Massive Gravitons in Curved Spacetimes

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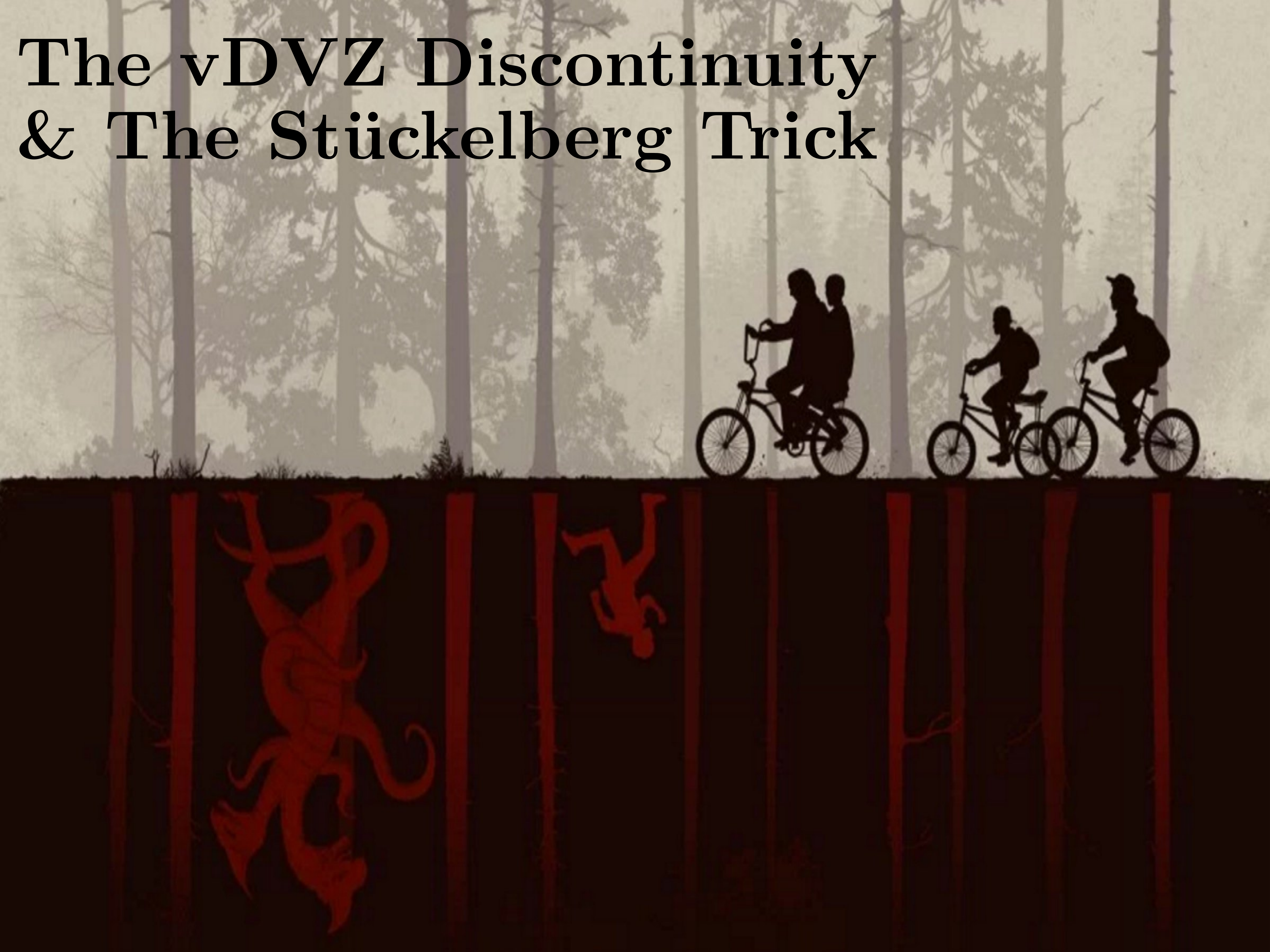
Outline

- Introduction to Massive Gravity and useful tools.
- Partially massless gravity in de Sitter
- Shift symmetries arising in anti-de Sitter
- Shift symmetries in flat space and soft subtracted recursion of their amplitudes

Massive Gravitons

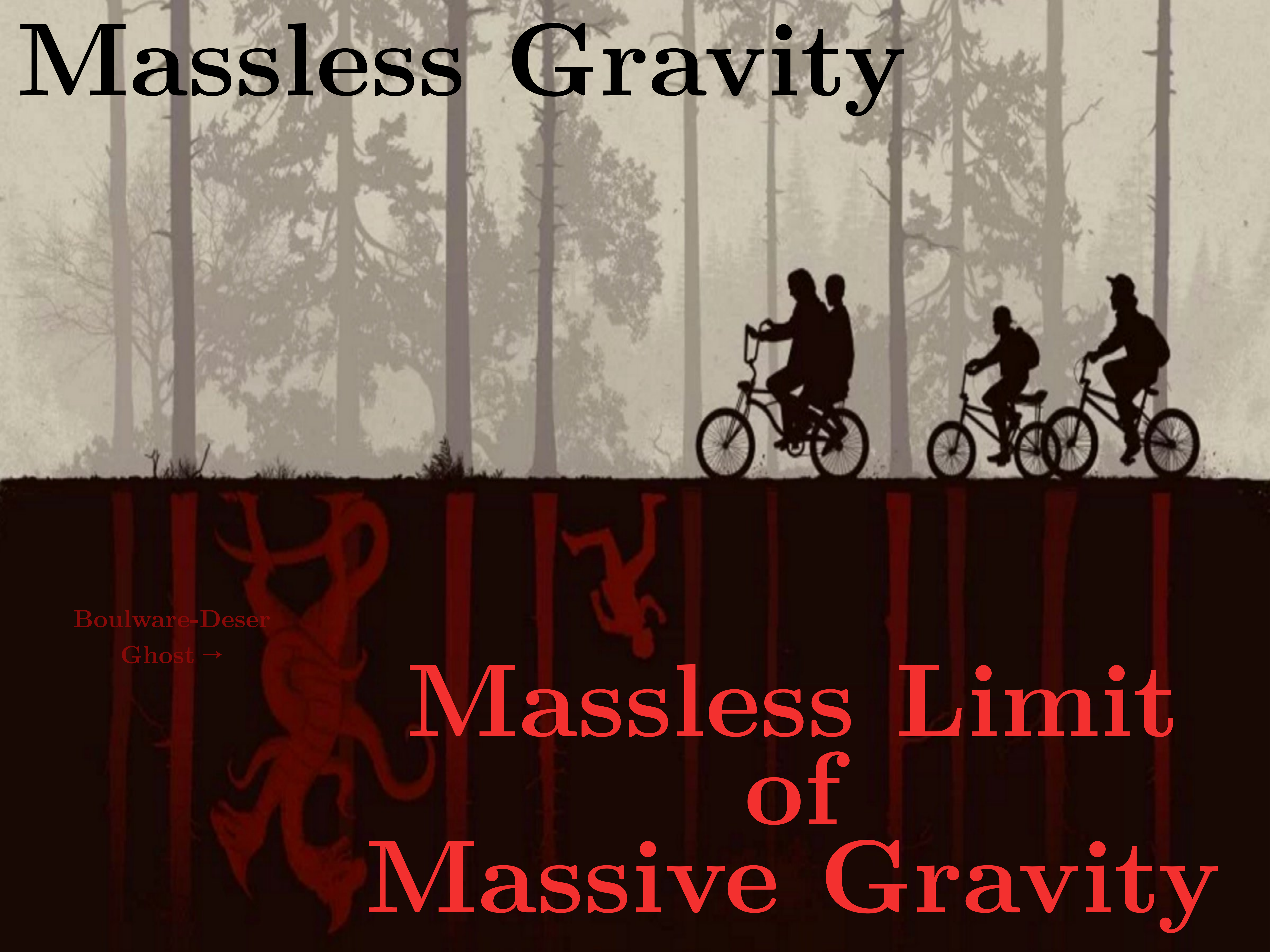
- Interesting cosmological phenomenology
- Appear in Kaluza-Klein reductions of gravity
- Has been used to describe the transport properties in materials with broken translational invariance via the AdS/CFT correspondence.
- Massive spin-2 resonances in QCD.

The vDVZ Discontinuity & The Stückelberg Trick



vDVZ Discontinuity

A discontinuity in the physical predictions of linear massless gravity and the massless limit of linear massive gravity.


Massless Gravity
Boulware-Deser Ghost →
Massless Limit of Massive Gravity

Linearized action for a massless spin 2 particle

$$S = \int d^4x - \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h$$

(2 degrees of freedom)

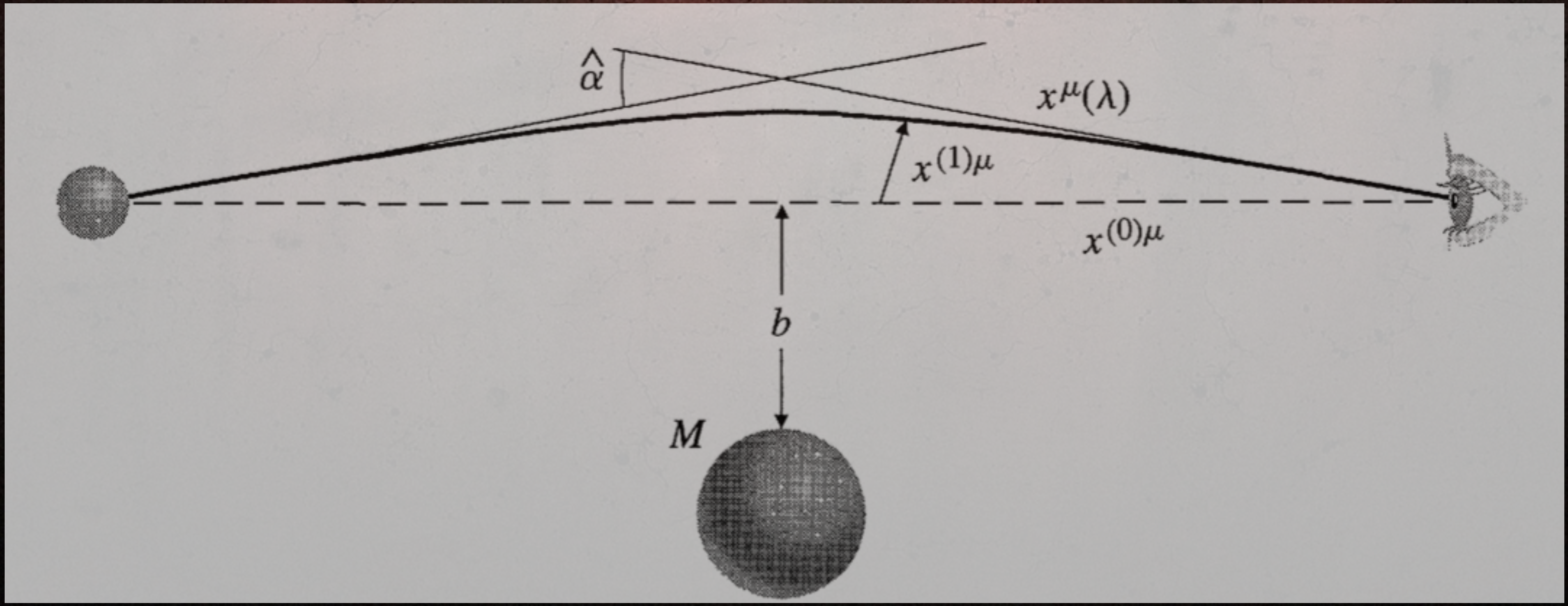
Linearized action for a massive spin 2 particle with Fierz-Pauli tuning

$$S = \int d^4x - \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h$$

$$-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

(5 degrees of freedom)

