Energy Condition, Modular Flow, and AdS/CFT

University of Michigan, Ann Arbor. Feb 20, 2019

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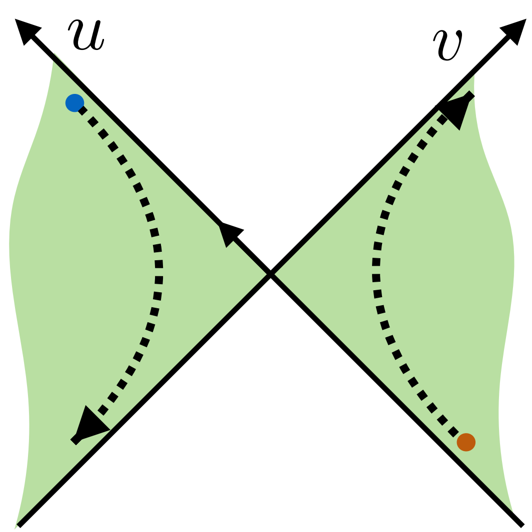
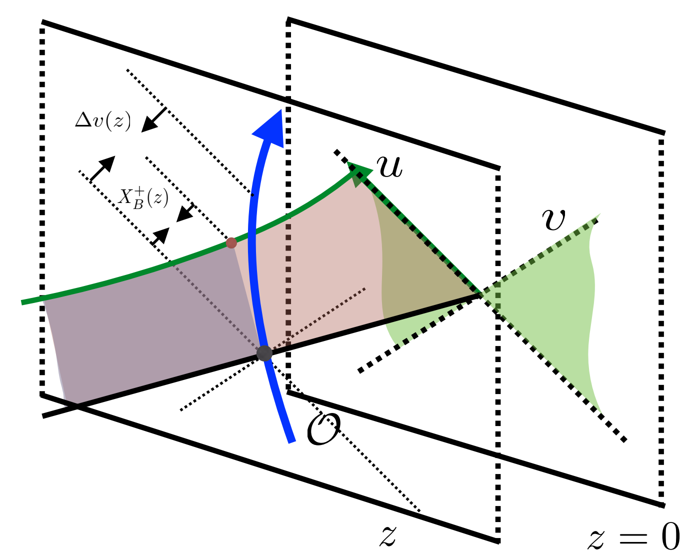
Kavli Institute for Theoretical Physics





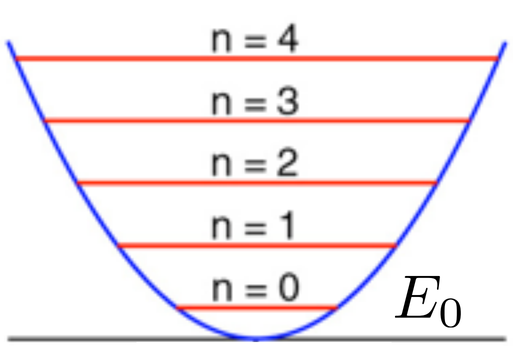
arXiv:1806.10560; arXiv:1706.09432

S. Balakrishnan, T. Faulkner, M. Li, Z. Khandker, H. Wang

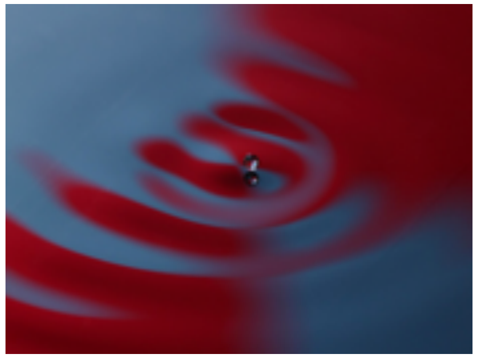
Energy Conditions

What are they?

- stability of QM: positivity of total energy
- extended systems (QFT): local energy/momentum density
- constraints on energy/momentum density



$$E \geq 0$$

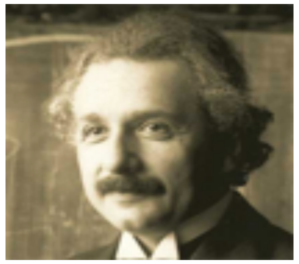


$$E = \int_{\mathcal{R}^n} dx^n \; \mathcal{E}(x) \geq 0$$

Energy Conditions

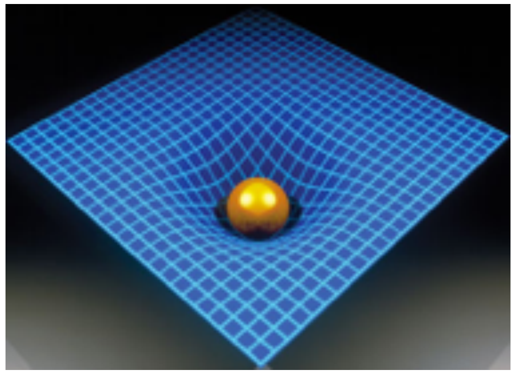
Why do we care?

- classical: important in general relativity
- energy-momentum = spacetime geometry
- energy conditions = constraints on spacetime



Einstein's equations:

$$G_{\mu\nu} = 8\pi\, T_{\mu\nu}$$



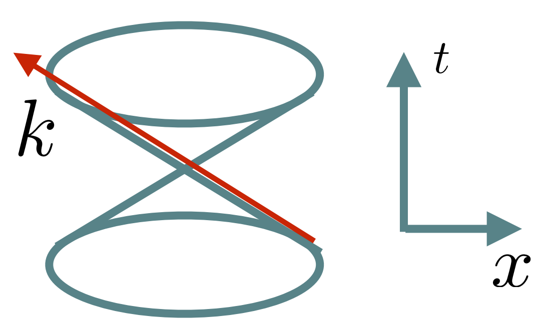
Energy Conditions

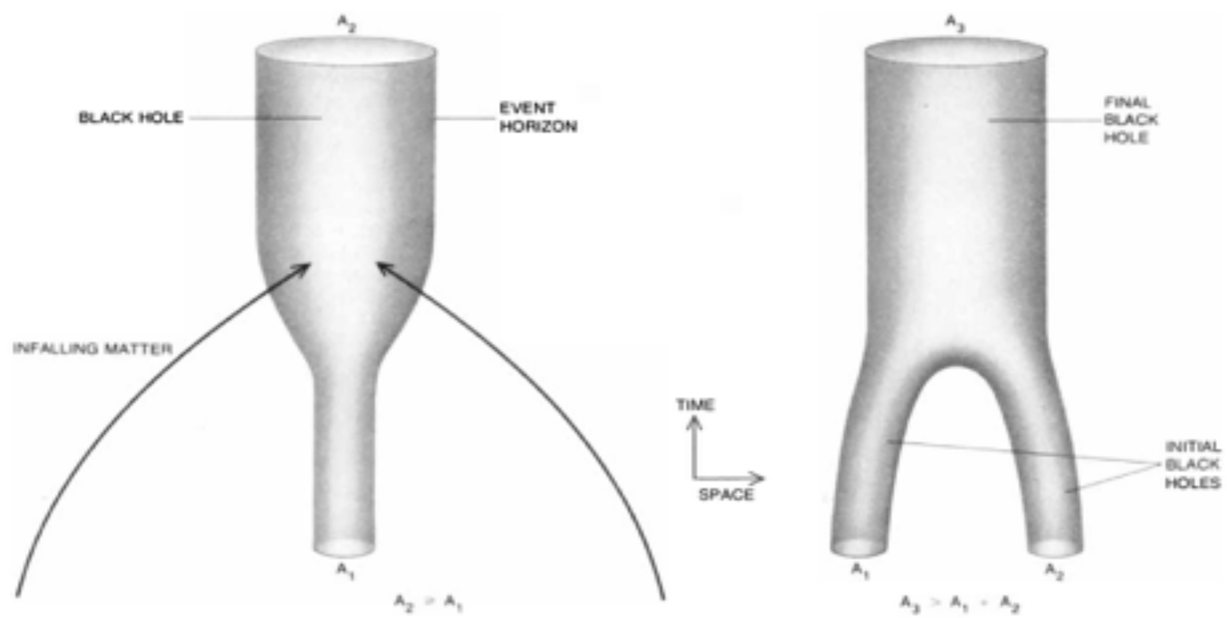
Why do we care?

examples: Hawking, Ellis, 1973

null energy condition (NEC) —> horizon area theorem

$$T_{ab}\, k^a k^b \geq 0$$





CERTAIN PROPERTIES OF BLACK HOLES suggest that there is a resemblance between the area of the event horizon of a black hole and the concept of entropy in thermodynamics. As matter and radiation continue to fall into a black hole (*space-time configuration at left*) the area of the cross section of the event horizon steadily increases. If two black holes collide and merge (*configuration at right*), the area of the cross section of the event horizon of the resulting black hole is greater than the sum of the areas of the event horizons of the initial black holes. The second law of thermodynamics says that the entropy of an isolated system always increases with passage of time.

Energy Conditions

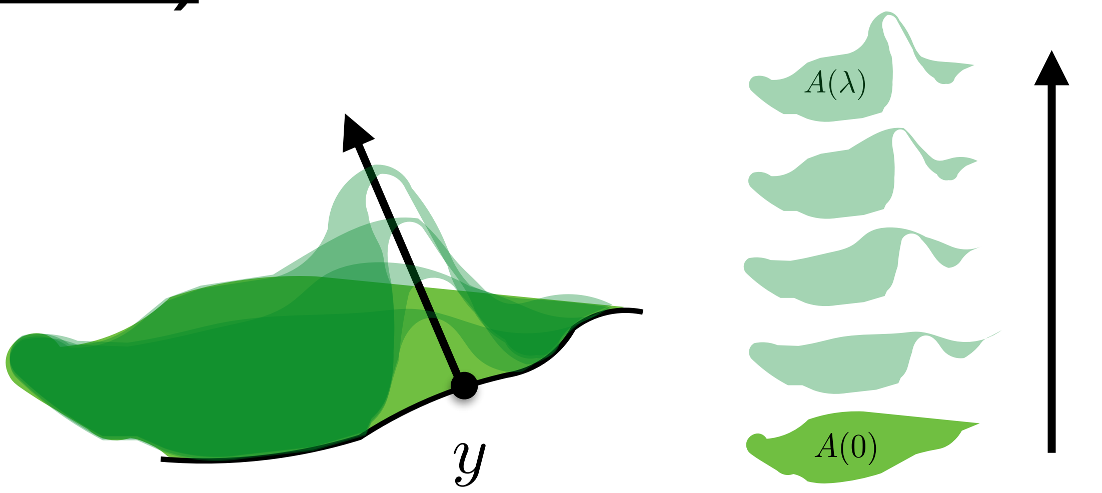
What do we want?

in QM: QFTs in fixed background spacetime

- do $\langle \hat{T}_{\mu\nu} \rangle_\psi$ satisfy NEC?
- violated by quantum effects: e.g. Casimir effect
- correct modification to NEC?

QUANTUM NULL ENERGY CONDITION (QNEC)

$$\langle \hat{T}_{\mu\nu}(y) \rangle_\psi k^\mu k^\nu \geq \partial_\lambda^2 S_{A(\lambda)}(\psi)$$



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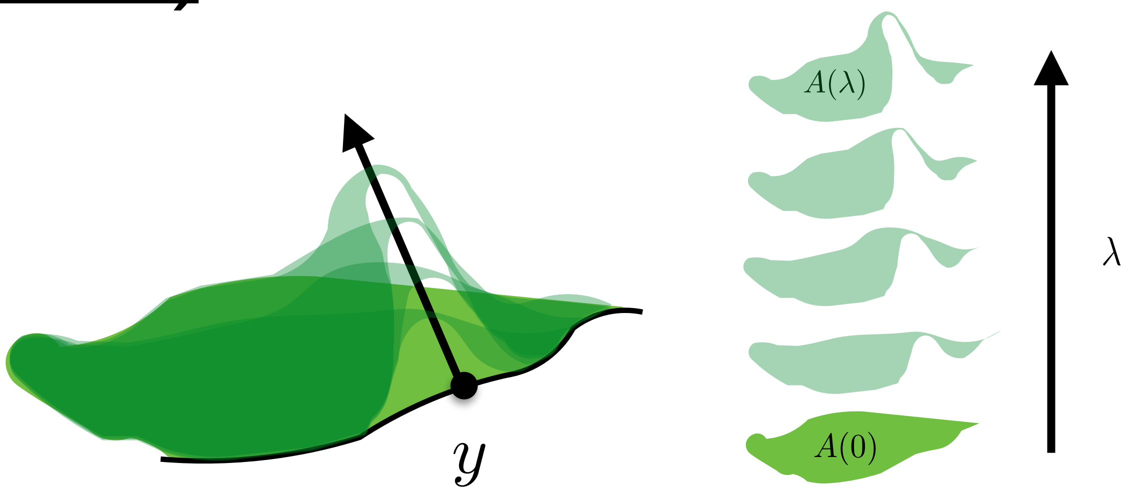
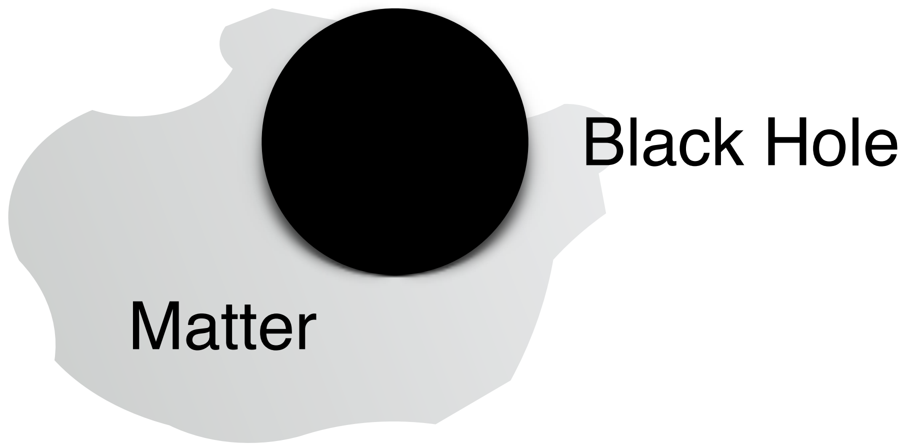


Motivation:

Generalized entropy: $S_{\text{gen}} = S_{EE} + \frac{\text{Area}}{4G}$ J. Bekenstein (1974)



