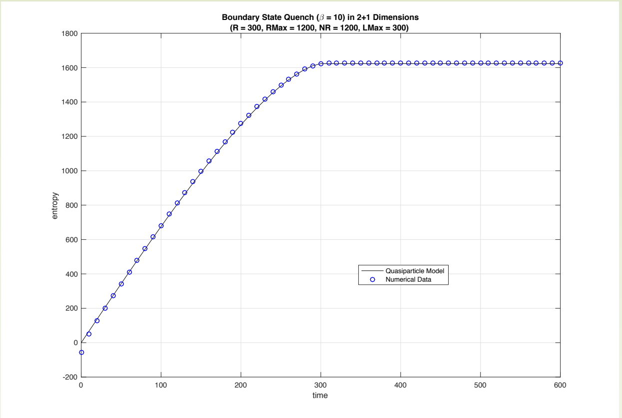
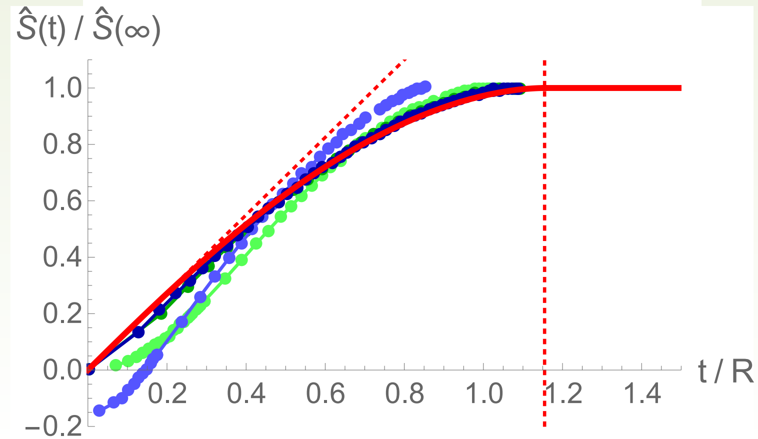


- EE in free scalar theory for a disk, dots are data points, line is quasiparticle theory [Cotler, Hertzberg, MM, Mueller]



- EE in holographic theories for a disk, data collapse, solid line is membrane theory, deviation is controlled by 1/R [MM_1]



AdS/CFT and entanglement entropy

AdS/CFT is an exact equivalence between QG theories in AdS_{d+1} spacetimes and CFT_d's.

- E.g. $\mathcal{N} = 4$ Super Yang-Mills theory = Type IIB string theory on $\mathrm{AdS}_5 \times S^5$
- Strongly coupled CFT (hard) = Semiclassical gravity (easy)
- Entanglement entropy of A = Area of extremal surface ending on Σ [Ryu, Takayanagi; Hubeny, Rangamani, Takayanagi]

$$S_A = \frac{\text{extremal area}}{4G_N}$$

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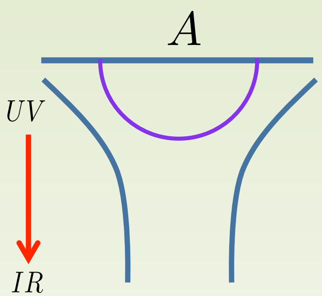
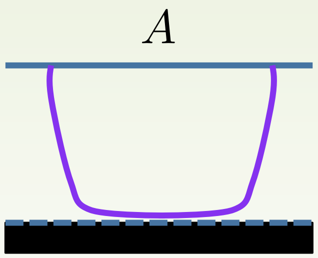


➢ Volume scaling at finite temperature



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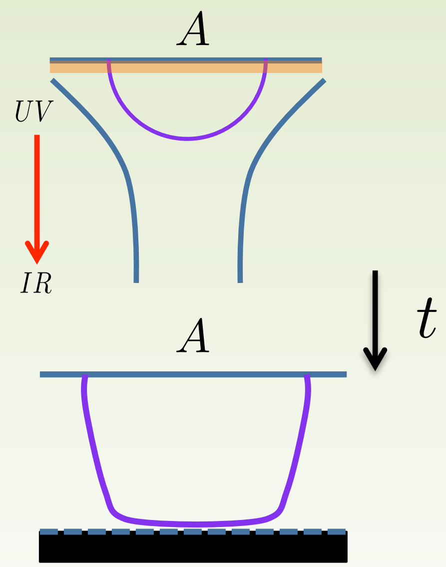
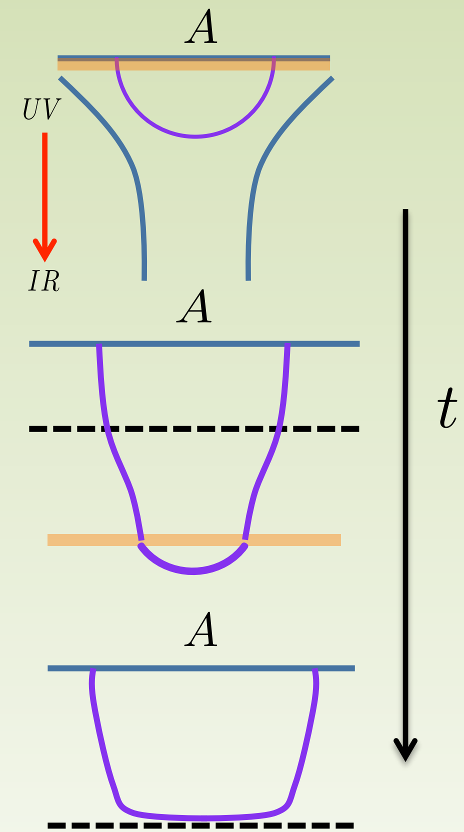


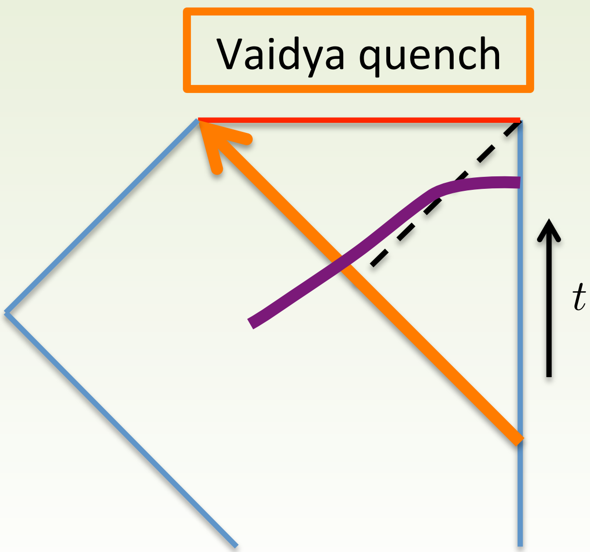
- Quench, thermalization $\equiv$ Black hole formation from collapse

Membrane theory of entanglement dynamics

Can reformulate holographic surface extremization in d+1 dimensions as membrane minimization in d dimensions in the limit $R, t \gg t_{\text{loc}}$. [MM$_2$]

- Detailed understanding of HRT surfaces. The surface has three parts: [MM$_1$]
  1. Outside the horizon part gives (divergent) area law.
  2. Behind the horizon region.
  3. Behind the shell part gives entropy in the vacuum.





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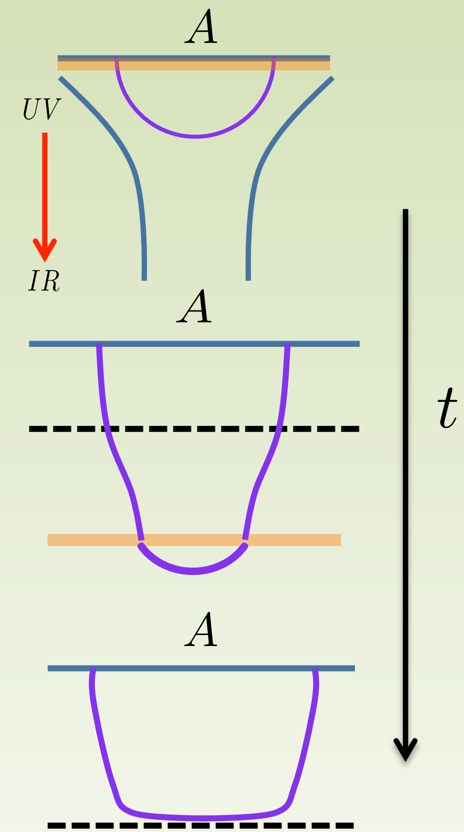
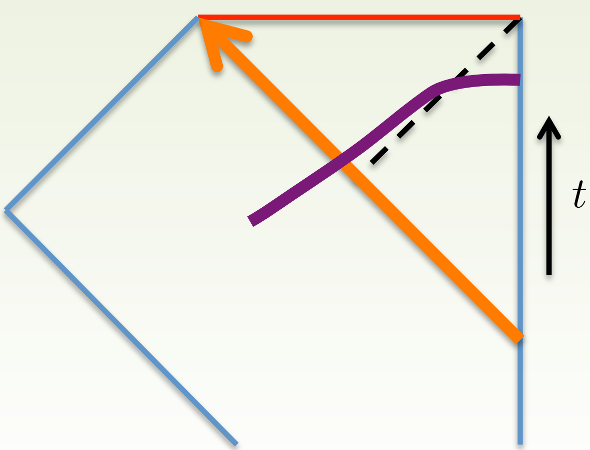
- Only the **2. part** contributes to the extensive part of the entropy.

$$S(t) = s_{\text{th}} R^{d-1} \mathcal{S}_{\text{ext}}\left(\frac{t}{R}\right) + \dots$$



Vaidya quench



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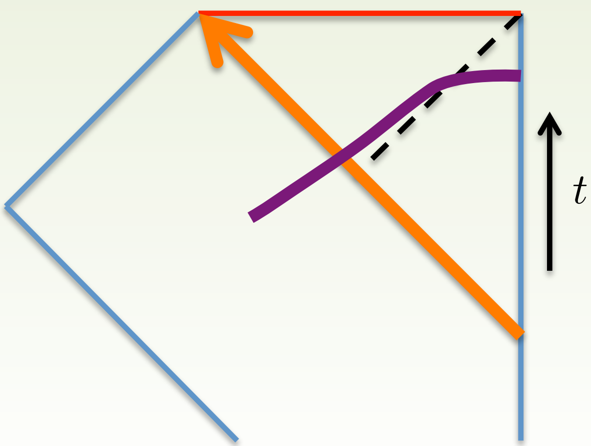
- Scaling limit: $x^\mu \to R\, x^\mu\,, \quad z \to z$
  Area functional independent of the derivatives of z. Solve algebraic EOM, plug back into action to derive membrane theory.

$$S[A] = s_{\rm th} \int d^{d-1}\xi \, \sqrt{\gamma}\, \frac{\mathcal{E}\,(v)}{\sqrt{1-v^2}}$$

Vaidya quench



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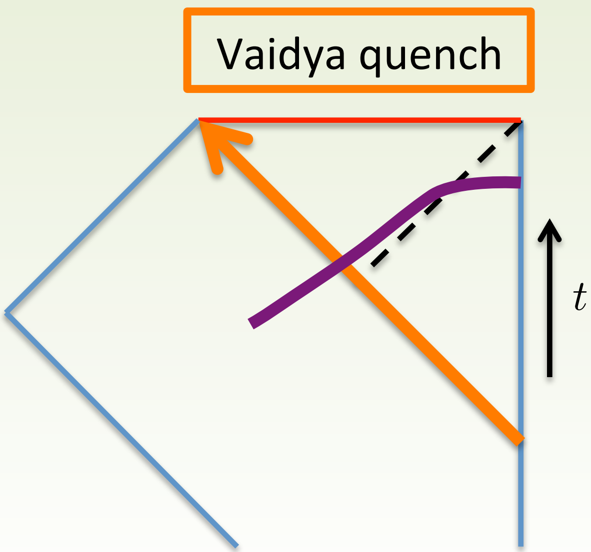

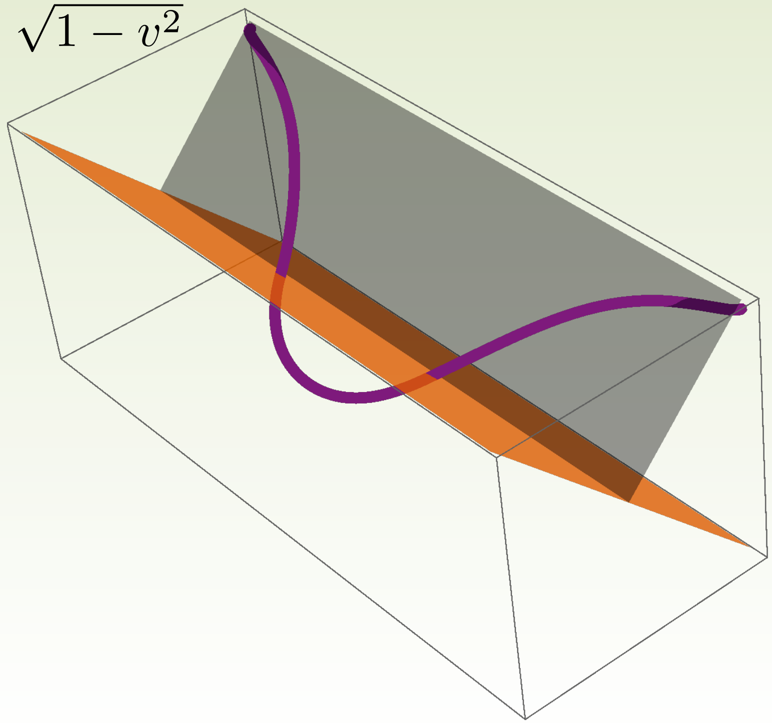

Membrane theory of entanglement dynamics

Can reformulate holographic surface extremization in d+1 dimensions as membrane minimization in d dimensions in the limit $R, t \gg t_{\rm loc}$. [MM₂]

