Searching for flavour symmetries: old data, new tricks

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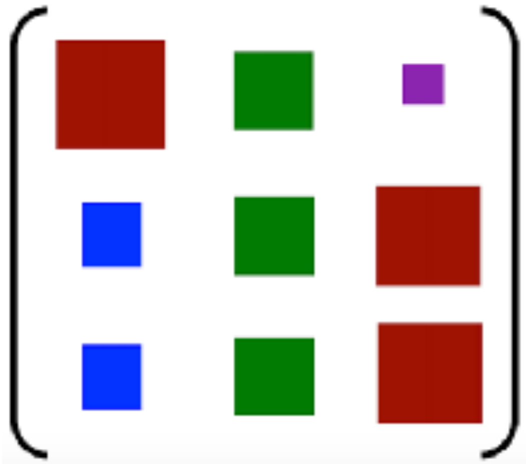
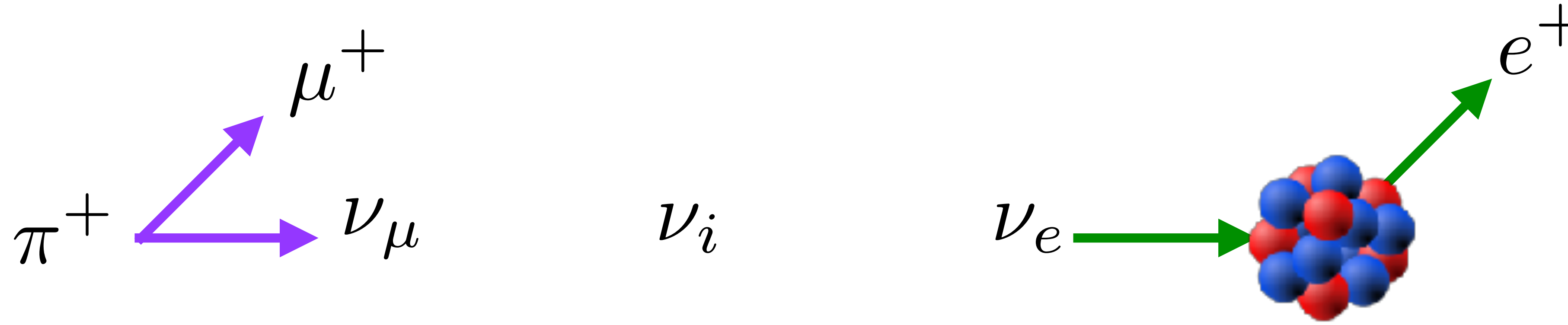
LCTP Seminar 27 Feb 2019

Based on arXiv:1810.05648 with
L. Heinrich, H. Schulz, Y. L. Zhou,

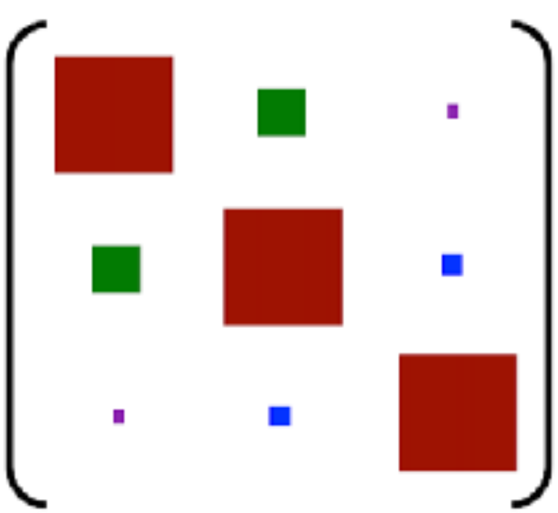
leptonic mixing

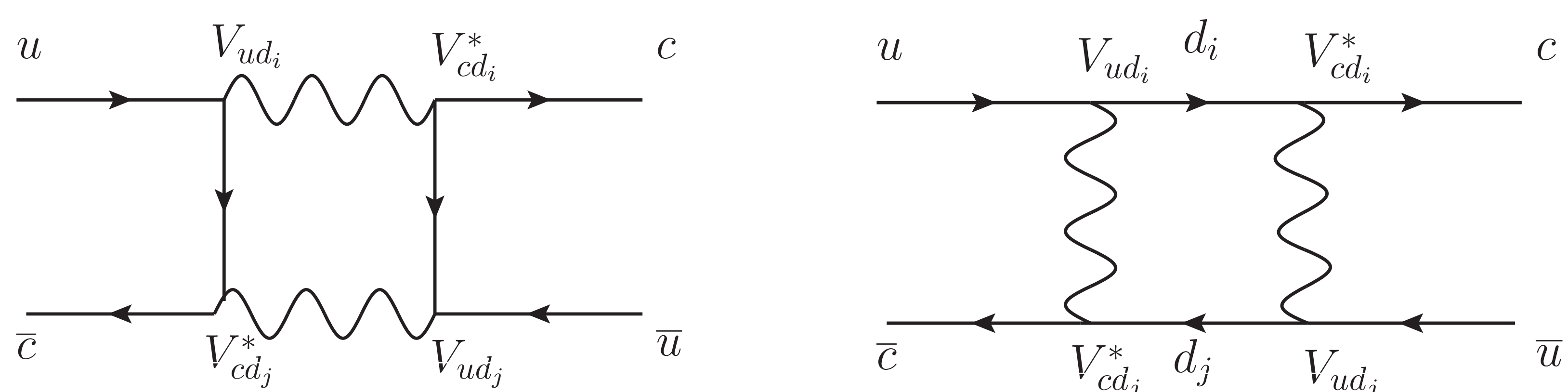




neutrino oscillations

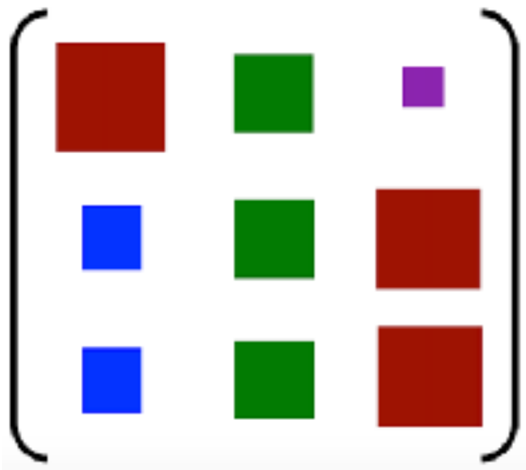
quark mixing

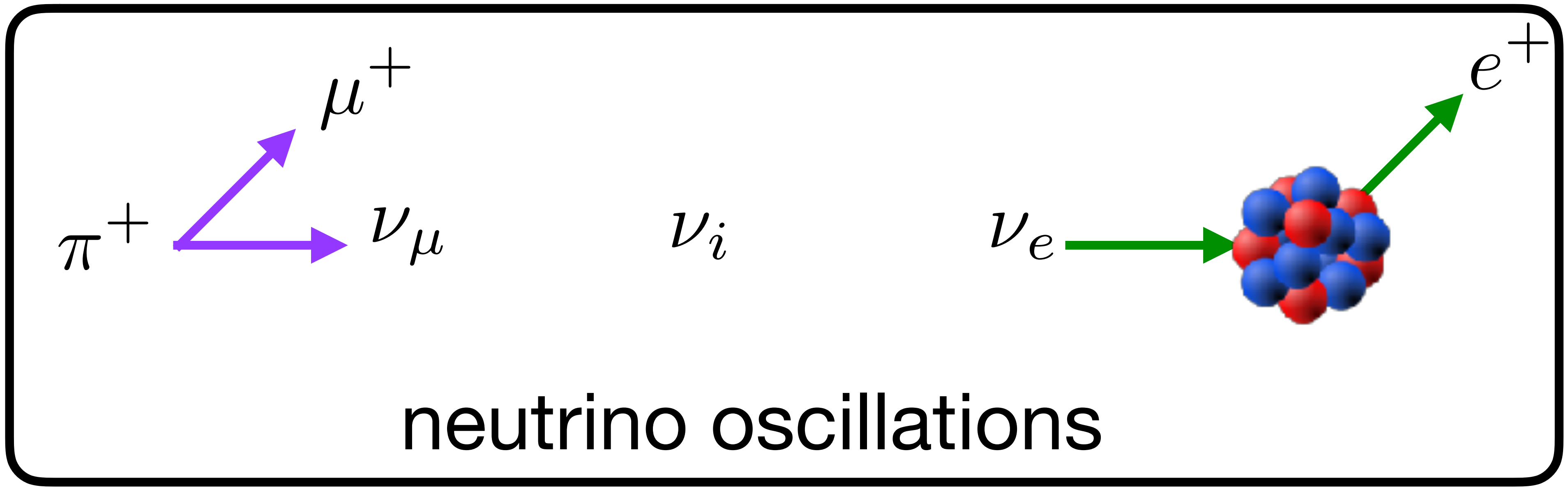




meson mixing

leptonic mixing





Why does the leptonic mixing matrix have its peculiar structure?

If a flavour symmetry is present, how can we test it?

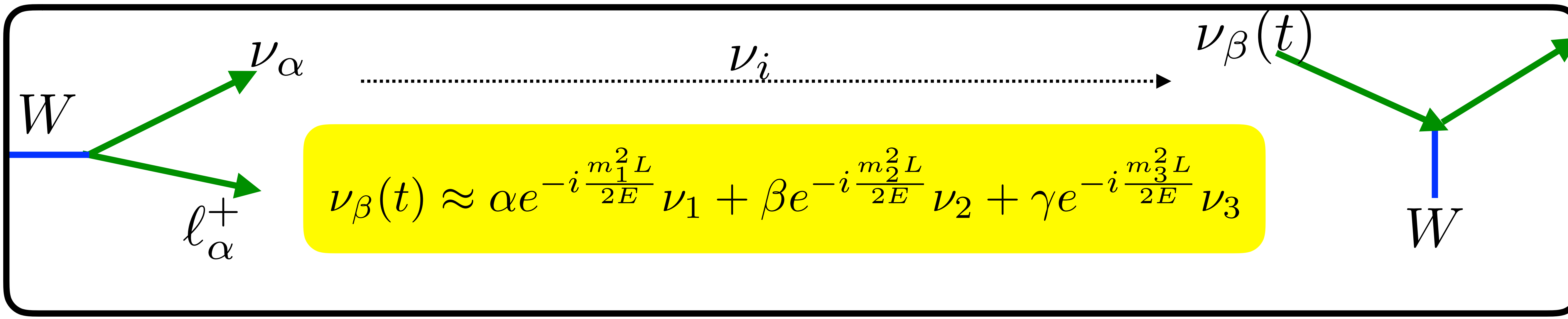
Outline

- Brief Overview of Neutrino Masses and Mixing
- Experimental Status of Leptonic Mixing Matrix
- Basic underlying paradigm and principles of flavour models
- Flavour Model, its parameter space and constraints
- Tool chain and how to calculate exclusion regions
- Results

Current Status of Neutrino Oscillation Parameters

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad U m_\nu U^\dagger = m_{\nu \mathrm{diag}}$$

flavour states | PMNS matrix | mass states



$$\nu_\beta(t) \approx \alpha e^{-i\frac{m_1^2 L}{2E}} \nu_1 + \beta e^{-i\frac{m_2^2 L}{2E}} \nu_2 + \gamma e^{-i\frac{m_3^2 L}{2E}} \nu_3$$

$$P(\nu_\mu \to \nu_\mu) \qquad P(\overline{\nu_e} \to \overline{\nu_e}) \qquad P(\nu_\mu \to \nu_e)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$46.40 \leq \theta_{23}(^\circ) \leq 52.40 \qquad 8.22 \leq \theta_{13}(^\circ) \leq 8.97 \qquad 31.54 \leq \theta_{12}(^\circ) \leq 36.16$$