Photon Trajectories



Massless Gravity

Newtonian Potential $$\phi = -\frac{GM}{r}$$

Deflection Angle $$\alpha = \frac{4GM}{b}$$

Massless Limit of Massive Gravity

Newtonian Potential $$\phi = -\frac{4}{3}\frac{GM}{r}$$

Deflection Angle $$\alpha = \frac{4GM}{b}$$

Stückelberg Trick

- As m→0,   5 dof → 2 dof
- The mass term breaks the gauge symmetry $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$
- Add a new field to the action in a way that creates a gauge symmetry and is still dynamically equivalent to original theory

Action for a massless spin 1 particle

$$S = \int d^4x \ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\mu J^\mu$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

(2 degrees of freedom)

Invariant under

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

Action for a massive spin 1 particle

$$S = \int d^4x \ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu + A_\mu J^\mu$$

(3 degrees of freedom)

Mass term breaks gauge symmetry

$$-\frac{1}{2}m^2 A_\mu A^\mu$$

Stückelberg Trick

- Introduce new field in a way patterned after the gauge symmetry

$$A_\mu \to A_\mu + \frac{1}{m}\partial_\mu \phi$$

- The kinetic term is unaffected and mass term becomes

$$-\frac{1}{2}m^2 A_\mu A^\mu \to -\frac{1}{2}m^2 (A_\mu + \frac{1}{m}\partial_\mu \phi)^2$$

Action for a massless spin 1 particle

$$S = \int d^4x \; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\mu J^\mu$$

Invariant under

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

Action for a massive spin 1 particle

$$S = \int d^4x \; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu$$

$$-mA_\mu \partial^\mu \phi - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + A_\mu J^\mu$$

Invariant under

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

$$\phi \to \phi - m\Lambda$$

Action for a massless spin 1 particle

$$S = \int d^4x \; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\mu J^\mu$$

(2 degrees of freedom)

Invariant under

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

Massless limit of massive spin 1 particle

$$S = \int d^4x - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + A_\mu J^\mu$$

(3 degrees of freedom)

Invariant under

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

Action for a massless spin 2 particle

$$S = \int d^4x \; \mathcal{L}_{m=0} + \kappa h_{\mu\nu} T^{\mu\nu}$$

(2 degrees of freedom)

Invariant under

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Action for a massive spin 2 particle

$$S = \int d^4x \; \mathcal{L}_{m=0} - \frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) + \kappa h_{\mu\nu} T^{\mu\nu}$$

(5 degrees of freedom)

Mass term breaks gauge symmetry

$$-\frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2)$$

Stückelberg Trick for Massive Graviton

- Introduce new field in a way patterned after the gauge symmetry

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \frac{1}{m}\partial_\mu A_\nu + \frac{1}{m}\partial_\nu A_\mu$$

- The massless piece is unaffected and only the mass term changes

Action for a massless spin 2 particle

$$S = \int d^4x \; \mathcal{L}_{m=0} + \kappa h_{\mu\nu} T^{\mu\nu}$$

(2 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Action for a massive spin 2 particle

$$S = \int d^4x \; \mathcal{L}_{m=0} - \frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) + \kappa h_{\mu\nu} T^{\mu\nu}$$
$$- \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 2m \left(h_{\mu\nu} \partial^\mu A^\nu - h \partial_\mu A^\mu \right)$$

(5 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$\delta A_\mu = -m \xi_\mu$$

Action for a massless spin 2 particle

$$S = \int d^4x \; \mathcal{L}_{m=0} + \kappa h_{\mu\nu} T^{\mu\nu}$$

(2 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Massless limit

$$S = \int d^4x \; \mathcal{L}_{m=0} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \kappa h_{\mu\nu} T^{\mu\nu}$$

(4 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$\delta A_\mu = \partial_\mu \Lambda$$

Stückelberg Trick for Massive Graviton

- Introduce new field in a way patterned after the gauge symmetry

$$A_\mu \to A_\mu + \frac{1}{m^2}\partial_\mu \phi$$

Action for a massless spin 2 particle

$$S = \int d^4x \; \mathcal{L}_{m=0} + \kappa h_{\mu\nu} T^{\mu\nu}$$

(2 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Action for a massive spin 2 particle

$$S = \int d^4x \; \mathcal{L}_{m=0} - \frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) + \kappa h_{\mu\nu} T^{\mu\nu}$$

$$-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 2m \left(h_{\mu\nu} \partial^\mu A^\nu - h \partial_\mu A^\mu \right)$$

$$-2(h_{\mu\nu} \partial^\mu \partial^\nu \phi - h \partial^2 \phi)$$

(5 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$\delta A_\mu = -m \xi_\mu$$

$$\delta A_\mu = \partial_\mu \Lambda$$

$$\delta \phi = -m \Lambda$$

Action for a massless spin 2 particle

$$S = \int d^4x \, \mathcal{L}_{m=0} + \kappa h_{\mu\nu} T^{\mu\nu}$$

(2 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Massless limit for a massive spin 2 particle

$$S = \int d^4x \, \mathcal{L}_{m=0} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \kappa h_{\mu\nu} T^{\mu\nu}$$

$$-2 \left(h_{\mu\nu} \partial^\mu \partial^\nu \phi - h \partial^2 \phi \right)$$

(5 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$\delta A_\mu = \partial_\mu \Lambda$$

Un-mixing

- Scalar is mixed with tensor
- Can un-mix by using the transformation

$$h_{\mu\nu} = h'_{\mu\nu} + \phi\eta_{\mu\nu}$$

- This will cause non-minimal coupling between the scalar and the stress-energy tensor.

Action for a massless spin 2 particle

$$S = \int d^4x \; \mathcal{L}_{m=0} + \kappa h_{\mu\nu} T^{\mu\nu}$$

(2 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Action for a massive spin 2 particle

$$S = \int d^4x \; \mathcal{L}_{m=0}(h') - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \kappa h'_{\mu\nu} T^{\mu\nu}$$

$$-3 \partial_\mu \phi \partial^\mu \phi + \kappa \phi T$$

(5 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$\delta A_\mu = \partial_\mu \Lambda$$

Stückelberg Trick in Curved Space

- Expand around a curved metric, instead of a flat metric
- Massless part will be the Einstein Hilbert action with a cosmological constant, expanded to second order

Massless spin 2 particle in curved space

$$S = \int d^4x \sqrt{-g}\Big[-\frac{1}{2}\nabla_\alpha h_{\mu\nu}\nabla^\alpha h^{\mu\nu} + \nabla_\alpha h_{\mu\nu}\nabla^\nu h^{\mu\alpha} - \nabla_\mu h \nabla_\nu h^{\mu\nu} + \frac{1}{2}\nabla_\mu h \nabla^\mu h$$
$$\frac{R}{4}\left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2\right) + \kappa h_{\mu\nu}T^{\mu\nu}\Big]$$

(2 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

Action for a massive spin 2 particle

$$S = \int d^4x \mathcal{L}_{m=0} + \sqrt{-g}\Big[-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \kappa h_{\mu\nu}T^{\mu\nu}\Big]$$

(5 degrees of freedom)

Mass term breaks gauge symmetry

$$-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

Stückelberg Trick in Curved Space

- Introduce new field in a way patterned after the gauge symmetry

$$h_{\mu\nu} \to h_{\mu\nu} + \frac{1}{m}\nabla_\mu A_\nu + \frac{1}{m}\nabla_\nu A_\mu$$

- The massless terms are unaffected and the mass term changes

Massless spin 2 particle in curved space

$$S = \int d^4x \, \sqrt{-g}\Big[-\frac{1}{2}\nabla_\alpha h_{\mu\nu}\nabla^\alpha h^{\mu\nu} + \nabla_\alpha h_{\mu\nu}\nabla^\nu h^{\mu\alpha} - \nabla_\mu h \nabla_\nu h^{\mu\nu} + \frac{1}{2}\nabla_\mu h \nabla^\mu h$$

$$\frac{R}{4}\left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2\right) + \kappa h_{\mu\nu}T^{\mu\nu}\Big]$$

(2 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

Action for a massive spin 2 particle

$$S = \int d^4x \, \mathcal{L}_{m=0} + \sqrt{-g}\Big[-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \kappa h_{\mu\nu}T^{\mu\nu}$$

$$-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}RA^\mu A_\mu - 2m\left(h_{\mu\nu}\nabla^\mu A^\nu - h\nabla_\mu A^\mu\right)\Big]$$

(5 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

$$\delta A_\mu = -m\xi_\mu$$

Massless spin 2 particle in curved space

$$S = \int d^4x \, \sqrt{-g}\Big[-\frac{1}{2}\nabla_\alpha h_{\mu\nu}\nabla^\alpha h^{\mu\nu} + \nabla_\alpha h_{\mu\nu}\nabla^\nu h^{\mu\alpha} - \nabla_\mu h \nabla_\nu h^{\mu\nu} + \frac{1}{2}\nabla_\mu h \nabla^\mu h$$

$$\frac{R}{4}\left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2 \right) + \kappa h_{\mu\nu}T^{\mu\nu} \Big]$$

(2 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

Action for a massive spin 2 particle

$$S = \int d^4x \; \mathcal{L}_{m=0} + \sqrt{-g}\left[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{R}{2}A^\mu A_\mu + \kappa h_{\mu\nu}T^{\mu\nu} \right]$$

(5 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

Stückelberg Trick in Curved Space

- No vDVZ discontinuity in curved space
- Massless limit preserves degrees of freedom without having to introduce the scalar field.
- If $R>0$, the vector mass term has the wrong sign and the theory is unstable.