- Only the **2. part** contributes to the extensive part of the entropy.

$$S(t) = s_{\rm th} R^{d-1} \mathcal{S}_{\rm ext}\left(\frac{t}{R}\right) + \ldots$$

- Scaling limit: $x^\mu \to R\, x^\mu \,, \quad z \to z$
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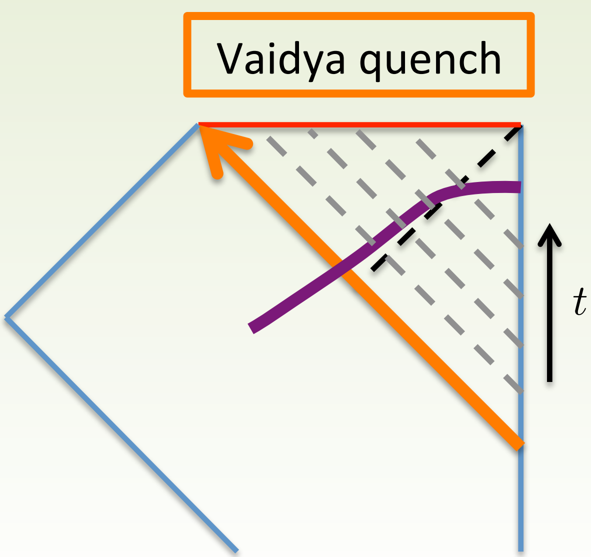

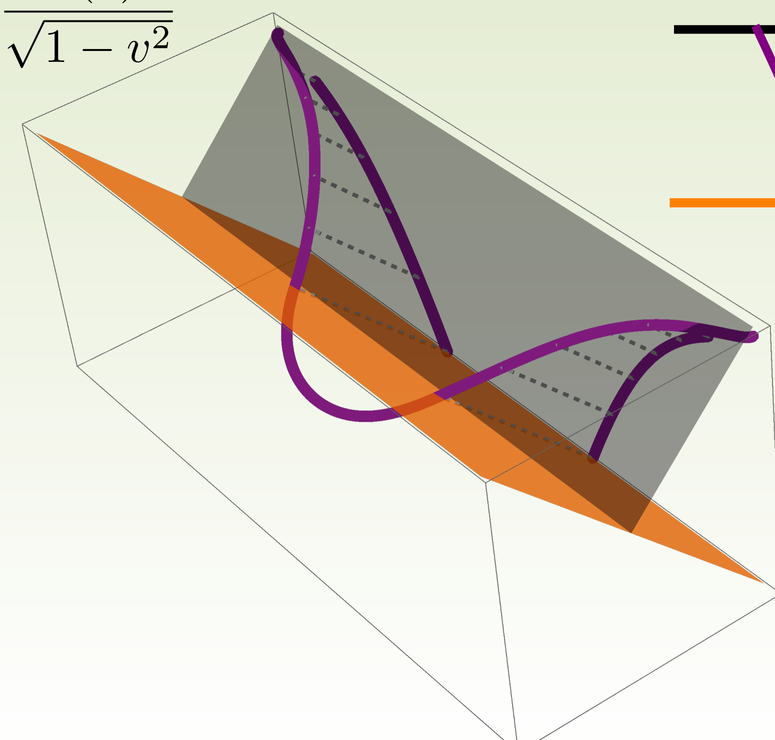

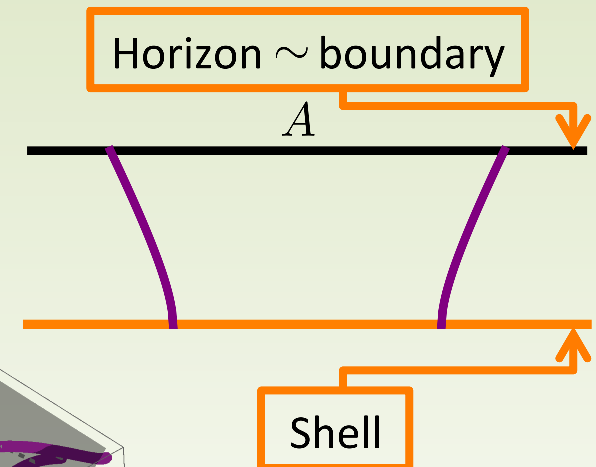

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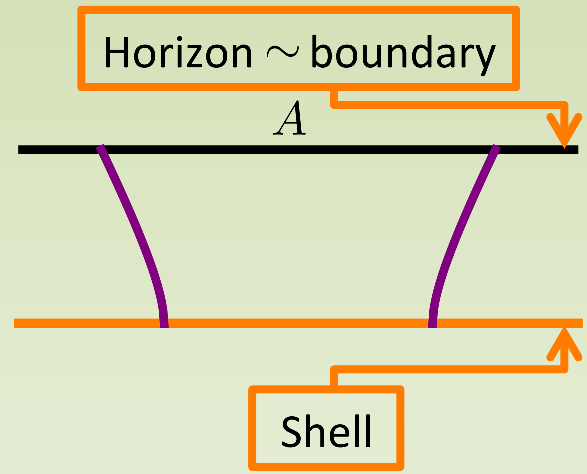
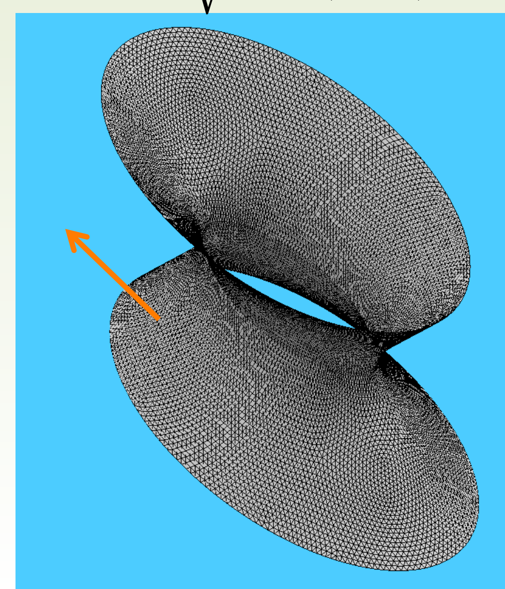
Membrane is projection of HRT to boundary along constant infalling time. It is independent of quench details.



$$v = \frac{(n \cdot \hat{t})}{\sqrt{1 + (n \cdot \hat{t})^2}}$$



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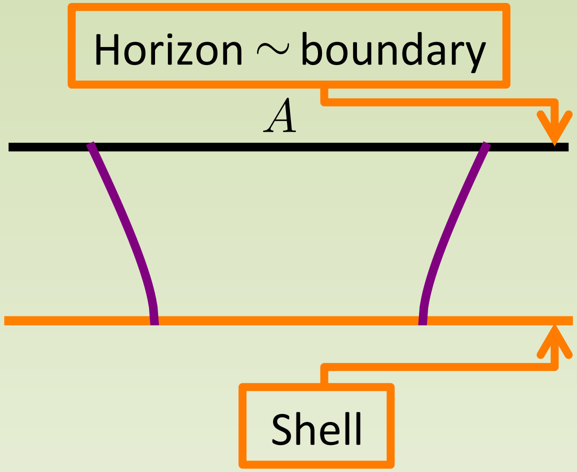

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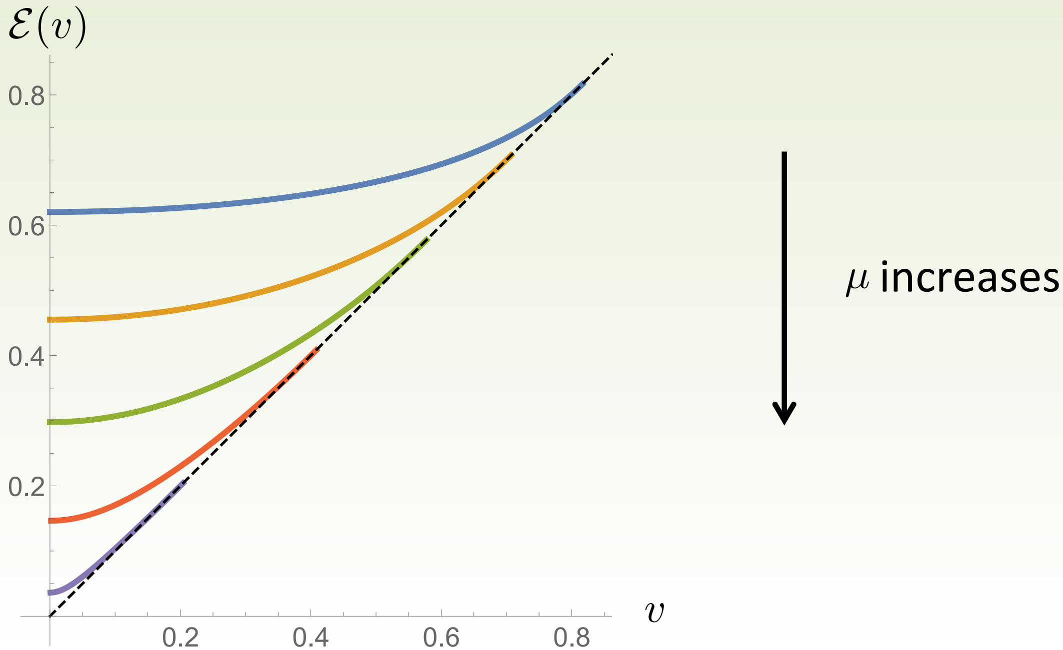

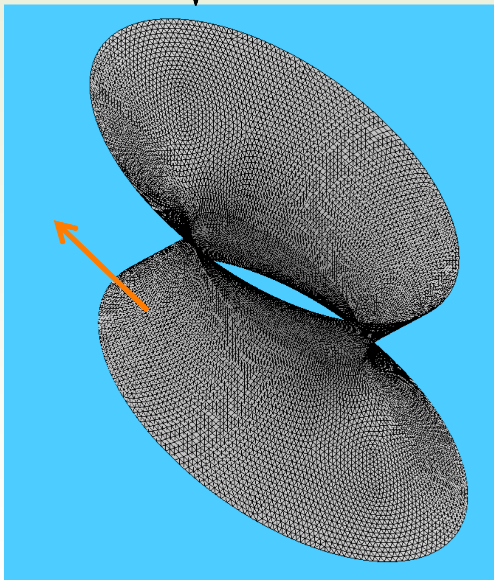

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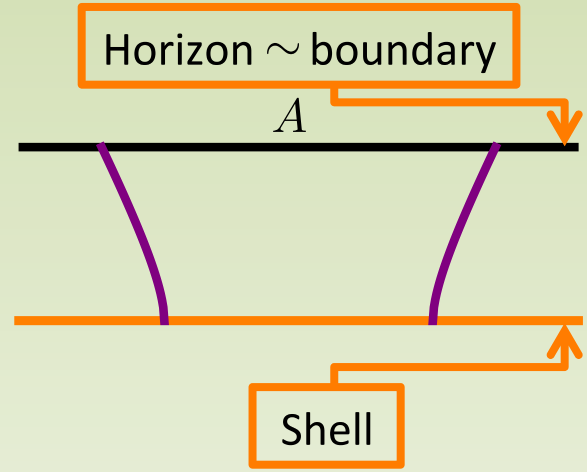