$$133 \leq \delta(^\circ) \leq 337$$

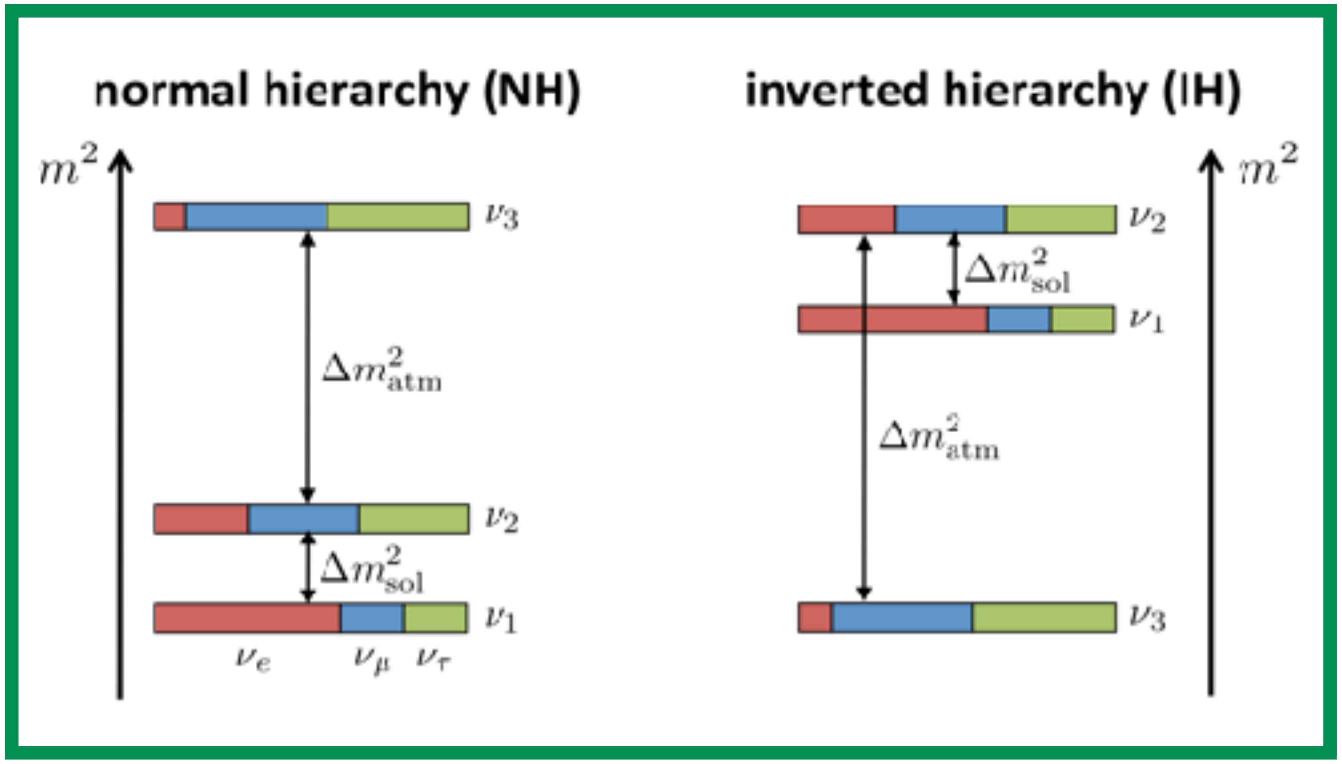
nufit 2018

Current Status of Neutrino Parameters

$$6.79 \leq \frac{\Delta m^2_{21}}{10^{-5}\mathrm{eV}^2} \leq 8.02$$

$$2.427 \leq \frac{\Delta m^2_{3\ell}}{10^{-3}\mathrm{eV}^2} \leq 2.625$$

nufit 2018



- 3 phases: 1 Dirac and 2 Majorana: $\delta, \alpha_{21}, \alpha_{31}$

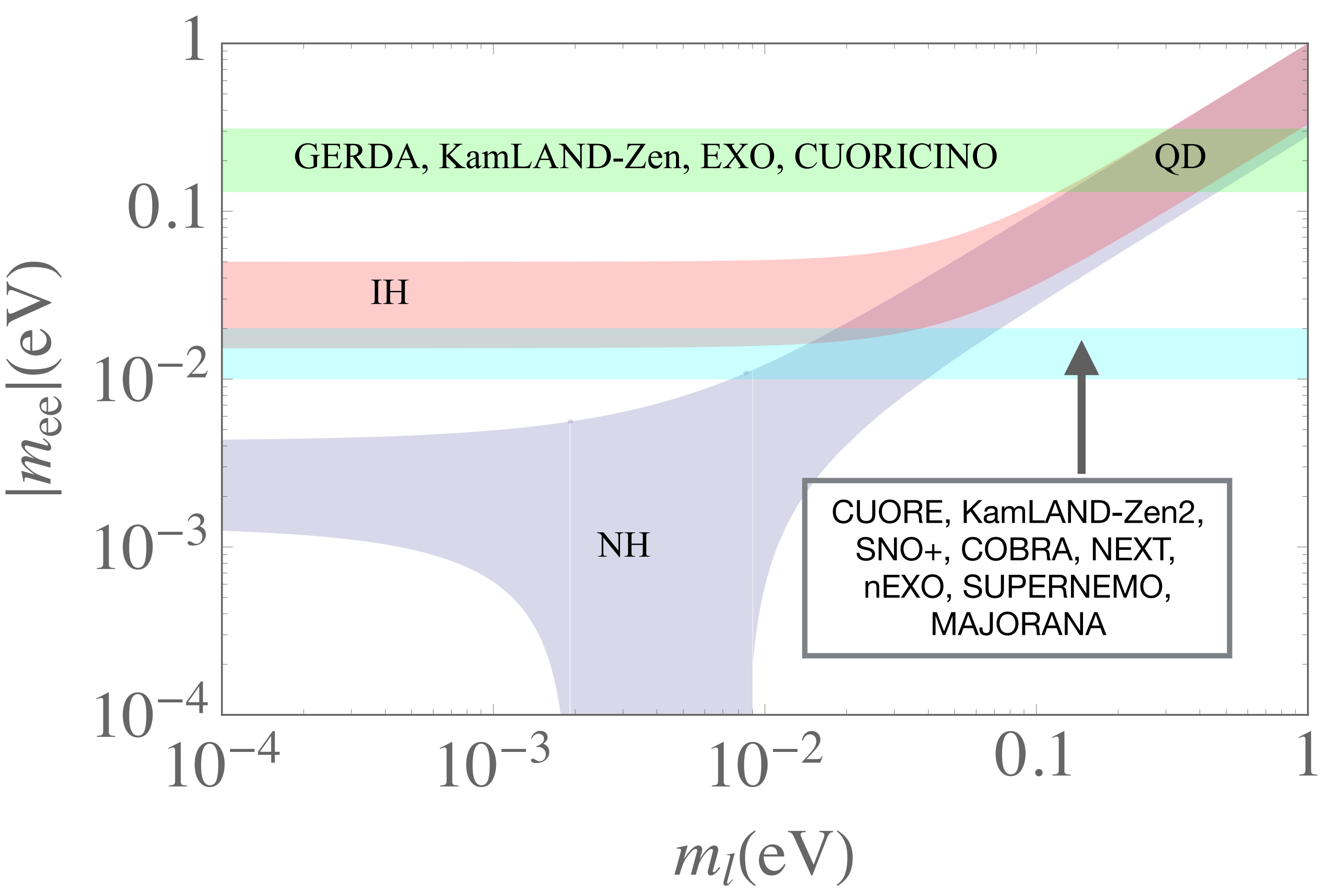
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

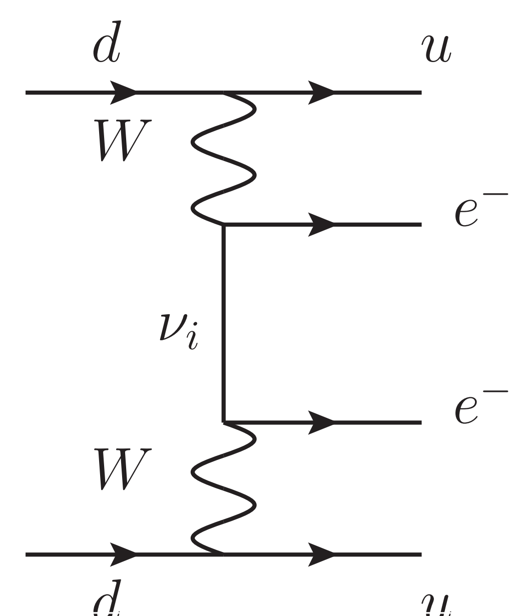
Current Status of Neutrino Oscillation Parameters

- Observation of LNV process e.g. neutrinoless double beta decay would indicate neutrinos are Majorana in nature.

- Half life proportionate to effective Majorana mass

$$|m_{ee}| = |m_1 \cos^2\theta_{12}\cos^2\theta_{13} + m_2 \sin^2\theta_{12}\cos^2\theta_{13}e^{i\alpha_{21}} + m_3 \sin^2\theta_{13}e^{i(\alpha_{31}-2\delta)}|$$

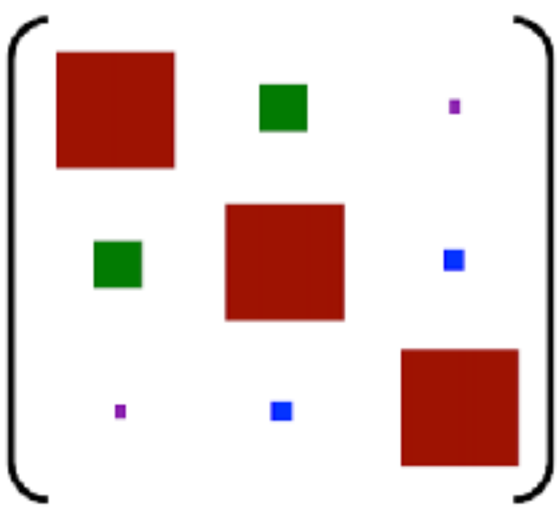




NDBD can, in principle, give information on neutrino mass ordering, absolute mass scale and CPV phases.

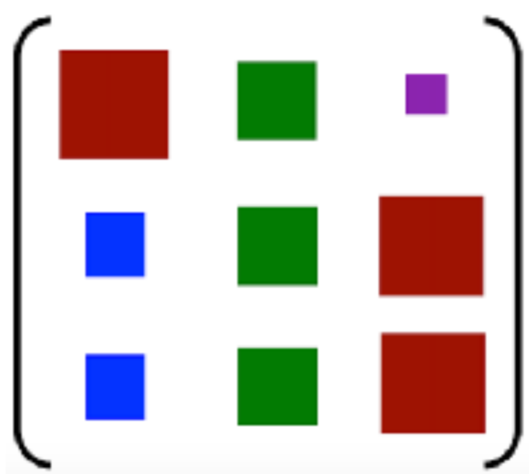
Motivation for Flavour Models

quark mixing



Perturbed Identity Matrix Small Mixing small CPV

leptonic mixing



entries resemble CG coefficient of discrete groups

Anarchy

PMNS matrix described as the result of a random draw from unbiased 3 x 3 unitary matrix

Does not work for CKM

Hall, de Gouvea, Murayama

Symmetry

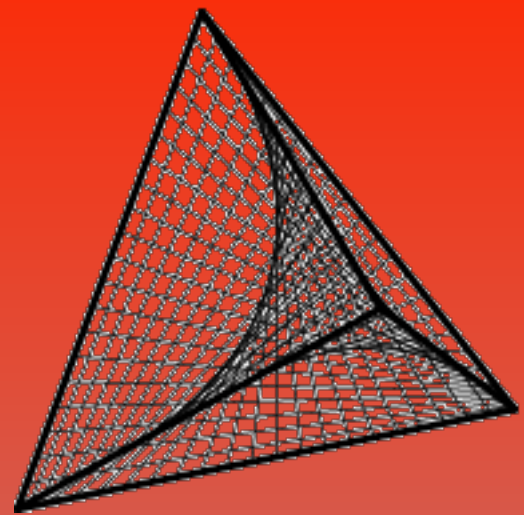
PMNS matrix results from the breaking of a non-Abelian symmetry at high energy scales

Difficult to apply to quark sector

Altarelli, Everett, Feruglio, King, Ding, Hagedorn, Petrov, M. C Chen, Harrison, Perkins, Scott, Luhn…….

ENERGY

A_4 unbroken



A_4

Flavour Symmetry Breaking

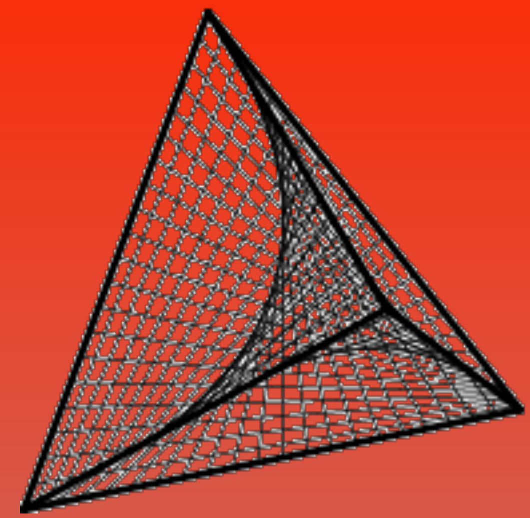
Charged Lepton

Neutrino Sector

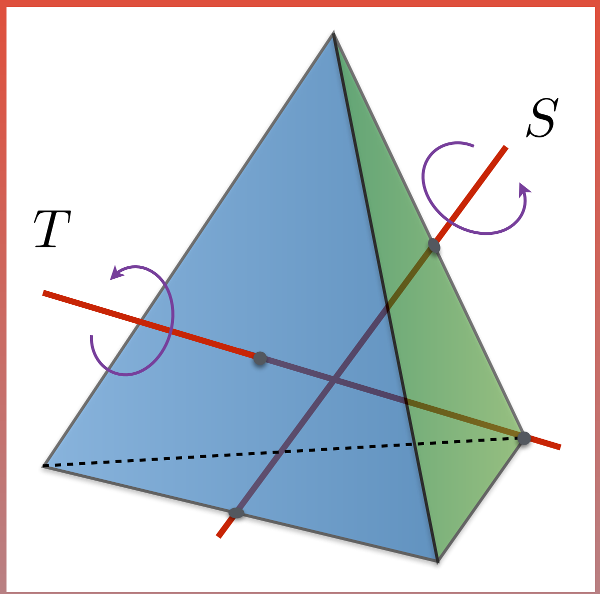
A_4 broken

ENERGY

A_4 unbroken



$$A_4$$



Flavour Symmetry Breaking

$$\ell_L \to T\ell_L$$

$$T^\dagger m_\ell m_\ell^\dagger T = m_\ell m_\ell^\dagger$$

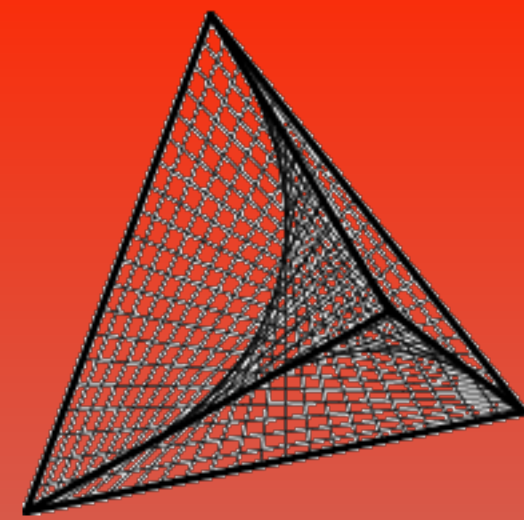
$$\nu_L \to S\nu_L$$

$$S^T m_\nu S = m_\nu$$

A_4 broken

ENERGY

A_4 unbroken



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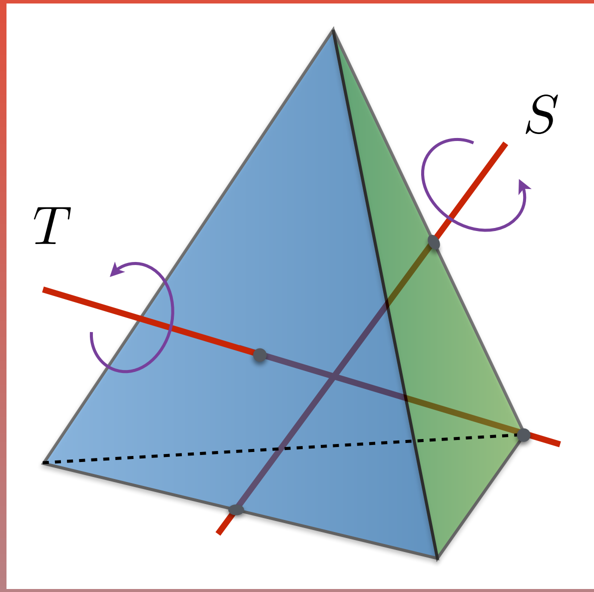
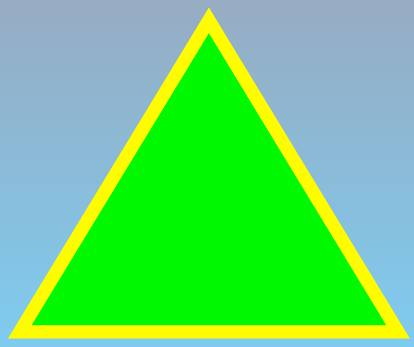