Stückelberg Trick in Curved Space

- Rescale $A_\mu \to mA_\mu$
- Introduce scalar patterned after gauge symmetry and make transformation to un-mix the scalar and tensor terms.

$$A_\mu \to A_\mu + \nabla_\mu \phi$$

$$h_{\mu\nu} = h'_{\mu\nu} + m^2 \phi g_{\mu\nu}$$

Massless spin 2 particle in curved space

$$S = \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2}\nabla_\alpha h_{\mu\nu}\nabla^\alpha h^{\mu\nu} + \nabla_\alpha h_{\mu\nu}\nabla^\nu h^{\mu\alpha} - \nabla_\mu h \nabla_\nu h^{\mu\nu} + \frac{1}{2}\nabla_\mu h \nabla^\mu h$$

$$\frac{R}{4}\left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2\right) + \kappa h_{\mu\nu}T^{\mu\nu}\Big]$$

(2 degrees of freedom)

Invariant under

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

Action for a massive spin 2 particle

$$S = \int d^4x\, \mathcal{L}_{m=0} + \sqrt{-g}\Big[-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \kappa h_{\mu\nu}T^{\mu\nu}$$

$$-\frac{1}{2}m^2 F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 R A^\mu A_\mu - 2m^2\left(h'_{\mu\nu}\nabla^\mu A^\nu - h'\nabla_\mu A^\mu\right)$$

$$+m^2\left(6m^2 - R\right)\left(\phi\nabla_\mu A^\mu + \frac{1}{2}h'\phi - \frac{1}{2}(\partial\phi)^2 + m^2\phi^2\right)\Big]$$

(5 degrees of freedom)

Invariant under

$$\delta h'_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu + \Lambda g_{\mu\nu}$$

$$\delta A_\mu = \nabla_\mu \Lambda - \xi_\mu$$

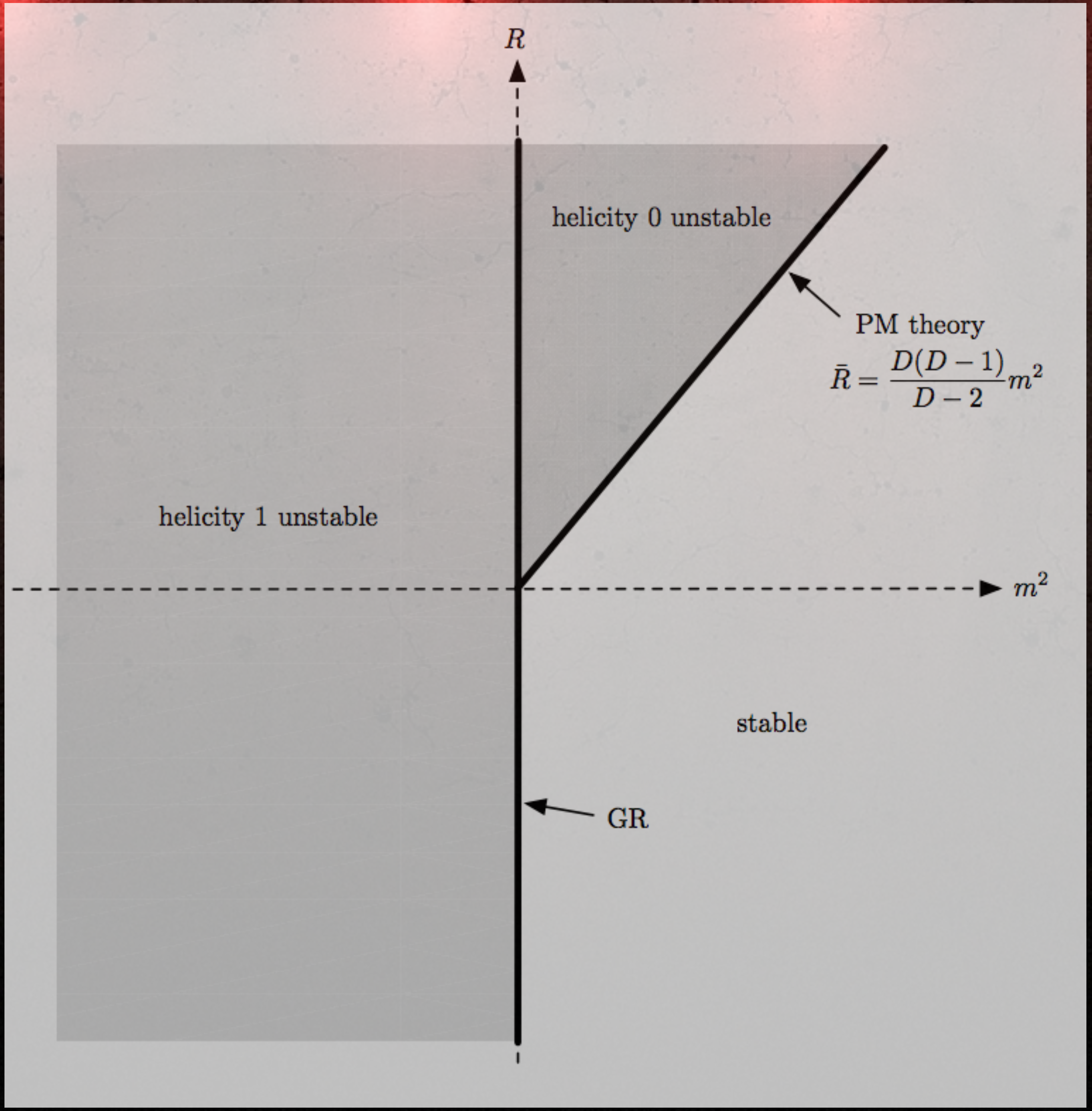
$$\delta \phi = -\Lambda$$

Partially Massless Symmetry

- When $R=6m^2$, the scalar field vanishes leaving 4 degrees of freedom.
- There is an additional gauge symmetry

$$\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \lambda + \frac{1}{2} m^2 \lambda g_{\mu\nu}$$

Partially Massless Symmetry



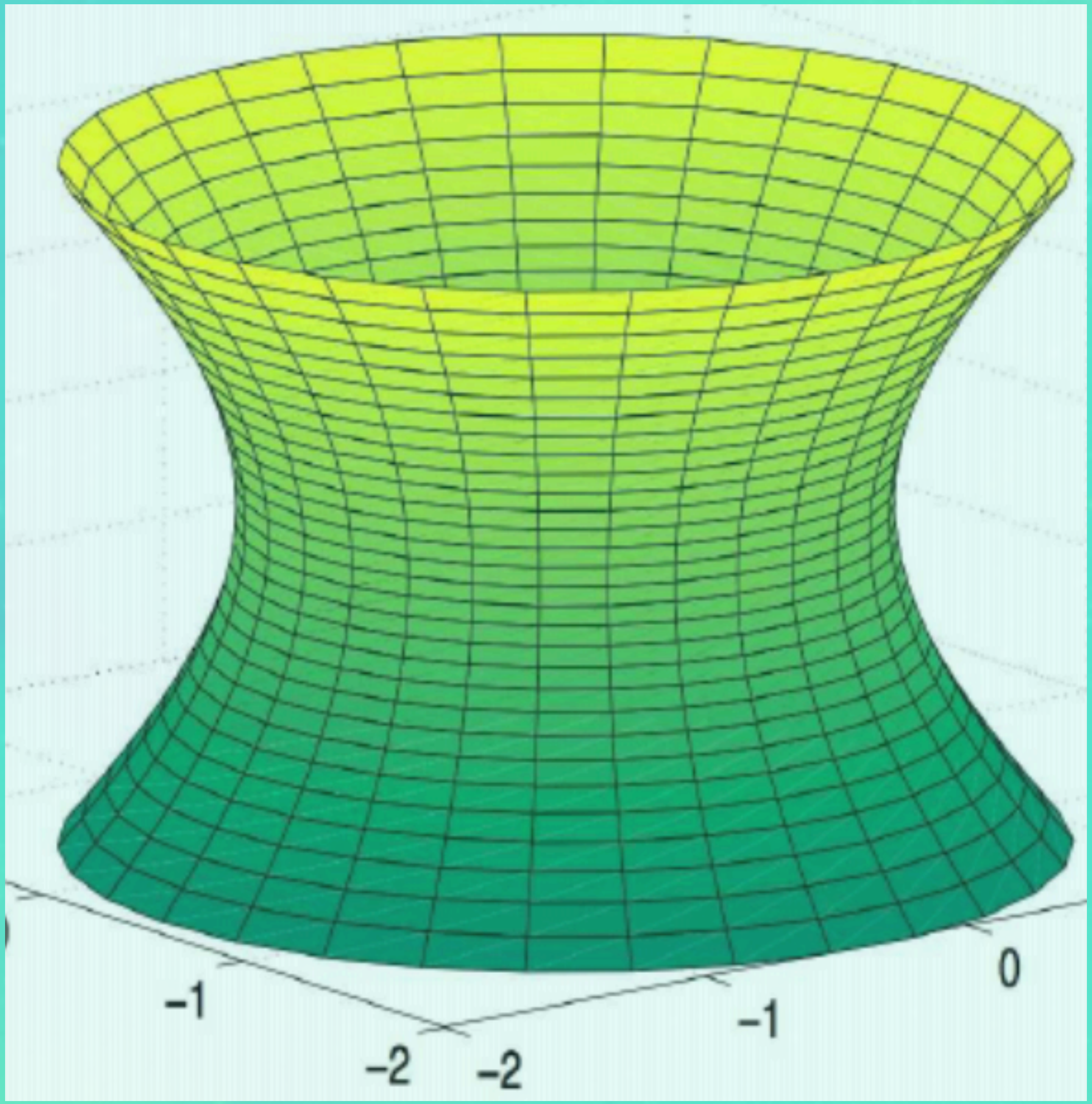
Summary

- The vDVZ discontinuity in Minkowski space is caused by non-minimal coupling of the scalar field to the stress energy tensor,
- In curved space, there is no vDVZ discontinuity.
- There is an additional symmetry when $R=6m^2$ leaving only 4 DOF.

Partially Massless Gravity in de Sitter

C. De Rham, K. Hinterbichler, and L. A. Johnson,
"On the (A)dS Decoupling Limits of Massive Gravity,"
JHEP 09 (2018) 154, arXiv:1807.08754 [hep-th].

Motivation



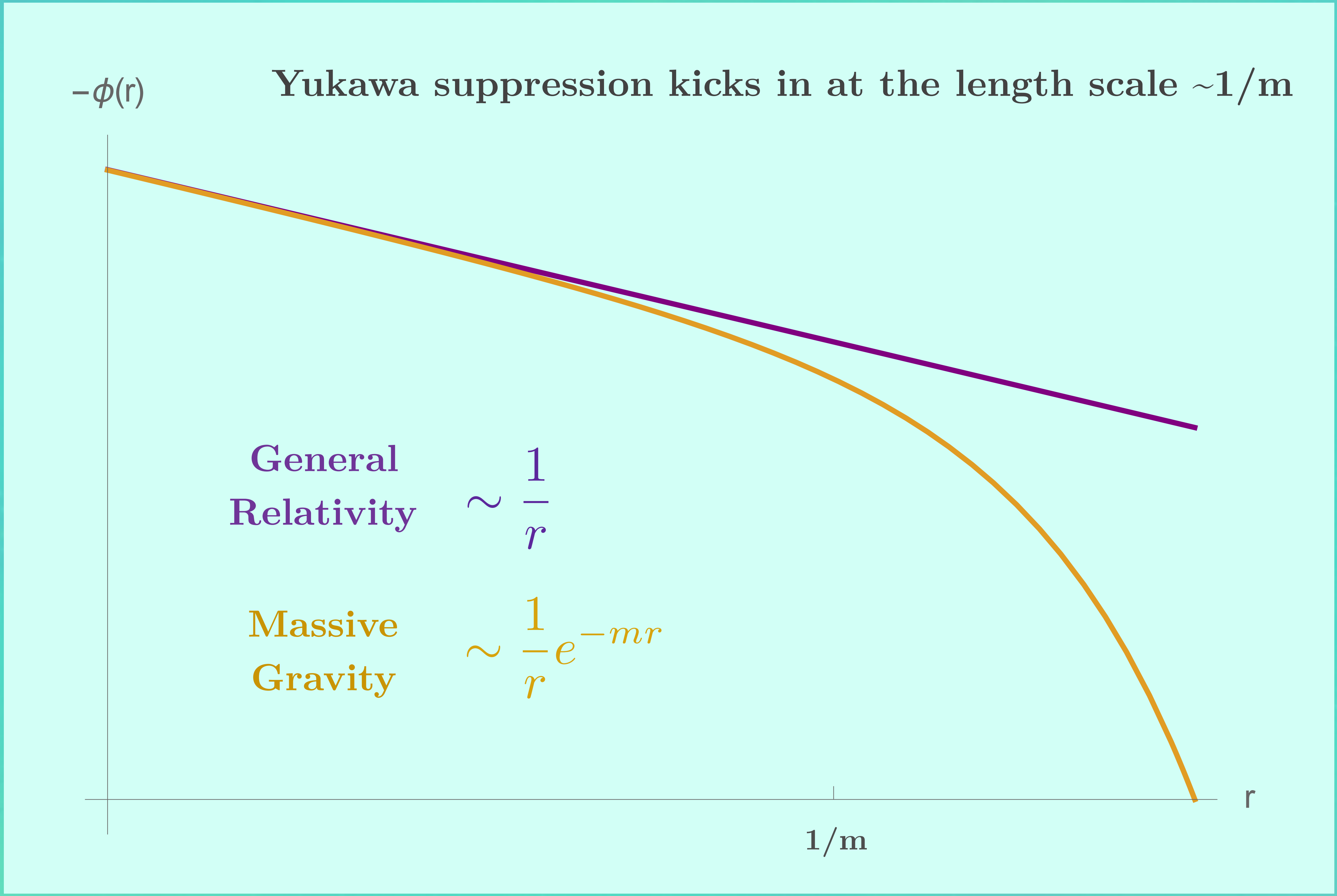
- De Sitter spacetime approximates our early universe during inflation and the phase our universe is currently entering into