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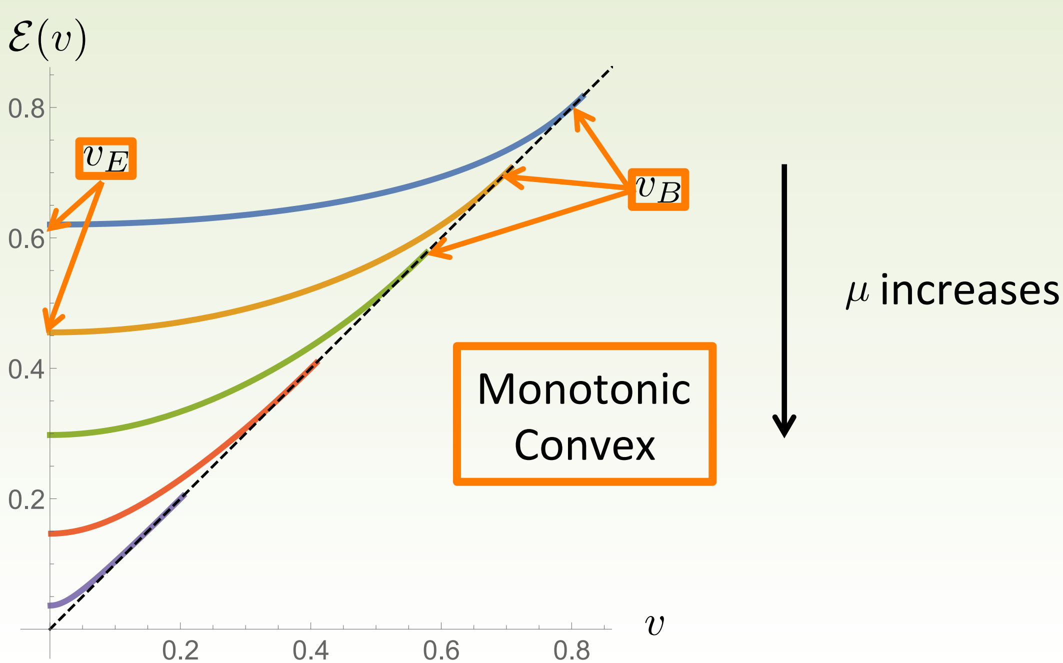
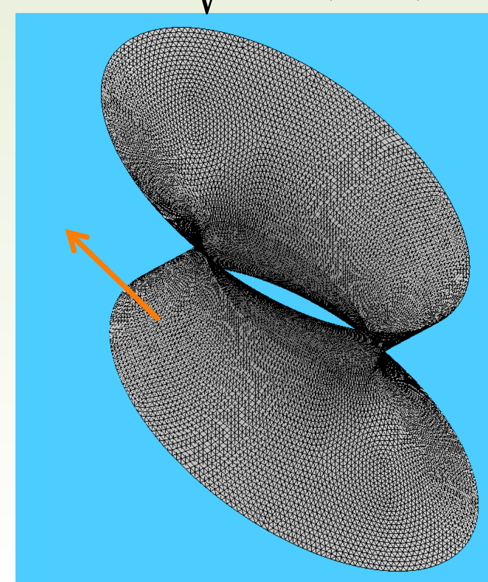
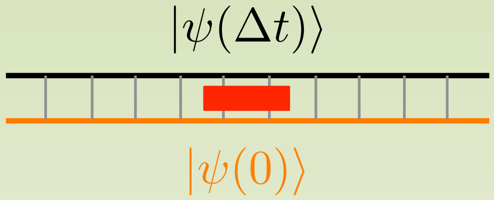




Membrane theory of entanglement dynamics

The same description of entanglement dynamics arises in CMT.

- Random quantum circuit model for the evolving wave function.



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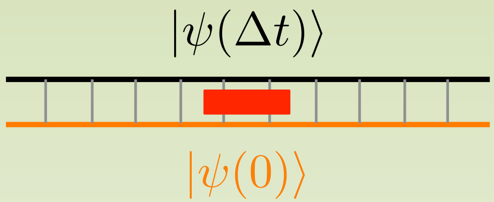

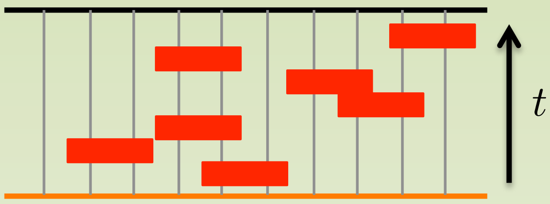

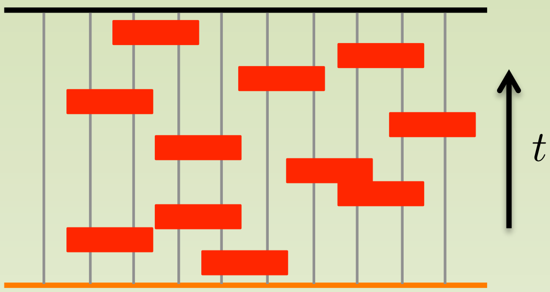

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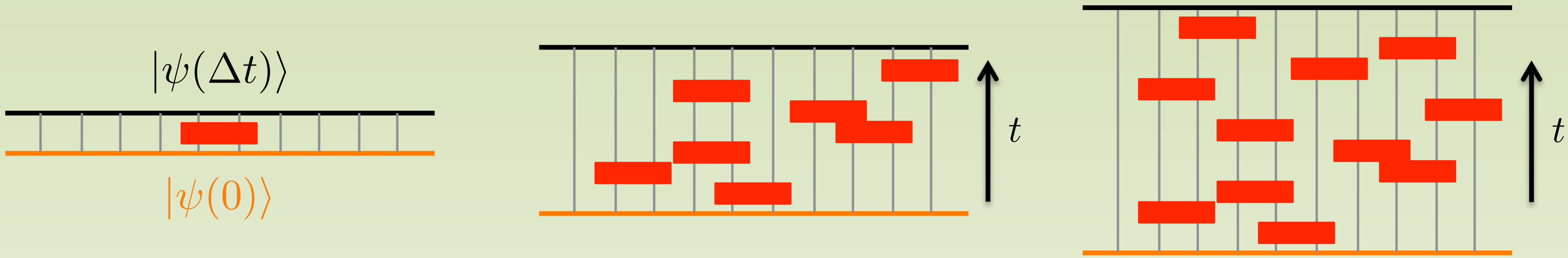
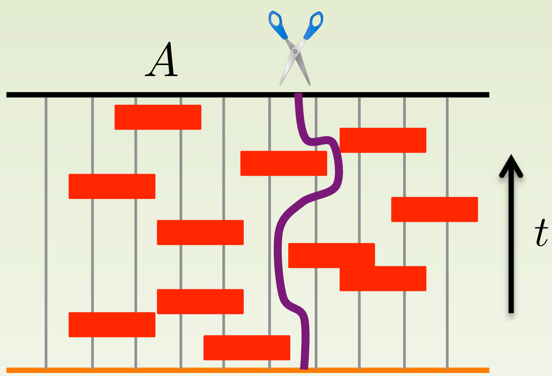
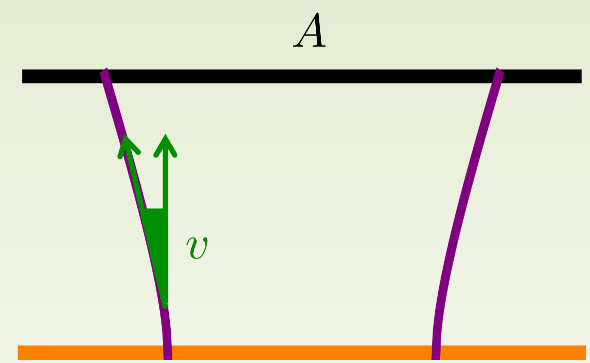


- Minimal cut computes the entropy. [Nahum, Ruhman, Vijay, Haah]



$$S = s_{\mathrm{th}} \int_0^t dt' \;\; \mathcal{E}(v)$$



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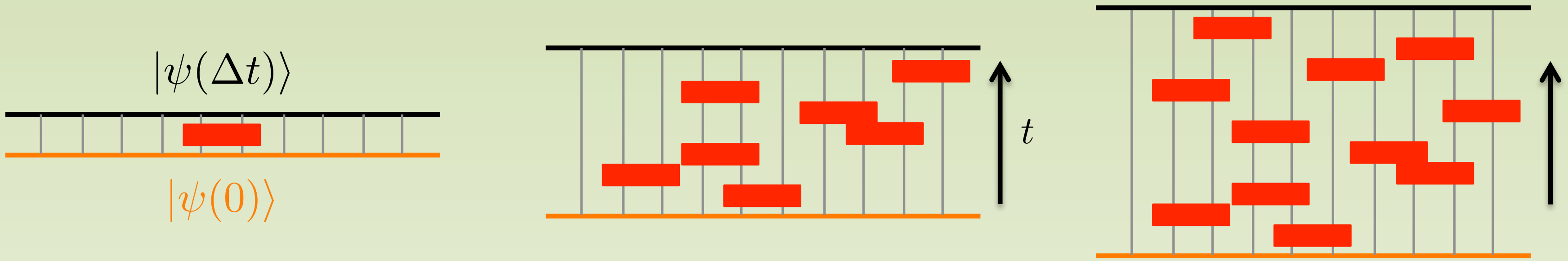

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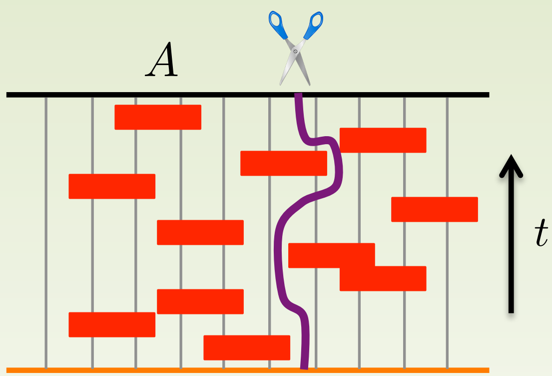
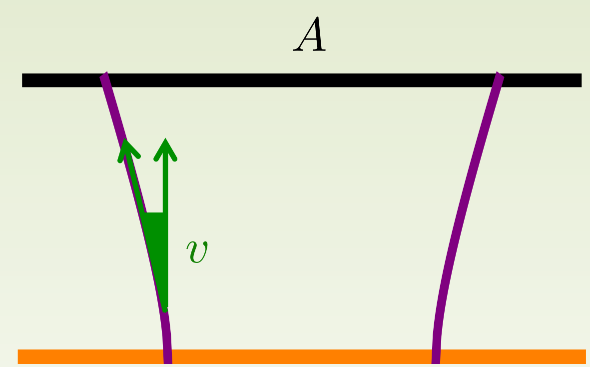


Coarse graining

$$S = s_{\text{th}} \int_0^t dt' \;\; \mathcal{E}(v)$$



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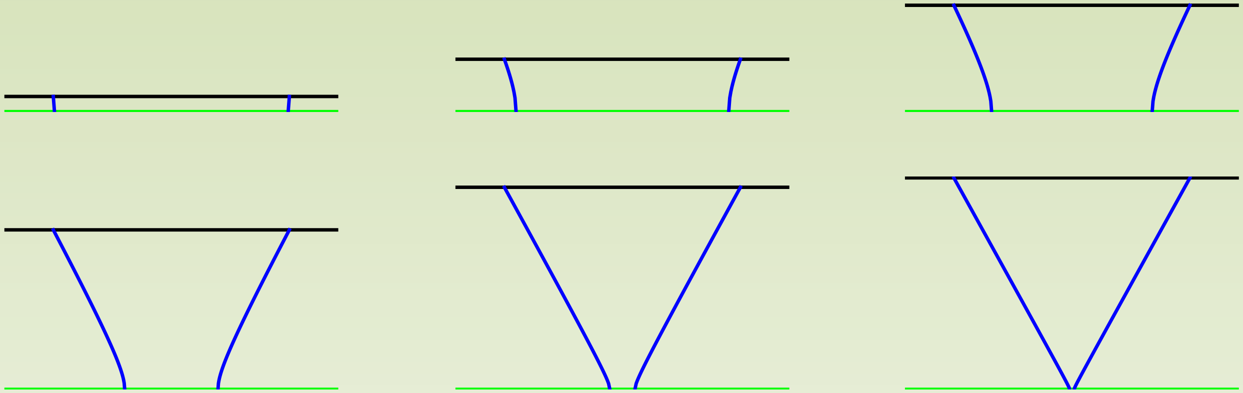
- Evidence in chaotic spin chains.
- Remarkable unification of CMT and HEP approaches: **Membrane description of EE growth in quenches.**

Applications

EE growth for spherical regions in the hydrodynamic limit is analytically solvable. [MM_1; MM_2]

- Representative membrane shapes.