Flavour Symmetry Breaking

$\mathbb{Z}_3$

$\mathbb{Z}_2$



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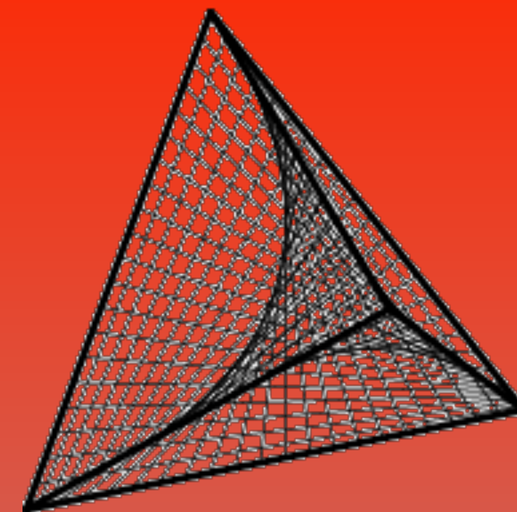
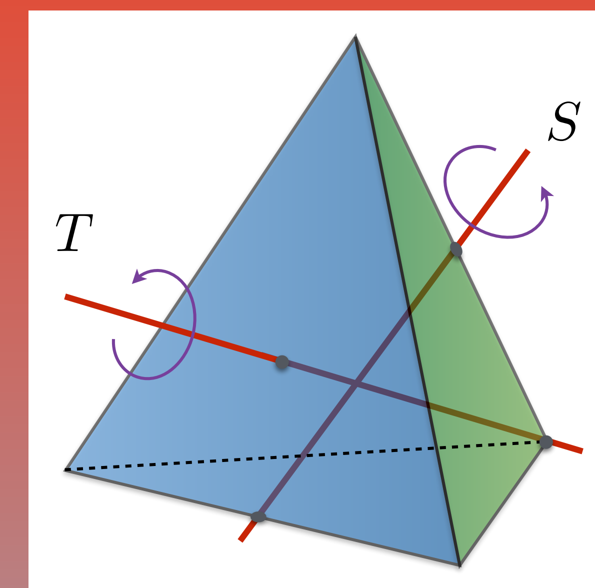


A_4



Alteralli-Feruglio Basis

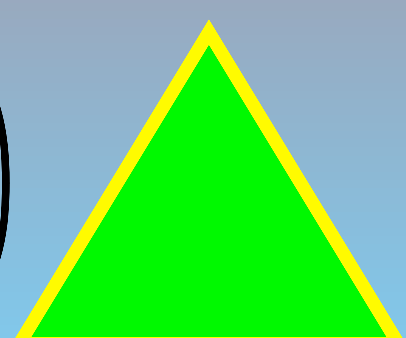
Flavour Symmetry Breaking

$\mathbb{Z}_3$

$\mathbb{Z}_2$

$$\omega = e^{\frac{2\pi i}{3}}$$

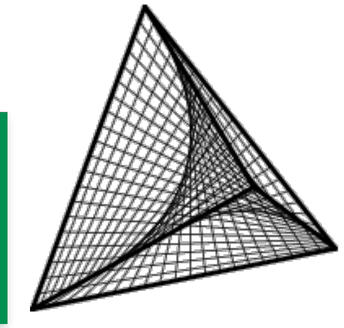
$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$



$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

Field Content

$$T\langle\varphi\rangle = \langle\varphi\rangle \qquad S\langle\chi\rangle = \langle\chi\rangle$$



$$\varphi = (\varphi_1, \varphi_2, \varphi_3)^T \sim 3,\ \chi = (\chi_1, \chi_2, \chi_3)^T \sim 3$$

flavon pseudo-real triplets

$$\ell_L = (\ell_{eL}, \ell_{\mu L}, \ell_{\tau L})^T \sim 3,\ e_R \sim 1,\ \mu_R \sim 1',\ \tau_R \sim 1''$$

SM fields

Lagrangian terms for charged lepton and neutrinos

$$-\mathcal{L}_l = \frac{y_e}{\Lambda}(\overline{\ell_L}\varphi)_{\mathbf{1}} e_R H + \frac{y_\mu}{\Lambda}(\overline{\ell_L}\varphi)_{\mathbf{1}''}\mu_R H + \frac{y_\tau}{\Lambda}(\overline{\ell_L}\varphi)_{\mathbf{1}'}\tau_R H + \text{h.c.}\,,$$

$$-\mathcal{L}_\nu = \frac{y_1}{2\Lambda\Lambda_{\rm W}}\left((\overline{\ell_L}\tilde{H}\tilde{H}^T\ell_L^c)_{\mathbf{3}_S}\chi\right)_{\mathbf{1}} + \frac{y_2}{2\Lambda_{\rm W}}(\overline{\ell_L}\tilde{H}\tilde{H}^T\ell_L^c)_{\mathbf{1}} + \text{h.c.}$$

Assume neutrino Majorana

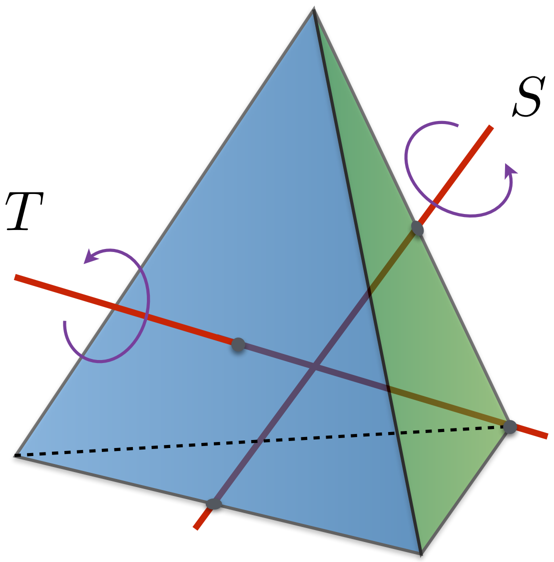
$$\langle\varphi\rangle = (1,0,0)^T\frac{v_\varphi}{\sqrt{n}} \qquad \langle\chi\rangle = (1,1,1)^T\frac{v_\chi}{\sqrt{3n}}$$

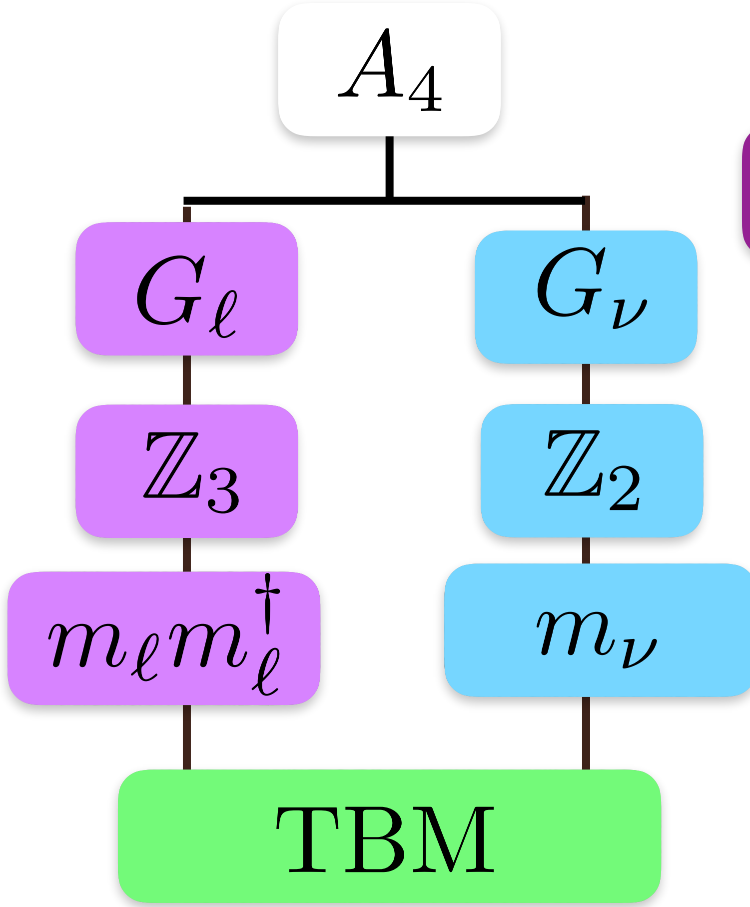
$$M_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}\frac{v v_\varphi}{\sqrt{2n}\Lambda} \qquad M_\nu = \begin{pmatrix} 2a+b & -a & -a \\ -a & 2a & -a+b \\ -a & -a+b & 2a \end{pmatrix}$$

Results in TBM mixing of PMNS matrix

$a \equiv y_1 v_\chi v^2/(4\sqrt{3n}\Lambda\Lambda_{\rm W})$

$b \equiv y_2 v^2/2\Lambda_{\rm W}$





Need corrections to TBM

break Z_2 or Z_3

modify mass matrices

sizeable θ_{13} and δ

$$U_{\rm TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$V_{\not{\mathbb{Z}}_3}(\varphi) = \frac{1}{2} A(\varphi_2^2 + 2\varphi_1\varphi_2^*) + \text{h.c.},$$

Parametrises EXPLICIT breaking of Z_3

How does this flavour sector communicate with us?

Scalar Sector

- Higgs-Flavon cross-coupling

$$V_{\text{cross}}(H, \varphi) = \frac{1}{2}\epsilon H^\dagger H(\varphi\varphi)_{\mathbf{1}}$$

$$\varphi_1 = v_\varphi + \tilde{\varphi}_1\,, \quad \varphi_2 = \epsilon_\varphi v_\varphi + \tilde{\varphi}_2\,. \qquad (\varphi\varphi)_{\mathbf{1}} = \left(\varphi_1^2 + 2\varphi_2\varphi_2^*\right)$$

- Flavon potential

$$V(\varphi) = \frac{1}{2}\mu_\varphi^2 I_{1\varphi} + \frac{g_1}{4} I_{1\varphi}^2 + \frac{g_2}{4} I_{2\varphi},$$

$$I_{1\varphi} = \varphi_1^2 + 2|\varphi_2|^2\,, \quad I_{2\varphi} = \frac{1}{3}\varphi_1^4 - \frac{2}{3}\varphi_1(\varphi_2^3 + \varphi_2^{*3}) + |\varphi_2|^4.$$

Scalar Sector

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6 real parameters

- Flavon potential

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Yukawa Sector

charged lepton flavour conserving

$$-\mathcal{L}_{\rm clfc}^{\tilde{h},\tilde{\varphi}_1} = \sum_{l=e,\mu,\tau} \frac{m_l}{v_H}\bar{l}l\tilde{h} + \frac{m_l}{v_\varphi}\bar{l}l\tilde{\varphi}_1 + \frac{m_l}{v_H v_\varphi}\bar{l}l\tilde{\varphi}_1\tilde{h}\,,$$

$$\begin{aligned}-\mathcal{L}_{\rm clfv}^{\tilde{\varphi}_2} = &\frac{m_e}{v_\varphi}\left(\overline{\mu_L}e_R\tilde{\varphi}_2 + \overline{\tau_L}e_R\tilde{\varphi}_2^*\right) + \frac{m_e}{v_H v_\varphi}\left(\overline{\mu_L}e_R\tilde{\varphi}_2 + \overline{\tau_L}e_R\tilde{\varphi}_2^*\right)\tilde{h}\\ &+ \frac{m_\mu}{v_\varphi}\left(\overline{\tau_L}\mu_R\tilde{\varphi}_2 + \overline{e_L}\mu_R\tilde{\varphi}_2^*\right) + \frac{m_\mu}{v_H v_\varphi}\left(\overline{\tau_L}\mu_R\tilde{\varphi}_2 + \overline{e_L}\mu_R\tilde{\varphi}_2^*\right)\tilde{h}\\ &+ \frac{m_\tau}{v_\varphi}\left(\overline{e_L}\tau_R\tilde{\varphi}_2 + \overline{\mu_L}\tau_R\tilde{\varphi}_2^*\right) + \frac{m_\tau}{v_H v_\varphi}\left(\overline{e_L}\tau_R\tilde{\varphi}_2 + \overline{\mu_L}\tau_R\tilde{\varphi}_2^*\right)\tilde{h} + \text{h.c.},\end{aligned}$$

Final state tau dominated

charged lepton flavour violating

Model Parameter Space

Parameter p	min(p)	max(p)
$\log_{10}(v_\varphi)$	1	3
$\log_{10}(\varepsilon)$	-3	0.5
$\log_{10}(g_1)$	-4	0
$-\log_{10}(g_2)$	-4	0
$\log_{10}(\|\epsilon_\varphi\|)$	-3	0.5
θ_φ	0	2π

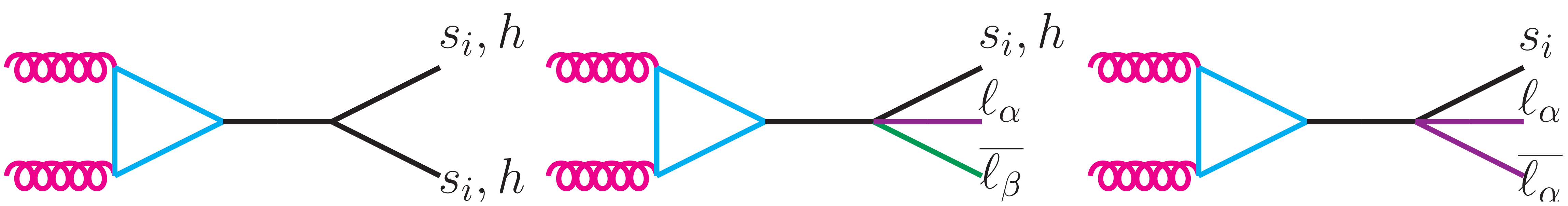
Table 1: Parameter sampling boundaries.

1. Any flavon mass is too light, i.e. $m(s_i) < 10$ GeV, $i = 1 \cdots 3$.
2. All flavon masses are > 1 TeV.
3. Any flavon mass is too close to the Higgs — $|m(s_i) - m_H| < 5$ GeV for $i = 1, 2, 3$.
4. Any flavon mass which is not the Higgs is close to degenerate – $|m(s_i) - m(s_j)| < 100$ MeV for $i, j = 1, 2, 3$.
5. $\lambda g < \frac{\varepsilon}{4}$
6. $g_1 + \frac{g_2}{3} < 0$

Conditions for Physicality

Collider Constraints

- Flavons mix with the Higgs and decay via CLFV and CLFC processes.