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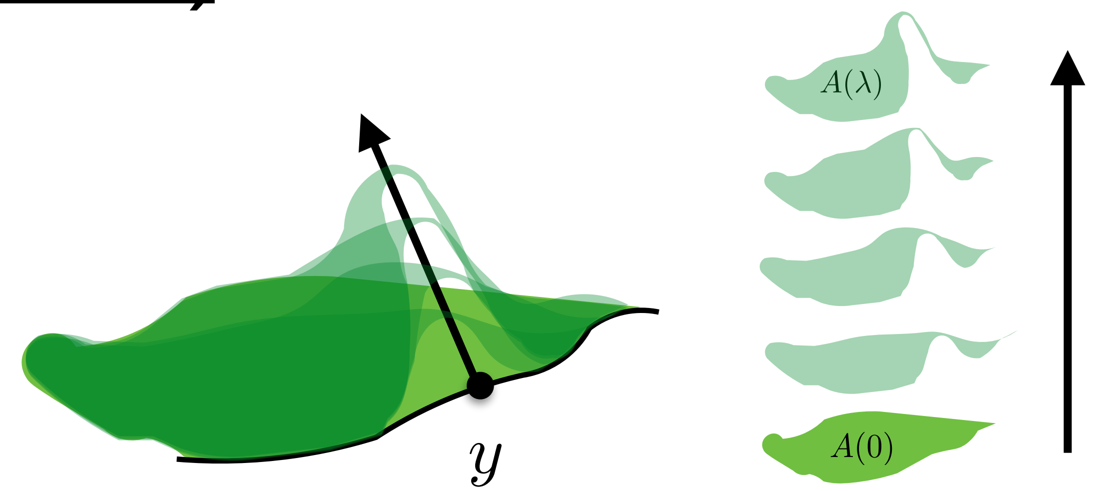

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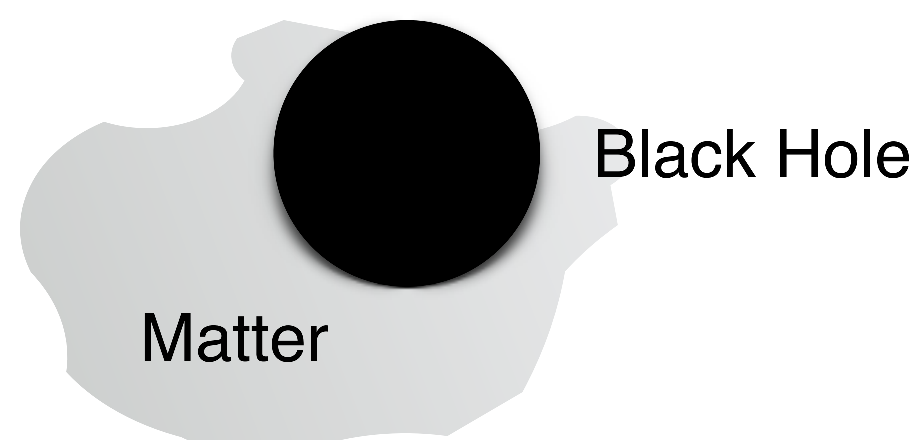
Generalized second law (GSL): $dS_{\text{gen}} \geq 0$



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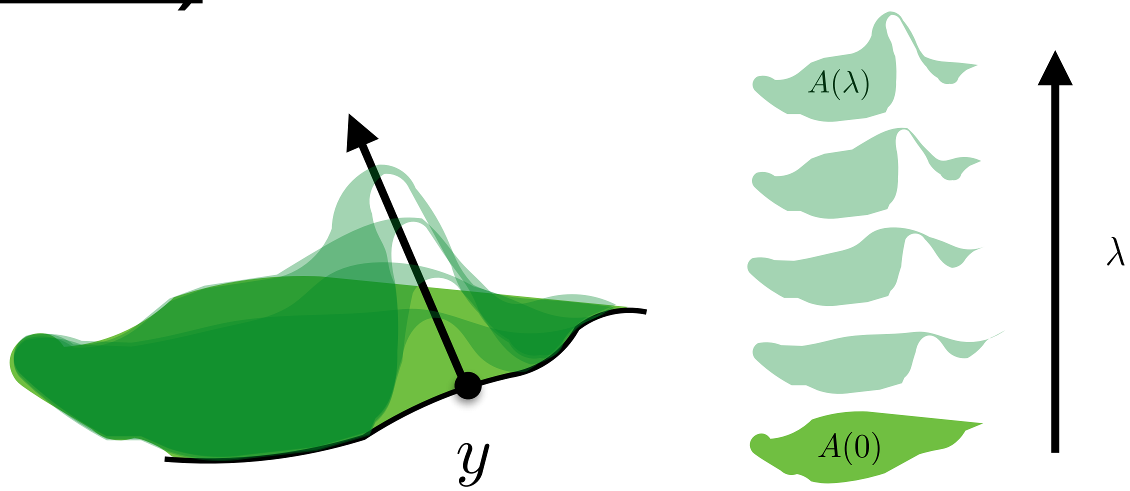

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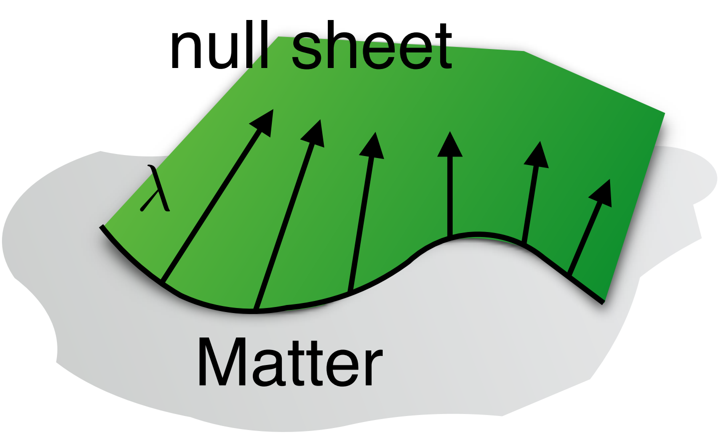
Quantum expansion: $\Theta(y, \lambda) = \lim_{A \to 0} \dfrac{4G}{A} \dfrac{dS_{\text{gen}}(y, \lambda)}{d\lambda}$



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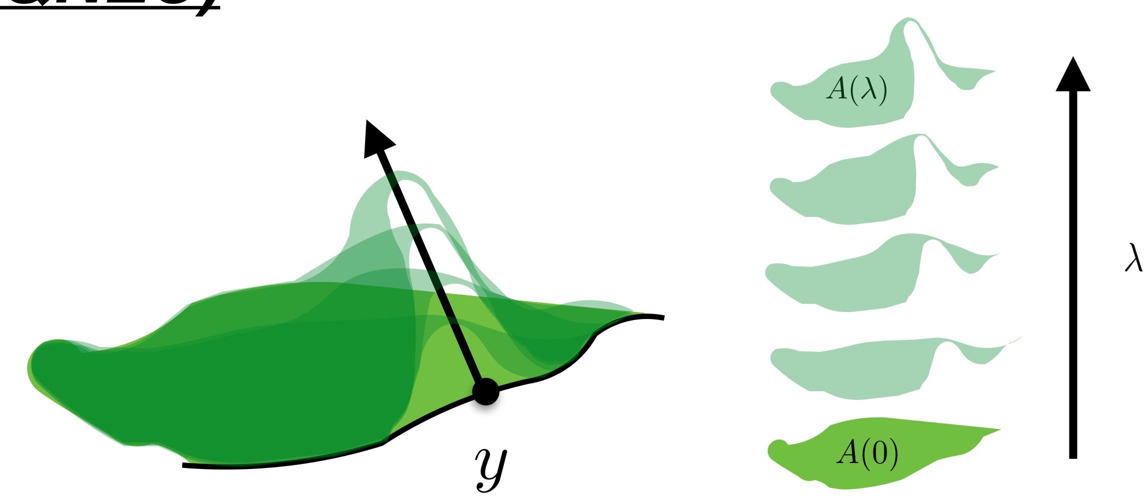

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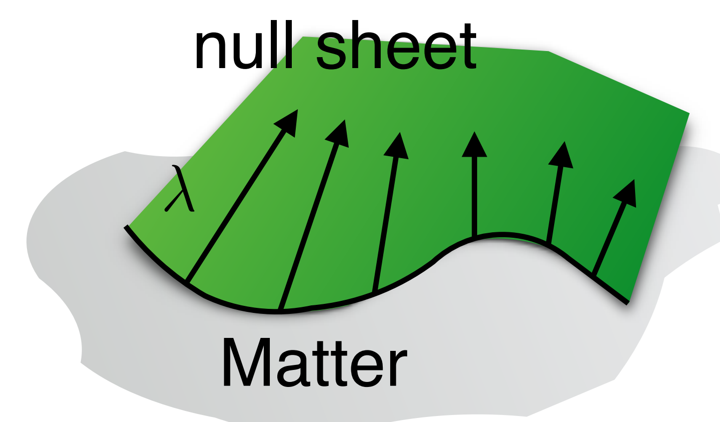
Quantum focusing conjecture (QFC): $\frac{d\Theta}{d\lambda} \leq 0$

R. Busso 2015



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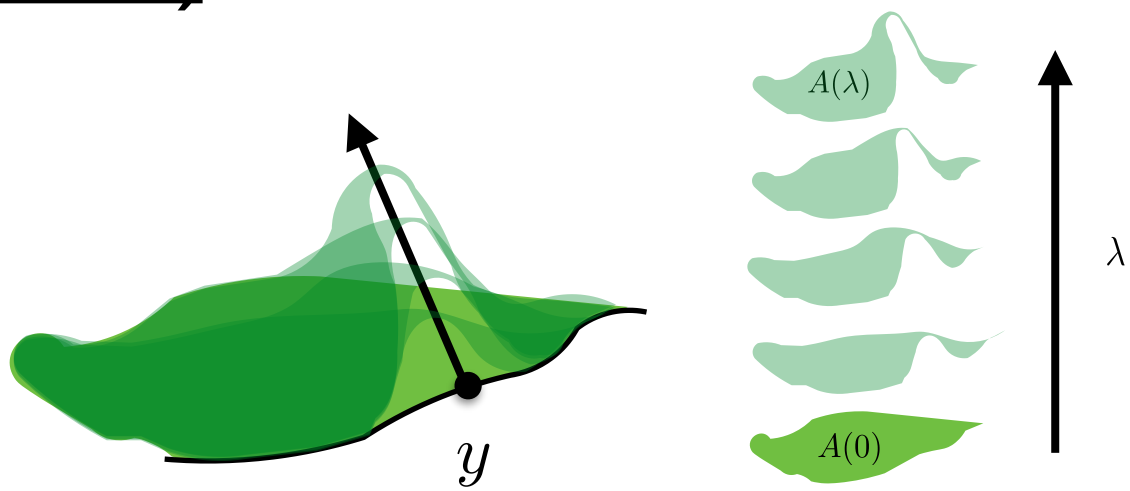

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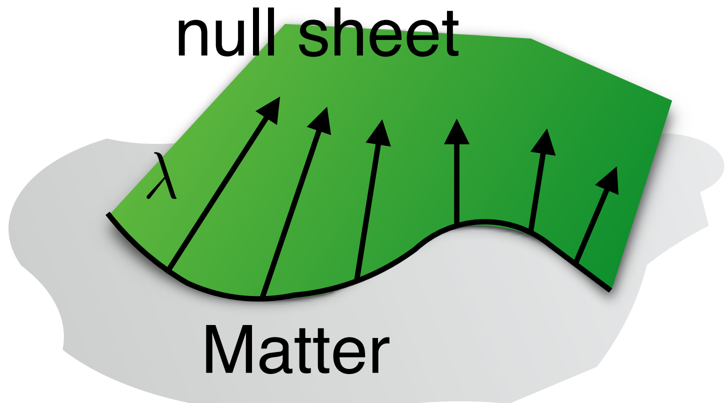

semi-classical limit $G \to 0$: QFC = QNEC.

Can we prove it in QFTs? How?

Can we prove it in QFTs? How?

- previous attempt: free or super-renormalizable QFTs R. Busso, et al (2015)
- recent progress: holographic proof in AdS/CFT J. Koeller, S. Leichenauer (2016)
- a general proof?

Plan of the talk:

- Proof in AdS/CFT (review)
- General proof in CFT
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Proving QNEC using AdS/CFT

$$\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$$

J. Koeller, S. Leichenauer, 2016; C. Akers. V. Chandrasekaran, S. Leichenaber, A. Levin, A. Moghaddam, 2017

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“bulk reconstruction on subregions”

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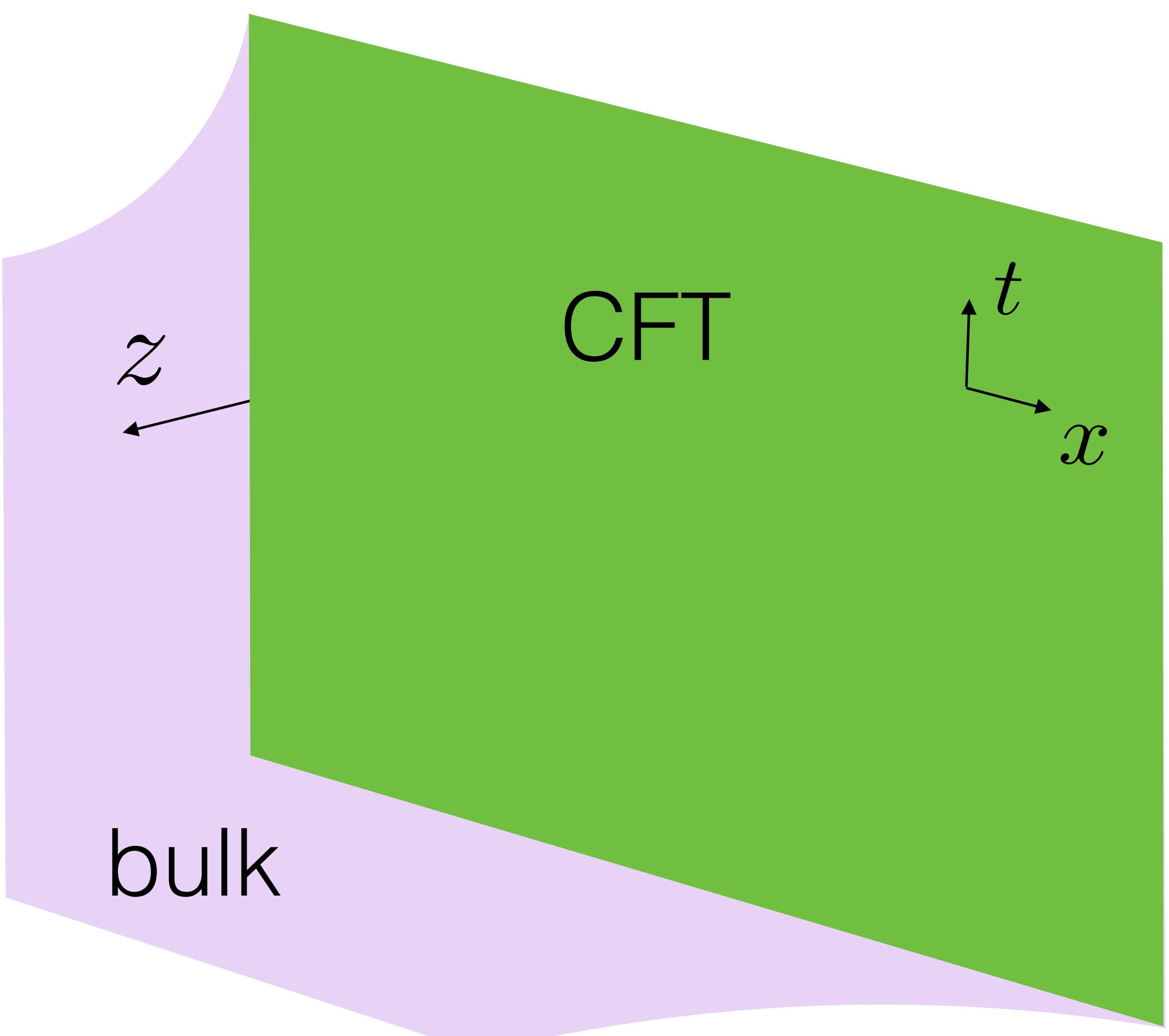