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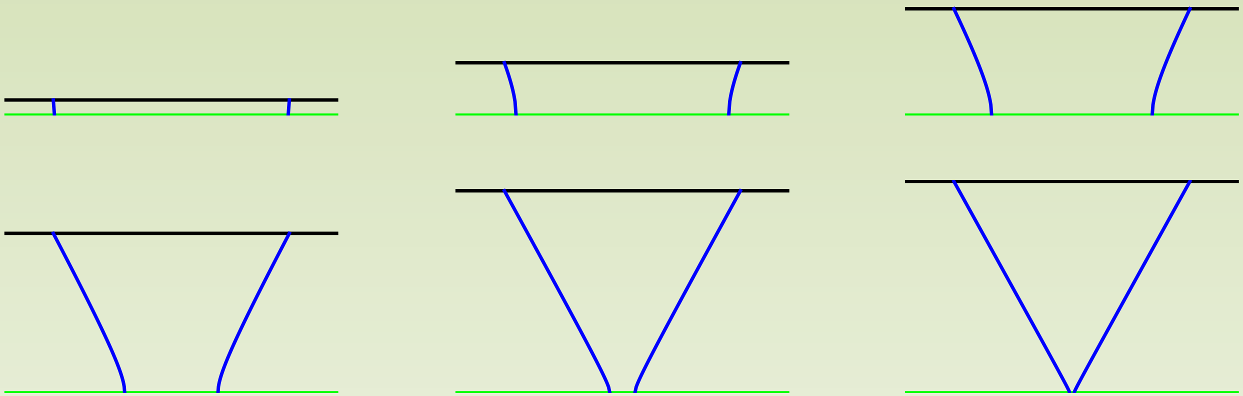
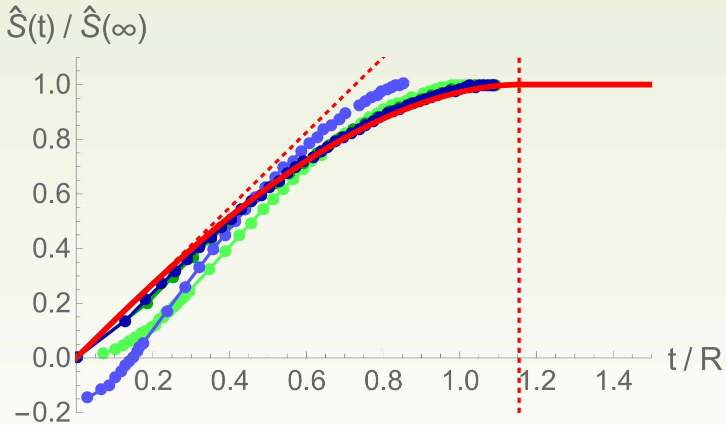
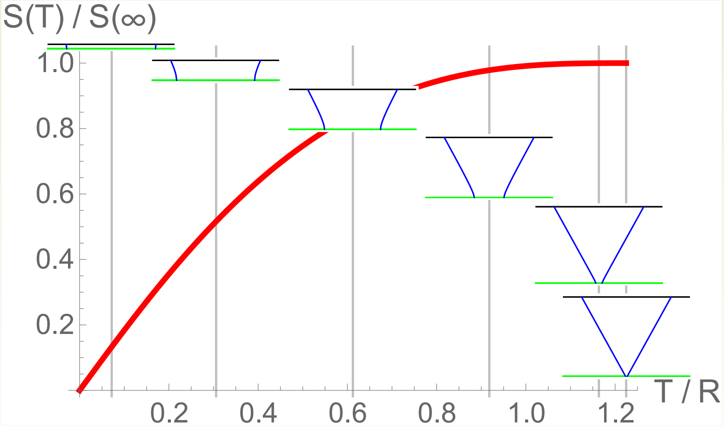


- Comparing numerical results for finite R and membrane theory predictions:





$$t_S = \frac{R}{v_B}$$

Extensions

The membrane theory is robust, can be generalized away from quenches. [MM, Virrueta]

- Fluid/gravity black brane dual to an inhomogenous state in local thermal equilibrium. To subleading order, we get the membrane coupled to hydrodynamics:

$$S = \int d\text{Area}\; s_{\text{th}}(x) \frac{\mathcal{E}(v)}{\sqrt{1-v^2}} \left[1 + \mathcal{F}_1(v)\left((\mathcal{A}\cdot n)(n\cdot u) - (\mathcal{A}\cdot u)\right) + \mathcal{F}_2(v)\sigma_{ab}n^a n^b + \ldots\right]$$

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- Adaptable to other setups, can incorporate β/R and $1/\lambda$ corrections without change in the structure of the membrane theory. $1/N$ corrections would be most interesting.
- New language opens a rich arena of applications in holographic EE.
  - Informs tensor network approaches to bulk reconstruction. [Swingle, Harlow et al.]
  - "Entropy cone" inequalities generalized to time dependent settings. [Hayden, Headrick, Maloney; Bao et al.; Bao, MM]
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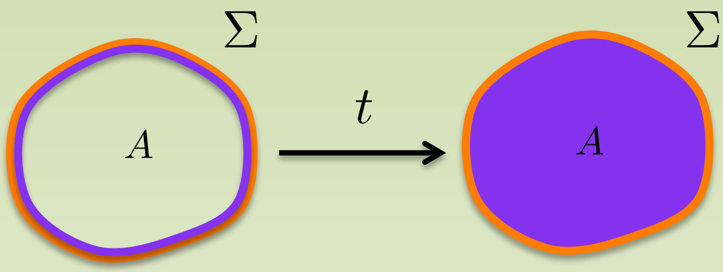
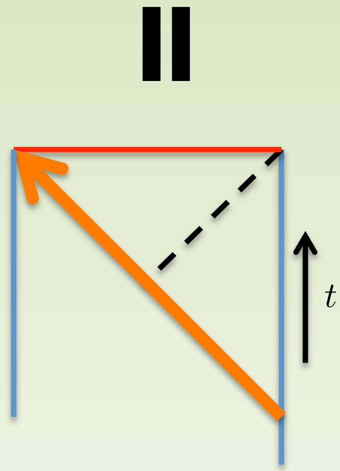
  - ➢ Numerical explorations, black holes (often) saturate entanglement entropy the fastest. [MM, van der Schee]
- **Membrane theory is versatile, has connections to operator growth and hydrodynamics, and has all the features to be a universal theory.**

Summary

Features of the thermalization:

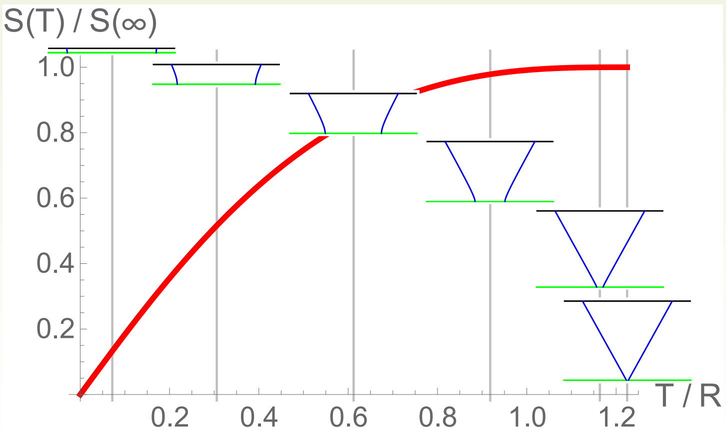
- Conserved densities described by hydro.
- State of the entire system cannot become thermal. Small subsystem thermalize by becoming entangled with the rest of the system.
  $S_A(t) \;\to\; S_A^{(\mathrm{eq})}(\beta) = s_{\mathrm{th}}(\beta)\,\mathrm{vol}(A)$
  Captures the essence of thermalization.





Goal: Find effective theory (akin to hydro) of entanglement dynamics.

- Insight into thermalization in isolated chaotic quantum systems.
- Alternative history method: Discovered membrane theory by studying AdS black holes, has structure applicable to all chaotic theories.
- In the future conduct further tests, give general derivation. Elucidate connections to other manifestations of chaotic dynamics.



Outline

Transport
- Hydro as an EFT
- Holography for real time dynamics

Thermalization
- Entanglement entropy as a probe
- Membrane theory is the EFT

Butterfly effect
- Butterfly effect, operator growth and OTOC
- Refinement of the chaos bound
- Towards an EFT

Summary and open questions

Butterfly effect, operator growth and OTOC

Butterfly effect in many-body systems: [Larkin, Ovchinnikov; Shenker, Stanford; Kitaev]

- In classical physics butterfly effect is sensitivity to initial data:

$$\delta q(t) = \delta q_0 \, e^{\lambda_L \, t}$$

- Quantum many-body context: simple operators (few-body) evolve into complex ones (many-body), one particle can have effect later in entire system.

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- Diagnostic is OTOC:

$$C(t,x) = -\frac{\langle [W(t,x), V(0)]^2 \rangle}{\langle V^2 \rangle \langle W^2 \rangle}$$

In its expansion both TO and OTO terms.

Chaotic operator growth

Effective size of an operator in a thermal state:

- Chaotic time evolution makes simple local operators complex. Size can be probed by the OTOC: [Roberts, Susskind, Stanford]

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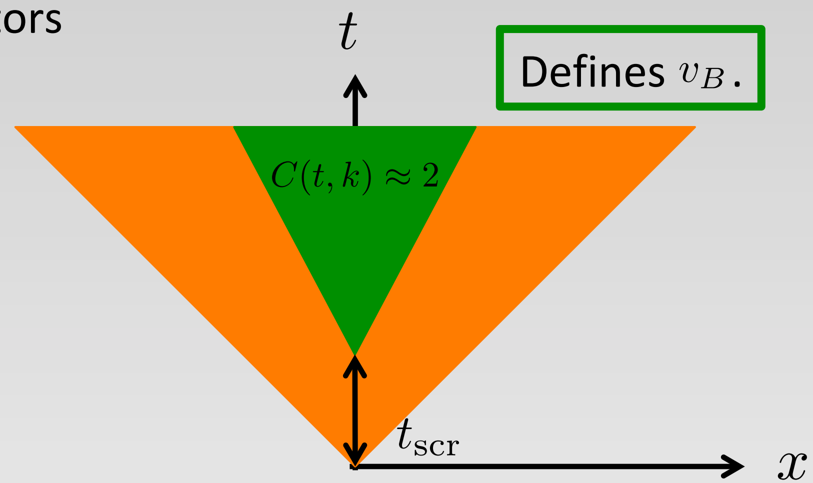

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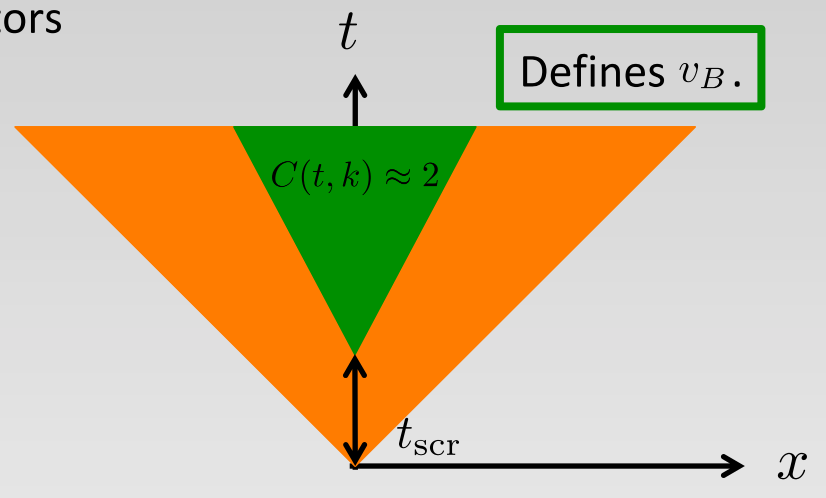
- Chaos bound: [Maldacena, Shenker, Stanford]

$$C(t,x) = \frac{\#}{N^2} \exp(\lambda_L t) + \ldots \qquad \beta \ll t \ll t_{\text{scr}}$$

$$\lambda_L \leq \frac{2\pi}{\beta}$$



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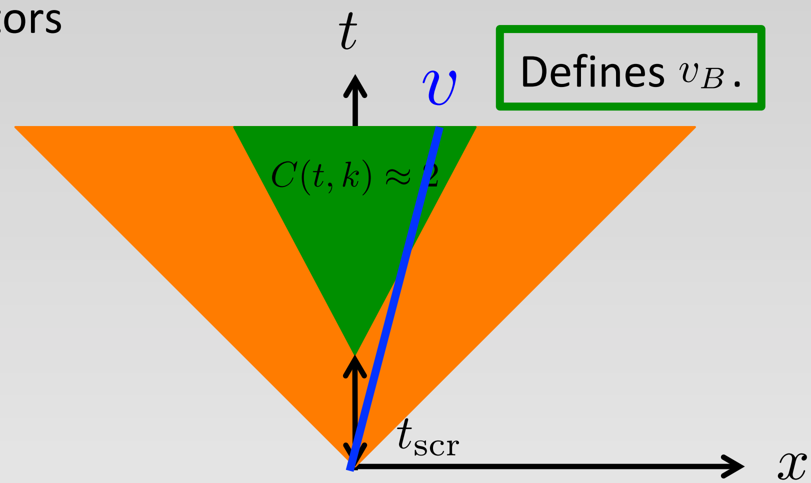

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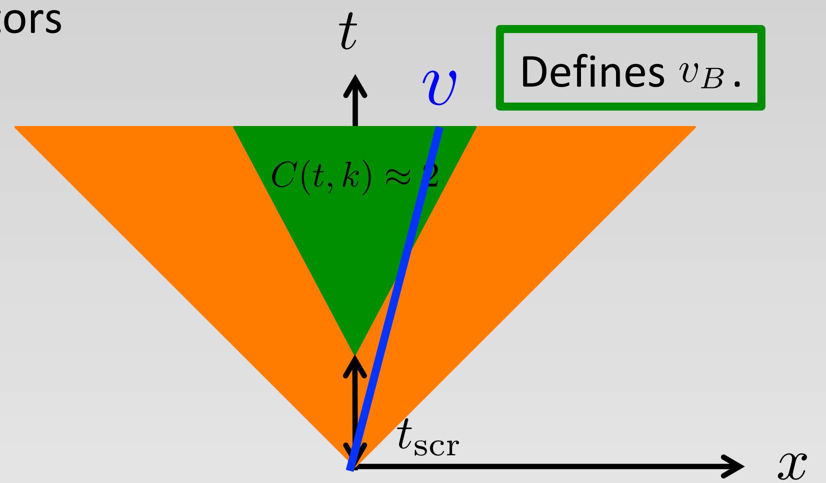