- Measured upper limits Higgs width ~ 22 MeV versus 4 MeV SM calculation

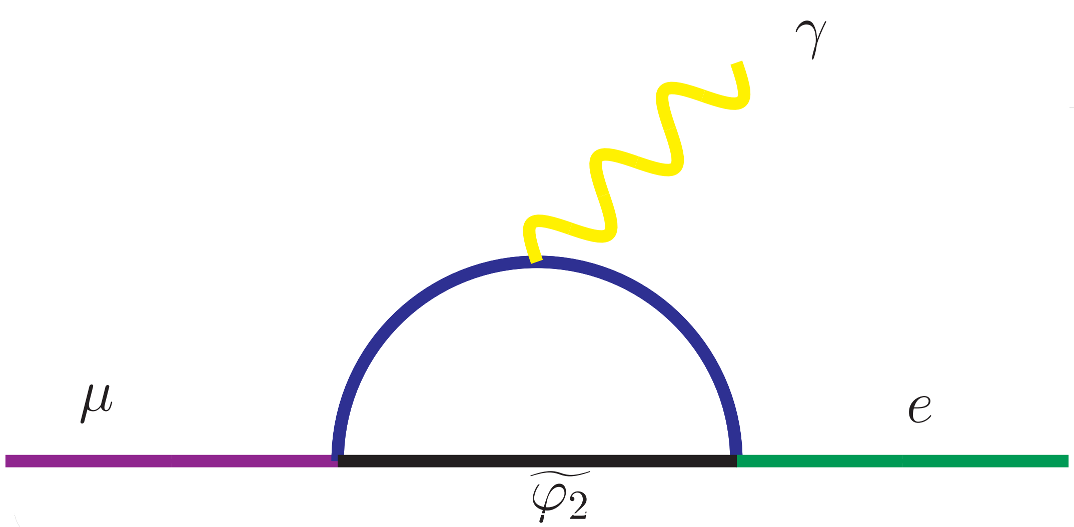
1405.3455

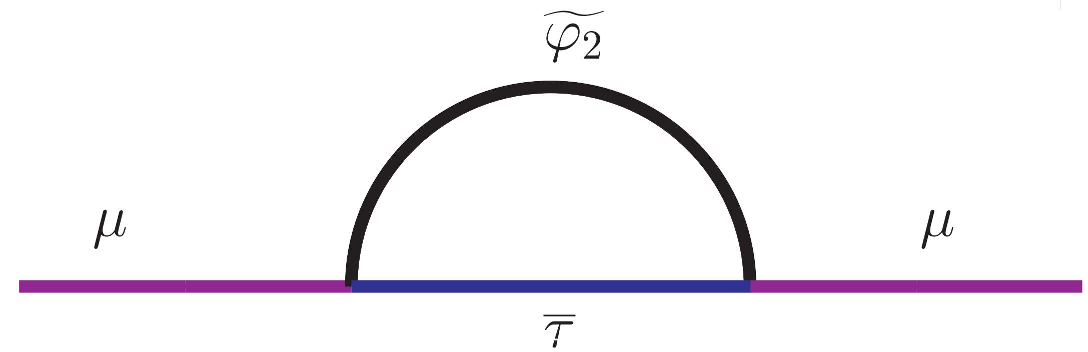
- Make sure the Higgs is mostly comprised of the Higgs mass eigenstate.

Robens, Stefaniak, Pruna,
Godunov, Roznanov, Vysotsky, Zhemchugov

1303.1150, 1501.02234,
1503.01618, 1502.01361

G-2 and MEG Constraints





E821 (BNL) measures muon anomalous magnetic moment

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (2.87 \pm 0.8) \times 10^{-9} \;\; (3.6\sigma)$$

0602035, 1311.2198

MEG experiment measures μ→eɣ

1605.05081

$$\mathrm{Br}\,(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$$ at 90% C.L

The Collider Analysis in a nutshell

ATLAS Analysis: 8 TeV

1411.2921

Search for new phenomena in events with three or more charged leptons in pp collisions at $\sqrt{s}$ = 8 TeV with the ATLAS detector

20.3^{-1} fb

no-OSSF

OSSF

distributions have not been corrected for detector effect i.e. not unfolded

no-OSSF ≥3e/μ

S1

no-OSSF 2e/μ≥1τ_{had}

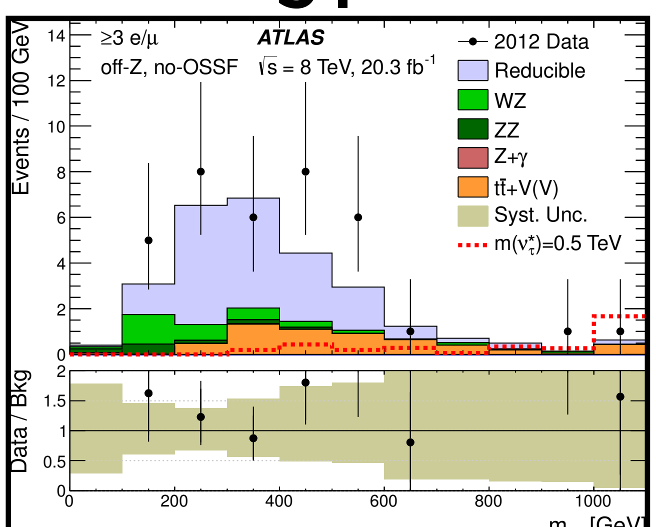
S2

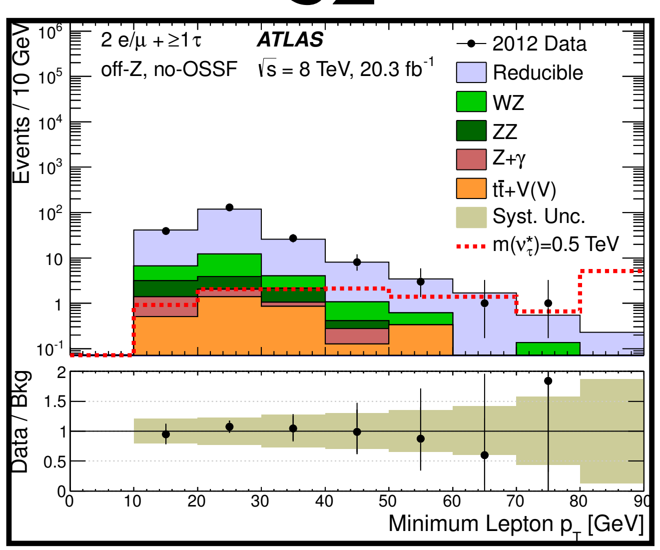
OSSF ≥3e/μ

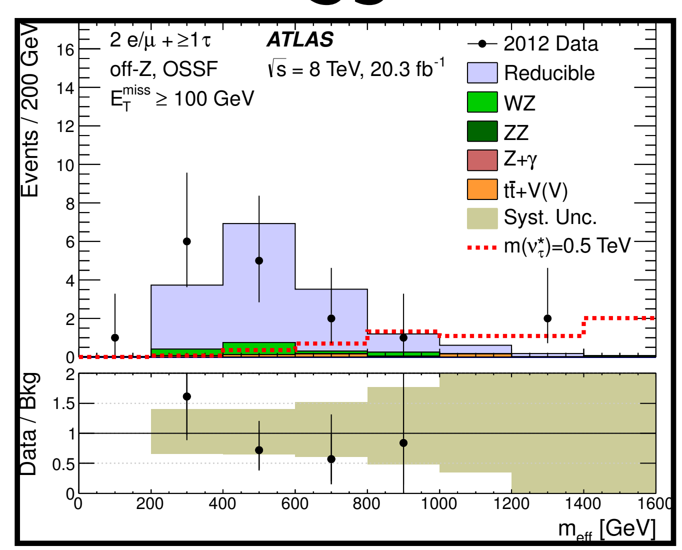
S3

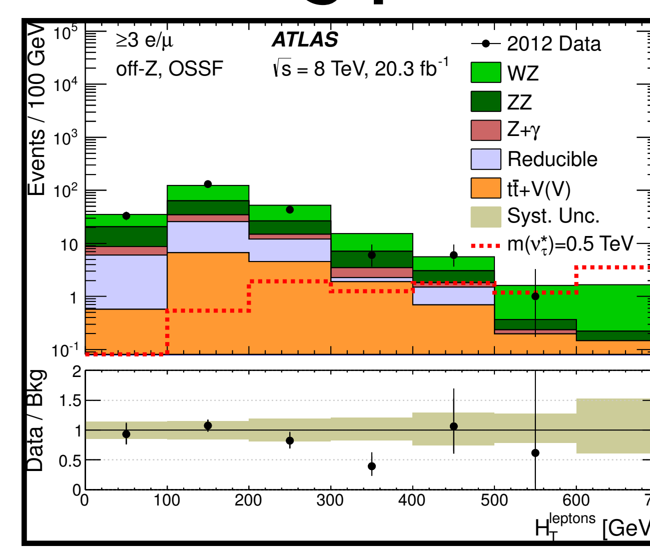
OSSF 2e/μ≥1τ_{had}

S4









m_{eff}: effective mass of event combining sum of jets, missing energy and lepton p_T

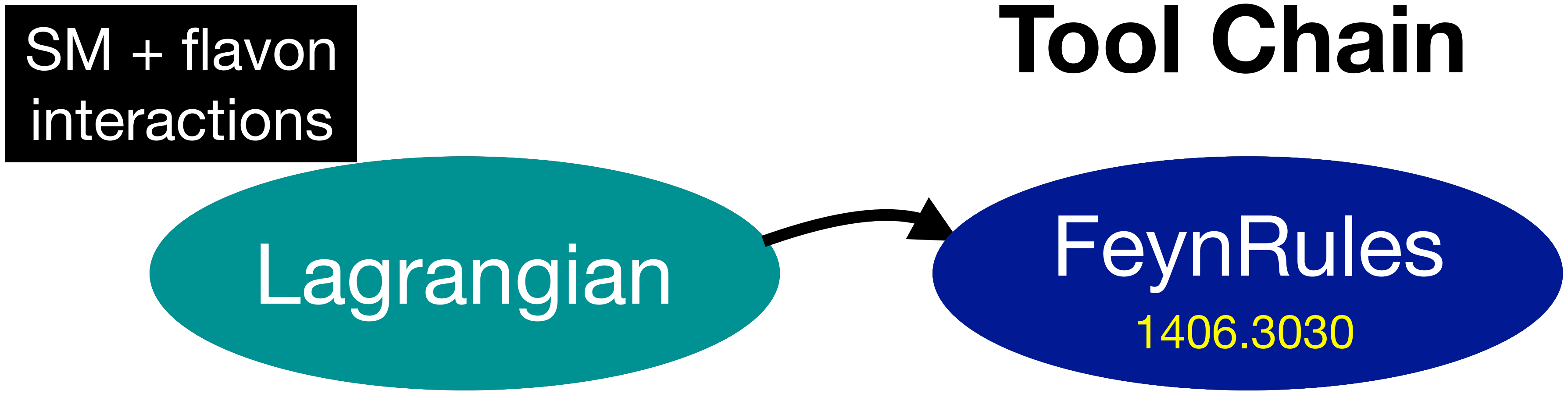
H_T^{lepton}: scalar sum of lepton p_T used to characterise event

Tool Chain

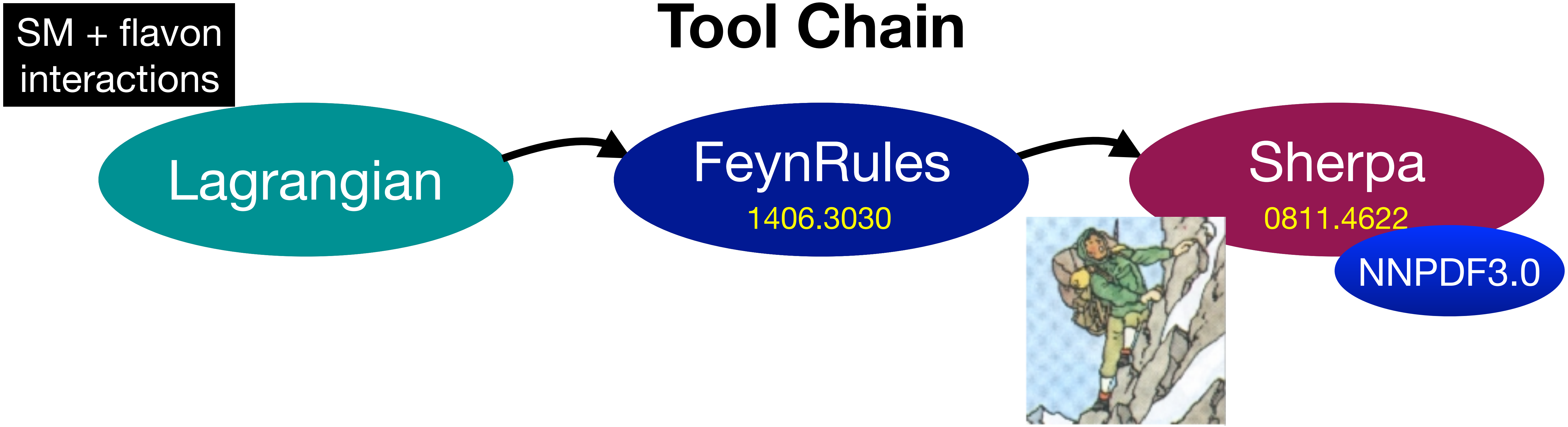
SM + flavon interactions

Lagrangian

Tool Chain



Tool Chain



Thanks to UK HEP Grid Computing for resources

Tool Chain

SM + flavon interactions

Lagrangian → FeynRules 1406.3030 → Sherpa 0811.4622

NNPDF3.0

10^6 MC events simulated per PS point: MPI, hard process, showering, hadronisation + hadronic decays

Tool Chain

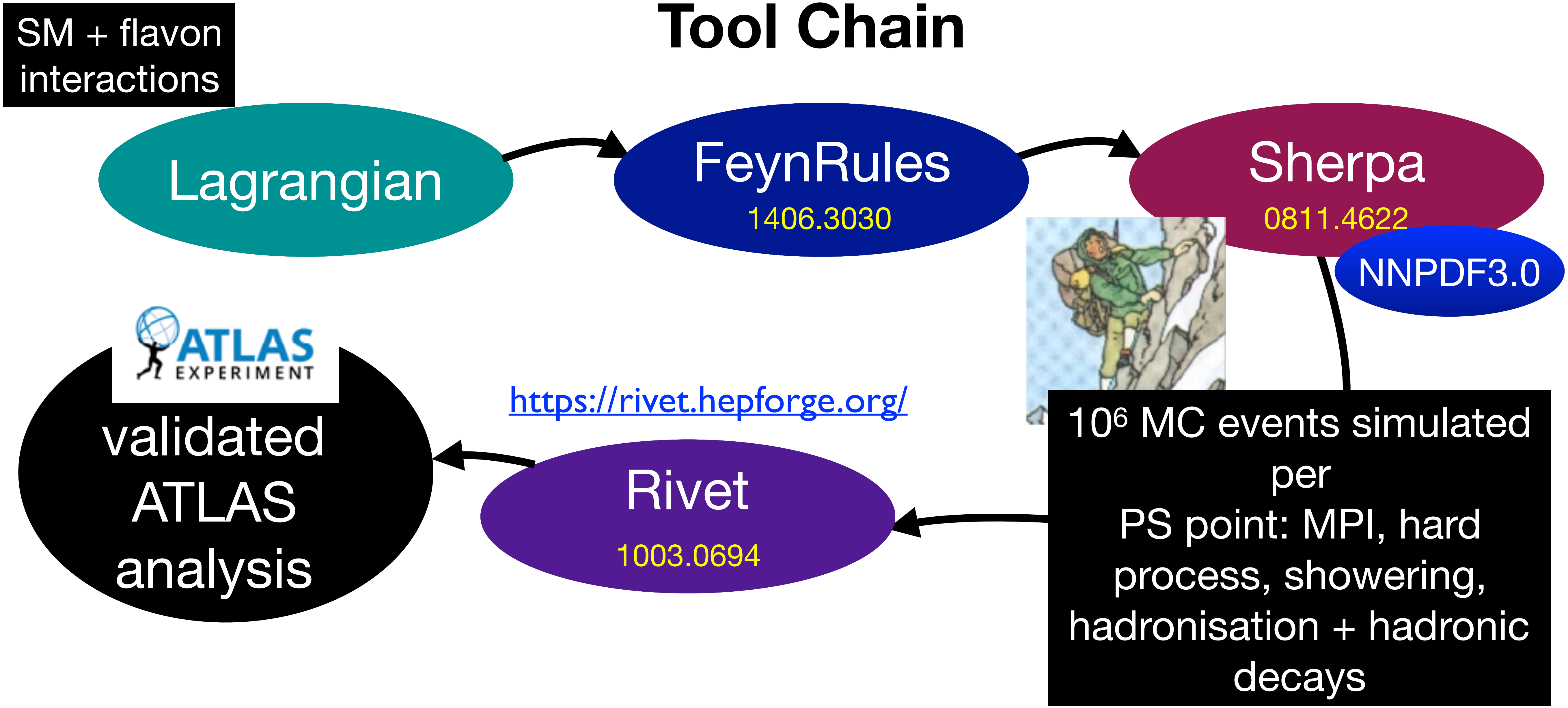
SM + flavon interactions

Lagrangian → FeynRules 1406.3030 → Sherpa 0811.4622 (NNPDF3.0) → Rivet 1003.0694

https://rivet.hepforge.org/

10^6 MC events simulated per PS point: MPI, hard process, showering, hadronisation + hadronic decays