AdS/CFT: bulk physics can be "reconstructed" from the boundary

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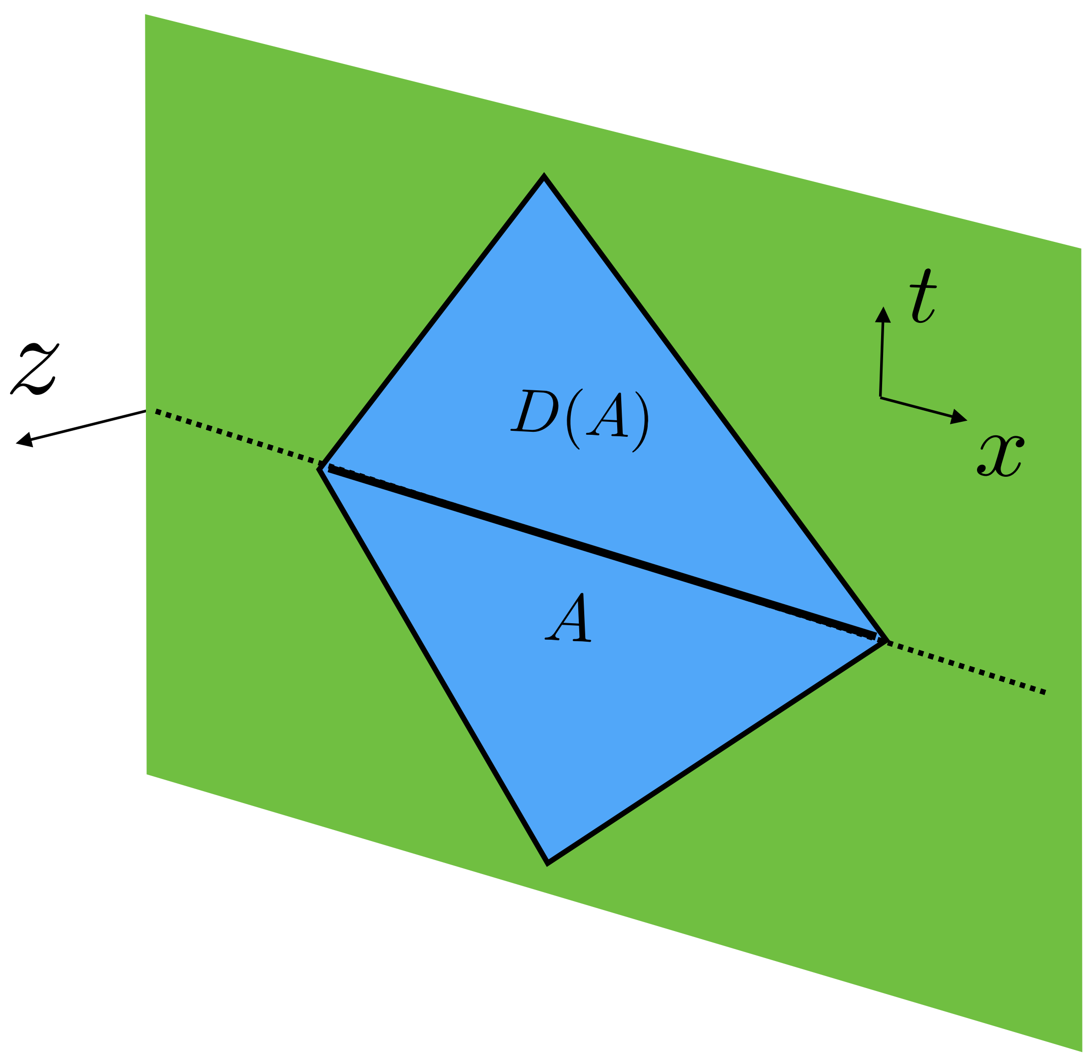

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restrict to D(A) on CFT,

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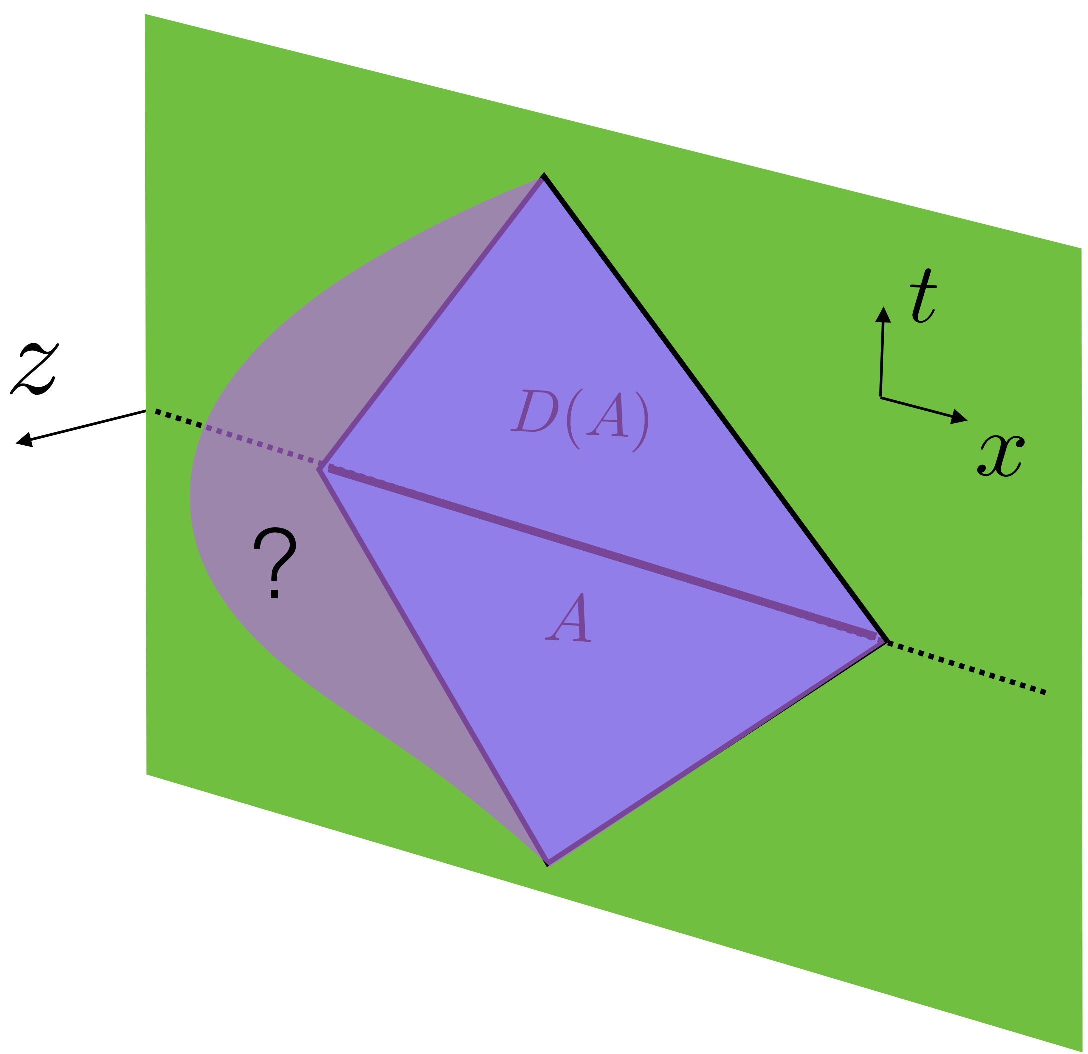

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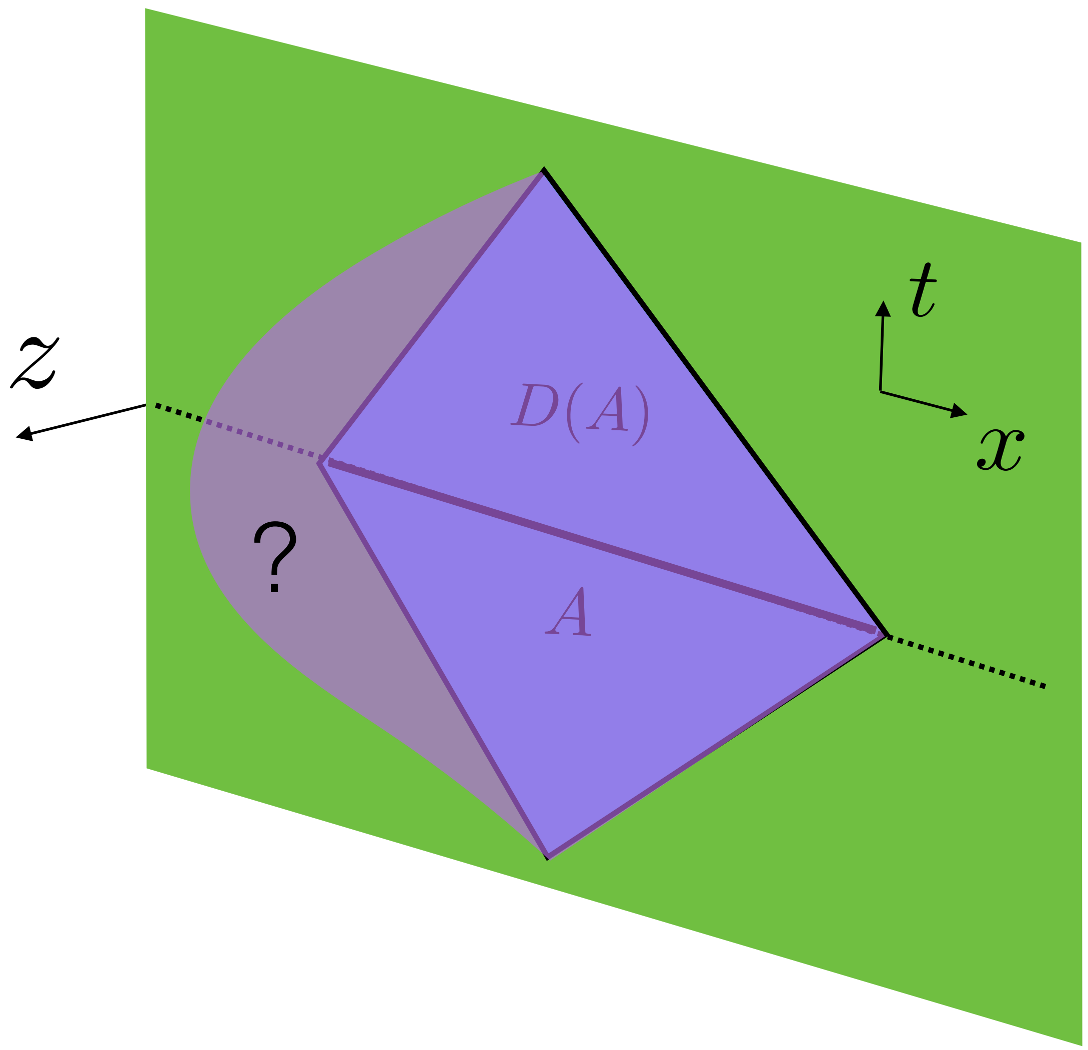