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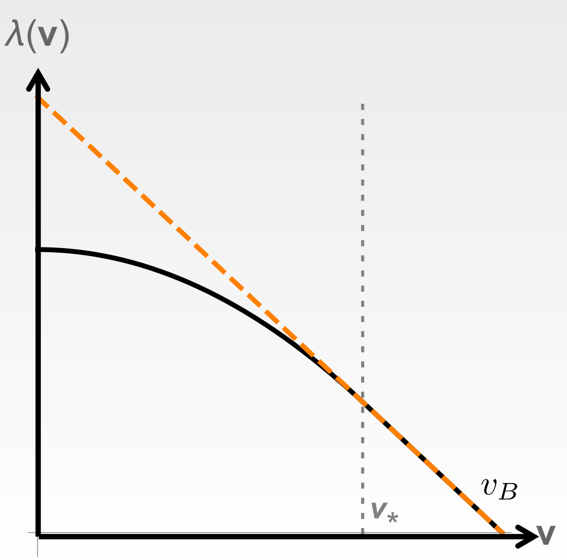
Generic behavior velocity dependence (SYK-like models, 2d CFT, higher-d CFT in hyperbolic space):



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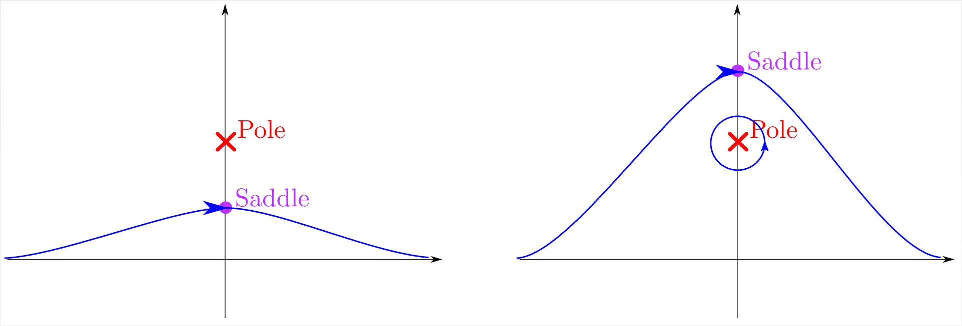

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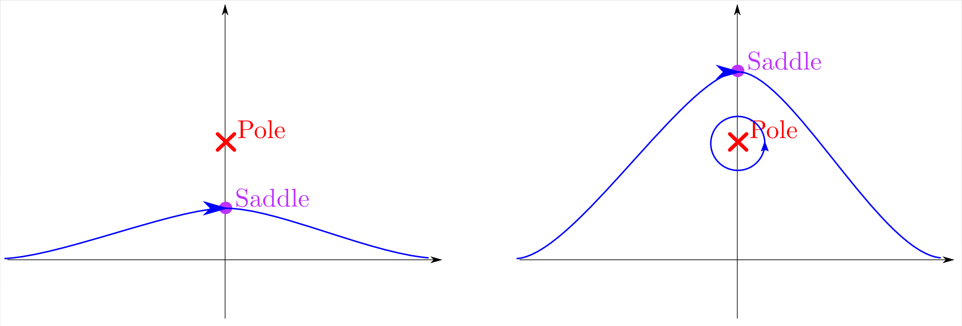
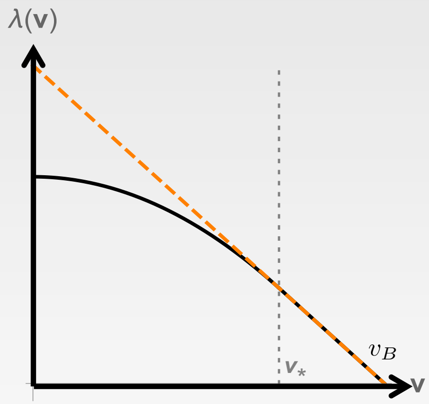


- For $v < v_*$, $\lambda(v)$ is the Legendre transform of $j(\nu)$.
  For $v \geq v_*$ stress tensor dominates, and chaos is maximal.

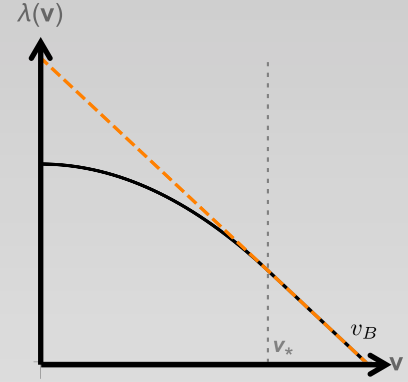


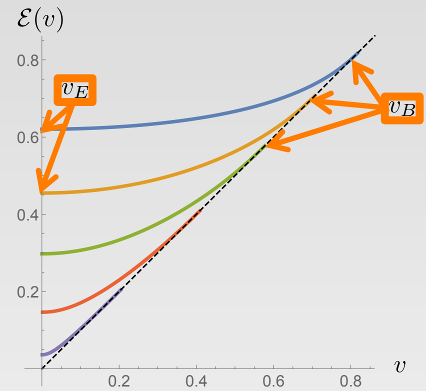
- How do we write a pomeron EFT?
  How is it related to the Schwarzian EFT of the SYK model? [Kitaev, Suh]
  What happens around $t \sim t_{\rm scr}$? [von Keyserlingk et al.; Xu, Swingle]

Interplay with entanglement dynamics

Hints at deep relation between operator growth and EE dynamics:

- v_B is special point in both $\lambda(v)$ and $\mathcal{E}(v)$, but relation between the two "transport coefficients" beyond this is unknown. (v_B also makes appearance in pole-skipping. [Grozdanov et al.])





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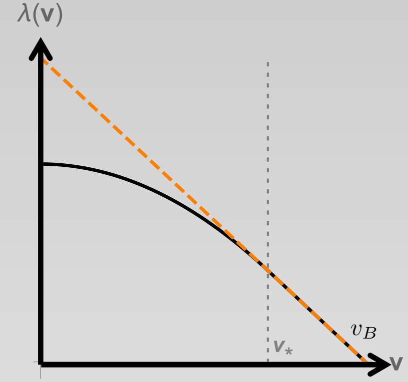
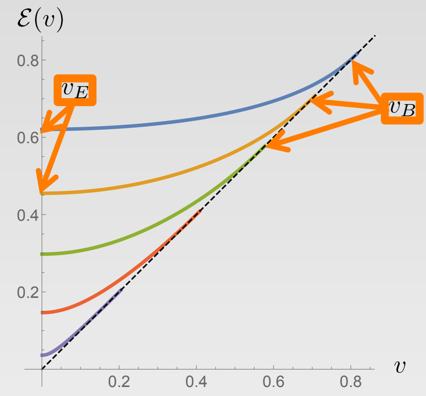
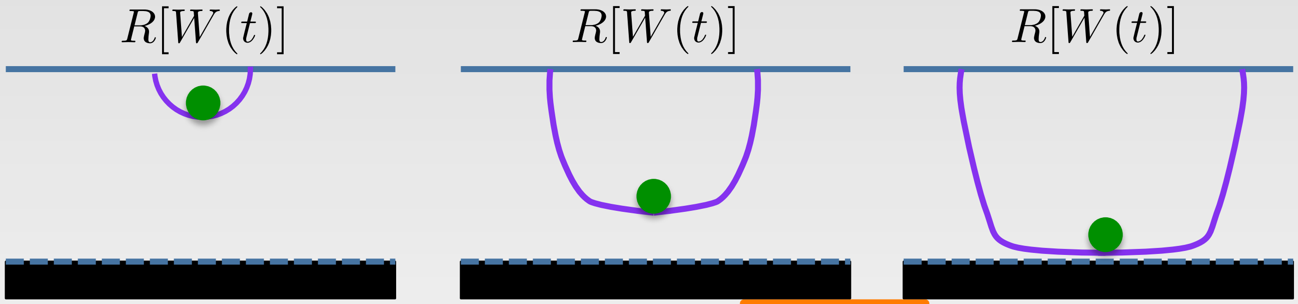


- Size of operator from entanglement wedge reconstruction: [MM, Stanford]





Gives an explanation of the relation: $t_S = \dfrac{R}{v_B}$

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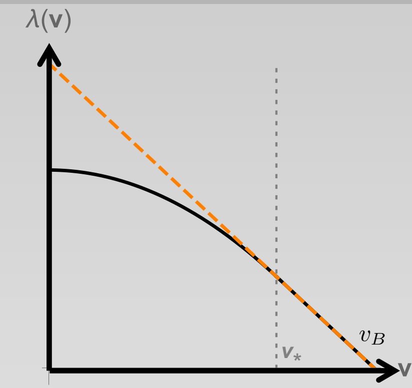

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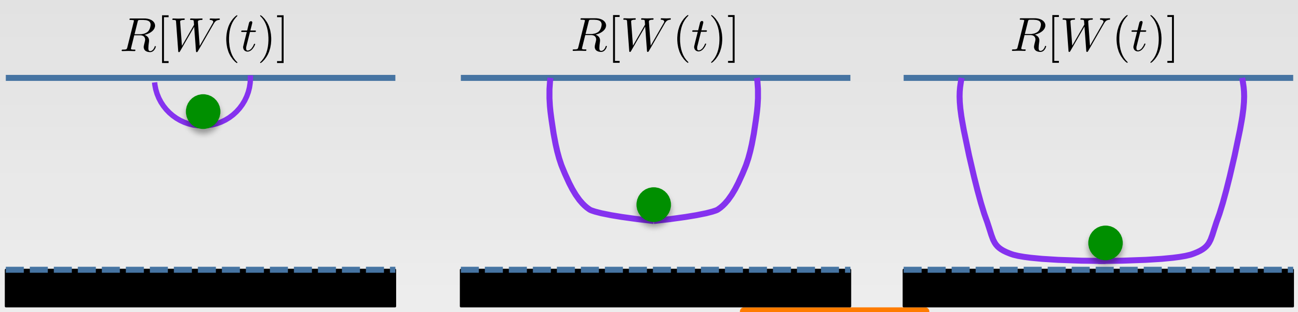

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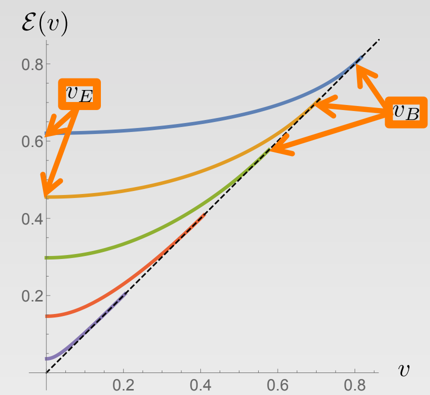

- Can use the emergent light cone to put tight bounds on the entropy. The membrane theory obeys these bounds. [MM, Stanford; Jonay, Huse, Nahum]

$$\mathcal{E}_{\max}(v) = v_E + \left(1 - \frac{v_E}{v_B}\right)|v| \qquad (|v| \leq v_B)$$

Outline

Transport
- Hydro as an EFT
- Holography for real time dynamics

Thermalization
- Entanglement entropy as a probe
- Membrane theory is the EFT

Butterfly effect
- Butterfly effect, operator growth and OTOC
- Refinement of the chaos bound
- Towards an EFT

Summary and open questions

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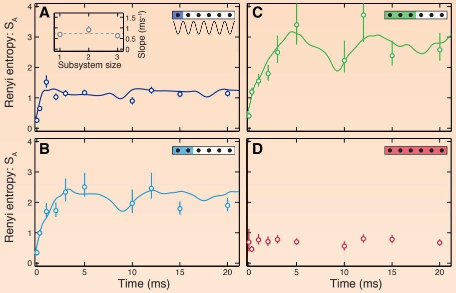

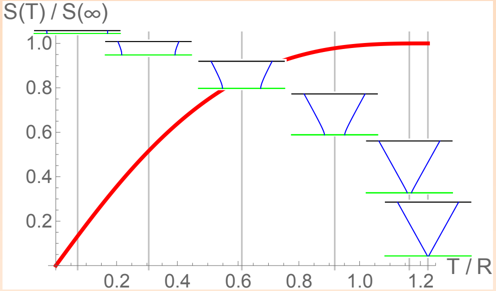

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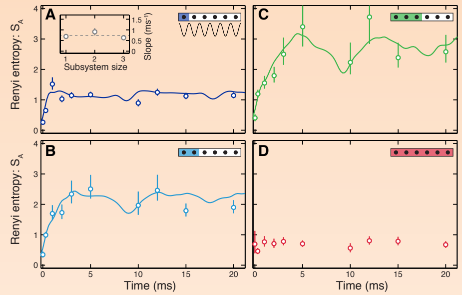