- observed cosmological constant is $\dfrac{\Lambda}{M_P^2} \sim 10^{-122}$

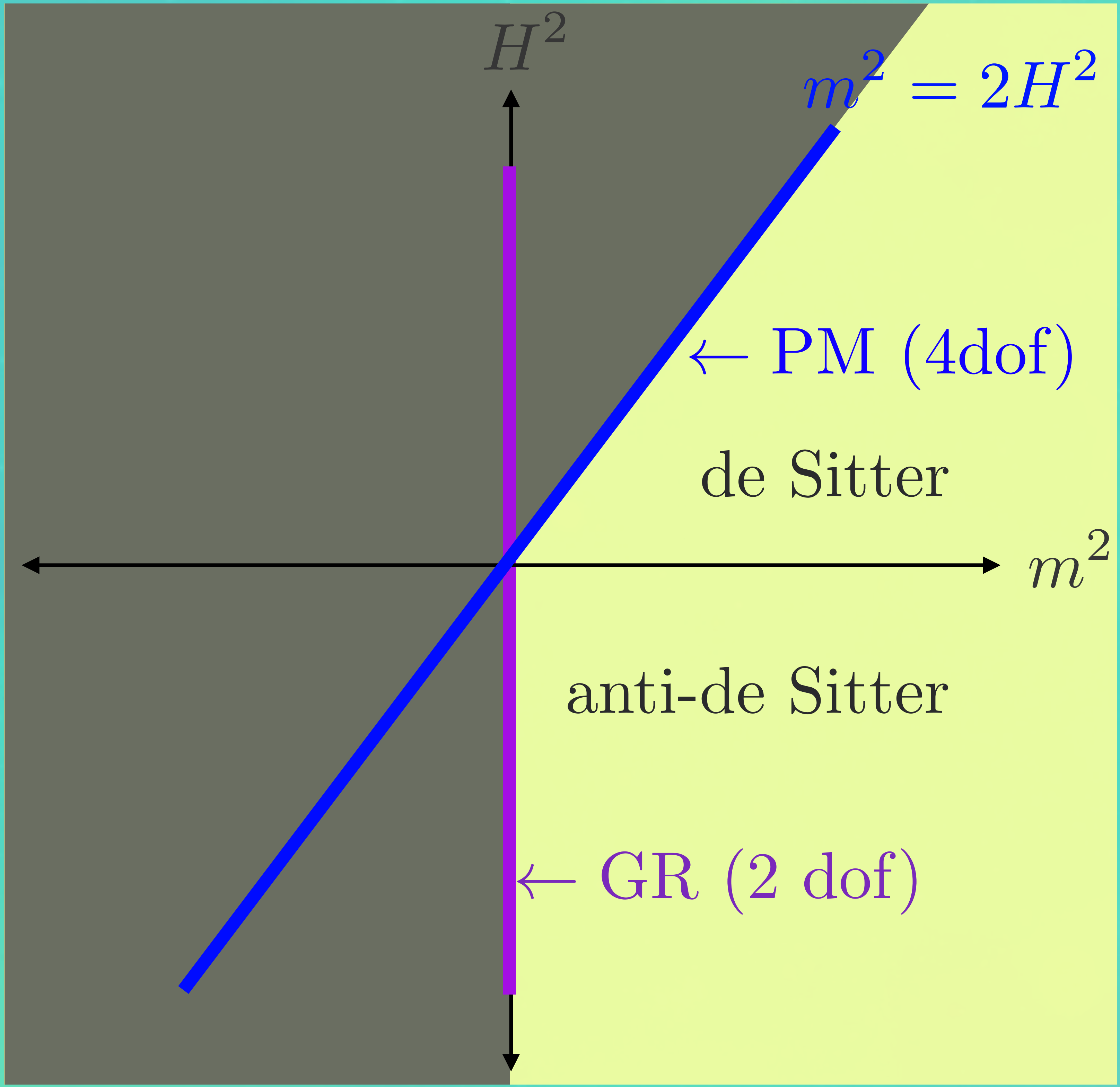
- Partially massless symmetry ties value of cosmological constant to graviton mass

$$m^2 = 2H^2$$

Potential in the static, weak-field limit



Curvature vs. Mass in Linear Massive Gravity



General Relativity 2 dof

Massive Gravity 5 dof

Partially Massless Gravity 4 dof

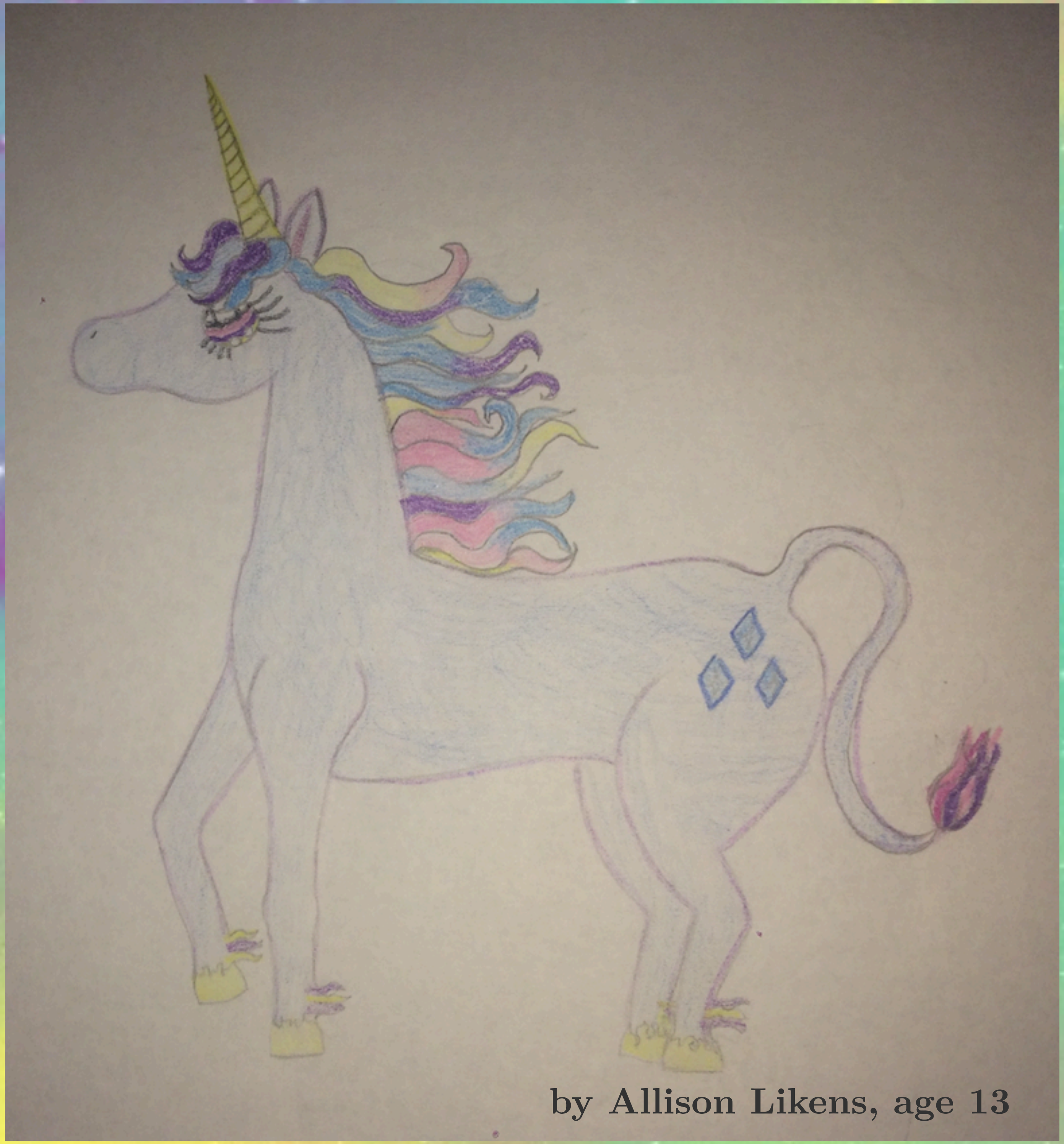
Gauge Symmetry

- When $m^2 = 2H^2$ the scalar field vanishes leaving 4 degrees of freedom.

- There is an additional gauge symmetry.

$$\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \chi + H^2 \chi \, g_{\mu\nu}$$

Non-linear Partially Massless Gravity (4 Degrees of Freedom)



- helicity-0 mode is absent giving only 4 degrees of freedom
- removes issues related to superluminalities associated with Galileon-like interactions
- no vDVZ discontinuity and no need for a Vainshtein mechanism.