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subregion duality

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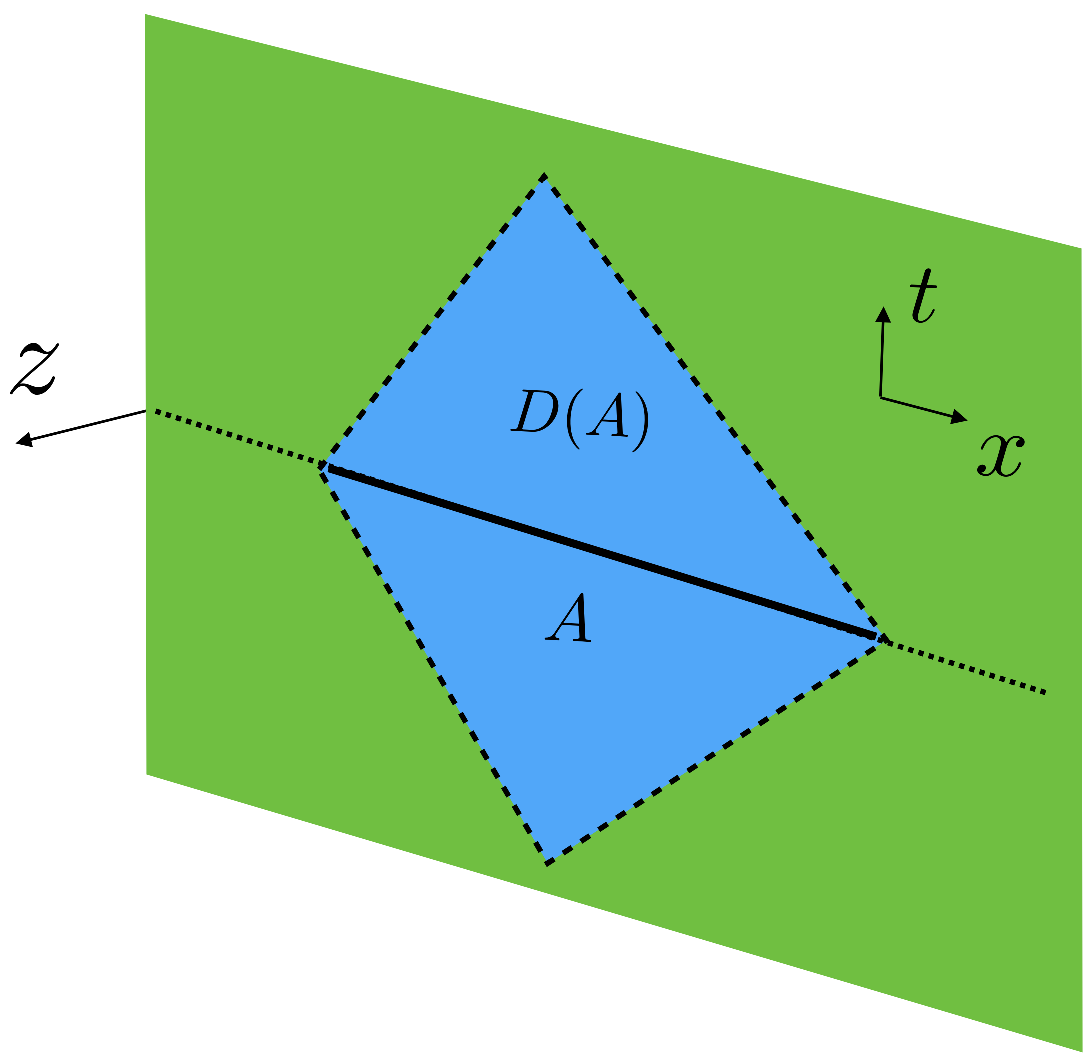

strong evidence:
entanglement wedge

X. Dong, D. Harlow, A. Wall, 2016

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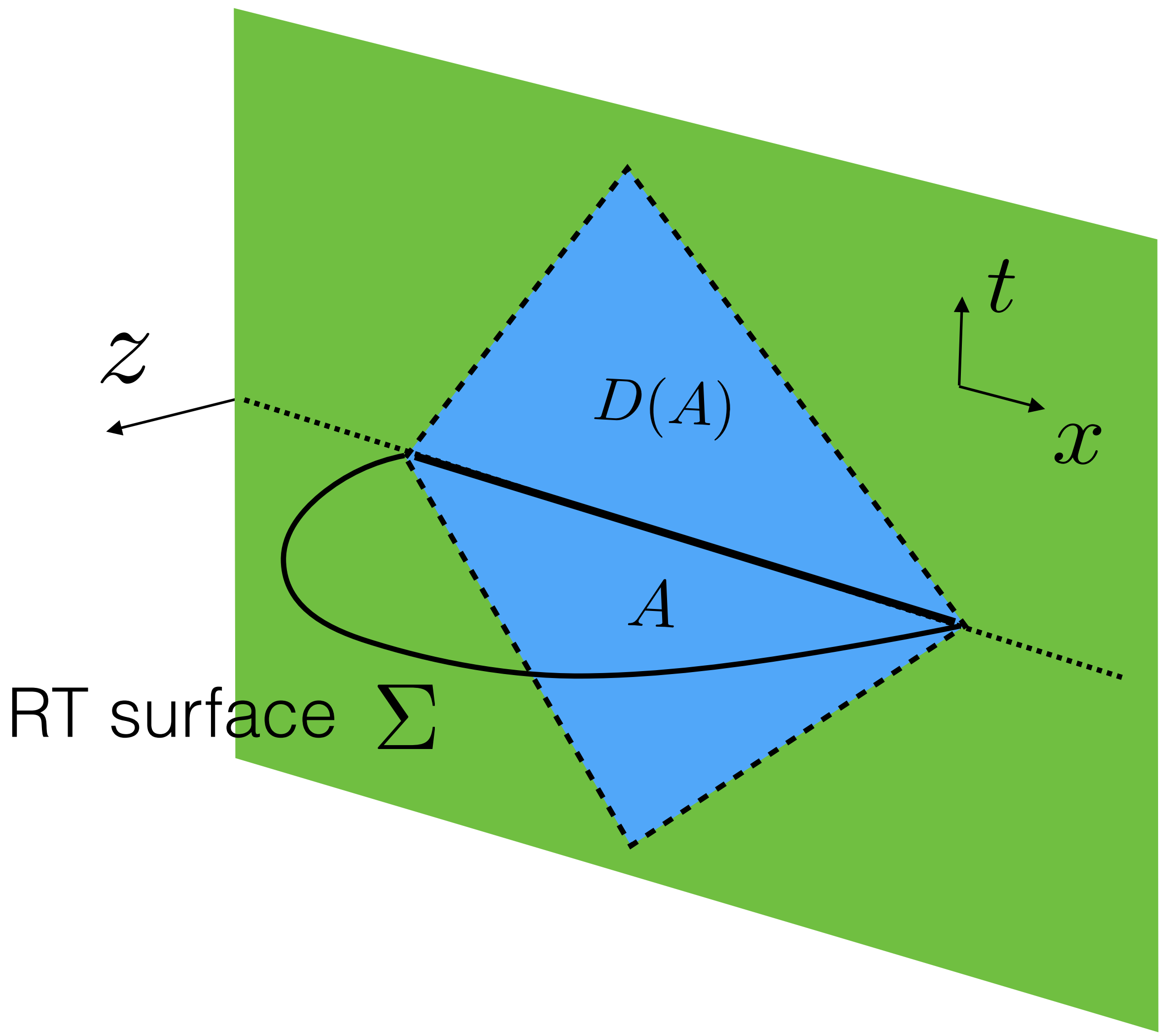

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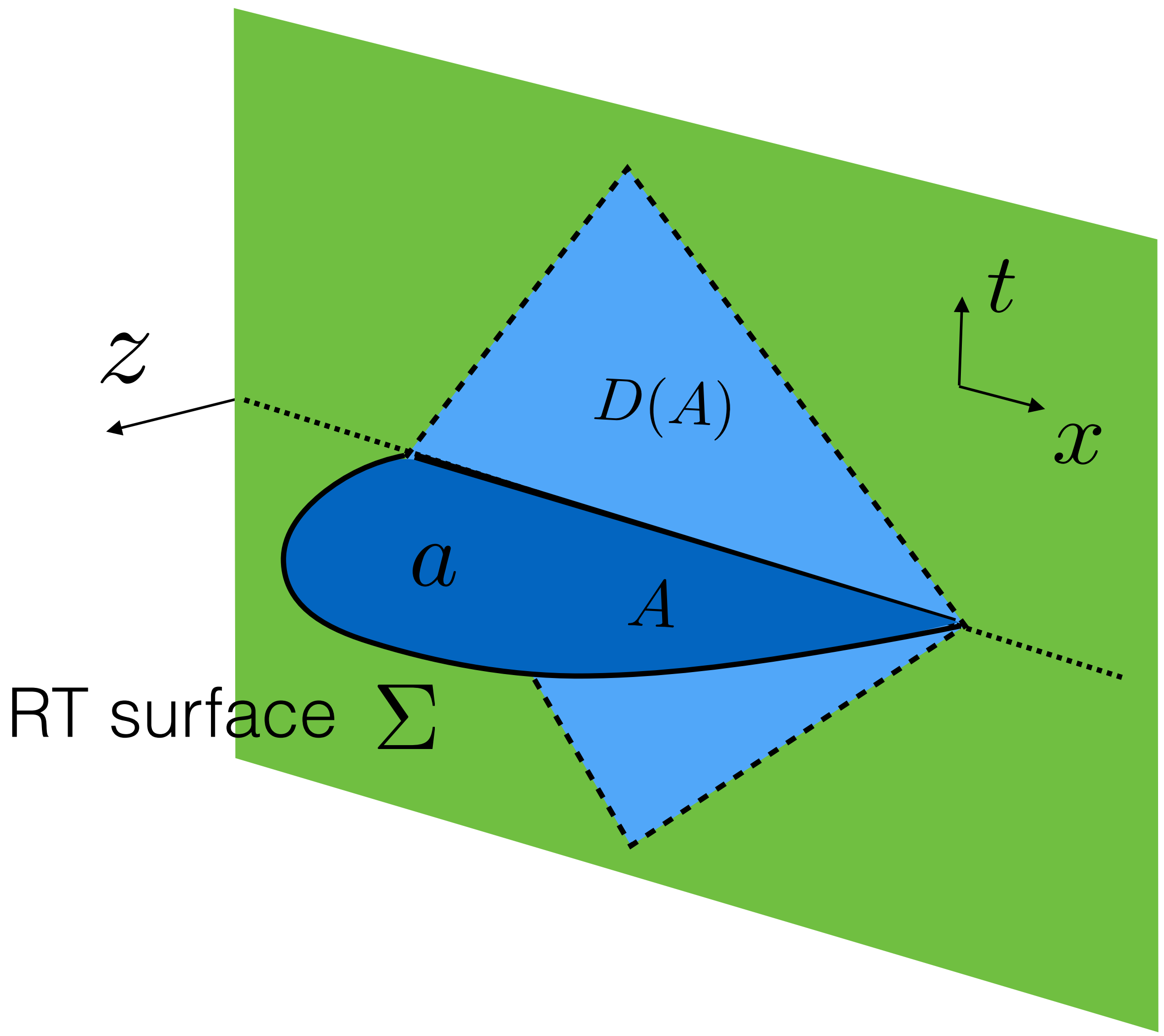

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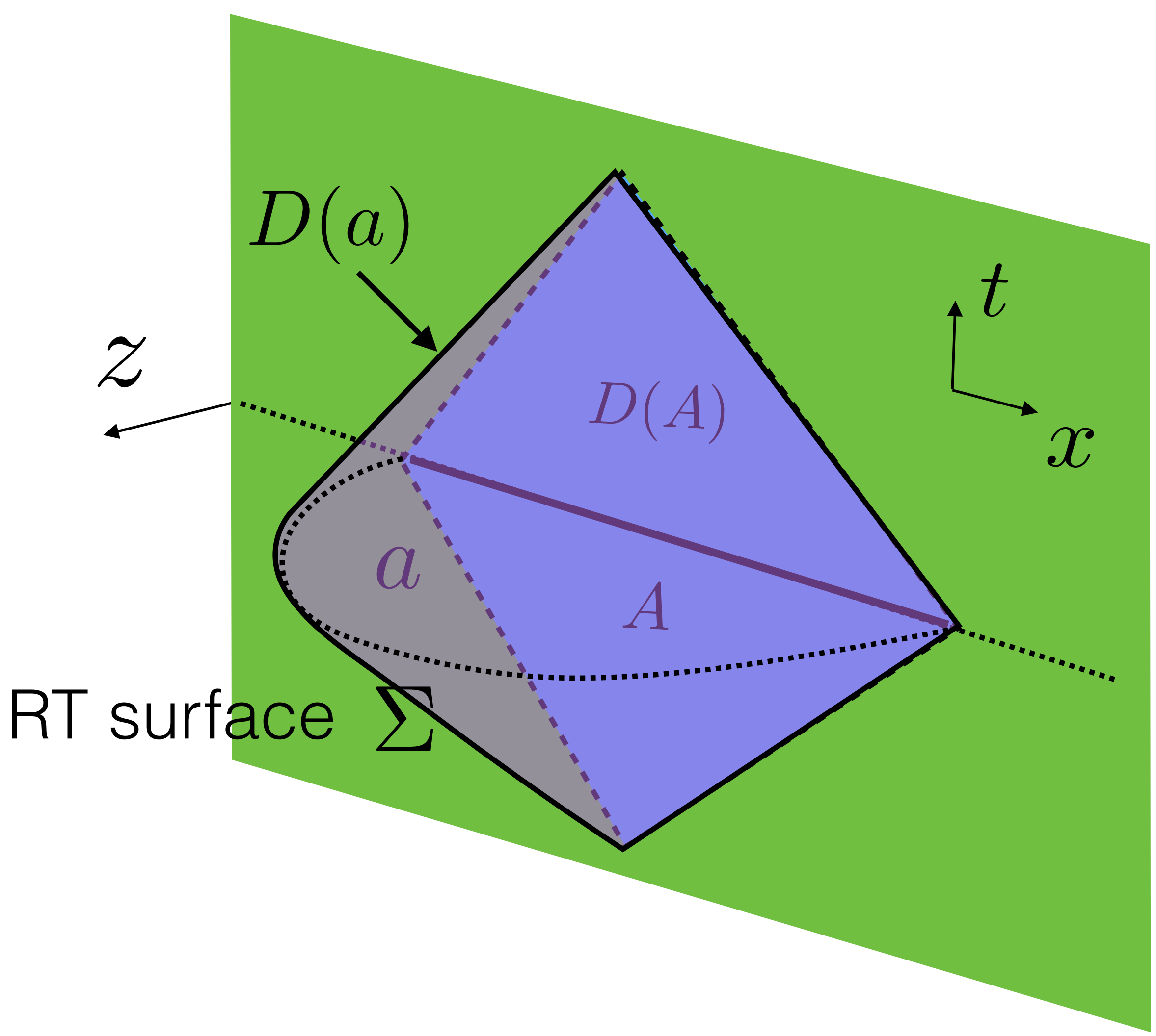