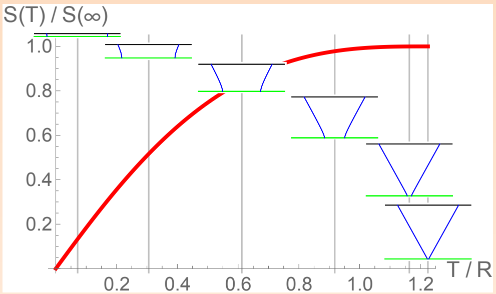

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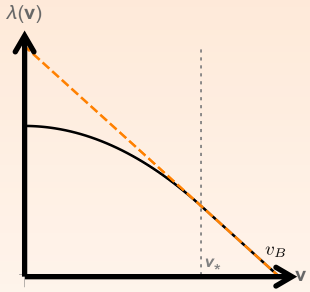

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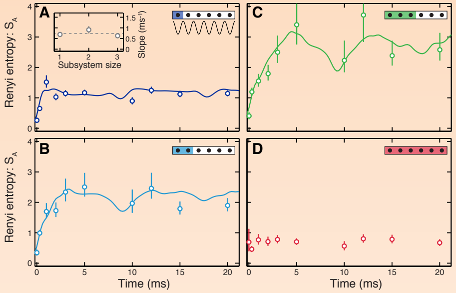

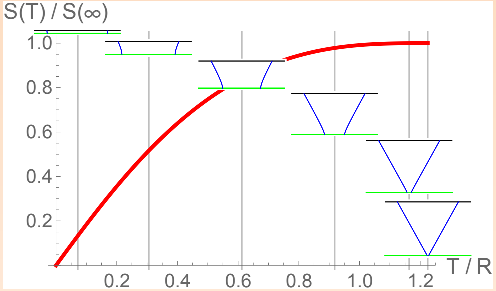

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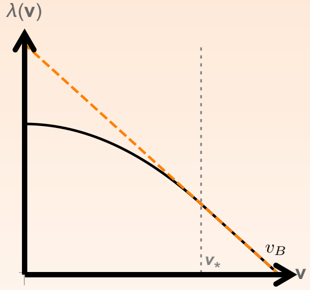
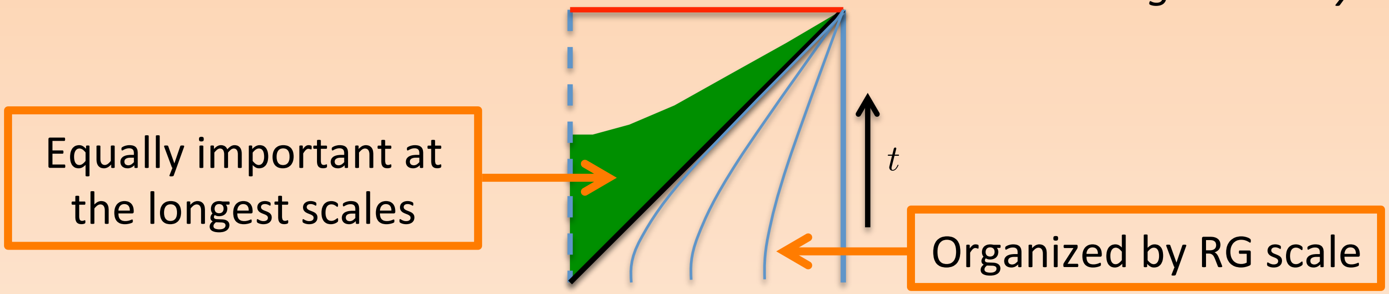


- Uncovered interplay between these phenomena:
  - ➢ Data of EE dynamics to chaotic correlators, membrane theory obeys general bounds, entanglement wedge argument
  - ➢ Membrane couples to hydrodynamics

Open questions and outlook for gravity

Open questions and some hints

- What does the membrane theory imply for holographic RG?
  Hint: The metric inside the horizon does not seems to be organized by scale.



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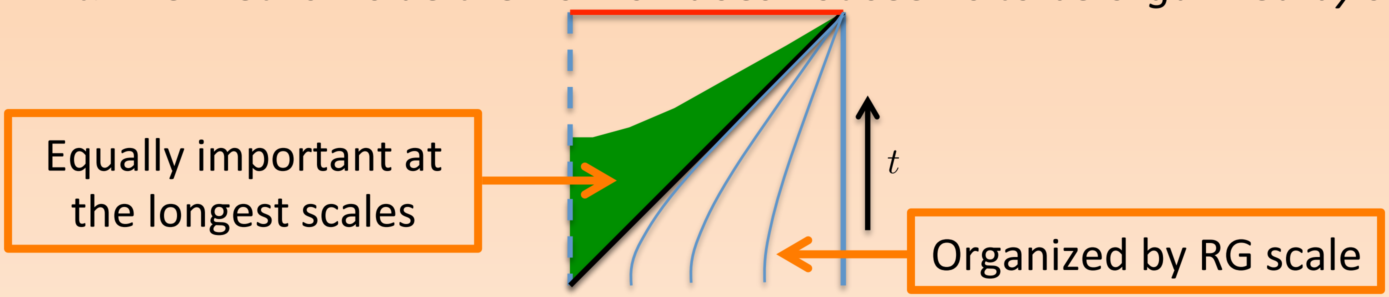

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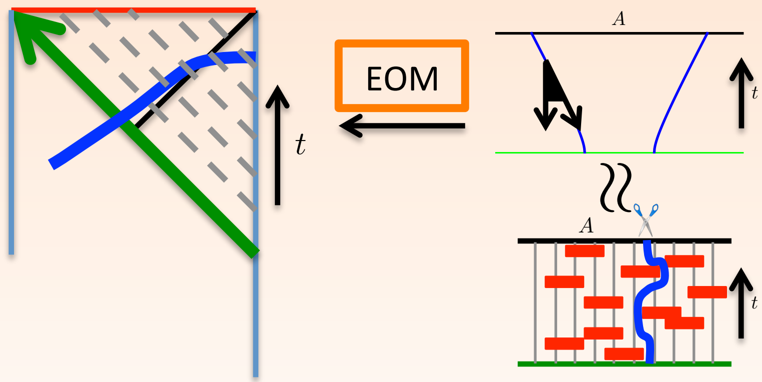
  "Gravity is the hydrodynamics of entanglement"?
  Hint: Found a quantitative tensor network-like description, after partially solving the EOMs.



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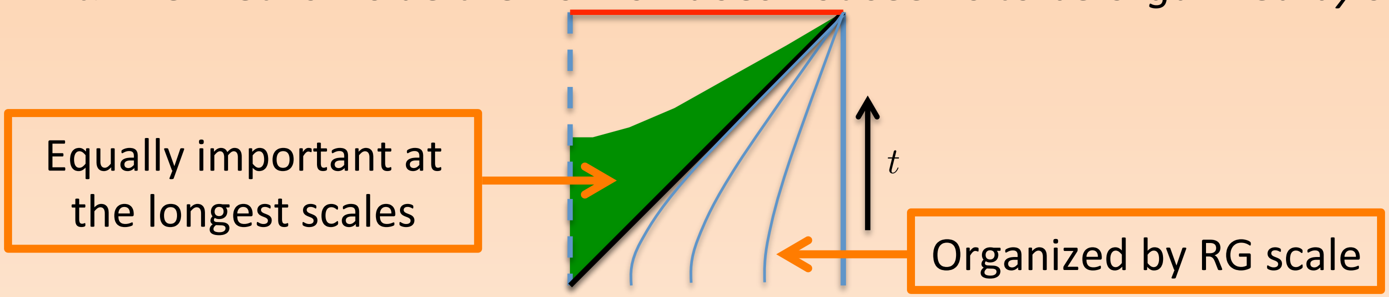

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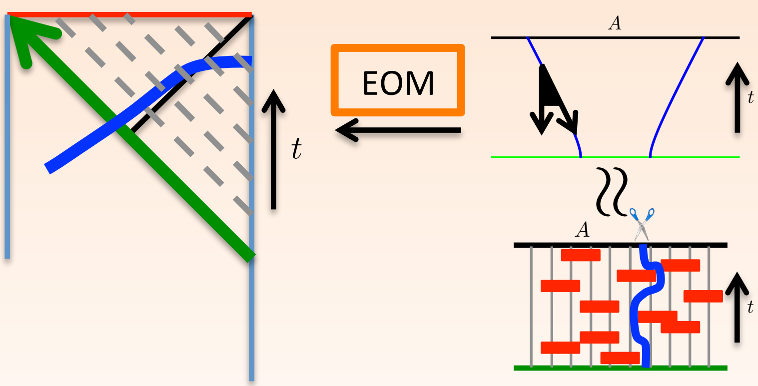
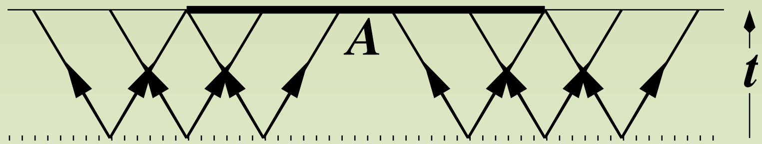


- Meaning of saturation of refined chaos bound?
  Hint: Boosting enhances chaos, stringy near-extremal rotating BHs likely give near-maximal growth, Schwarzian universality.

Backup slides

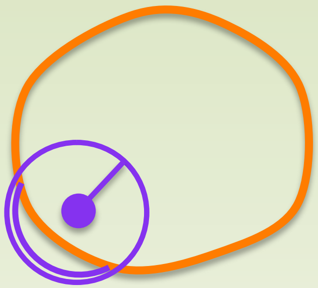
Quasiparticle model

Calabrese-Cardy model: energy injection from quench creates a finite density of EPR pairs, subsequently travel freely at the speed of light isotropically.



- Leads to linear growth with $v_E = 1$ in 2d.

- Higher dimensions: entanglement spreading depends on entanglement pattern on the light cone $\mu[L_\Sigma]$. Contribution from each light cone has to be added. [Casini, Liu, MM]



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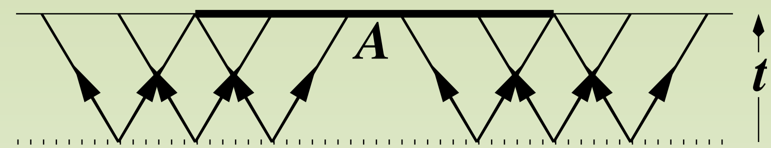

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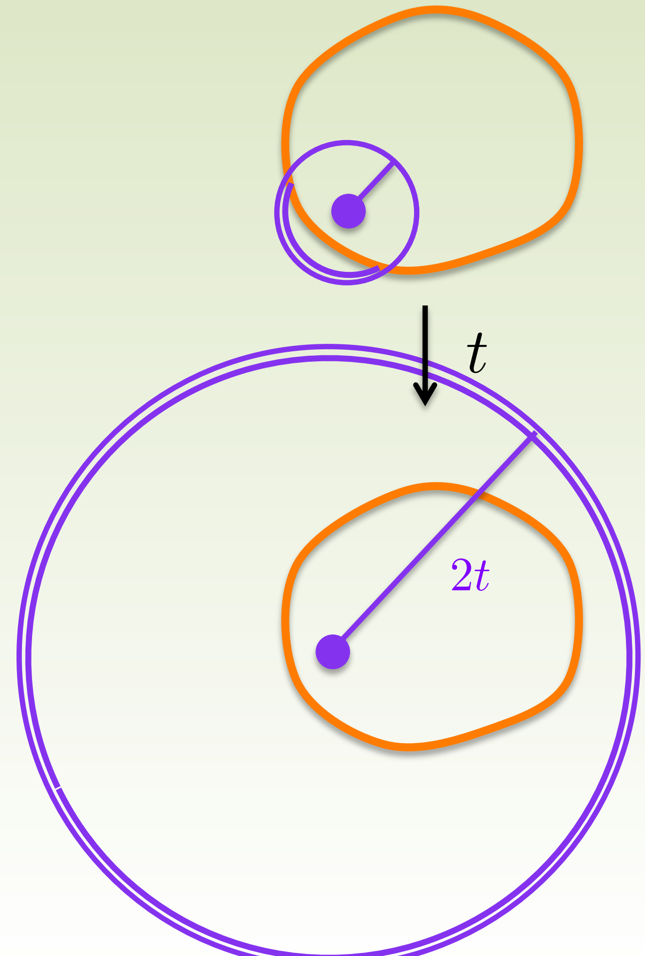

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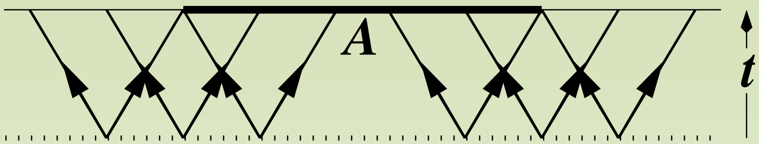