Massive Gravity Non-Linear Interactions

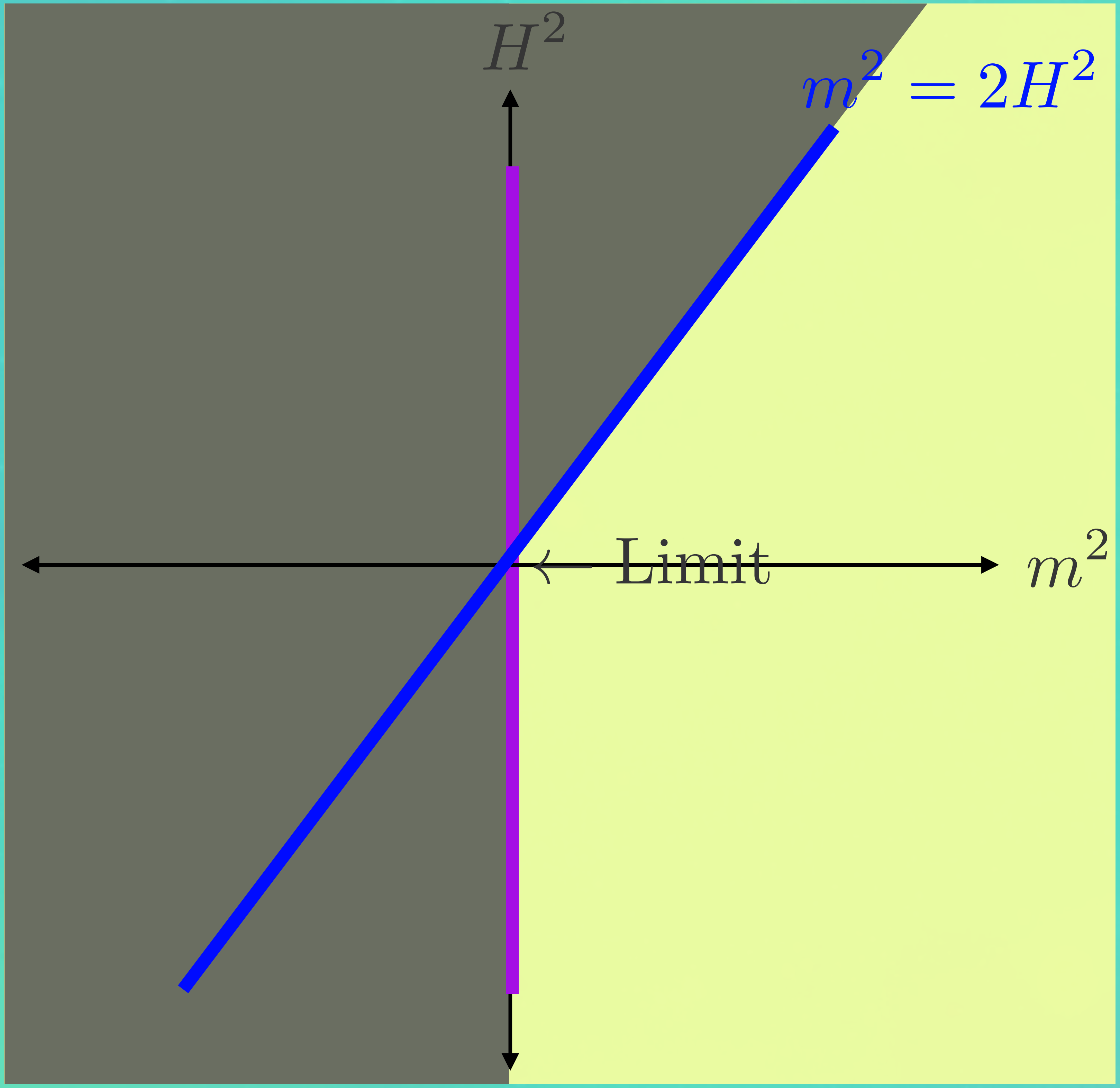
$$S = \frac{1}{2} M_P^2 \int d^4x \sqrt{-g} \left[(R - 2\Lambda) - \frac{1}{4} m^2 V(g,h) \right]$$

$$V(g,h) = V_2(g,h) + V_3(g,h) + V_4(g,h) + V_5(g,h) + \cdots ,$$

$$\begin{aligned}
V_2(g,h) &= \langle h^2 \rangle - \langle h \rangle^2, \\
V_3(g,h) &= +c_1 \langle h^3 \rangle + c_2 \langle h^2 \rangle \langle h \rangle + c_3 \langle h \rangle^3, \\
V_4(g,h) &= +d_1 \langle h^4 \rangle + d_2 \langle h^3 \rangle \langle h \rangle + d_3 \langle h^2 \rangle^2 + d_4 \langle h^2 \rangle \langle h \rangle^2 + d_5 \langle h \rangle^4, \\
V_5(g,h) &= +f_1 \langle h^5 \rangle + f_2 \langle h^4 \rangle \langle h \rangle + f_3 \langle h^3 \rangle \langle h \rangle^2 + f_4 \langle h^3 \rangle \langle h^2 \rangle + f_5 \langle h^2 \rangle^2 \langle h \rangle \\
&\quad + f_6 \langle h^2 \rangle \langle h \rangle^3 + f_7 \langle h \rangle^5, \\
&\vdots
\end{aligned}$$

S. Hassan, R. A. Rosen, and A. Schmidt-May, "Ghost-free Massive Gravity with a General Reference Metric," arXiv:1109.3230 [hep-th].

Massless Limit in Non-Linear Theory



- Take limit as $m \to 0$

Λ_5 Theory

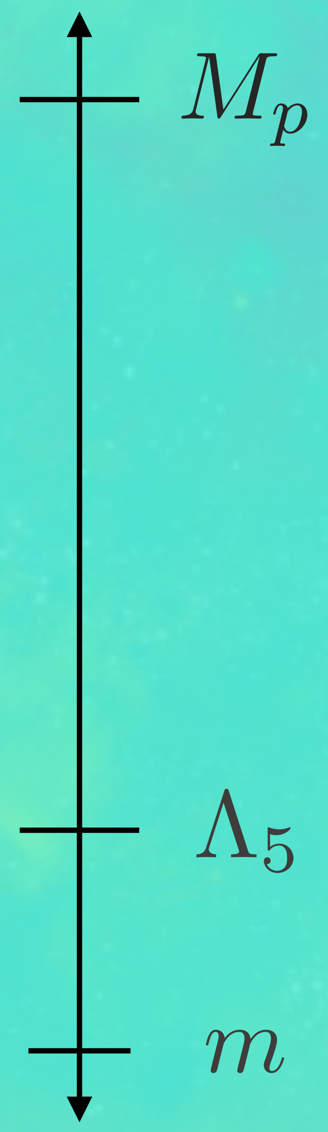
- Generic massive gravity has a cutoff scale of Λ_5

$$\sim \frac{(\partial^2\phi)^3}{\Lambda_5^5}, \quad \Lambda_5 = (M_p m^4)^{1/5}$$

- Has an extra scalar degree of freedom with a wrong sign kinetic term, known as the Boulware-Deser ghost, giving the theory 6 degrees of freedom.

- Best bounds on the graviton mass are given by the lunar laser ranging experiment. $m_g < 10^{-30} eV$

- For reference, if we pick $m_g \sim 10^{-32} eV$, the cutoff scale is $\frac{1}{\Lambda_5} \sim 10^{10} km$



Tuning Coefficients to Raise Cutoff (dRGT)

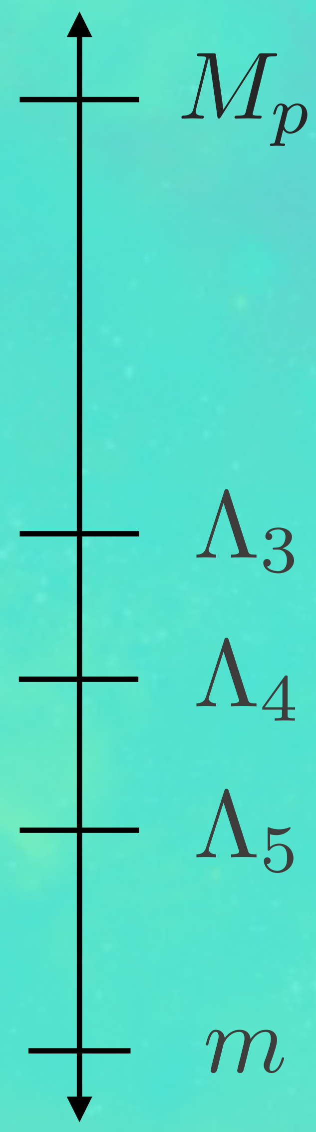
- Can tune the parameters to remove interactions coming in at:

$$\sim \frac{(\partial^2\phi)^3}{\Lambda_5^5}, \quad \Lambda_5 = (M_p m^4)^{1/5}$$

$$\sim \frac{h(\partial^2\phi)^4}{\Lambda_4^8}, \frac{\partial A(\partial^2\phi)^2}{\Lambda_4^4}, \quad \Lambda_4 = (M_p m^3)^{1/4}$$

- This raises the cutoff scale to Λ_3

$$\sim \frac{h(\partial^2\phi)^n}{\Lambda_3^{3(n-1)}}, \frac{\partial A(\partial^2\phi)^n}{\Lambda_3^n}, \quad \Lambda_3 = (M_p m^2)^{1/3}$$



C. de Rham, G. Gabadadze, and A. J. Tolley, "Resummation of Massive Gravity," arXiv:1011.1232 [hep-th].

S. F. Hassan and R. A. Rosen, "Resolving the Ghost Problem in non-Linear Massive Gravity," arXiv:1106.3344 [hep-th].

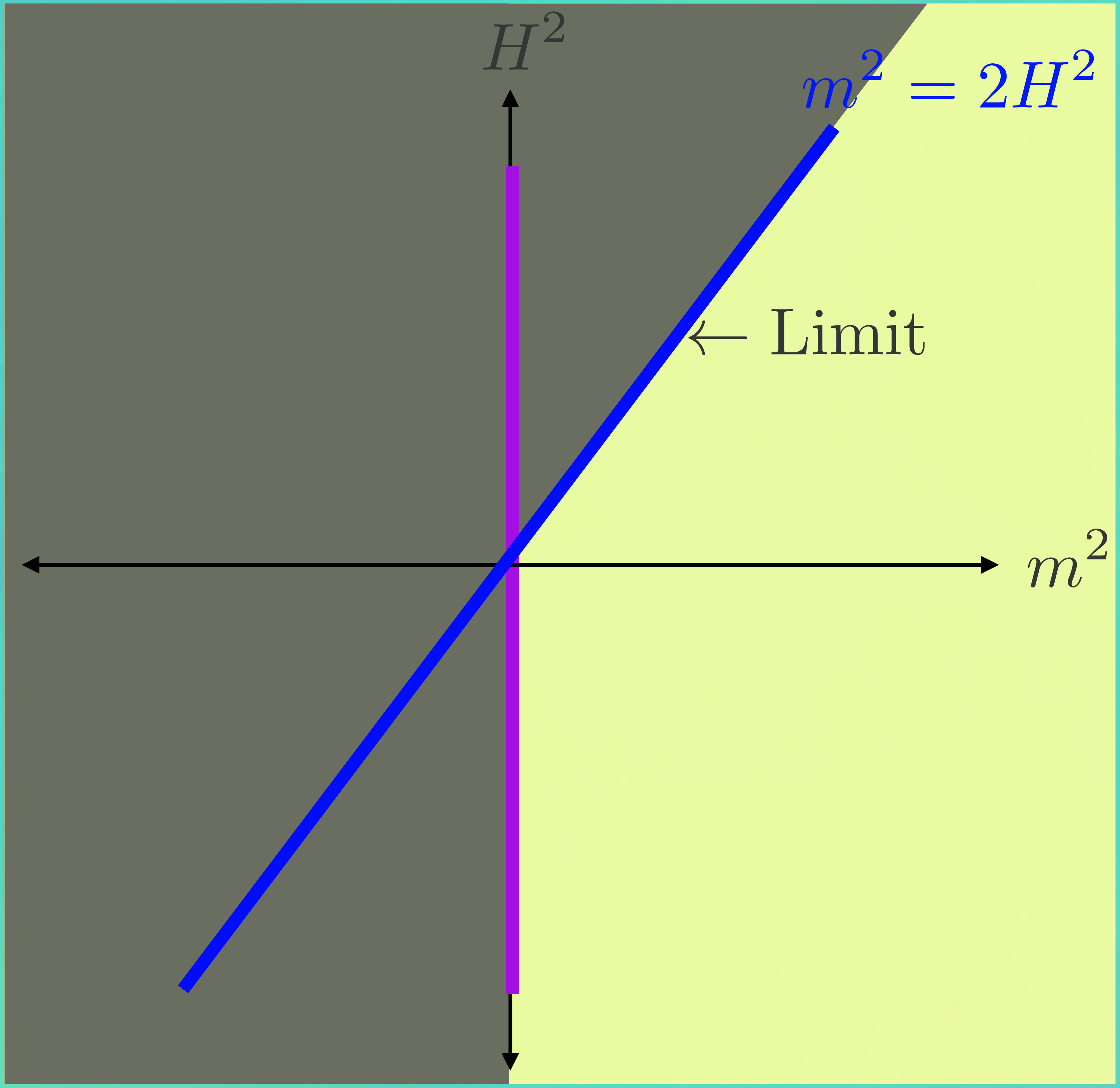
Λ_3 Theory

- The ghost is removed, leaving only 5 dof.
- For a graviton mass, $m_g \sim 10^{-32} eV$, the cutoff scale is $\dfrac{1}{\Lambda_3} \sim 10^3 km$
- At cubic order, a field redefinition can be performed to decouple the scalar and tensor and galileons emerge.

$$\mathcal{L}_2 = -\frac{1}{2}(\partial\phi)^2$$
$$\mathcal{L}_3 = -\frac{1}{2}(\partial\phi)^2 \partial_\mu \partial^\mu \phi$$
$$\mathcal{L}_4 = -\frac{1}{2}(\partial\phi)^2 \left((\partial_\mu \partial^\mu \phi)^2 - \partial_\mu \partial_\nu \phi \partial^\mu \partial^\nu \phi \right)$$

- They have a shift symmetry: $\delta\phi(x) = c + b_\mu x^\mu$
- In spite of higher derivatives terms appearing in the Lagrangian, their EOM are purely second order.