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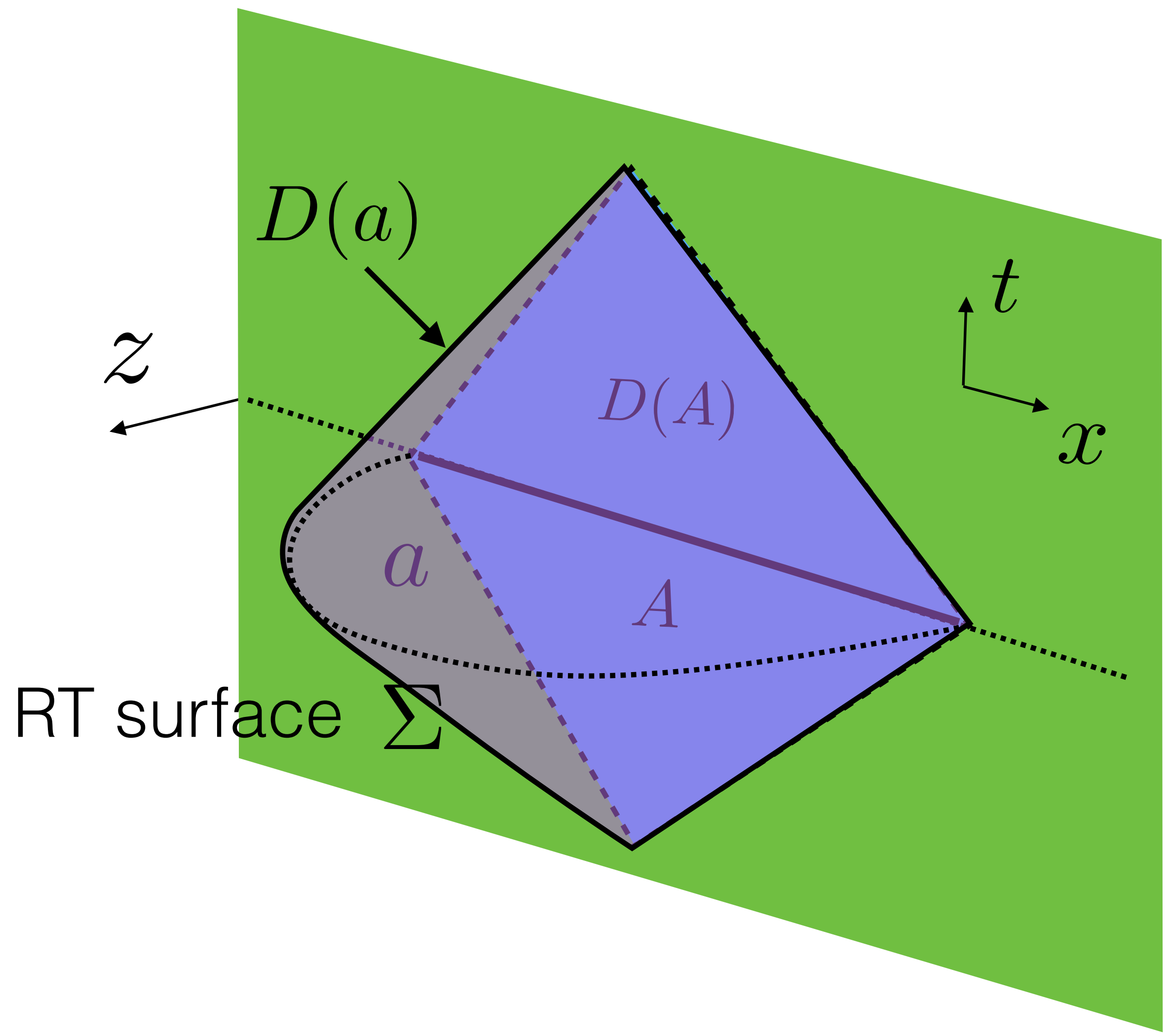

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$$D(a) \text{``} \approx \text{''} D(A)$$

Proving QNEC using AdS/CFT $\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$

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Entanglement Wedge Nesting (EWN): $D(\tilde{A}) \subseteq D(A) \rightarrow D(\tilde{a}) \subseteq D(a)$

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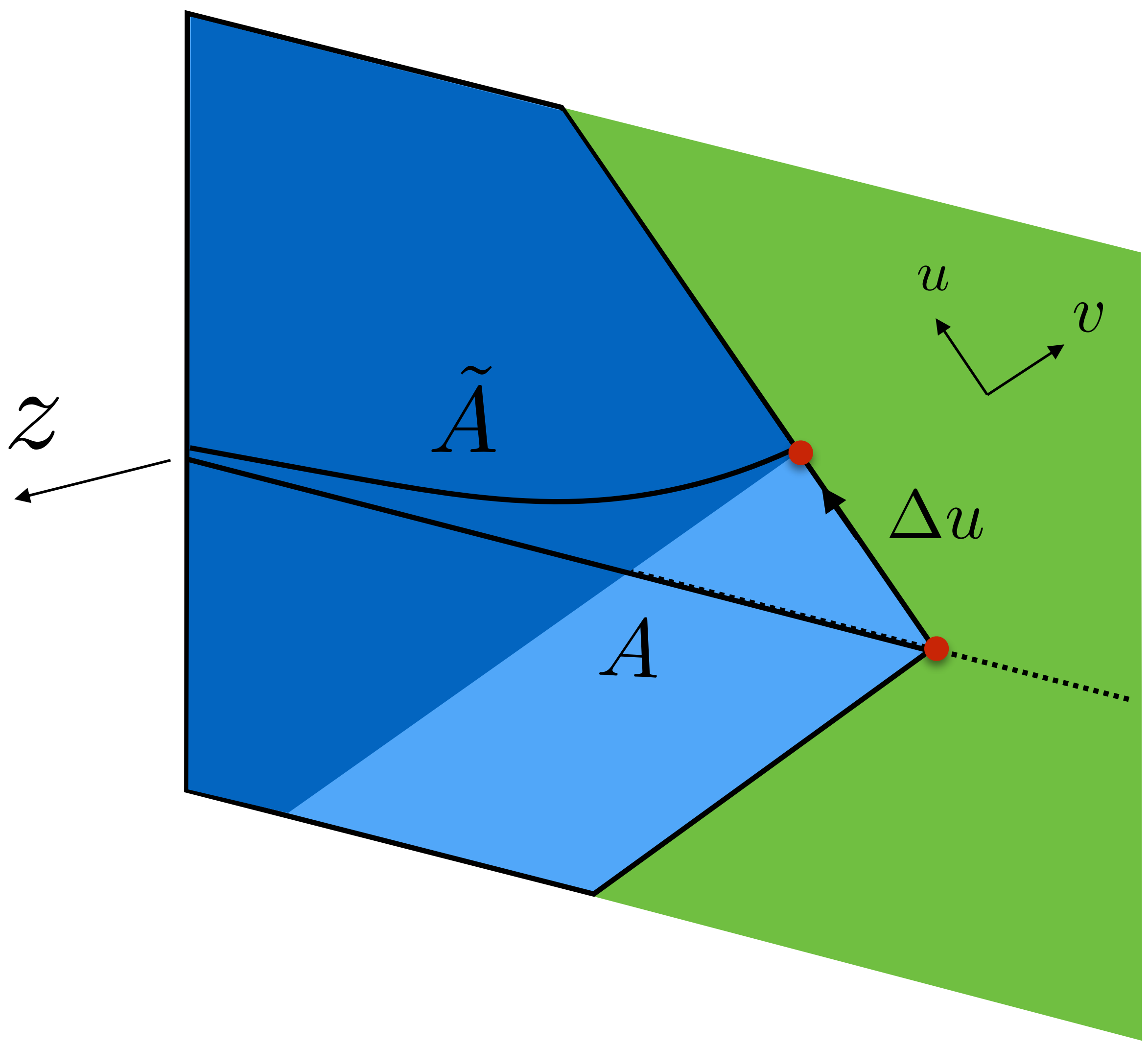

at the boundary:

$\Delta u \geq 0$: null deformation

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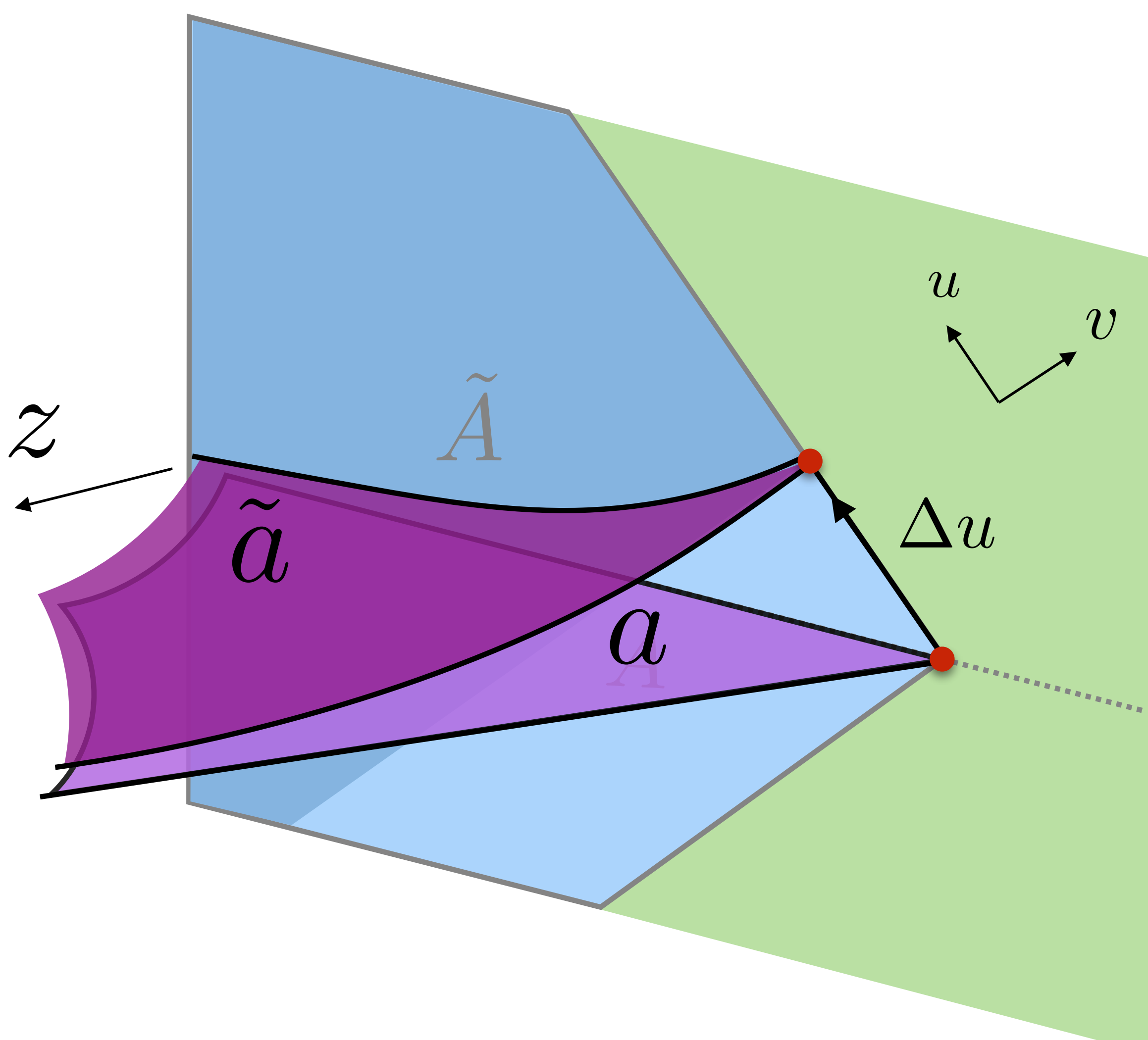

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into the bulk:

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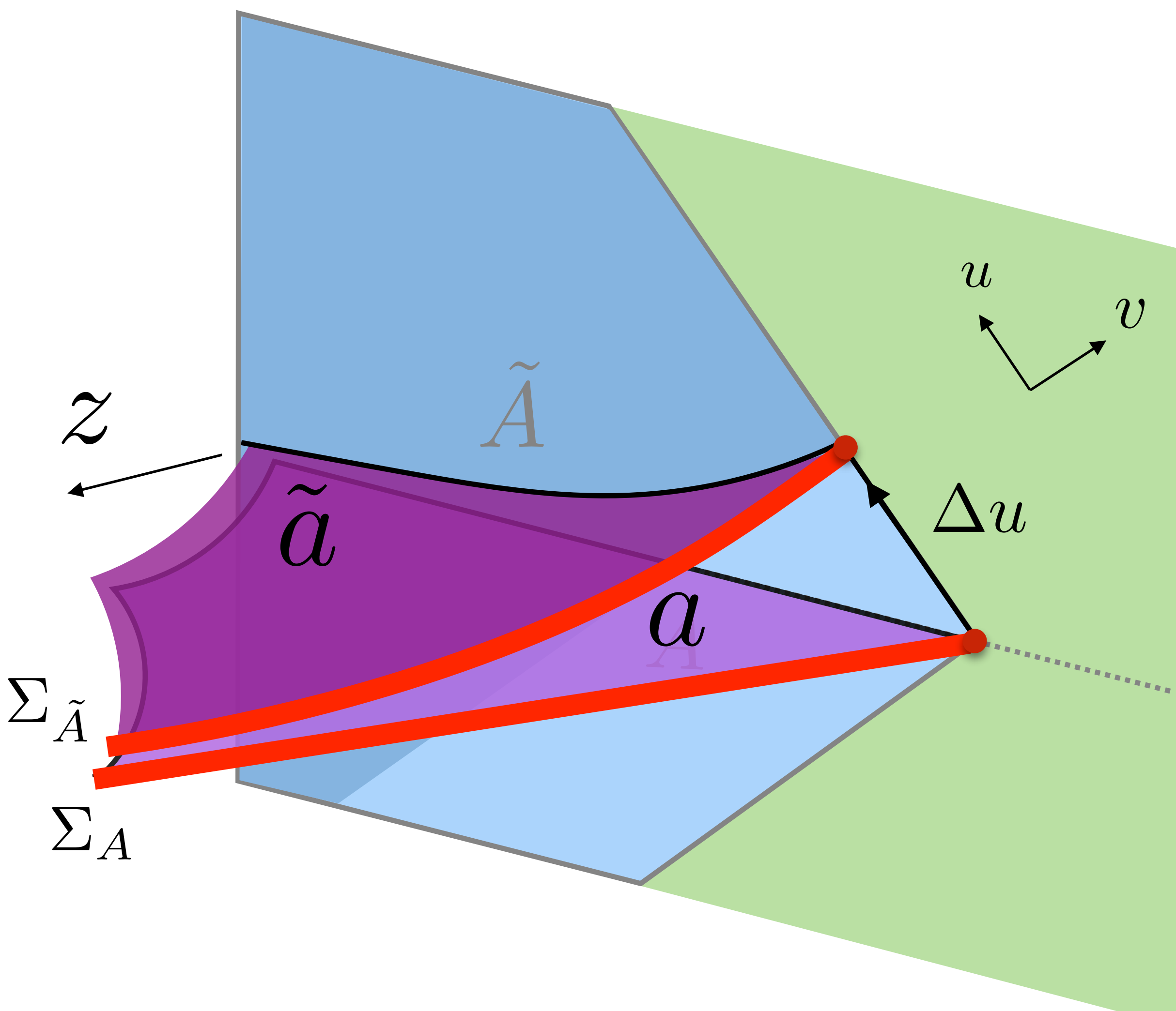