Tool Chain


SM + flavon interactions
Lagrangian
FeynRules
1406.3030
Sherpa
0811.4622
NNPDF3.0
ATLAS EXPERIMENT
validated ATLAS analysis
https://rivet.hepforge.org/
Rivet
1003.0694
10⁶ MC events simulated per PS point: MPI, hard process, showering, hadronisation + hadronic decays

Tool Chain

SM + flavon interactions

Lagrangian

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Calculate CLs

https://github.com/diana-hep/pyhf

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g-2 μ→eɣ

Higgs width + mixing

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ATLAS EXPERIMENT

validated ATLAS analysis

Calculate CLs

https://github.com/diana-hep/pyhf

Constraining our 6D model PS

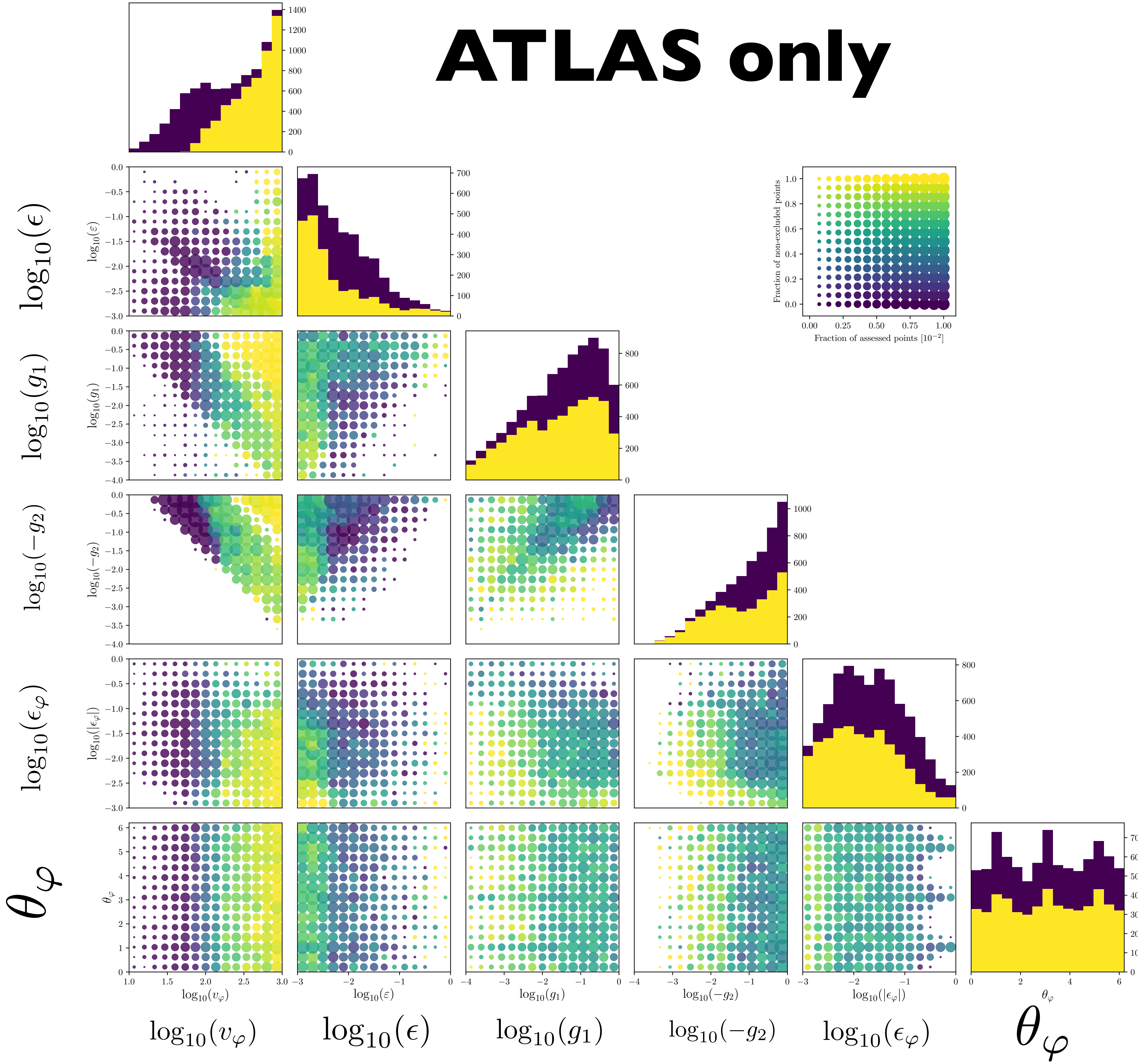
g-2   μ→eɣ

Higgs width + mixing

Thanks to UK HEP Grid Computing for resources

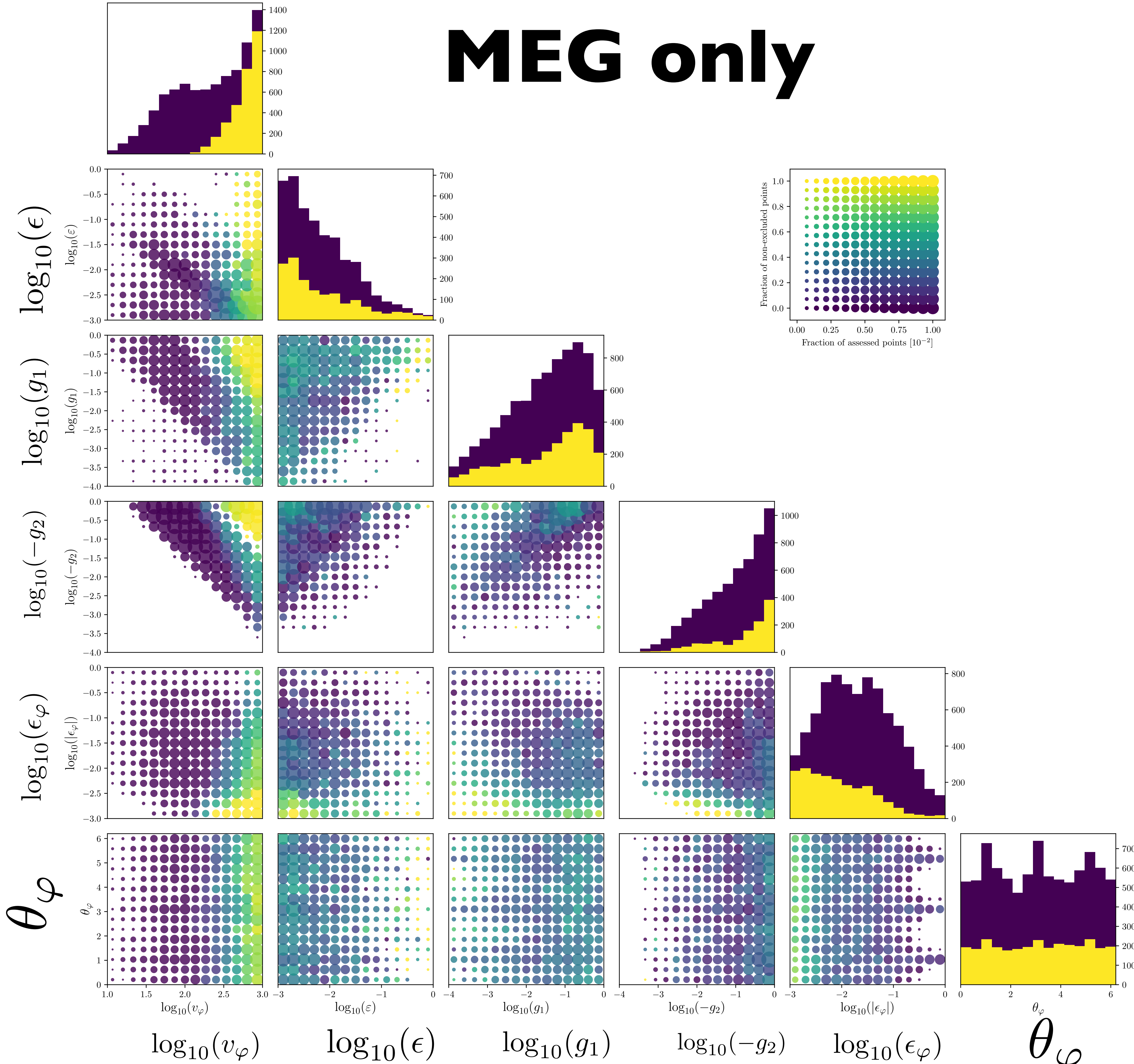
Results

ATLAS only



Fraction of non-excluded points
Fraction of assessed points $[10^{-2}]$
$\log_{10}(\epsilon)$
$\log_{10}(g_1)$
$\log_{10}(-g_2)$
$\log_{10}(\epsilon_\varphi)$
θ_φ
$\log_{10}(v_\varphi)$
$\log_{10}(|\epsilon_\varphi|)$

MEG only



$\log_{10}(\epsilon)$
$\log_{10}(g_1)$
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θ_φ
$\log_{10}(v_\varphi)$
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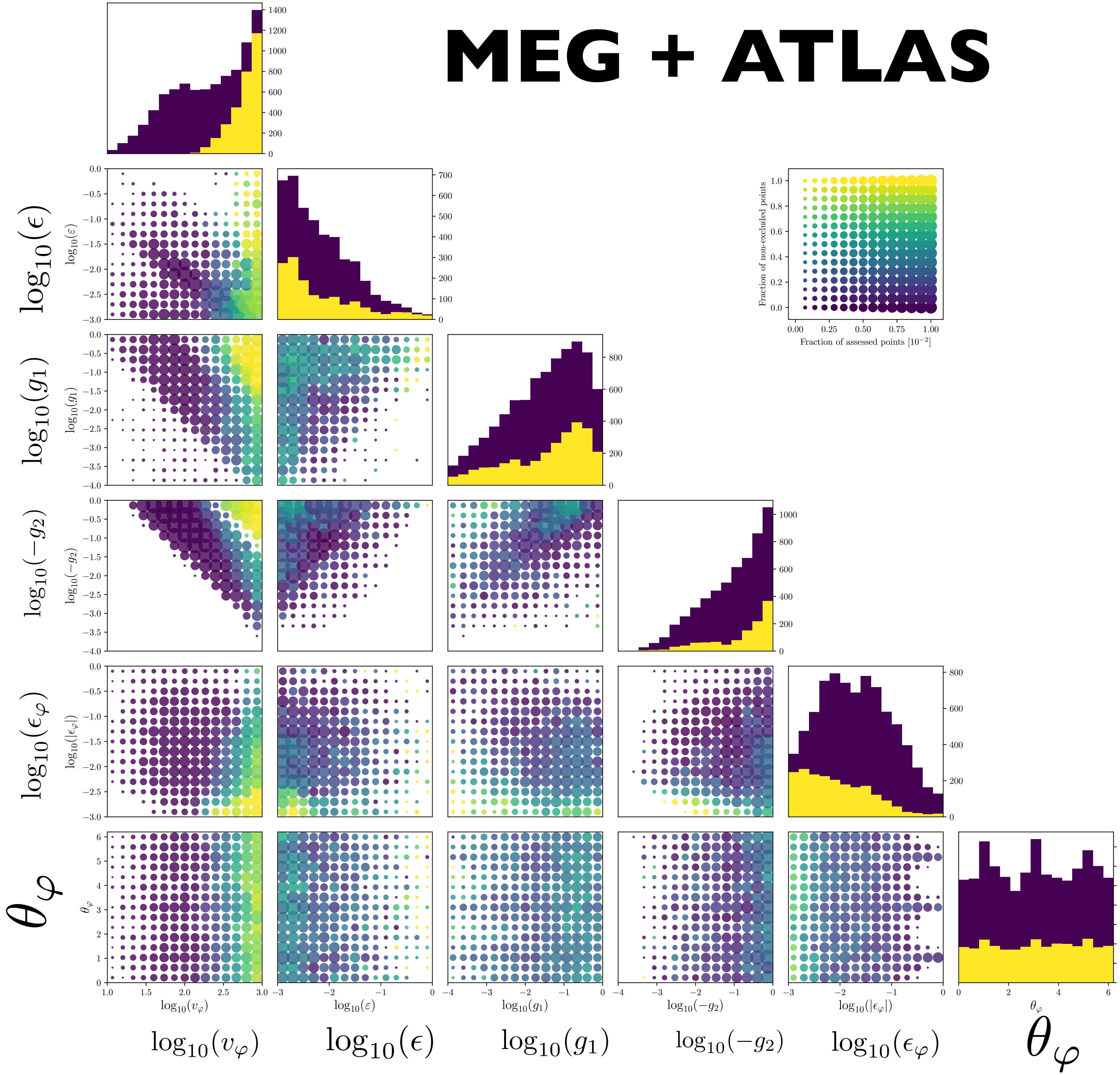
Fraction of assessed points $[10^{-2}]$