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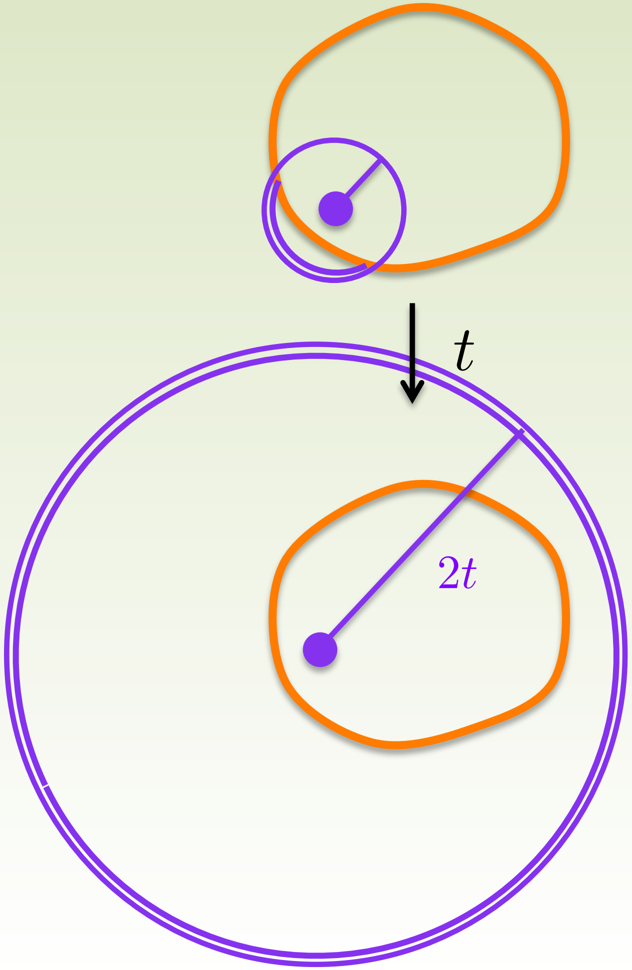

Bound on the entanglement speed from SSA:

$$v_E \leq v_E^{(\mathrm{EPR})} = \frac{\Gamma\left(\frac{d-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{d}{2}\right)} < v_E^{(\mathrm{SBH})}$$

Slower than holography.

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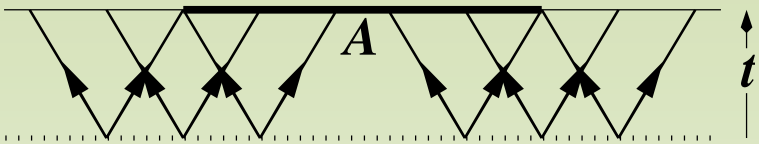

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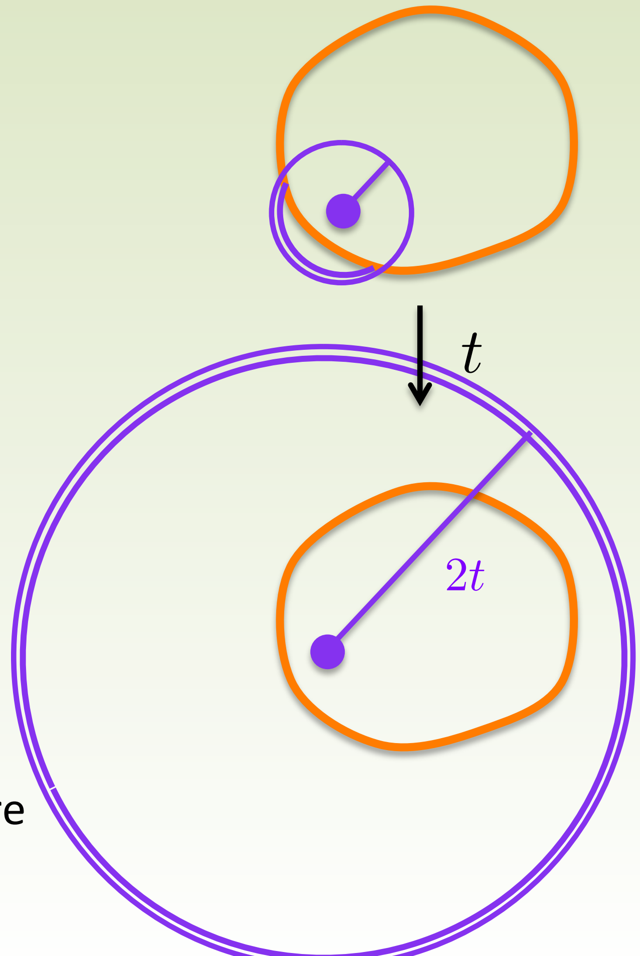

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- In strongly coupled systems, entanglement grows faster than what's possible for free particles streaming at the speed of light!

- Consider the effect of interactions: tensor network picture emerging from scattering particles is natural. [Hartman, Maldacena; Casini, Liu, MM]

Free field theory and the quasiparticle model

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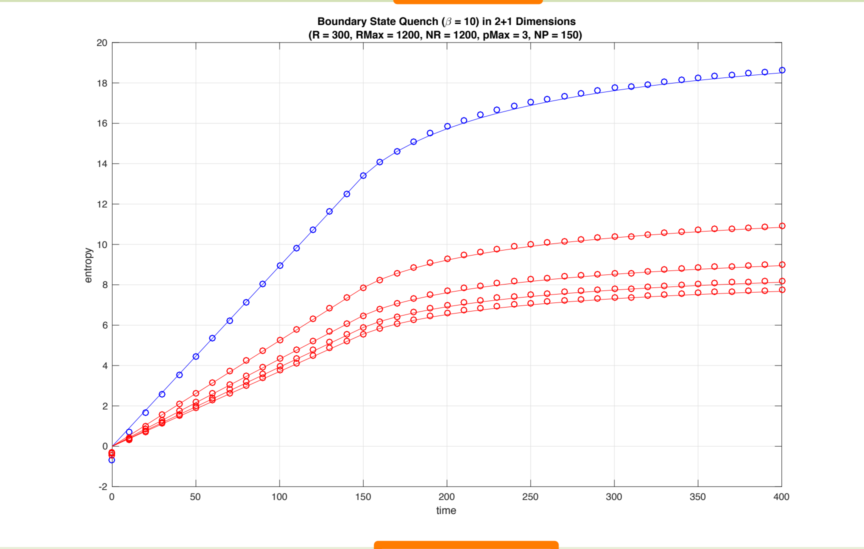
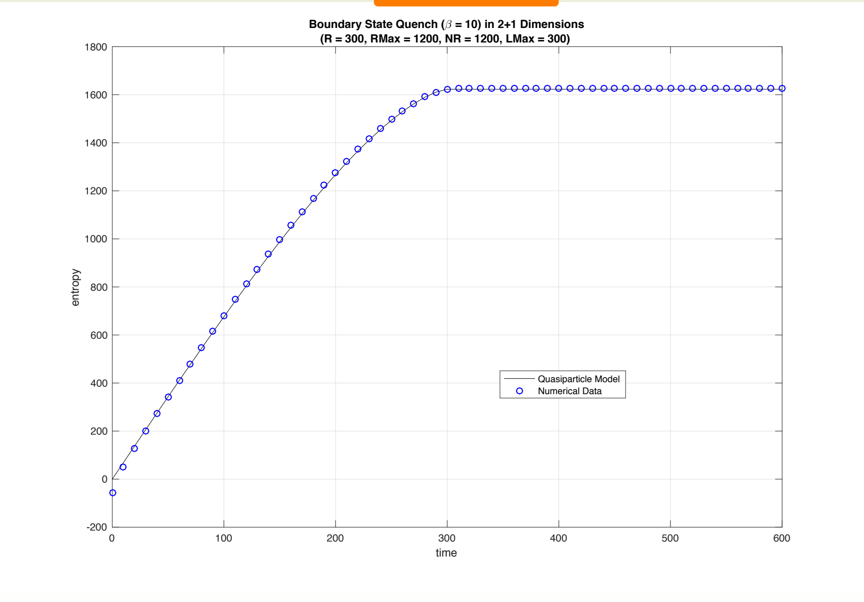
- Numerical results for 3d boundary state quench for scalar field. [Cotler, Hertzberg, MM, Mueller]

Strip



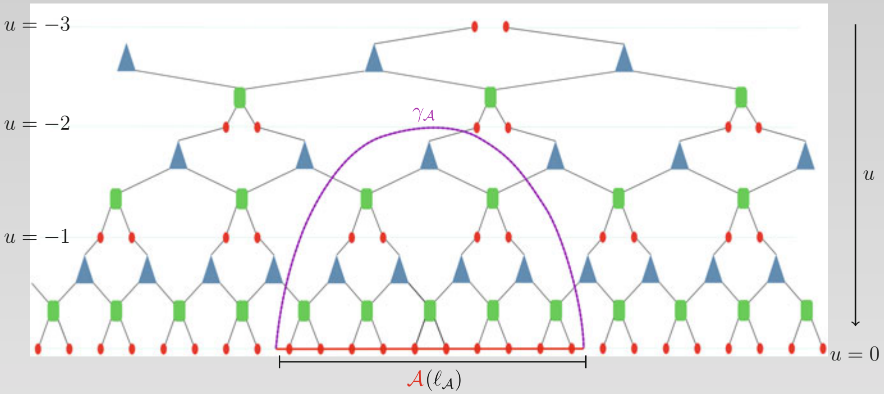
Sphere



Tensor networks and holography

The analogy between minimal cuts and the RT surface computing entanglement entropy has inspired toy models of holography.

- AdS/MERA analogy [Swingle]



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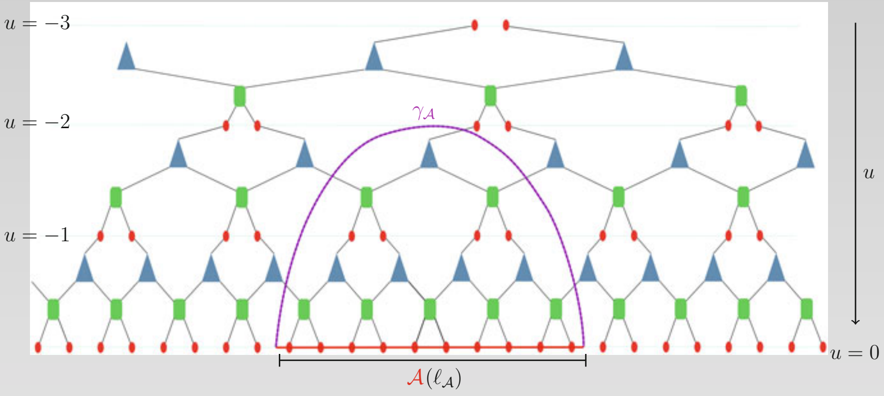

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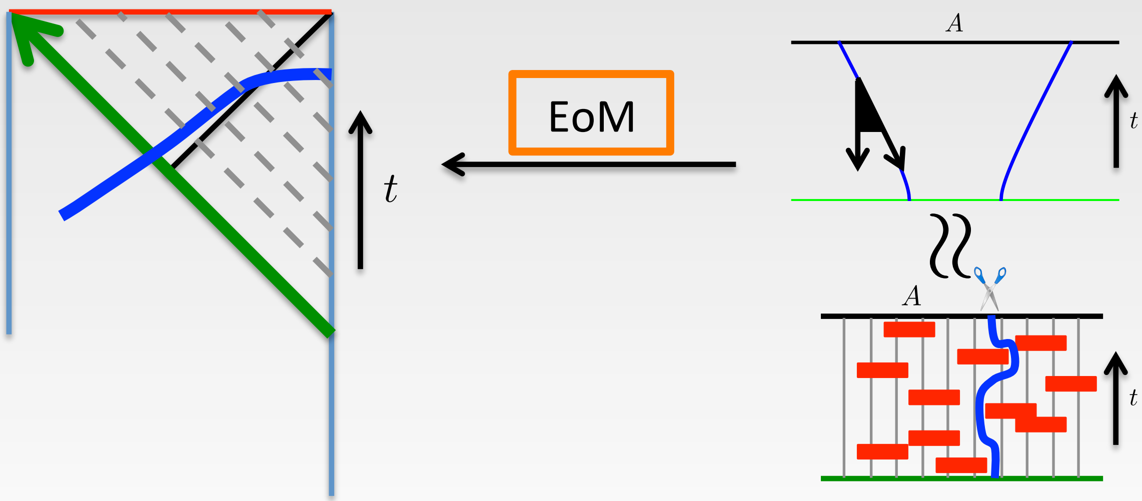

Realization of slogan: "Gravity is the hydrodynamics of entanglement"

Entropy cone

Entanglement entropy in static holographic states obeys inequalities, that are not true in general in QM.

- The best known one is the monogamy of mutual information. [Hayden, Headrick, Maloney] It can be proven using the same steps as in the proof of SSA.