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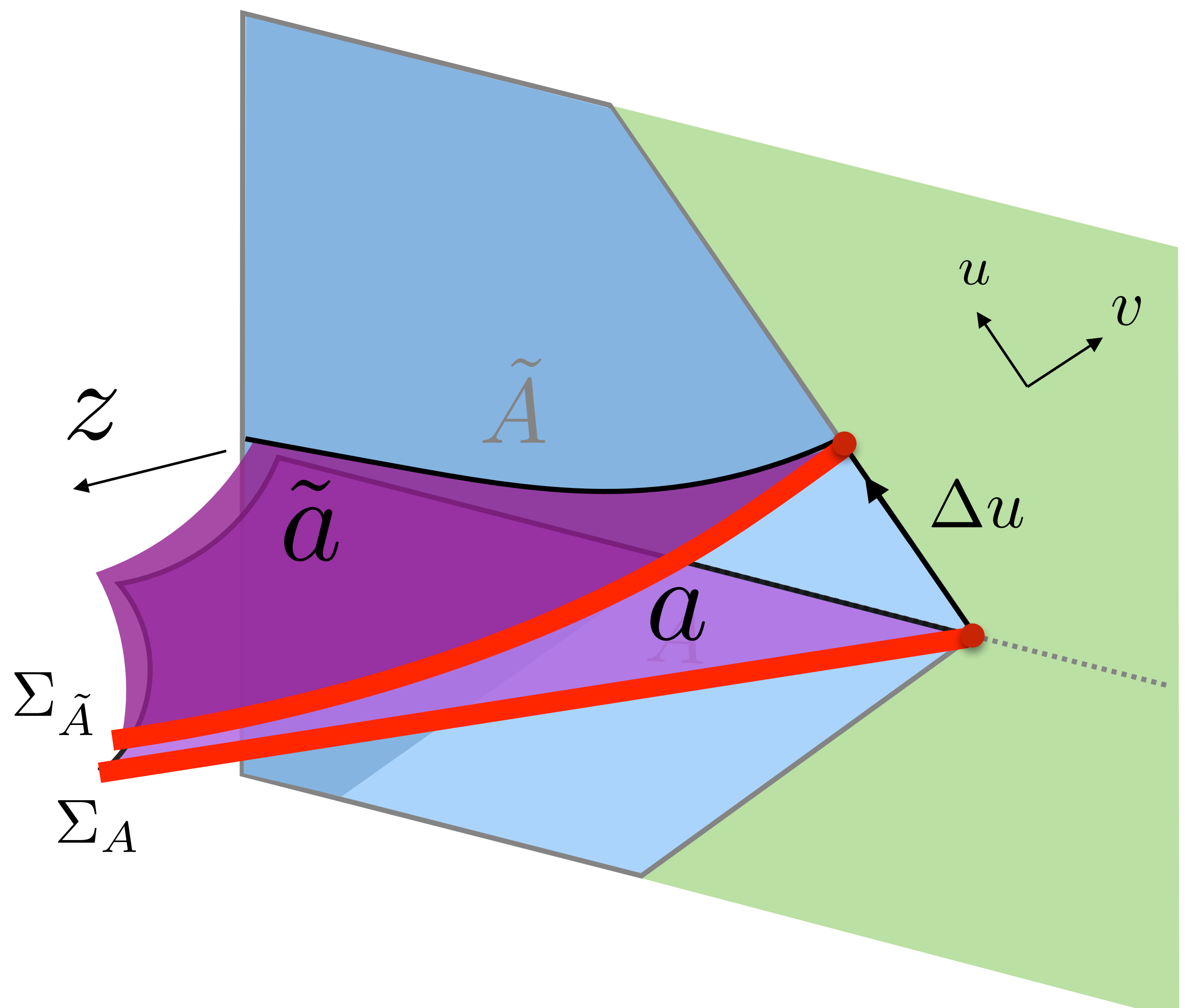

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RT surfaces dynamics

Proving QNEC using AdS/CFT $\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$

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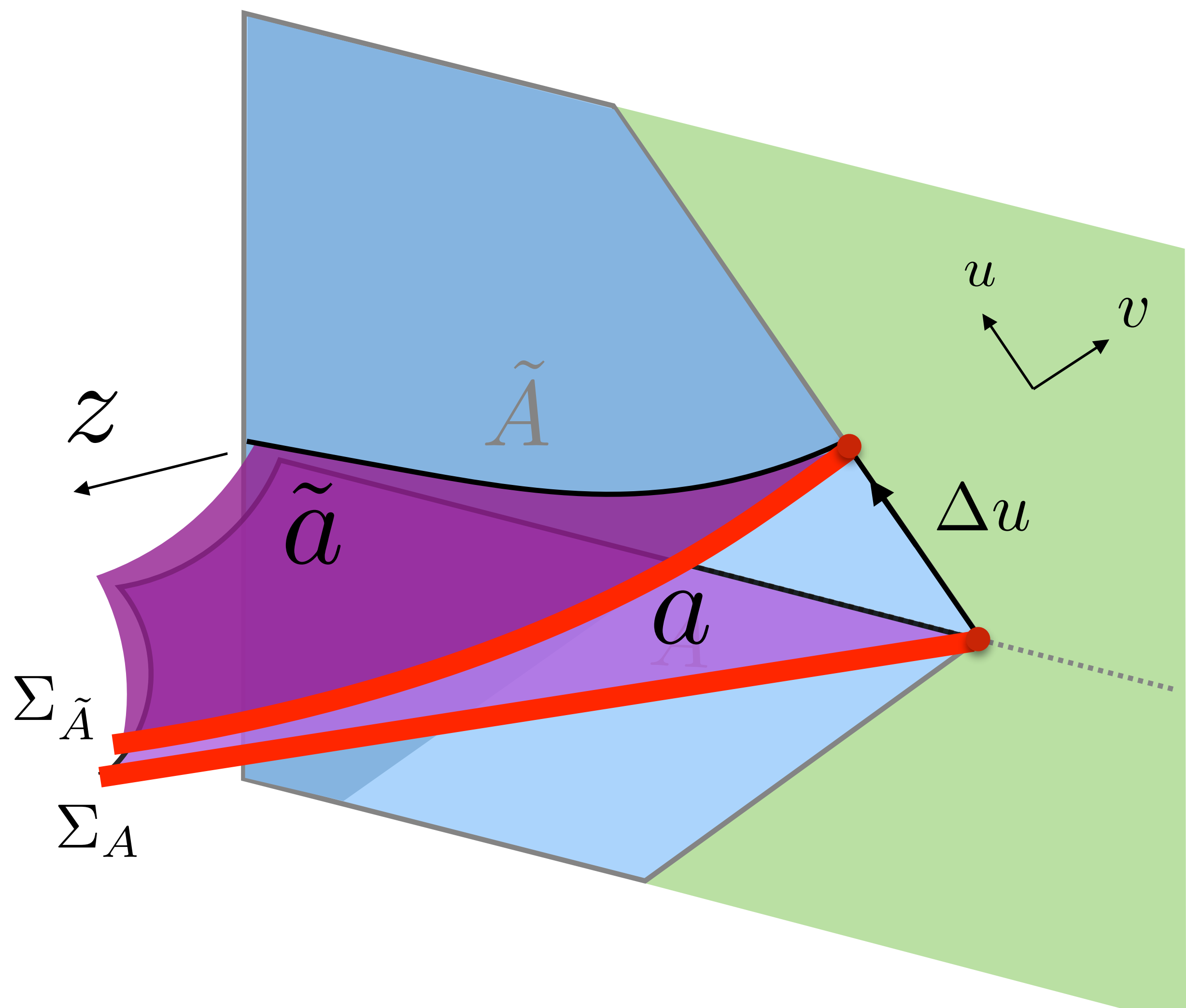

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near boundary expansion:

(F-G gauge)

$$g_{uu} = \frac{16\pi G}{dR^{d-3}} z^{d-2}\langle T_{ab}\rangle_\psi + \mathcal{O}(z^d)$$

$$X^i_{\Sigma_A}(z) = X^i_{\partial A} + \frac{4G}{dR^{d-1}} z^d \partial_i S_{EE}(A) + \mathcal{O}(z^{d+1})$$

$$X^i_{\Sigma_{\tilde{A}}}(z) = X^i_{\partial \tilde{A}} + \frac{4G}{dR^{d-1}} z^d \partial_i S_{EE}(\tilde{A}) + \mathcal{O}(z^{d+1})$$

Proving QNEC using AdS/CFT

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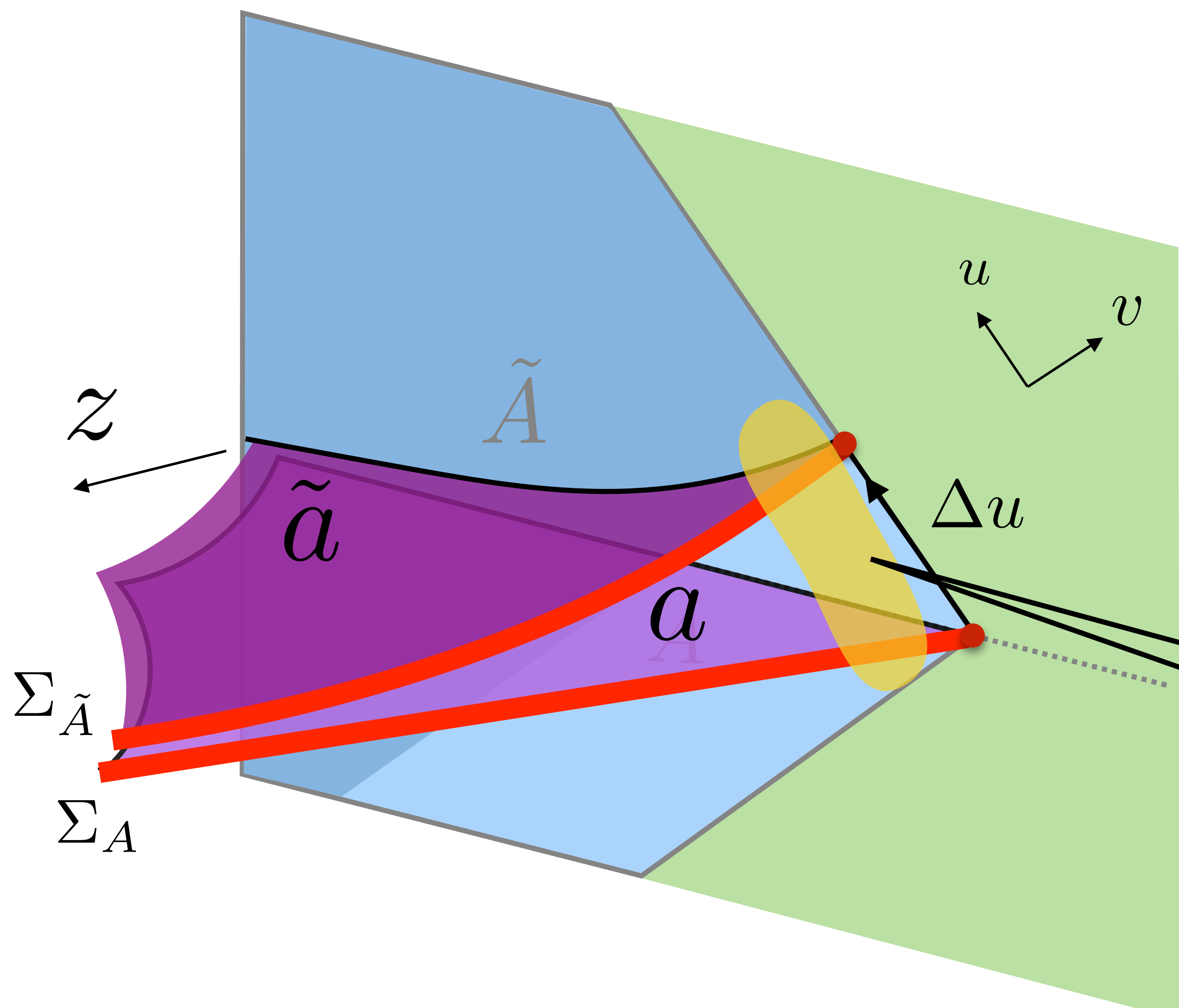

$\Sigma_{\tilde{A}}$ spacelike/null Σ_A

$z \rightarrow 0$

$\Delta u \rightarrow 0$

$$\langle T_{uu} \rangle_\psi - \partial_u^2 S_{EE} \geq 0$$

boundary QNEC ✓

Plan of the talk:

- Proof in AdS/CFT (review)
- General proof in CFT
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Proving QNEC in general CFTs $\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017

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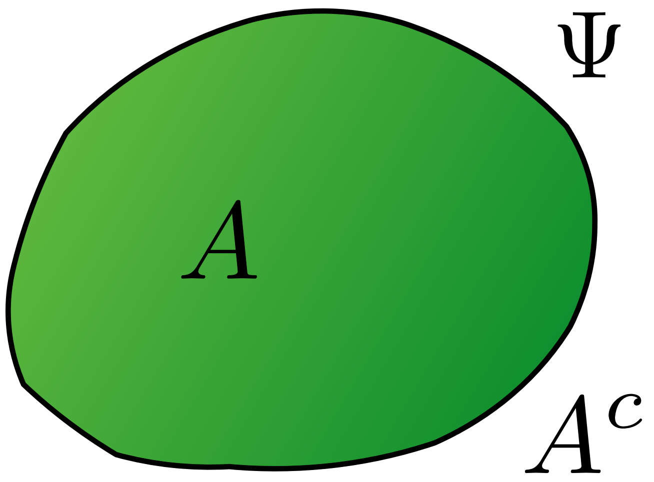
Modular Hamiltonian:

$$K_A^\Psi = -\ln\rho_A^\Psi \otimes \mathbb{1}_{A^c} + \mathbb{1}_A \otimes \ln\rho_{A^c}^\Psi = H_A^\Psi - H_{A^c}^\Psi$$