Partially Massless Limit in Non-Linear Theory



- Set $m^2 = 2H^2 + \Delta^2$
- Take limit as $\Delta \to 0$

Non-linear Interactions

- Ghost free massive gravity (dRGT) has two free parameters, $\alpha_3, \ \alpha_4$

- and a cutoff of Λ_4

$$\sim \frac{\partial^5 \hat{\phi}^3}{\Lambda_4^4}, \quad \Lambda_4 = (M_P \Delta^3)^{1/4}$$

- Using the same mass as before,

$$\frac{1}{\Lambda_4} \sim 10^7 km$$

Tuning Coefficients to Raise Cutoff

- Tuning the parameters to their PM values removes interactions $\alpha_3 = -\frac{1}{2}, \quad \alpha_4 = \frac{1}{8}$

$$\sim \frac{\partial^5 \hat{\phi}^3}{\Lambda_4^4}, \quad \Lambda_4 = (M_P \Delta^3)^{1/4}$$

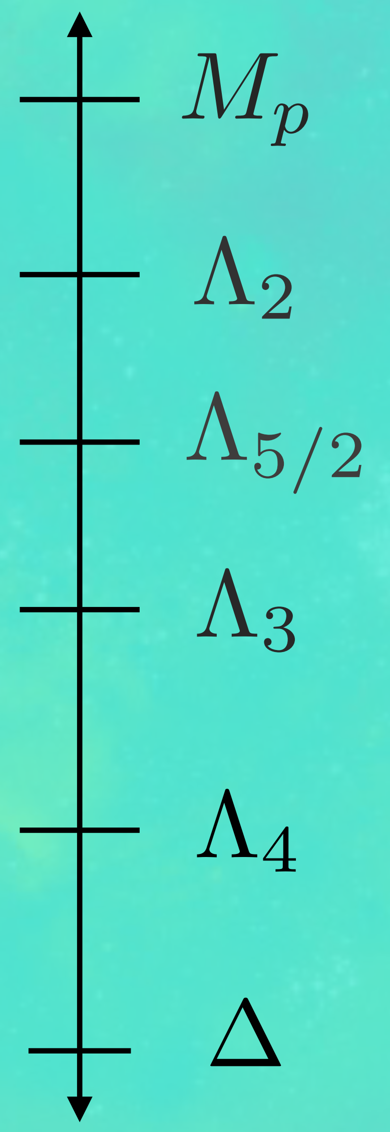
$$\sim \frac{\partial^4 \hat{h} \hat{\phi}^2}{\Lambda_3^3}, \frac{\partial^6 \hat{\phi}^4}{\Lambda_3^6}, \quad \Lambda_3 = (M_P \Delta^2)^{1/3}$$

$$\sim \frac{\partial^5 \hat{h} \hat{\phi}^3}{\Lambda_{5/2}^5}, \quad \Lambda_{5/2} = (M_P^2 \Delta^3)^{1/5}$$

- This raises the cutoff scale to Λ_2

$$\sim \frac{(\nabla^2 \phi)^n}{\Lambda_2^{n-2}}, \frac{h^2 (\nabla^2 \phi)^{n-2}}{\Lambda_2^{n-2}}, \quad \Lambda_2 = (M_p \Delta)^{1/2}$$

- Using the same mass as before, $\frac{1}{\Lambda_2} \sim 10 \mu m$



Partially Massless Limit of Massive Gravity

$$\mathcal{L}_{dSGal} = -\frac{3}{16}\Big((\partial\phi)^2 - 4H^2\phi^2)\Big) - \frac{3}{64}\frac{1}{\Lambda_2}\Big((\partial\phi)^2\Box\phi + 6H^2\phi(\partial\phi)^2 - 8H^4\phi^3\Big)$$
$$+ \frac{1}{256}\frac{1}{\Lambda_2^2}\Big[(\partial\phi)^2\Big([\Pi^2] - [\Pi]^2\Big) - 6H^2\phi(\partial\phi)^2\Box\phi - \frac{1}{2}H^2(\partial\phi)^4$$
$$- 18H^4\phi^2(\partial\phi)^2 + 12H^6\phi^4\Big]$$

invariant under the shift symmetry $\delta_B\phi(x) = B_A Z^A(x)$

$$\mathcal{L}_{h^2} = -\frac{1}{4}|detV|\Big[\frac{1}{2}F_{\mu\alpha a}F_{\nu\beta b}(V^{-2})^{\mu\nu}(V^{-2})^{\alpha\beta}\gamma^{\alpha\beta}$$
$$- (2F_{\mu ab}F_{\nu\alpha\beta} - F_{\mu\alpha a}F_{\nu b\beta})(V^{-2})^{\mu\nu}(V^{-1})^{\alpha\beta}(V^{-1})^{ab}\Big]$$

where
$$F_{\mu\nu\lambda} = \nabla_\mu h_{\nu\lambda} - \nabla_\nu h_{\mu\lambda}$$
$$V_{\mu\nu} = \gamma_{\mu\nu} + \frac{1}{\Lambda_2}\left(\nabla_\mu\nabla_\nu\phi + H^2\phi\gamma_{\mu\nu}\right)$$

invariant under the PM symmetry $\delta h_{\mu\nu} = \nabla_\mu\nabla_\nu\chi + H^2\chi\, g_{\mu\nu}$

Summary for de Sitter

- scalar mode in the full non-linear theory does not completely decouple (still have 5 dof)

- strong coupling scale is raised, increasing the range of applicability of the theory

- remaining Lagrangian has a cutoff of Λ_2 and enjoys the partially massless symmetry

$$\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \chi + H^2 \chi \, g_{\mu\nu}$$

Works in Progress

- Look for spherical solutions (black holes)
- See how Vainshtein radius is affected (radius inside which general relativity is restored)
- See what sort of implications this theory would have for cosmology

Massive Gravity in AdS & Shift Symmetries

James Bonifacio, Kurt Hinterbichler, Laura A. Johnson, and Austin Joyce. Shift-Symmetric Spin-1 Theories. JHEP, 09:029, 2019.

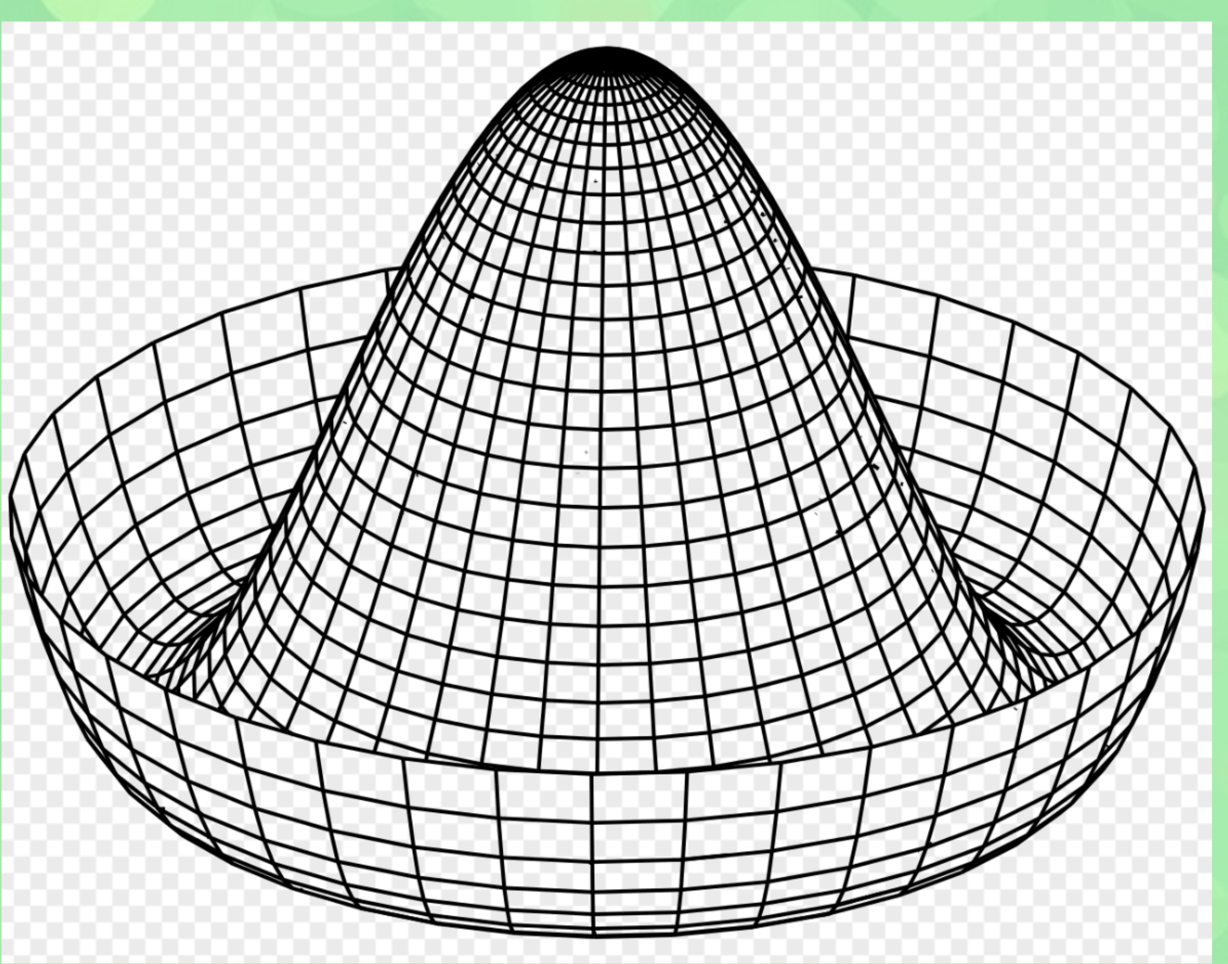
Galileons

- Have higher derivative Lagrangians, however their EOMs are still second order.
- Have a shift symmetry $\delta\phi = c + b_\mu x^\mu$
- Appealing for model building in cosmology.
- Have non-renormalization theorems

Motivation

- Shift symmetries provide a useful classification of low energy EFTs.
- Shift symmetries have played an important role in the development of theories, for example in chiral perturbation theory in explaining pion physics.
- Shift symmetries imply amplitudes vanish as the momentum of an external Goldstone line vanishes.
- They lead to enhanced soft limits and in exceptional cases, can be used to bootstrap the theories.
- In (A)dS, analogous shift symmetries occur. Goldstone modes have fixed curvature-dependent masses.