MEG + ATLAS



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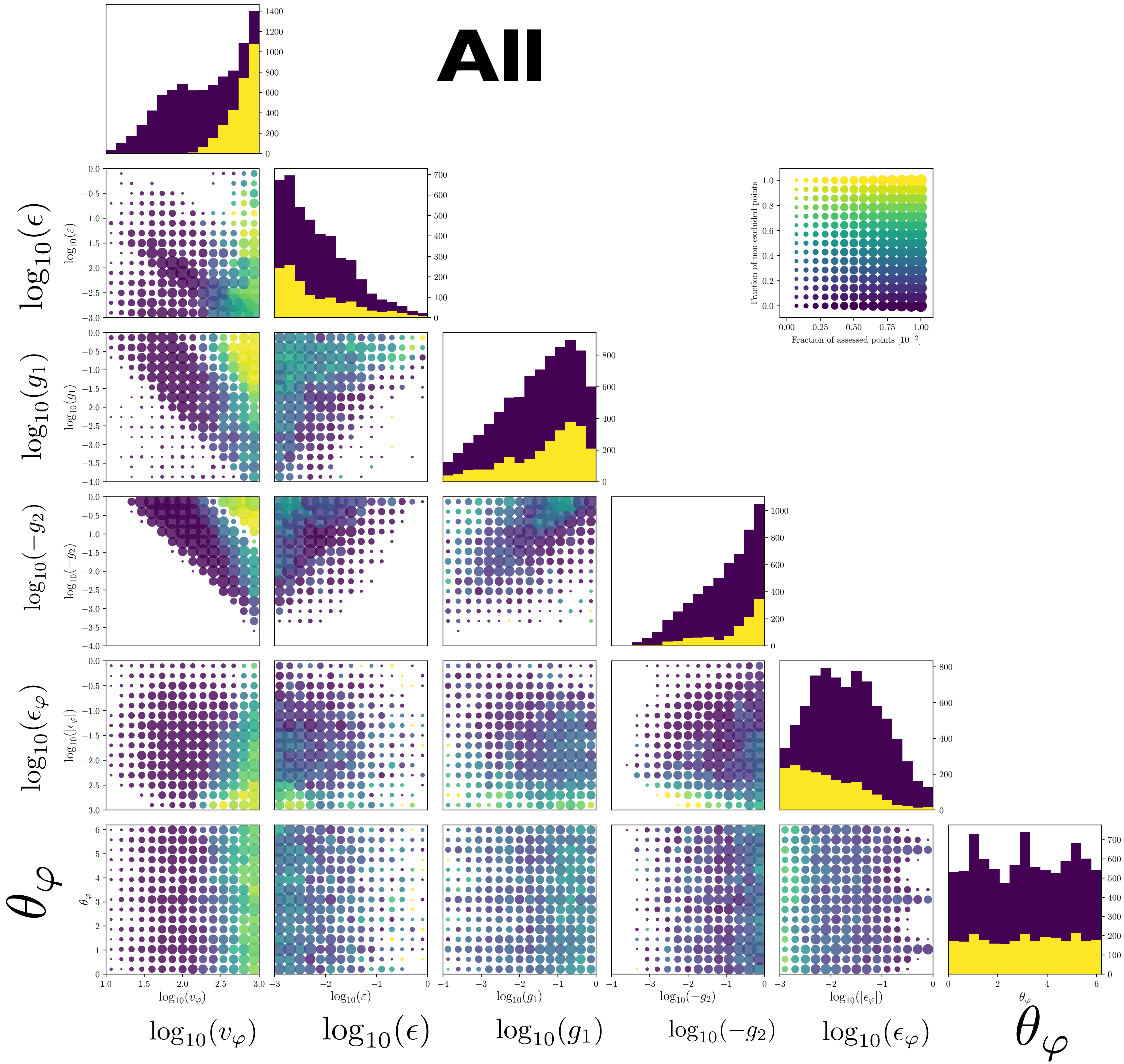
Fraction of assessed points $[10^{-2}]$

All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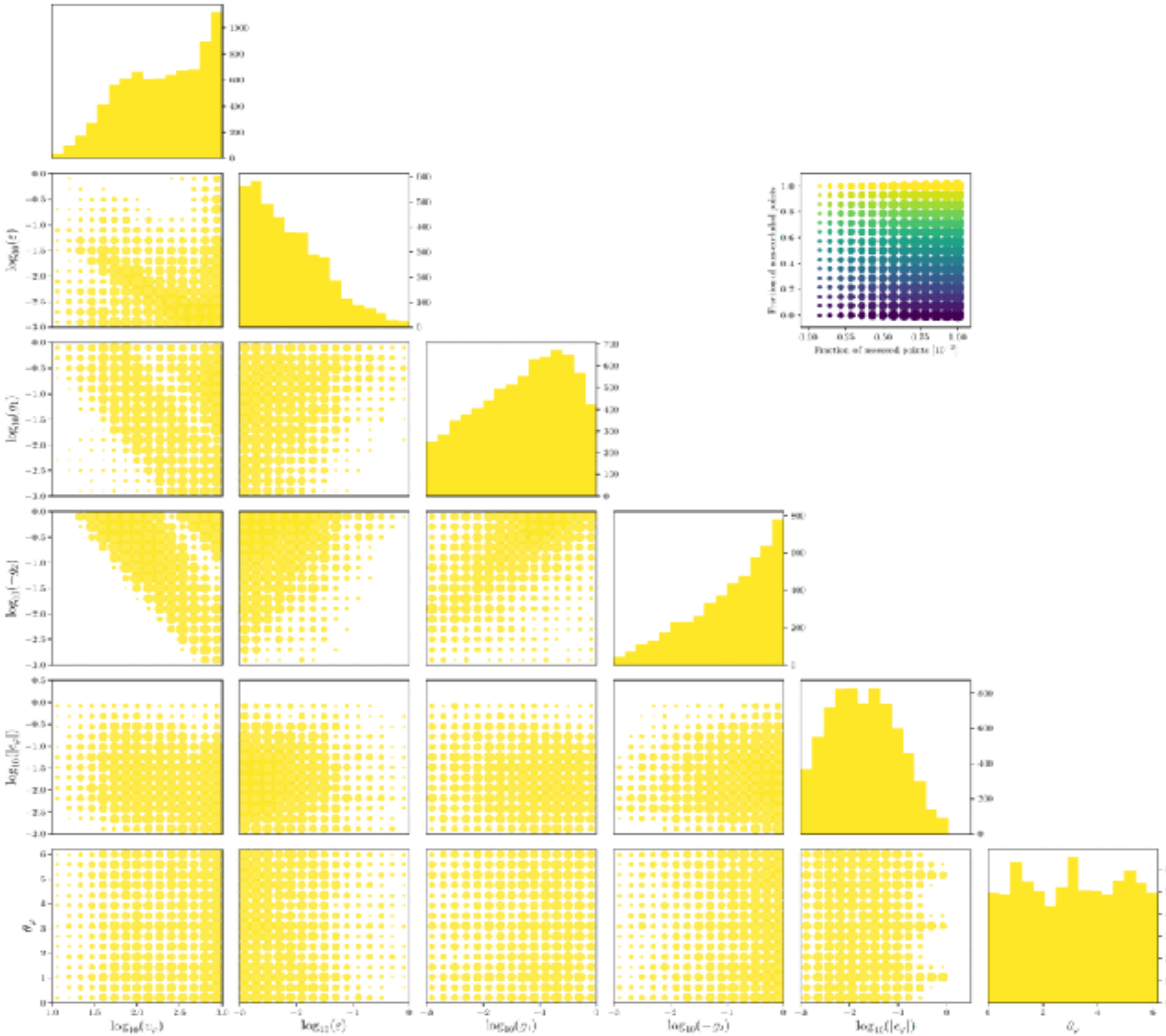
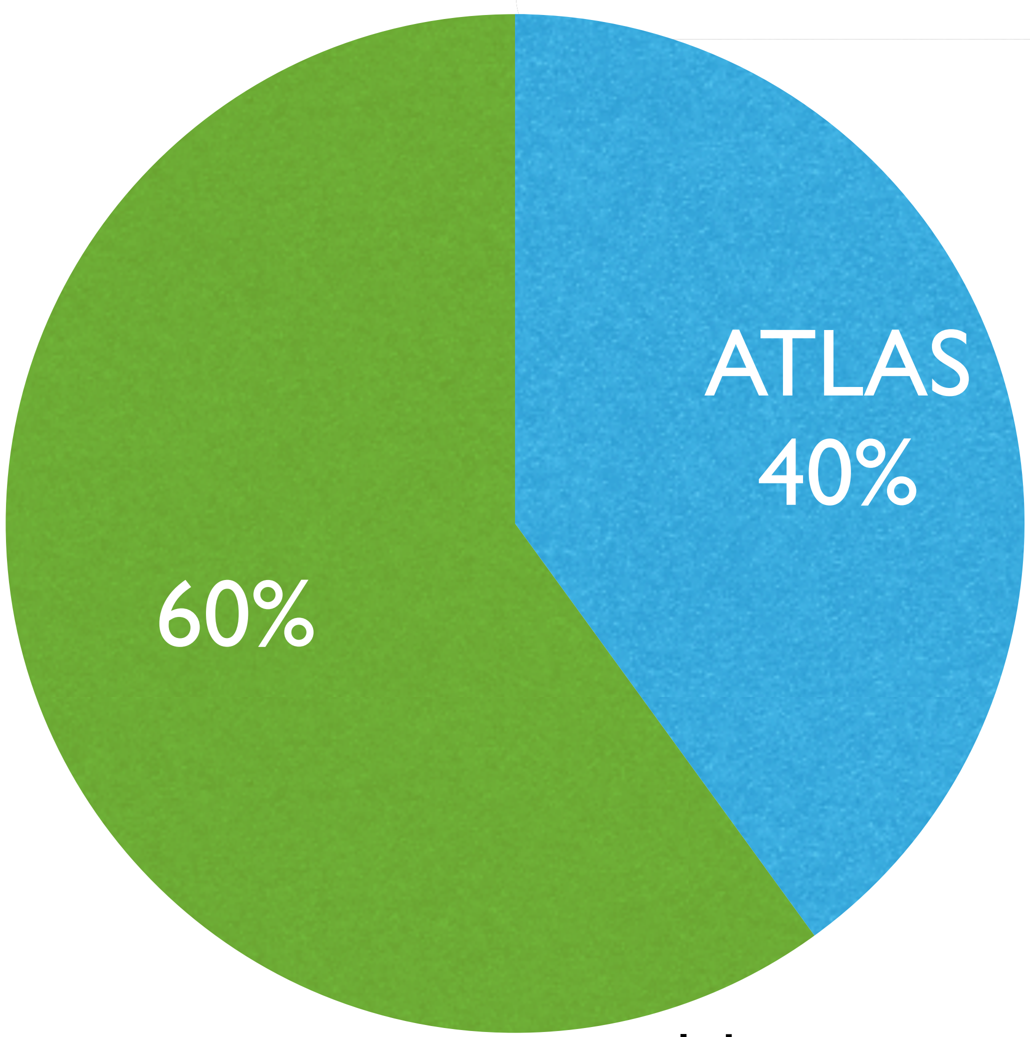
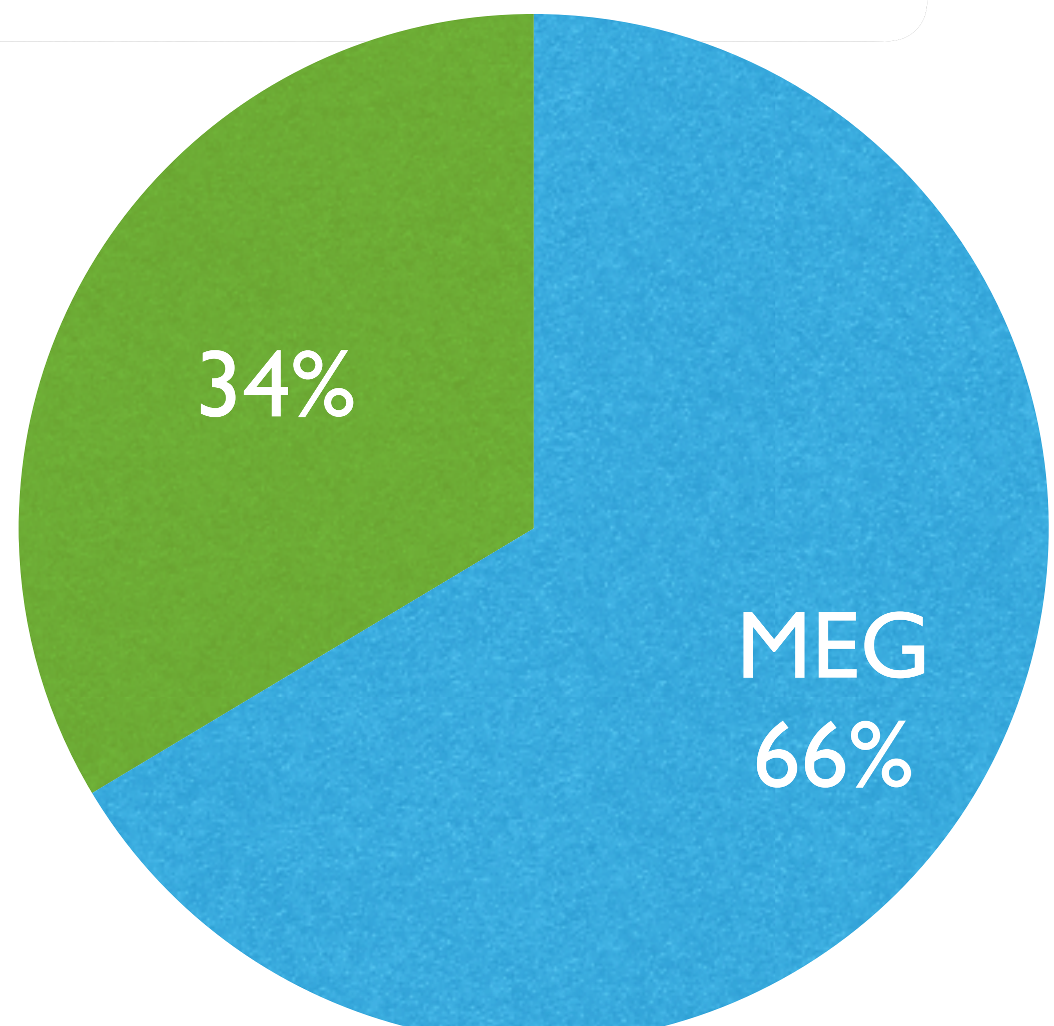


Exclusionary Power

$$\text{exclusion power} = \frac{N_{\text{tot}} - N_{\text{pass}}}{N_{\text{tot}}}$$

Experimental data	Exclusion power [%]
MEG	65.6
ATLAS	40.0
Higgs-width	6.0
Higgs-mixing	1.7
$g-2$	0.7