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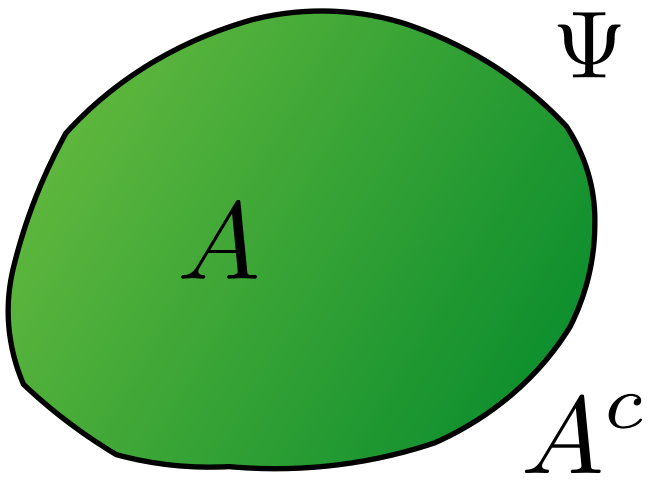
$$K_A^\Psi \ : \ \mathcal{H}_{\text{full}} \to \mathcal{H}_{\text{full}} \qquad K_A^\Psi |\Psi\rangle = 0$$



Proving QNEC in general CFTs $\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017

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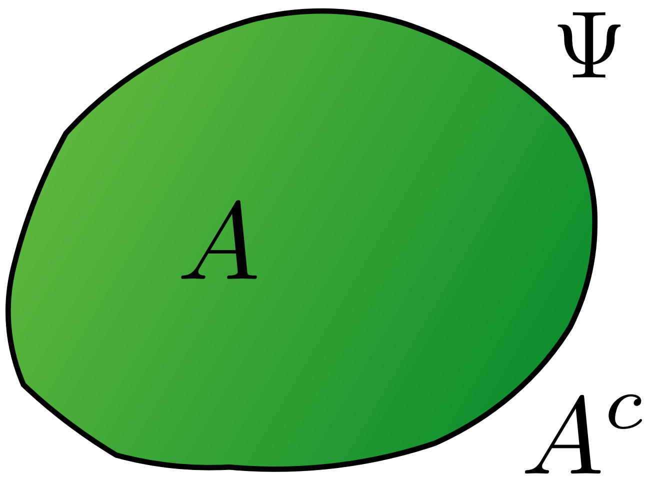

- encodes more detailed entanglement data
- in general, complicated and non-local

Proving QNEC in general CFTs $\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017

- Averaged Null Energy Condition (ANEC): $\int_{-\infty}^{\infty} dx^+ \langle \hat{T}_{++} \rangle_\psi \geq 0$
- modular hamiltonian (entanglement structure) used to prove ANEC
  T. Faulkner, R. Leigh, O. Parrikar, H. Wang, 2016
- alternative proof of ANEC from causality of correlation function
  T. Hartman, S. Kundu, A. Tajdini, 2016
- combine entanglement structure + causality?
- proof of QNEC (stronger conjecture)!

Proving QNEC in general CFTs $\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$

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causality of correlation function: $f(u,v) \propto \langle\psi|\mathcal{O}(u,v)\mathcal{O}(-u,-v)|\psi\rangle$

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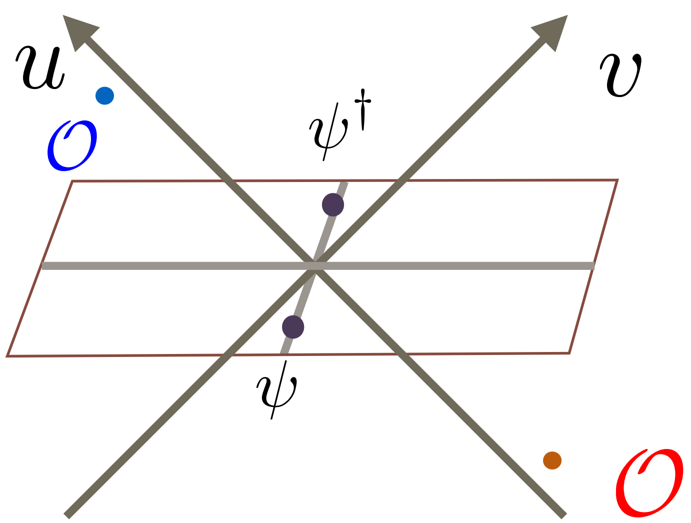

Causality: $\langle\psi|\,[\mathcal{O},\mathcal{O}]\,|\psi\rangle = 0$ for $uv < 0$

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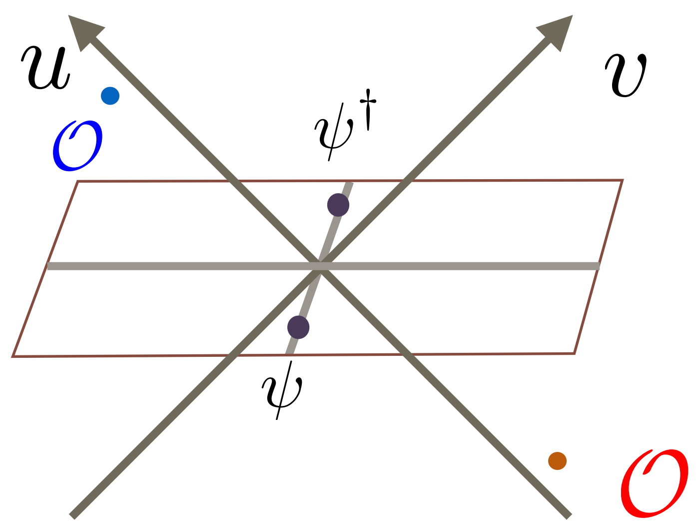

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"dress" the correlator to probe entanglement structure?

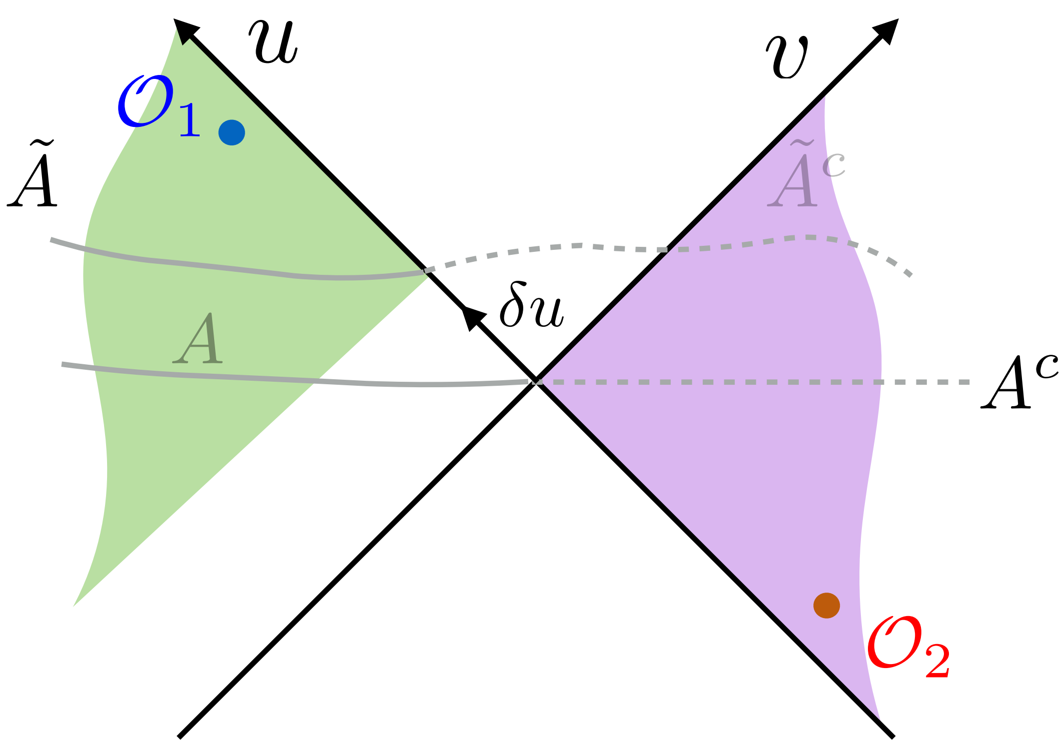
modular flow: $\mathcal{O} \to \mathcal{O}^A(s) \equiv e^{is\, K_A^\psi} \mathcal{O} e^{-is\, K_A^\psi}$

in general: highly non-local!

Proving QNEC in general CFTs

$$\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017



consider:

$$f(s) = \mathcal{N}^{-1}\langle\psi|\mathcal{O}_1^{\tilde{A}}(s)\mathcal{O}_2^{A}(s)|\psi\rangle$$

$$\mathcal{O}_1^{\tilde{A}}(s) = e^{is\,K_{\tilde{A}}^{\psi}}\mathcal{O}_1 e^{-is\,K_{\tilde{A}}^{\psi}}$$

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Proving QNEC in general CFTs $\langle T_{uu}\rangle_\psi \geq \partial_u^2 S_{EE}$

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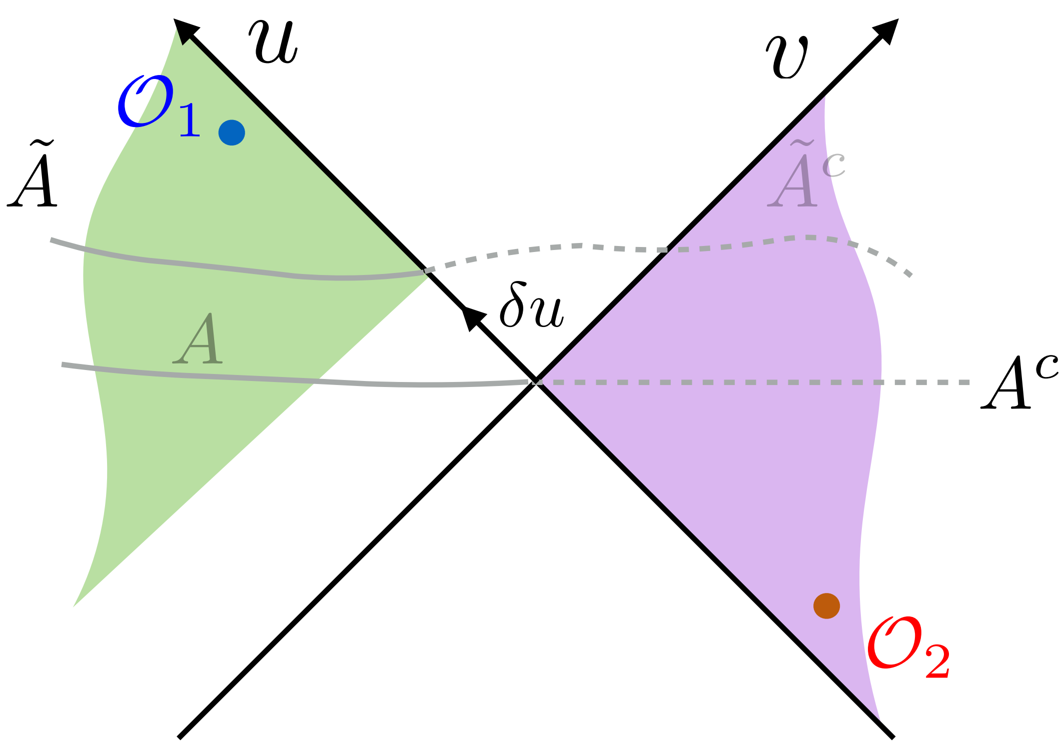

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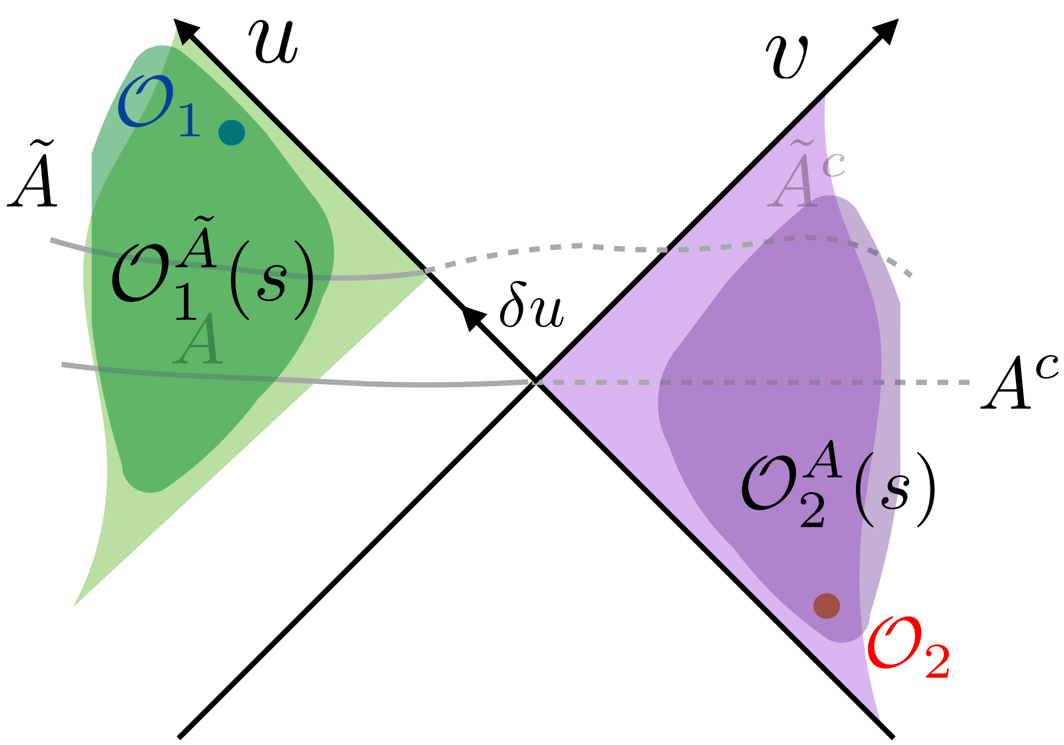
Tomita-Takesaki theory (in algebraic QFT):

$$\mathcal{O} \in \mathcal{M}_A \to \mathcal{O}^A(s) \in \mathcal{M}_A, \quad s \in \mathbb{R}$$

$\mathcal{M}_A$: von Neumann algebra associated with A, i.e. operators supported in D(A)

Proving QNEC in general CFTs $\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017



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Tomita-Takesaki theory (in algebraic QFT):