Symmetry Breaking



- unbroken symmetry (preserves vacuum $\phi = 0$)

$$\delta\phi = \mathcal{O}(\phi) + \mathcal{O}(\phi^2) + ...$$

- broken symmetry (doesn't preserve vacuum $\phi = 0$)

$$\delta\phi = c + \mathcal{O}(\phi) + \mathcal{O}(\phi^2) + ...$$

Simple Example from Massive Vector in AdS

- Starting with massive vector

$$S = \int d^4x\sqrt{-g}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2A^2\Big]$$

- Restore gauge invariance using Stückelberg trick $A_\mu \to A_\mu + \frac{1}{m}\nabla_\mu\phi$

$$S = \int d^4x\sqrt{-g}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2(A_\mu + \frac{1}{m}\nabla_\mu\phi)^2\Big]$$

$$\delta A_\mu = \nabla_\mu\Lambda, \quad \delta\phi = -m\Lambda$$

Massless Limit of Massive Vector

- Taking the massless limit

$$S = \int d^4x \sqrt{-g} \Big[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi \Big]$$

- The gauge symmetry becomes

$$\delta A_\mu = \nabla_\mu \Lambda, \quad \delta\phi = -m\Lambda \quad \rightarrow \quad \delta A_\mu = \nabla_\mu \Lambda, \quad \phi = 0$$

Emergence of Shift Symmetry

- If Λ is such that $\nabla_\mu \Lambda = 0$, then a shift symmetry survives the massless limit.

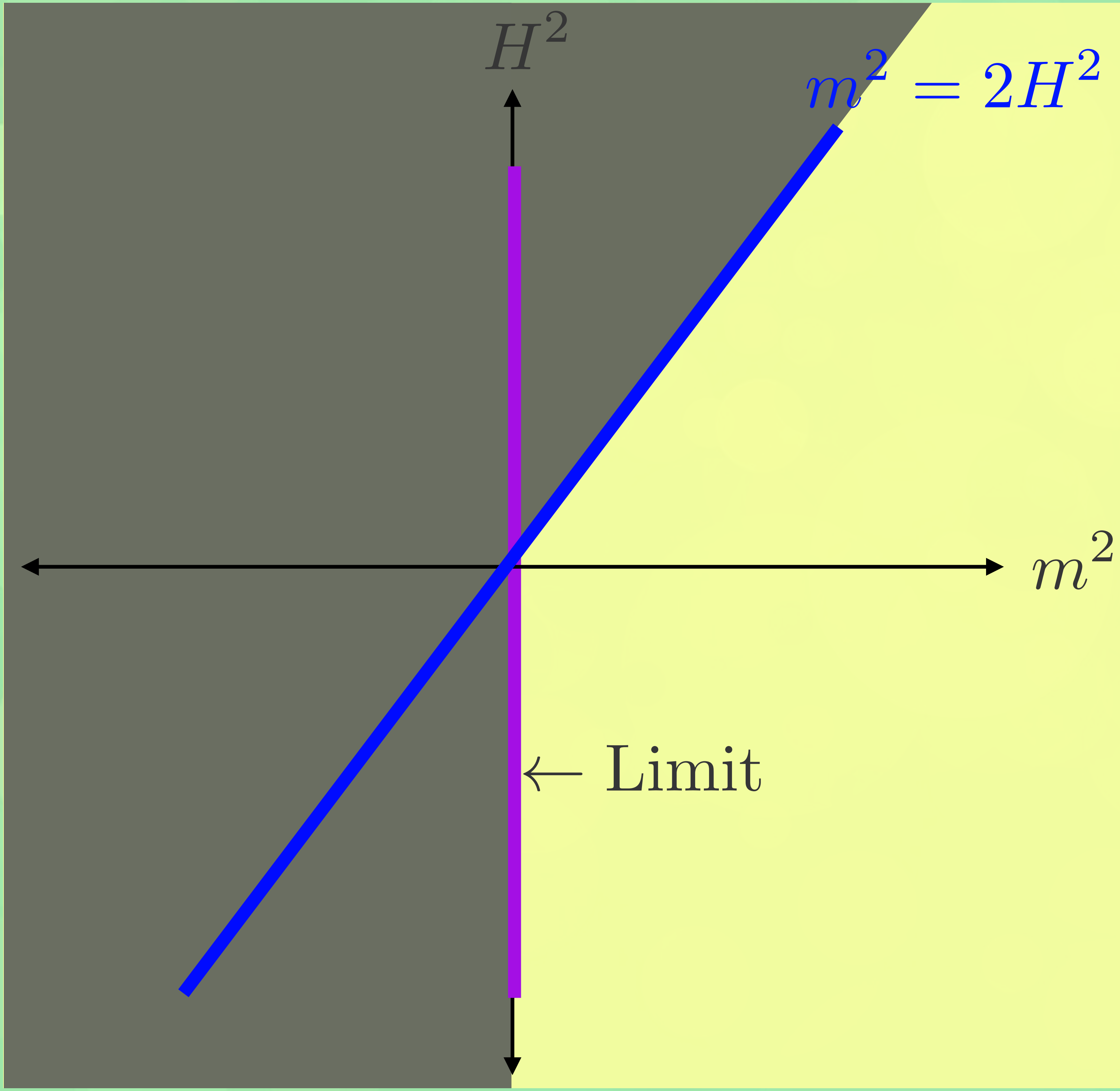
$\hat{\Lambda} = m\Lambda$

$$\delta A_\mu = \nabla_\mu \Lambda, \quad \delta\phi = -m\Lambda \quad \to \quad \delta A_\mu = 0, \quad \delta\phi = \hat{\Lambda}$$

- The scalar field gets a shift symmetry and has a symmetry breaking pattern

$$\mathfrak{so}(2,3) \oplus \mathfrak{u}(1) \to \mathfrak{so}(2,3)$$

Massless Limit of Massive Gravity in AdS



- Keep AdS radius fixed
- Take limit as

$$m \to 0$$

Linearized Massive Gravity in AdS

- Starting with linearized massive graviton

$$S = \int d^4x \mathcal{L}_{m=0} + \sqrt{-g}\left[-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\right]$$

- Restore gauge invariance using Stückelberg trick $\quad h_{\mu\nu} \to h_{\mu\nu} + \frac{1}{m}(\nabla_\mu A_\nu + \nabla_\nu A_\mu)$

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, \quad \delta A_\mu = -m\xi_\mu$$

Massless Limit of Linearized Massive Gravity in AdS

- The tensor modes decouple from a massive vector with mass $m_A^2 = \frac{6}{L^2}$

$$S = \int d^4x \mathcal{L}_{m=0} + \sqrt{-g}\left[-\frac{1}{2} F_{\mu\nu}^2 - \frac{6}{L^2} A^2 \right]$$

- The gauge symmetry becomes

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, \quad \delta A_\mu = 0$$

Emergence of Shift Symmetry

- If ξ_μ is a Killing vector $\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$, then a shift symmetry survives the massless limit.

$\hat{\xi}_\mu = m\xi_\mu$

$$\delta h_{\mu\nu} = 0, \quad \delta A_\mu = -\hat{\xi}_\mu$$

- The massive vector with mass $m_A^2 = \dfrac{6}{L^2}$ gets a shift symmetry and has a symmetry breaking pattern.

Non-Linear Massive Gravity in AdS

- Starting with massive graviton

$$S = \frac{1}{2}M_P^2 \int d^4x\sqrt{-g}\left[R(g) + \frac{6}{L^2} + m^2 V(g,\gamma)\right]$$

- Restore gauge invariance using Stückelberg trick

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu + \frac{2}{M_P}\mathcal{L}_\xi h_{\mu\nu}$$

$$\delta A_\mu = -m\xi_\mu + \frac{2}{M_P}\xi^\rho \nabla_\rho A_\mu + m\xi_\mu \left(1 - \sqrt{1 + \frac{4A^2}{(mM_P L)^2}}\right)$$

Massless Limit of Non-Linear Massive Gravity in AdS

- The theory decouples into a massless graviton and a non-linear massive vector

$$\sim \frac{(\partial A)^n}{\Lambda_2^{2n-4}}, \quad \Lambda_2 = (M_P m)^{1/2}$$

- The gauge symmetry becomes

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, \quad \delta A_\mu = 0$$

Emergence of Shift Symmetry

- If ξ_μ is a Killing vector $\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$, then a shift symmetry survives the massless limit.

$$\delta A_\mu = -\frac{2}{\Lambda_2^2}\nabla_\mu \xi^\nu A_\nu - \xi_\mu \sqrt{1 + \frac{4A^2}{(\Lambda_2^2 L)^2}}$$

- The non-linear Proca theory has a symmetry breaking pattern

$$\mathfrak{so}(5) \oplus \mathfrak{so}(5) \to \mathfrak{so}(5)_{\text{diag}}$$

Non-linear Proca Theory

- The shift symmetry fixes the non-linear structure and ensures that it is ghost free, propagating only 3 degrees of freedom, rather than 4.

$$\mathcal{L}_{\Lambda_2} = \mathcal{L}^{(2)}_{\Lambda_2}(A) + \frac{1}{\Lambda_2^2}\mathcal{L}^{(3)}_{\Lambda_2}(A) + \frac{1}{\Lambda_2^4}\mathcal{L}^{(4)}_{\Lambda_2}(A) + \cdots$$