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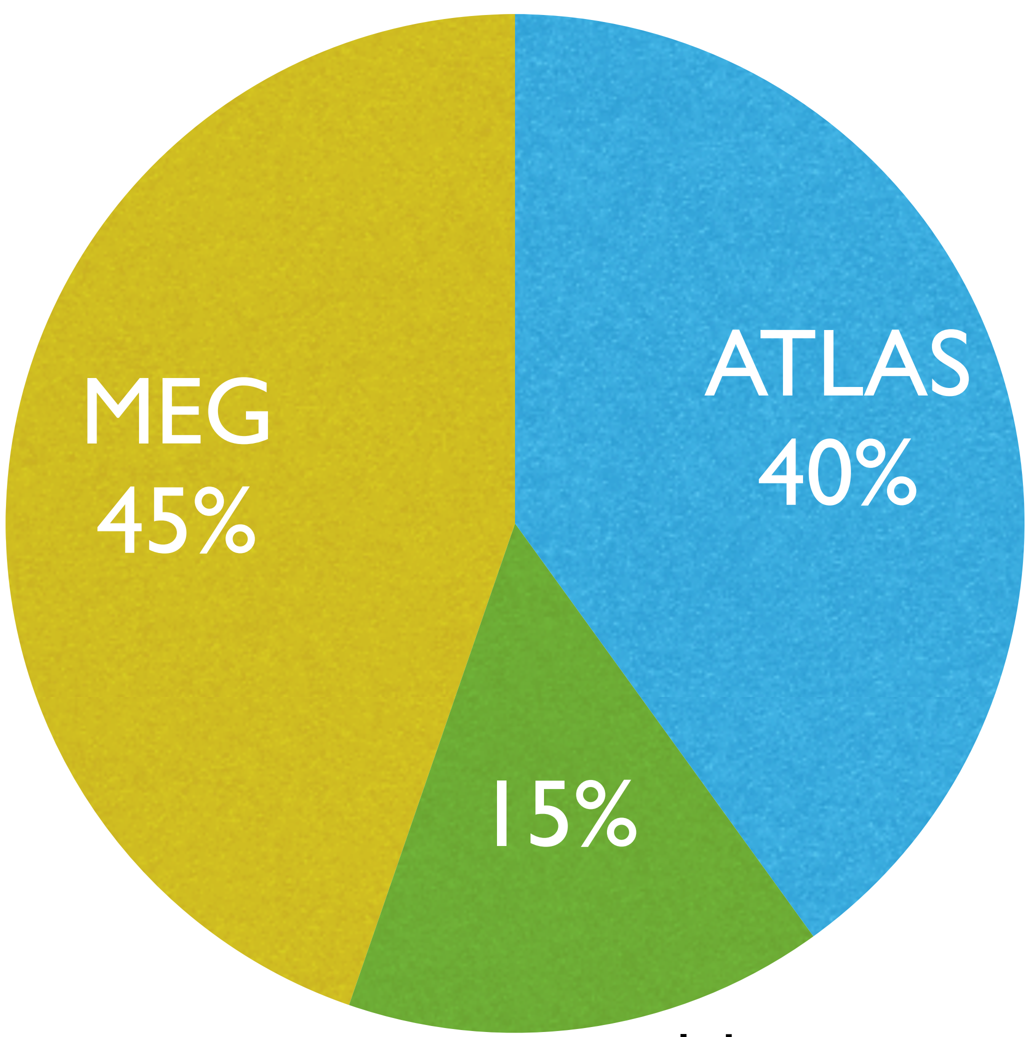
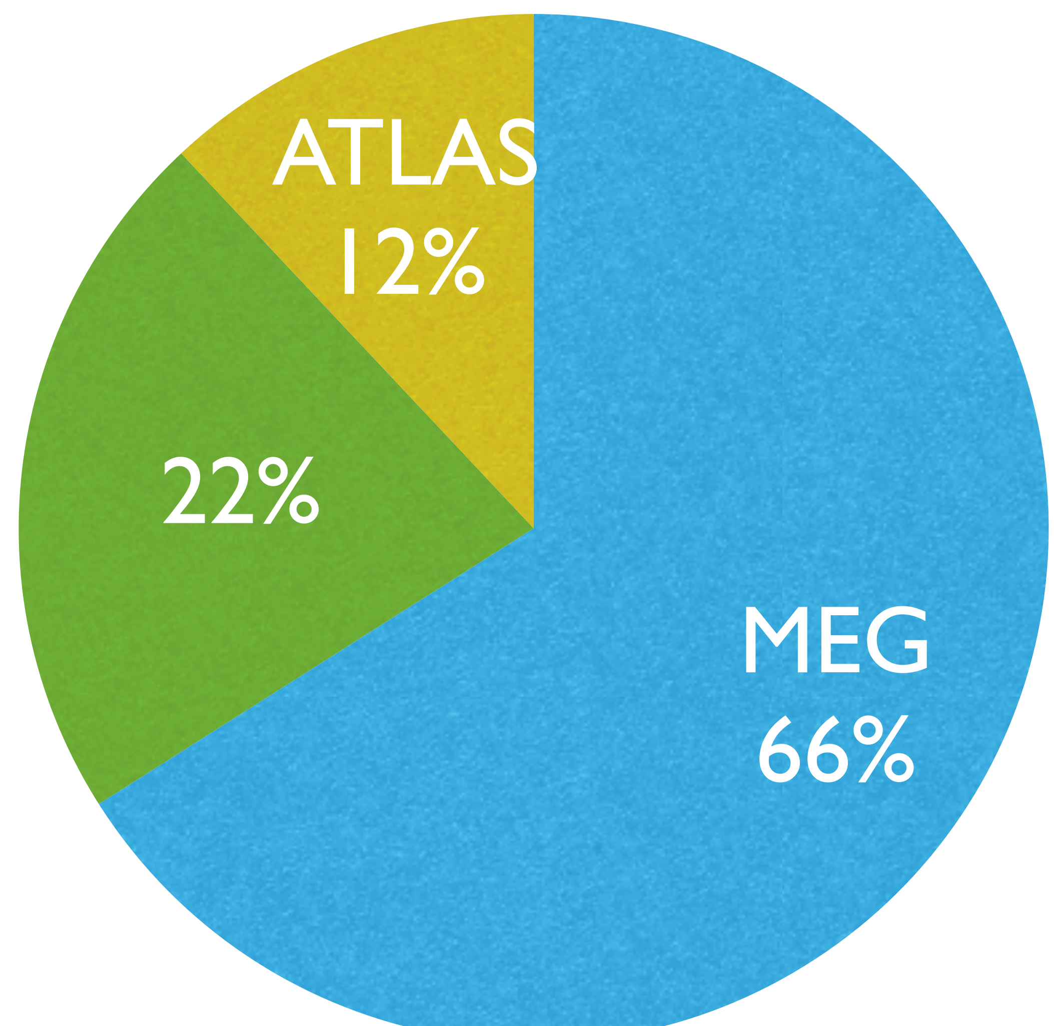




Conclusions

- A priori it is not clear the flavour breaking scale should be close to the GUT scale. Can we exclude a lower value of this scale?
- Experiments such as MEG place highly competitive constraints on flavour model P.S (we were skeptical the collider would be able to compete!)
- We demonstrated **collider searches** for high multiplicity leptonic final states **can compete and complement** MEG and g-2 experimental constraints.
- Why? The collider has sensitivity to flavon coupling to Higgs, MEG and g-2 are not.
- The chosen model P.S is largely excluded through synergy of these experiments.

Thank you!

Back-up Slides

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \qquad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$\begin{aligned}
(ab)_{\mathbf{1}} &= a_1b_1 + a_2b_3 + a_2b_3 \\
(ab)_{\mathbf{1}'} &= a_3b_3 + a_1b_2 + a_2b_1 \\
(ab)_{\mathbf{1}''} &= a_2b_2 + a_1b_3 + a_3b_1 \\
(ab)_{\mathbf{3}_S} &= \frac{1}{2}\begin{pmatrix} 2a_1b_1 - a_2b_3 - a_3b_2 \\ 2a_3b_3 - a_1b_2 - a_2b_1 \\ 2a_2b_2 - a_3b_1 - a_1b_3 \end{pmatrix}, (ab)_{\mathbf{3}_A} = \frac{1}{2}\begin{pmatrix} a_2b_3 - a_3b_2 \\ a_1b_2 - a_2b_1 \\ a_3b_1 - a_1b_3 \end{pmatrix}.
\end{aligned}$$

Back-up Slides

Minimise the flavon and Higgs potential

$$\mu_H^2 + \lambda v_H^2 + \frac{1}{2}\epsilon v_\varphi^2(1+2|\epsilon_\varphi|^2) = 0\,,$$

$$\mu_\varphi^2 + g_1 v_\varphi^2(1+2|\epsilon_\varphi|^2) + \frac{1}{3}g_2 v_\varphi^2[1-\mathrm{Re}(\epsilon_\varphi^3)] + \frac{1}{2}\epsilon v_H^2 + A\epsilon_\varphi^* + A^*\epsilon_\varphi = 0\,,$$

$$\mu_\varphi^2\epsilon_\varphi + g_1 v_\varphi^2(1+2|\epsilon_\varphi|^2)\epsilon_\varphi + \frac{1}{2}g_2 v_\varphi^2[-\epsilon_\varphi^{*2} + |\epsilon_\varphi|^2\epsilon_\varphi] + \frac{1}{2}\epsilon\epsilon_\varphi v_H^2 + A + A^*\epsilon_\varphi^* = 0\,.$$

$$A\epsilon_\varphi^* + A^*\epsilon_\varphi^{*2} + 2\mathrm{Re}(A^*\epsilon_\varphi)|\epsilon_\varphi|^2 = \underbrace{-\frac{1}{2}g_2 v_\varphi^2\epsilon_\varphi^{*3} + \frac{1}{3}g_2 v_\varphi^2|\epsilon_\varphi|^2\left[1-\mathrm{Re}(\epsilon_\varphi^3) - \frac{3}{2}|\epsilon_\varphi|^2\right]}_{x}$$

$$A = \frac{\left(\epsilon_\varphi^*\right)^2 x^* - \epsilon_\varphi\left(x + 2i\,|\epsilon_\varphi|^2\,\Im\,[x]\right)}{|\epsilon_\varphi|^2\left(-\,|\epsilon_\varphi|^2 + {\epsilon_\varphi^*}^3 + \epsilon_\varphi^3 - 1\right)}.$$

Back-up Slides

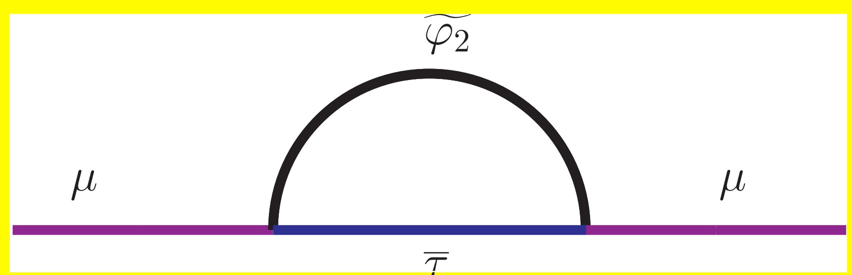
$$
\begin{aligned}
(M^2_{\tilde{\Phi}})_{11} &= 2\lambda v_H^2 \,, \\
(M^2_{\tilde{\Phi}})_{22} &= 2g v_\varphi^2 + \frac{1}{3} g_2 v_\varphi^2 \mathrm{Re}(\epsilon_\varphi^3) - 2\mathrm{Re}(A\epsilon_\varphi^*) \,, \\
(M^2_{\tilde{\Phi}})_{33} &= -\frac{1}{3} g_2 v_\varphi^2 [1 - \mathrm{Re}(\epsilon_\varphi^3)] + \frac{1}{2} g_2 v_\varphi^2 |\epsilon_\varphi|^2 - 2\mathrm{Re}(A\epsilon_\varphi^*) + \mathrm{Re}\left(-g_2 v_\varphi^2 (\epsilon_\varphi^* - \frac{1}{2}\epsilon_\varphi^2) + 2g_1 v_\varphi^2 \epsilon_\varphi^2 + A^*\right) \,, \\
(M^2_{\tilde{\Phi}})_{44} &= -\frac{1}{3} g_2 v_\varphi^2 [1 - \mathrm{Re}(\epsilon_\varphi^3)] + \frac{1}{2} g_2 v_\varphi^2 |\epsilon_\varphi|^2 - 2\mathrm{Re}(A\epsilon_\varphi^*) - \mathrm{Re}\left(-g_2 v_\varphi^2 (\epsilon_\varphi^* - \frac{1}{2}\epsilon_\varphi^2) + 2g_1 v_\varphi^2 \epsilon_\varphi^2 + A^*\right) \,, \\
(M^2_{\tilde{\Phi}})_{12} &= v_H v_\varphi \epsilon \,, \\
(M^2_{\tilde{\Phi}})_{13} &= \sqrt{2} v_H v_\varphi \epsilon \mathrm{Re}(\epsilon_\varphi) \,, \\
(M^2_{\tilde{\Phi}})_{14} &= \sqrt{2} v_H v_\varphi \epsilon \mathrm{Im}(\epsilon_\varphi) \,, \\
(M^2_{\tilde{\Phi}})_{23} &= \sqrt{2}\mathrm{Re}\left(2g_1 v_\varphi^2 \epsilon_\varphi - \frac{1}{2} g_2 v_\varphi^2 \epsilon_\varphi^{*2} + A\right) \,, \\
(M^2_{\tilde{\Phi}})_{24} &= \sqrt{2}\mathrm{Im}\left(2g_1 v_\varphi^2 \epsilon_\varphi - \frac{1}{2} g_2 v_\varphi^2 \epsilon_\varphi^{*2} + A\right) \,, \\
(M^2_{\tilde{\Phi}})_{34} &= \mathrm{Im}\left(-g_2 v_\varphi^2 (\epsilon_\varphi^* - \frac{1}{2}\epsilon_\varphi^2) + 2g_1 v_\varphi^2 \epsilon_\varphi^2 + A^*\right) \,, \qquad (2.19)
\end{aligned}
$$

Diagonalise mass matrix ensuring (1,1) entry is the Higgs mass

Relating gauge to mass basis

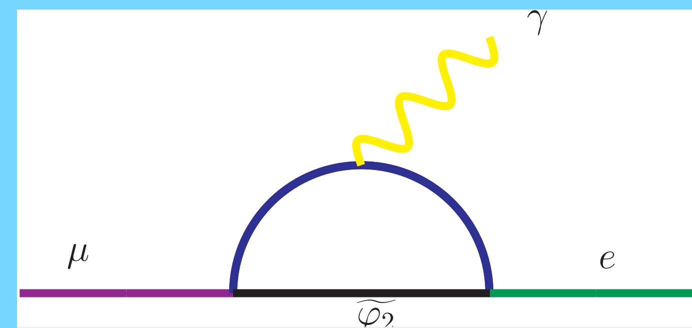
$$\begin{pmatrix} \tilde{h} \\ \tilde{\varphi}_1 \\ \sqrt{2}\mathrm{Re}\left(\varphi_2\right) \\ \sqrt{2}\mathrm{Im}\left(\varphi_2\right) \end{pmatrix} = \begin{pmatrix} W_{00} & W_{01} & W_{02} & W_{03} \\ W_{10} & W_{11} & W_{12} & W_{13} \\ W_{20} & W_{21} & W_{22} & W_{23} \\ W_{30} & W_{31} & W_{32} & W_{33} \end{pmatrix} \begin{pmatrix} h \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

g-2 Constraint



$$\Delta a_\mu = \frac{m_\mu^2 m_\tau^2}{24\pi^2 v_\varphi^2}\Big[\frac{\left(|W_{13}|^2 - |W_{14}|^2\right)}{m_h^2} + \frac{\left(|W_{23}|^2 - |W_{24}|^2\right)}{m_{s_1}^2} + \frac{\left(|W_{33}|^2 - |W_{34}|^2\right)}{m_{s_2}^2} + \frac{\left(|W_{43}|^2 - |W_{44}|^2\right)}{m_{s_3}^2}\Big].$$