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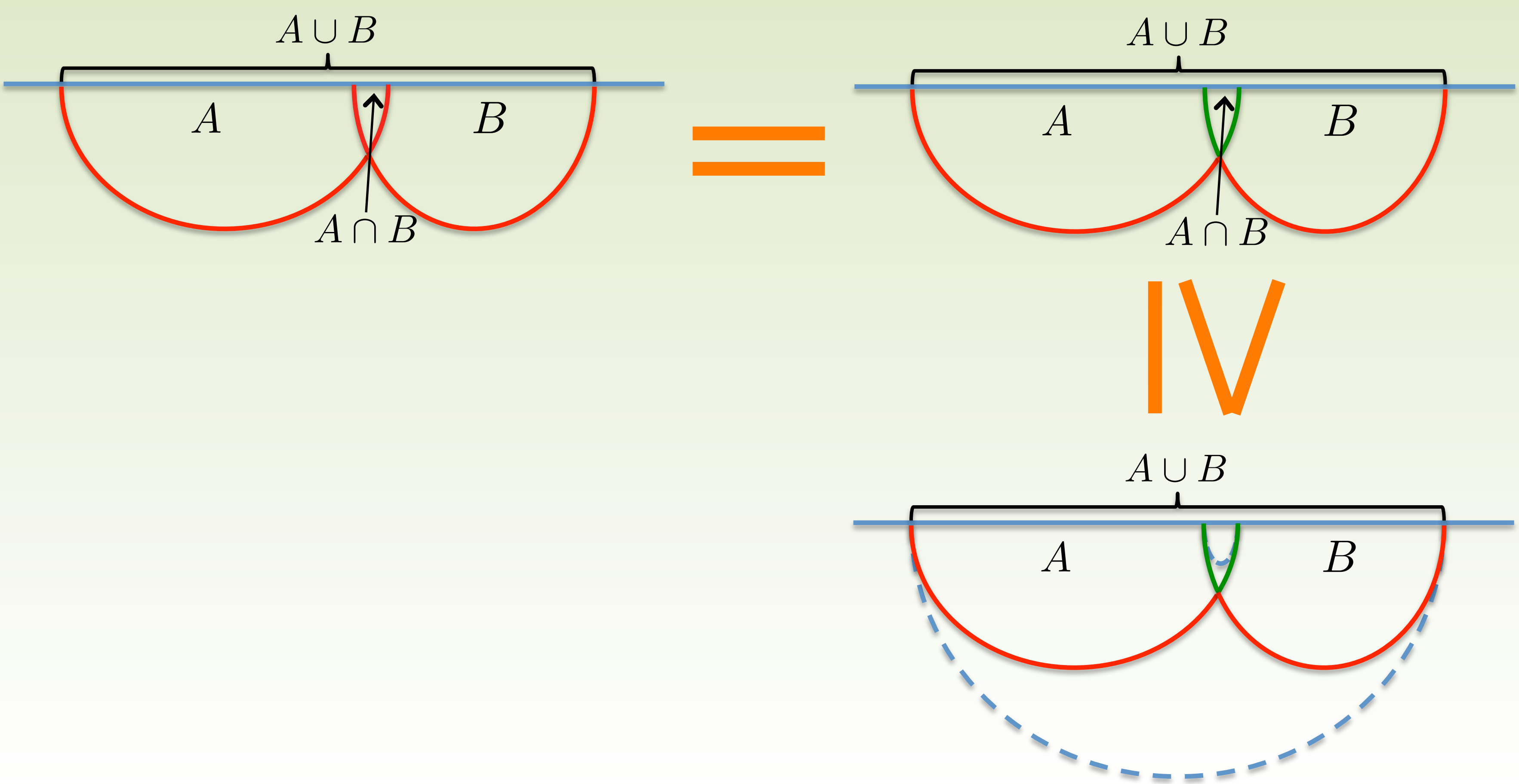

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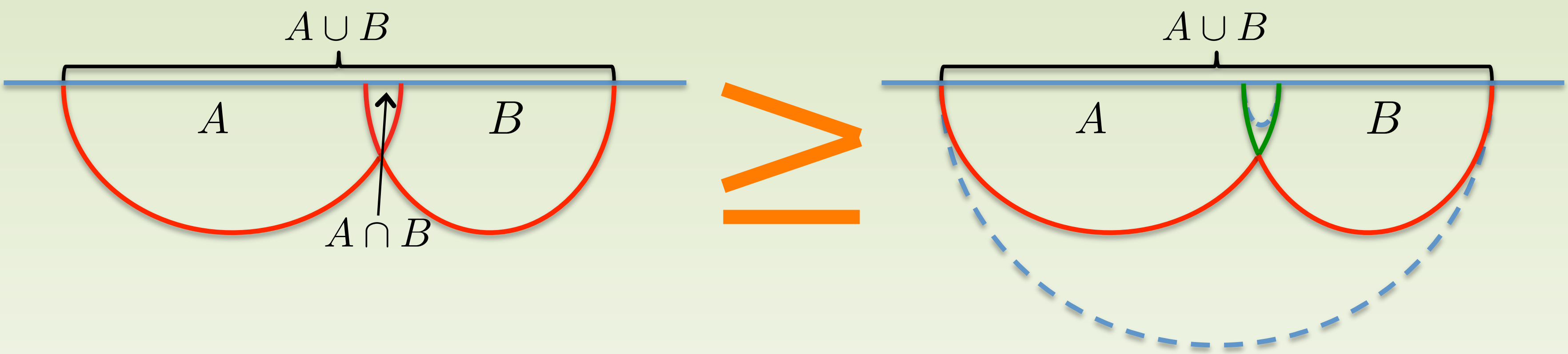


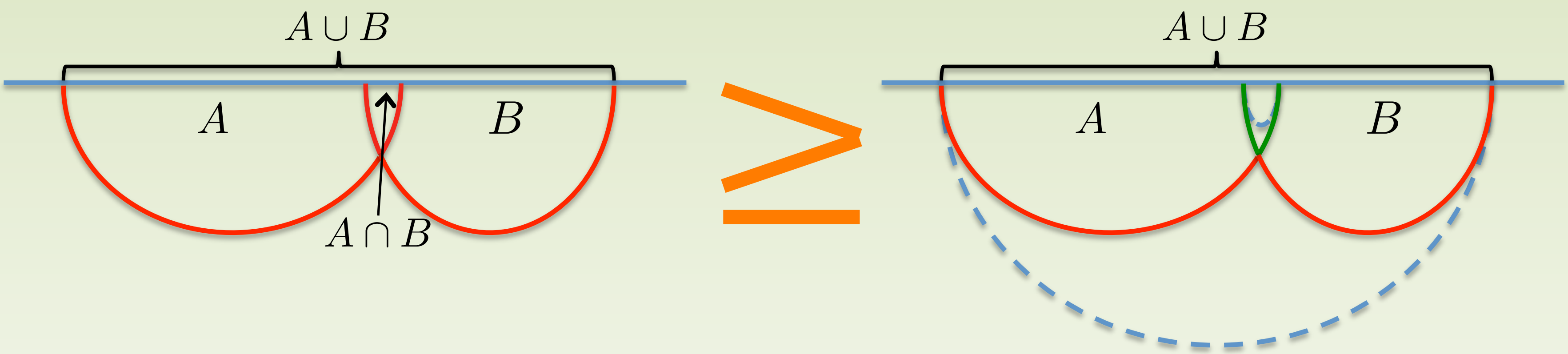
- The inclusion-exclusion proof method can be used to derive many-party inequalities. [Bao et al.] Holography is not essential, only need that the entropy is proportional to a partionable geometric minimization problem.

Entropy cone

Entanglement entropy in static holographic states obeys inequalities, that are not true in general in QM.

- The best known one is the monogamy of mutual information. [Hayden, Headrick, Maloney] It can be proven using the same steps as in the proof of SSA.

$$S(AB) + S(BC) + S(AC) \geq S(A) + S(B) + S(C) + S(ABC)$$



- The inclusion-exclusion proof method can be used to derive many-party inequalities. [Bao et al.] Holography is not essential, only need that the entropy is proportional to a partionable geometric minimization problem.
- HRT is an extremization of codimension-2 surface, no proof (or counterexample) is known for many-party inequalities. Inclusion-exclusion applies to the membrane theory, hence proof for time dependent states (large regions, late times). [Bao, MM]

Bit threads

The Ryu-Takayanagi prescription can be reformulated in the language of bit threads. [Freedman, Headrick]

- Maximize $\int_A \sqrt{h}\, n_\mu w^\mu$

  Constraints: $\nabla_\mu w^\mu = 0\,, \quad 1 - |w^\mu| \geq 0$

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  $$\nabla_\mu w^\mu = 0\,, \quad H(w_t) - |\vec{w}| \geq 0$$

  $H(w_t)$ is the Legendre transform of $\mathcal{E}(v)$:

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- The map that reconstructs the HRT surface from the minimal membrane can be used to push the membrane theory bit thread into the bulk.
- **Membrane theory is versatile, has connections to operator growth and hydrodynamics, and has all the features to be a universal theory.**

Bounds

Entanglement entropy obeys inequalities, natural to consider bounds in the quench setup.

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- Monotonicity of (thermal) relative entropy for subsystems combined with emergent v_B light cones at finite temperature in chaotic systems:

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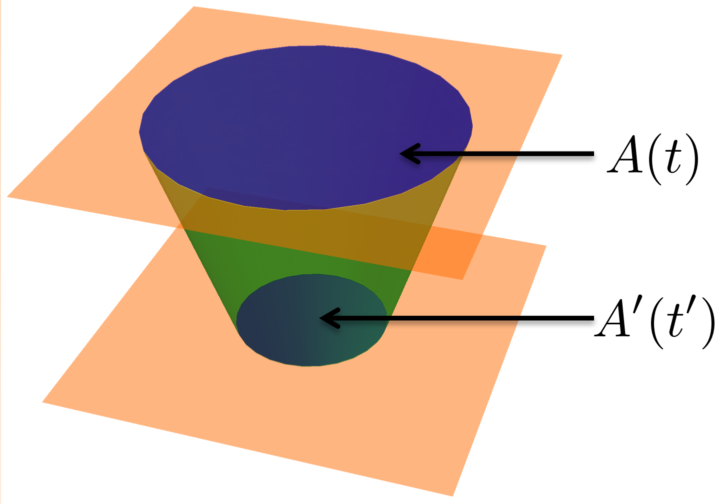
  Gives bound for all times. Can be combined with another proposed inequality. [MM, Stanford]



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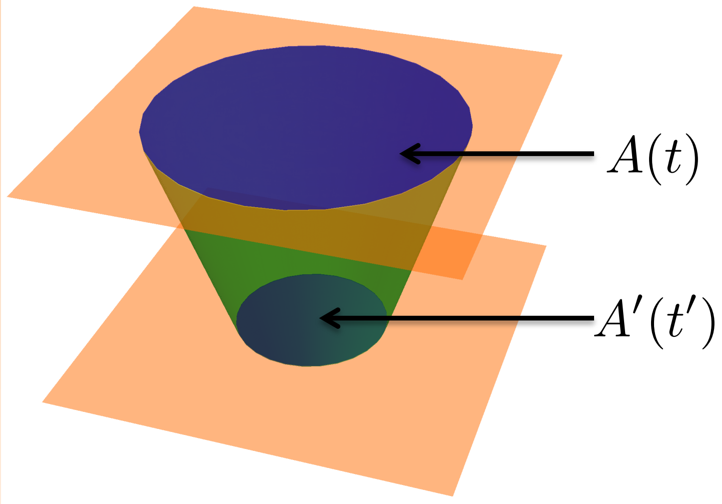

- Consequences:

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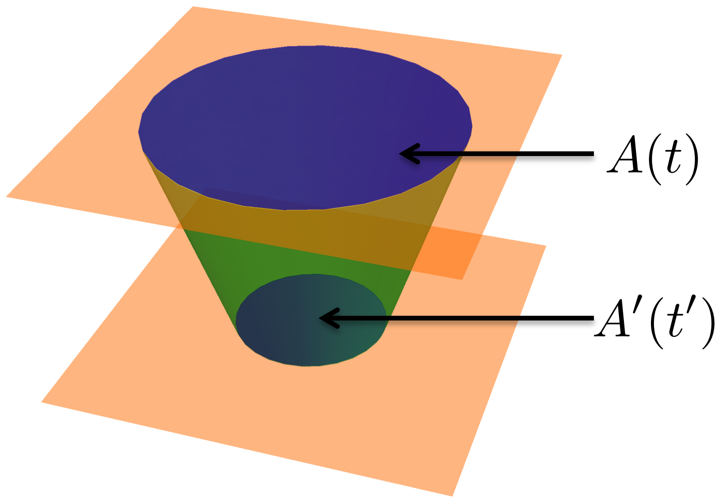

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- Membrane theory proof: there exists a maximal membrane tension compatible with the general properties discussed before.

  $$\mathcal{E}_{\max}(v) = v_E + \left(1 - \frac{v_E}{v_B}\right)|v| \qquad (|v| \leq v_B)$$

  The resulting minimal membrane is a combination of a cylinder and the cone saturating the combined inequalities. [MM_2]