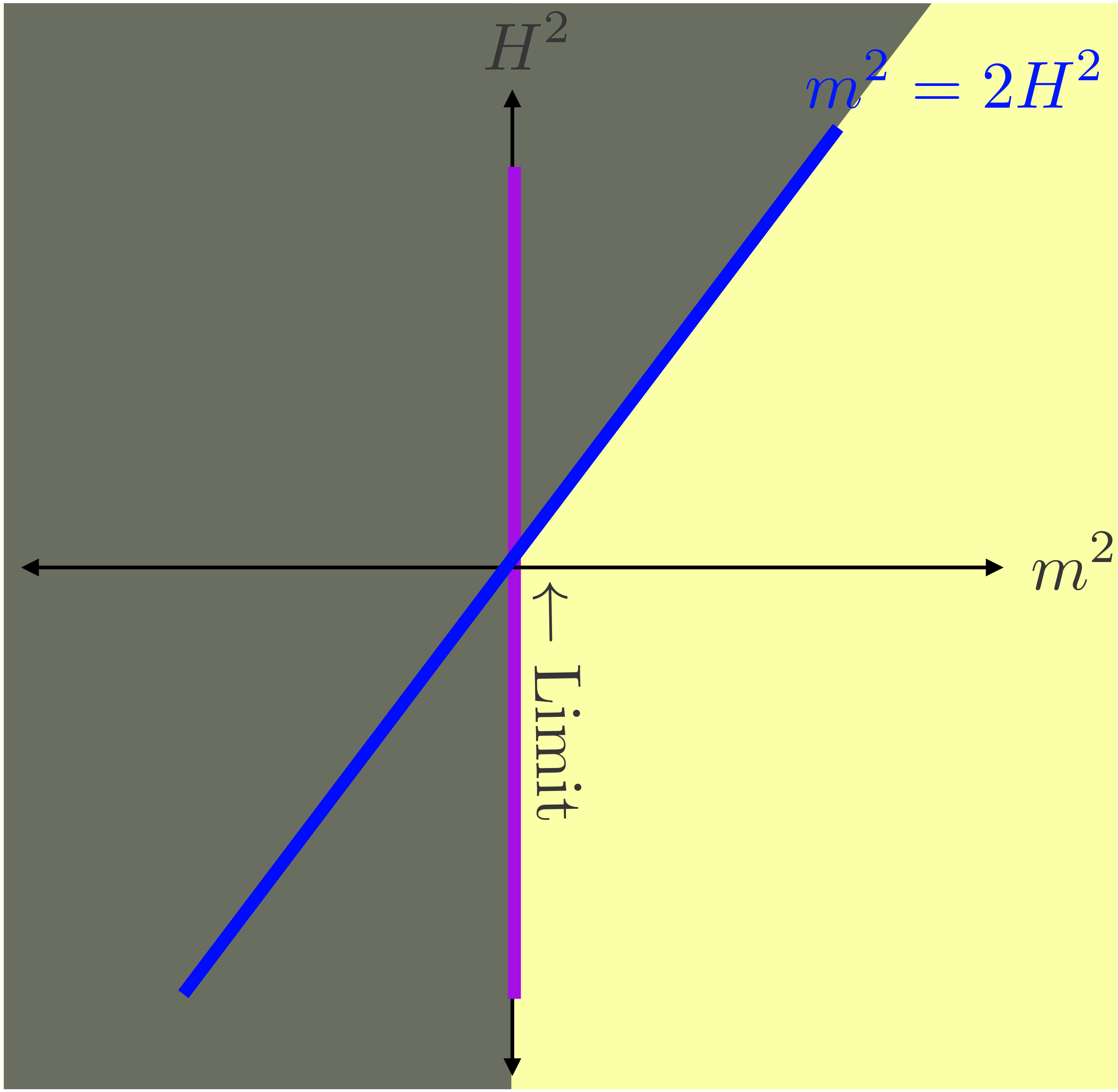
$$\frac{1}{\sqrt{-\gamma}}\mathcal{L}^{(2)}_{\Lambda_2}(A) = -\frac{1}{2}F^2_{\mu\nu} - \frac{6}{L^2}A^2$$

$$\frac{1}{\sqrt{-\gamma}}\mathcal{L}^{(3)}_{\Lambda_2}(A) = \frac{\alpha_3}{2}S_3(B) - \frac{1}{2}F^{\mu\alpha}F^{\nu}{}_{\alpha}X^{(1)}_{\mu\nu}(B) - \frac{3}{L^2}A^2 B$$

$$\begin{aligned}\frac{1}{\sqrt{-\gamma}}\mathcal{L}^{(4)}_{\Lambda_2}(A) = {} & \frac{1}{8}((F_{\mu\nu}F^{\mu\nu})^2 - F_{\mu\nu}F^{\nu\alpha}F_{\alpha\beta}F^{\beta\mu}) + \frac{\alpha_4}{2}S_4(B) - \frac{3\alpha_3}{4}F^{\mu\alpha}F^{\nu}{}_{\alpha}X^{(2)}_{\mu\nu}(B) \\ & + \frac{1}{4}F^{\mu\alpha}F^{\nu\beta}B_{\mu\nu}B_{\alpha\beta} + \frac{1}{4}F^{\mu\alpha}F^{\nu}{}_{\alpha}B^2_{\mu\nu} - \frac{1}{2}F^{\mu\alpha}F^{\nu\alpha}BB_{\mu\nu} \\ & - \frac{1+6\alpha_3}{L^2}A^2 S_2(B) + \frac{1+3\alpha_3}{L^2}A^\mu A^\nu X^{(2)}_{\mu\nu}(B) \\ & + \frac{2}{L^2}(A^2F_{\mu\nu}F^{\mu\nu} - A^\mu A^\nu B_{\mu\alpha}F_\nu{}^\alpha + \frac{1}{2}A^\mu A^\nu F_{\mu\alpha}F_\nu{}^\alpha) + \frac{12}{L^4}A^4\end{aligned}$$

Massive Gravity in Flat Space

Flat Limit of Non-Linear Proca Theory



- Take limit as

  $L \to \infty$

Scalar-Vector Theory in Flat Space

- Starting with non-linear Proca theory in AdS $\sim \frac{(\partial A)^n}{\Lambda_2^{2n-4}}, \quad \Lambda_2 = (M_P m)^{1/2}$

- Restore U(1) gauge invariance using Stückelberg trick $A_\mu \to A_\mu + L\nabla_\mu\phi$

$$\delta A_\mu = \partial_\mu \Lambda, \quad \phi = -\frac{1}{L}\Lambda$$

Flat Space Limit
New Symmetry arises

- The theory becomes a coupled scalar-vector theory

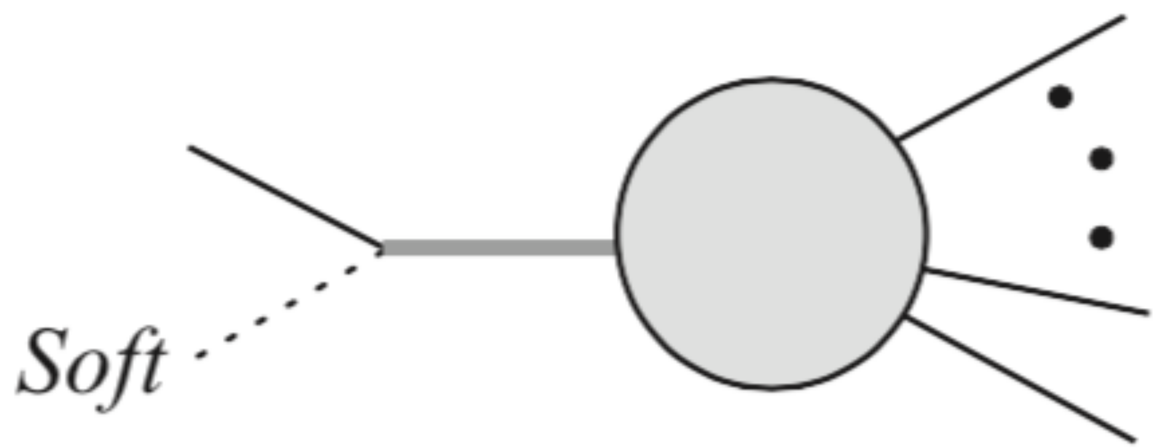
$$\sim \frac{\partial^{2n+2} A^2 \phi^n}{\Lambda_3^{3n}}, \quad \Lambda_3 = (\Lambda_2^2/L)^{1/3}$$

- The shift symmetry becomes

$$\delta A_\mu = -\xi_\mu - \frac{2}{\Lambda_3^3} \partial_\mu \xi^\nu \partial_\nu \phi$$

Soft Behavior

- Can look at soft behavior of amplitudes in our scalar-vector theory.



$$\mathcal{A}_n(\epsilon p_1, p_2, \ldots) = \epsilon^\sigma \, \mathcal{S}_n^{(0)} + O(\epsilon^{\sigma+1}) \quad \text{as} \;\; \epsilon \to 0$$

Soft Behavior

- Tells us if the amplitudes are constructible using soft subtracted recursion.

$$\mathcal{A}_n = \sum_I \sum_{|\psi^{(I)}\rangle} \left(\frac{\hat{\mathcal{A}}_L^{(I)}(0)\hat{\mathcal{A}}_R^{(I)}(0)}{P_I^2} + \sum_{i=1}^{n} \text{Res}_{z=\frac{1}{a_i}} \frac{\hat{\mathcal{A}}_L^{(I)}(z)\hat{\mathcal{A}}_R^{(I)}(z)}{z\, F(z)\, \hat{P}_I^2} \right)$$

Validity Criterion

- For amplitudes with one fundamental coupling to be constructed via soft subtracted recursion, they need to satisfy the criterion:

Dimension of operator

$$4 - n + (n-2)\left(\frac{\Delta[\mathcal{O}] - 4}{N[\mathcal{O}] - 2}\right) - \sum_{i=1}^{n} s_i - \sum_{i=1}^{n} \sigma_i < 0$$

Number of external legs · **Number of fields in operator** · **Spin** · **Soft Weight**

H. Elvang, M. Hadjiantonis, C. R. T. Jones, and S. Paranjape, "Soft Bootstrap and Supersymmetry," JHEP 01 (2019) 195, arXiv:1806.06079 [hep-th].

Scalar-Vector from Massive Gravity

- In this case, the validity criterion reduces to $\quad -2 + (2 - \sigma_\phi)n_\phi + (1 - \sigma_A)n_A < 0$
- In this theory, $\sigma_\phi = 2, \quad \sigma_A = 0$, giving

$$n_A < 2$$

- Amplitudes are not constructible via soft subtracted recursion because they will have at least two vectors.

Pseudo-Linear Massive Gravity

- Linearized Massive Gravity

$$S = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}\nabla_\lambda h_{\mu\nu}\nabla^\lambda h^{\mu\nu} + \nabla_\mu h_{\nu\lambda}\nabla^\nu h^{\mu\lambda} - \nabla_\mu h^{\mu\nu}\nabla_\nu h + \frac{1}{2}\nabla_\lambda h\nabla^\lambda h + \frac{R}{4}\left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2\right) - \frac{1}{2}m^2\left(h^{\mu\nu}h_{\mu\nu} - h^2\right) + m^2M_p^2V(h/M_p)\Big]$$

- with potential

$$V(h) = -\frac{\alpha_4}{2}\epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}$$

Shift Symmetry from Pseudo-Linear Massive Gravity

- Do Stückelberg procedure and take proper decoupling limits to get coupled vector-scalar theory with shift symmetry when ξ_μ is a Killing vector.

$$\mathcal{L}_{\rm pl}\left(A,\phi\right) = -\frac{1}{2}F_{\mu\nu}^2 + \epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4}\left(\frac{3\alpha_3}{2\Lambda_3^3}F_{\mu_1\mu_2}F_{\nu_1\nu_2}\partial_{\mu_3}\partial_{\nu_3}\phi\,\eta_{\mu_4\nu_4} + \frac{6\alpha_4}{\Lambda_3^6}F_{\mu_1\mu_2}F_{\nu_1\nu_2}\partial_{\mu_3}\partial_{\nu_3}\phi\,\partial_{\mu_4}\partial_{\nu_4}\phi\right)$$

$$\delta A_\mu = -\xi_\mu$$

Scalar-Vector from Pseudo-Linear Massive Gravity

- The validity criterion is

$$-2 + (2 - \sigma_\phi) n_\phi + (1 - \sigma_A) n_A < 0$$

- In this theory, $\sigma_\phi = 2, \quad \sigma_A = 1$ giving

$$-2 < 0$$

- Amplitudes are constructible via soft subtracted recursion!

Summary

- Theories with shift symmetries arise in decoupling limits of massive gravity.
- We discovered a non-linear Proca theory in AdS that is ghost-free, but cannot be put into the form of generalized Proca theories previously studied and it has an interesting shift symmetry that fixes the non-linear structure.
- We found a new shift symmetry of the vector-scalar sector of the massless limit of dRGT in flat space.
- We discovered a vector-scalar theory whose amplitudes are constructible by soft subtracted recursion.