μ→eγ Constraint



$$A(h) = \frac{1}{128\pi^2}\frac{1}{m_h^2 v_\varphi^2}\left[m_\mu m_\tau^2 G_2\left(\frac{m_\tau^2}{m_H^2}\right)(W_{13}+iW_{14})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2\left(\frac{m_\tau^2}{m_H^2}\right)\left(|W_{13}|^2+|W_{14}|^2\right)\right],$$

$$A(s_1) = \frac{1}{128\pi^2}\frac{1}{m_{s_1}^2 v_\varphi^2}\left[m_\mu m_\tau^2 G_2\left(\frac{m_\tau^2}{m_1^2}\right)(W_{23}+iW_{24})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2\left(\frac{m_\tau^2}{m_1^2}\right)\left(|W_{23}|^2+|W_{24}|^2\right)\right],$$

$$A(s_2) = \frac{1}{128\pi^2}\frac{1}{m_{s_2}^2 v_\varphi^2}\left[m_\mu m_\tau^2 G_2\left(\frac{m_\tau^2}{m_2^2}\right)(W_{33}+iW_{34})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2\left(\frac{m_\tau^2}{m_2^2}\right)\left(|W_{33}|^2+|W_{34}|^2\right)\right],$$

$$A(s_3) = \frac{1}{128\pi^2}\frac{1}{m_{s_3}^2 v_\varphi^2}\left[m_\mu m_\tau^2 G_2\left(\frac{m_\tau^2}{m_3^2}\right)(W_{43}+iW_{44})^2 - m_\mu m_\tau^2 \epsilon_\varphi^* G_2\left(\frac{m_\tau^2}{m_3^2}\right)\left(|W_{43}|^2+|W_{44}|^2\right)\right].$$

$$G_2(x) = -\log x - \frac{11}{6}$$

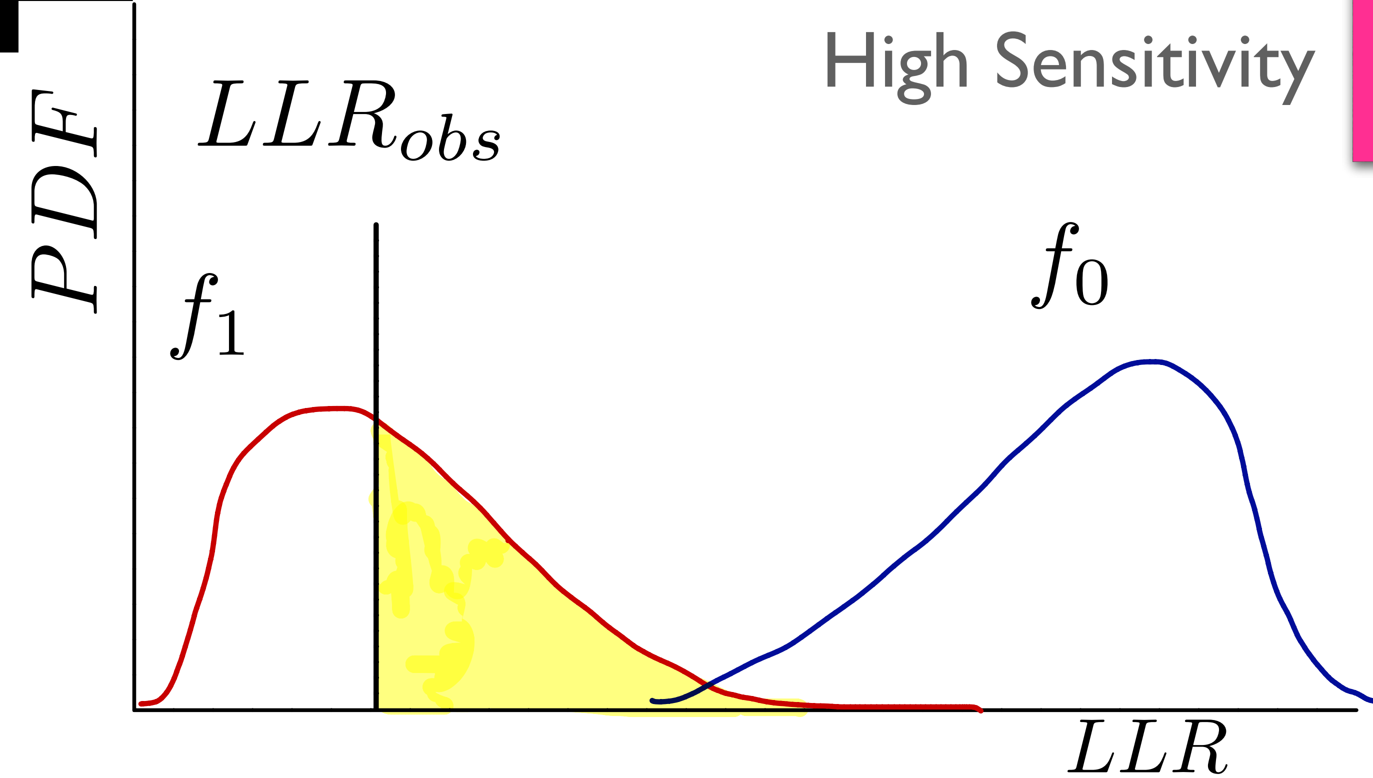
$$\Gamma(\mu \to e\gamma) = \frac{m_\mu^3 |A|^2}{16\pi}, \quad \Gamma(\mu \to e\overline{\nu}_e \nu_\mu \gamma) = \frac{G_F^2 m_\mu^5}{192\pi^3},$$

CLs Method for Recast

PDF generated through possible fluctuations (Asimov data set) 1007.1727

Calculated using PyHF:

https://github.com/diana-hep/pyhf



$$1 - CL_b \equiv \int_{-\infty}^{LLR_{obs}} f_0(LLR)dLLR$$

$$CL_{s+b} \equiv \int_{LLR_{obs}}^{\infty} f_1(LLR)dLLR$$

$$CL_s = \frac{CL_{s+b}}{CL_b}$$

Frequentist is CL_{s+b} only

CLs Method for Recast

PDF generated through possible fluctuations (Asimov data set) 1007.1727

High Sensitivity

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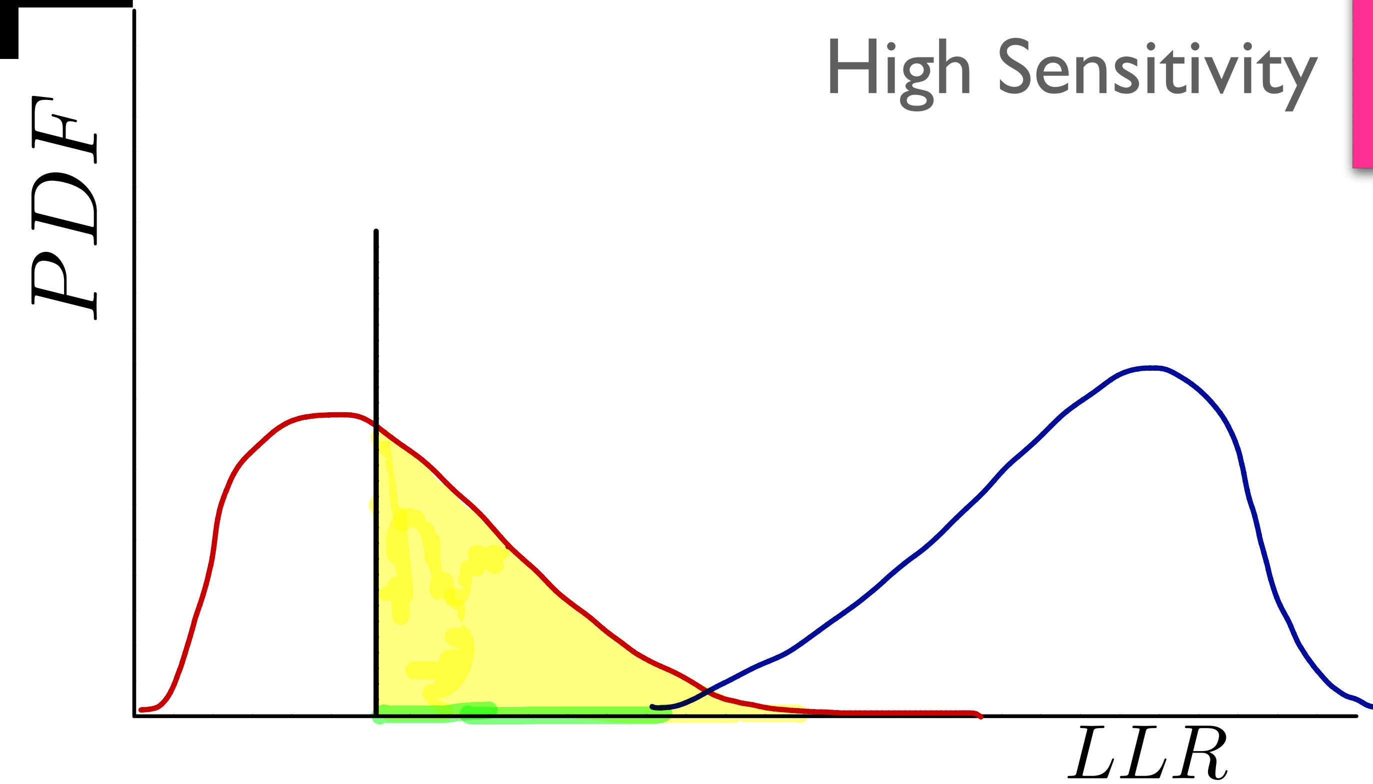

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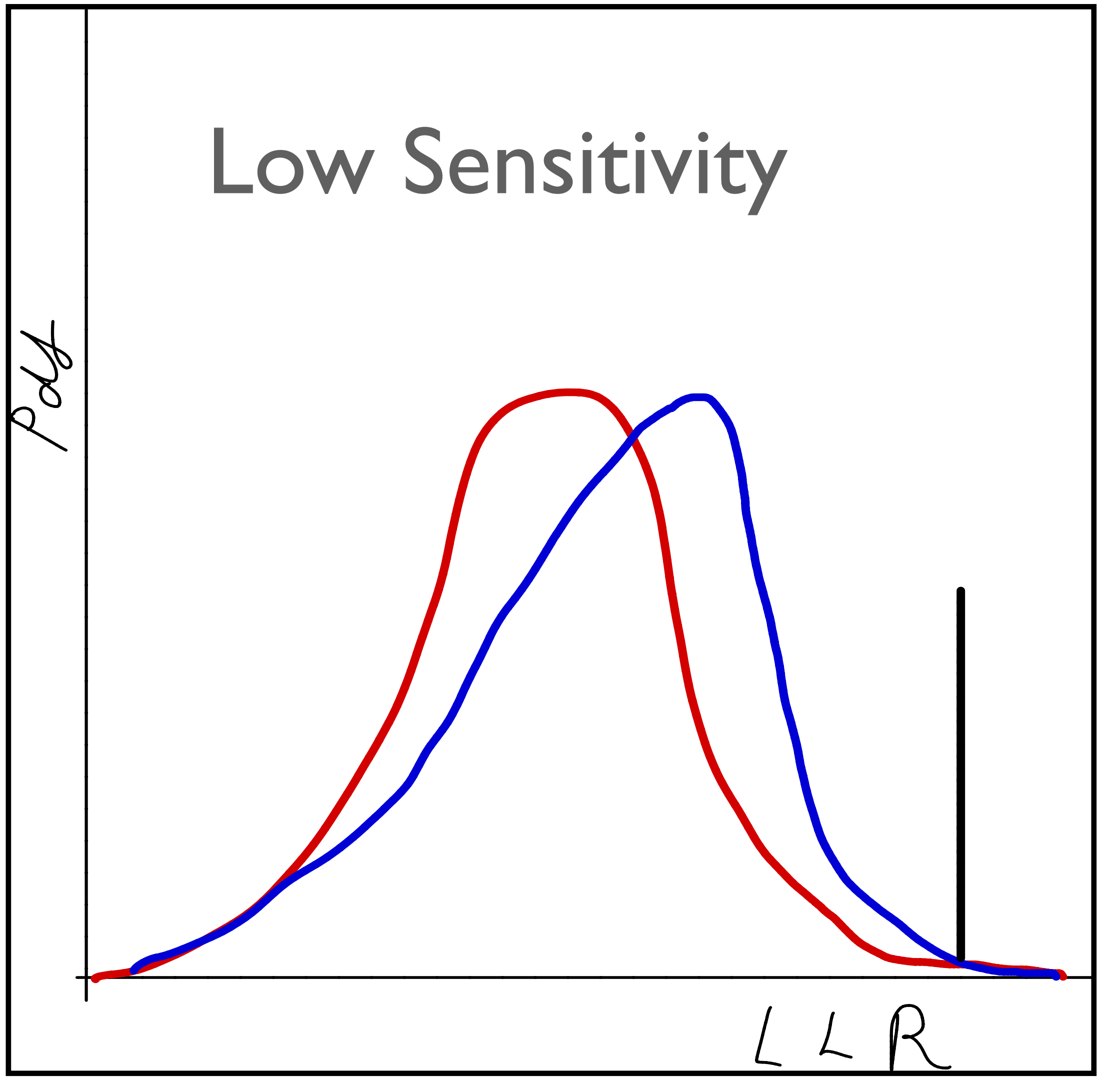
Frequentist is CL_{s+b} only

$CL_S > 0.05$, H_1 is not excluded 95% C.L.

CL_S Method for Recast



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CL_S is conservative against overestimating exclusionary power in case of low signal sensitivity

CL_b becomes small therefore CL_s becomes large and H_1 cannot be excluded

$$1 - CL_b \equiv \int_{-\infty}^{LLR_{obs}} f_0(LLR) dLLR \qquad CL_{s+b} \equiv \int_{LLR_{obs}}^{\infty} f_1(LLR) dLLR$$

$$CL_s = \frac{CL_{s+b}}{CL_b}$$

Frequentist is CL_{s+b} only

$CL_S < 0.05$, H_1 is excluded 95% C.L.