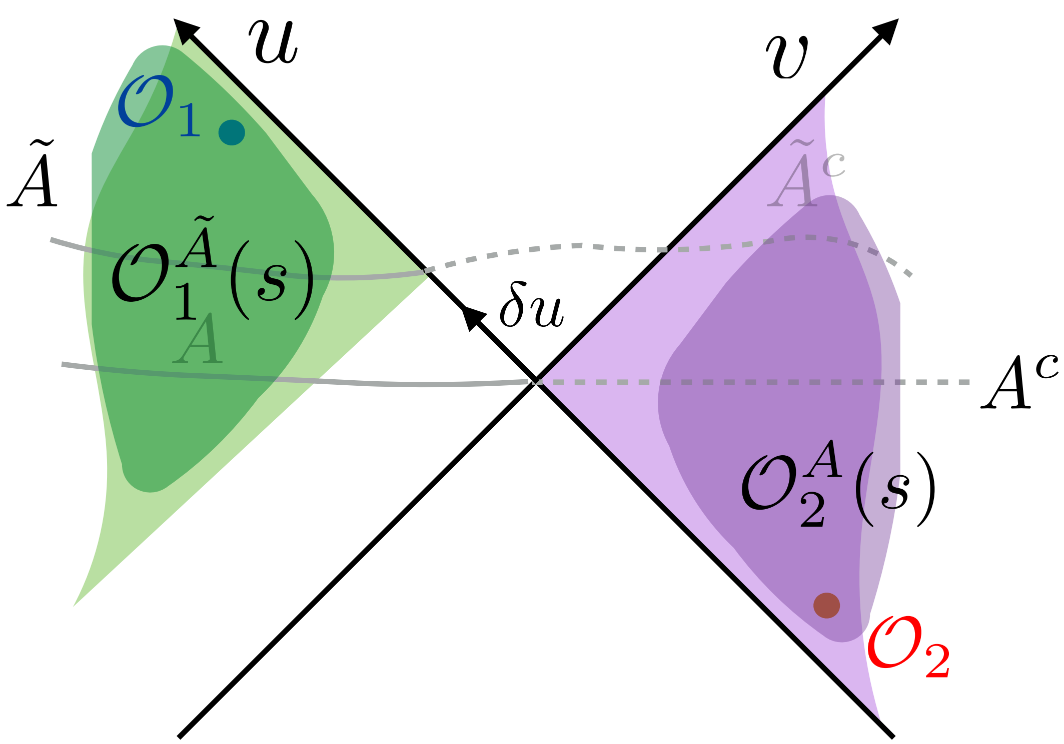
$\mathcal{O}_1^{\tilde{A}}(s)$ is supported only in $D(\tilde{A})$

$\mathcal{O}_2^{A}(s)$ is supported only in $D(A^c)$

Proving QNEC in general CFTs

$$\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017



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Tomita-Takesaki theory (in algebraic QFT):

$$\left[\, \mathcal{O}_1^{\tilde{A}}(s) \,,\, \mathcal{O}_2^{A}(s) \,\right] = 0 \quad \text{for} \quad s \in \mathbb{R}$$

a subtler notion of causality:

hidden in entanglement structure!

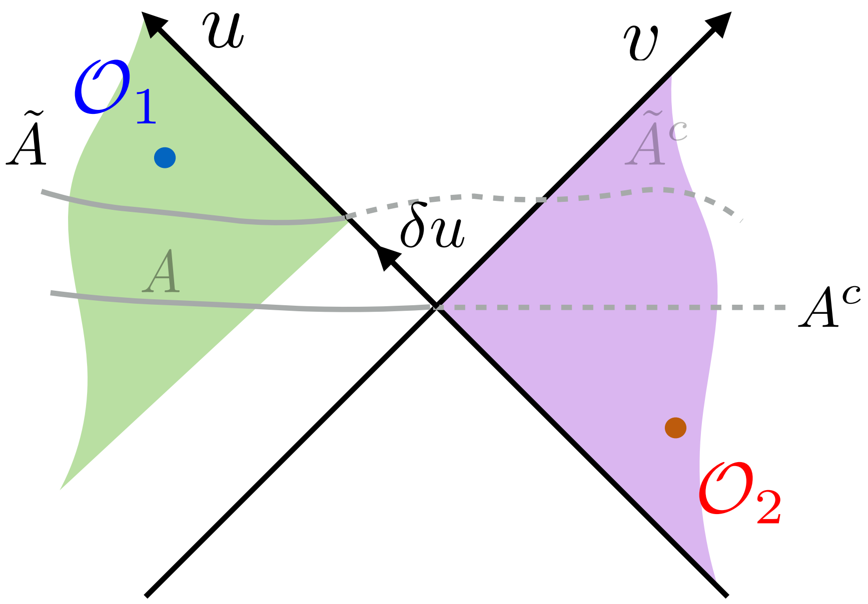
Proving QNEC in general CFTs $\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$

S. Balakrishna, T. Faulkner, Z. Khandker, H. Wang, 2017

Outline of the proof:

1. Unitarity + Cauchy-Schwarz inequality:

$$\mathrm{Re} f(s) \leq 1, \ \mathrm{Im}\ s = \pm\pi/2$$



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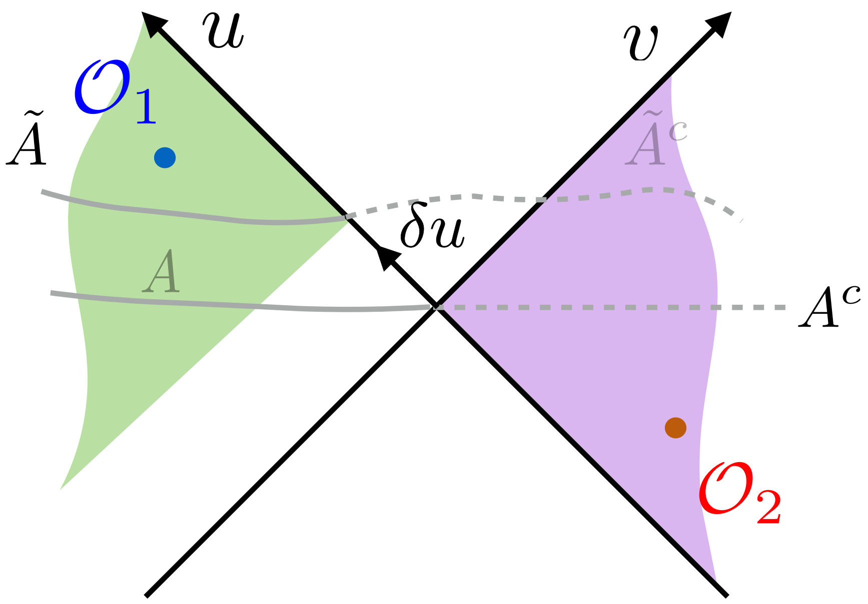

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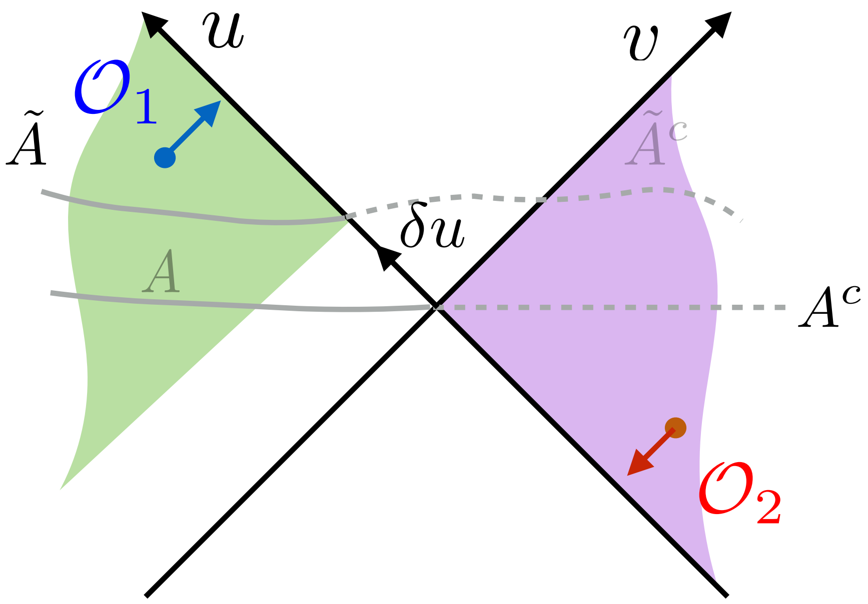

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3. Light-cone limit expansion: $v \to 0, \ u$ fixed

$$f(s) = 1 + C_T^{-1} e^s u(-uv)^{\frac{d-2}{2}} \mathcal{I}_Q + \ldots$$

$$\mathcal{I}_Q = \int_0^{\delta u} du' T_{uu}(u') + \left(\frac{\delta S_{EE}(A)}{\delta u} - \frac{\delta S_{EE}(\tilde{A})}{\delta u} \right)$$

Proving QNEC in general CFTs $\langle T_{uu} \rangle_\psi \geq \partial_u^2 S_{EE}$

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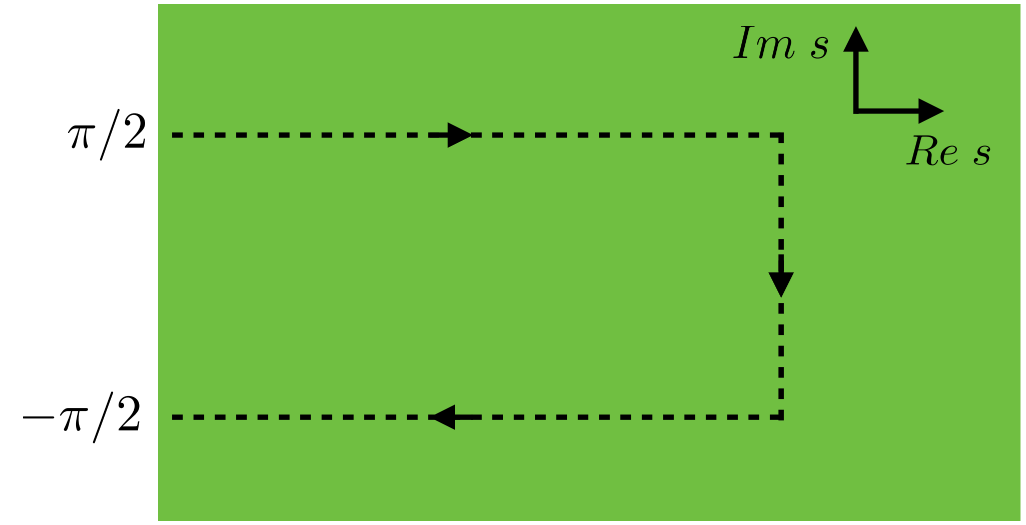

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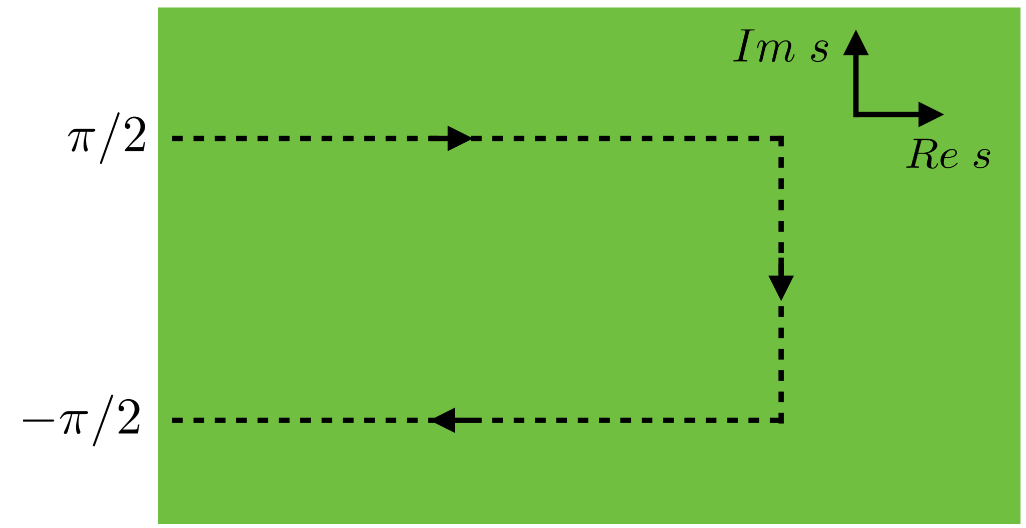

$$\mathcal{I}_Q = \int_0^{\delta u} du' T_{uu}(u') + \left(\frac{\delta S_{EE}(A)}{\delta u} - \frac{\delta S_{EE}(\tilde{A})}{\delta u} \right) \approx \delta u \left(\langle T_{uu} \rangle_\psi - \partial_u^2 S_{EE} \right) \geq 0$$

$$(\lim \delta u \to 0) \to \left(\langle T_{uu} \rangle_\psi - \partial_u^2 S_{EE} \right) \geq 0$$ QNEC

Plan of the talk:

- Proof in AdS/CFT (review)
- General proof in CFT
- Bulk modular flow in AdS/CFT
- Conclusion/outlooks

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

- in holography, EWN near boundary = QNEC

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- understand this connection more explicitly

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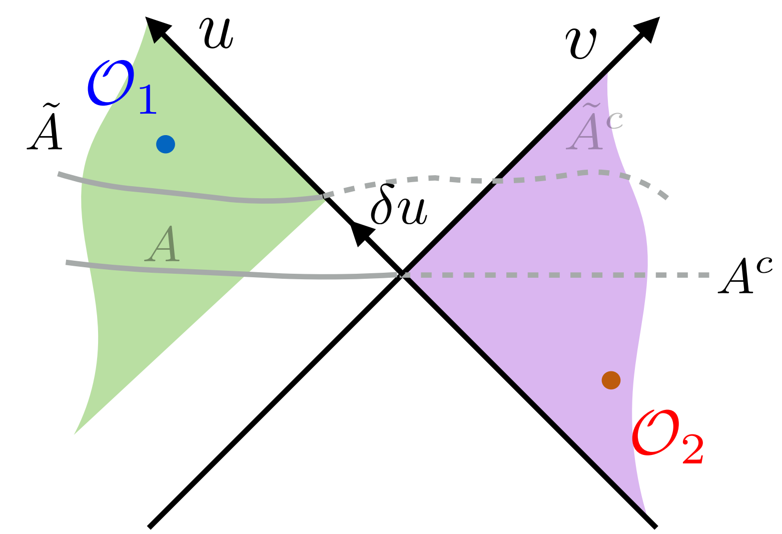
- understand this connection more explicitly
- a concrete step: bulk approach for computing $f(s)$

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

Revisit $f(s) \propto \langle\psi|\mathcal{O}_1^{\tilde{A}}(s)\mathcal{O}_2^{A}(s)|\psi\rangle$



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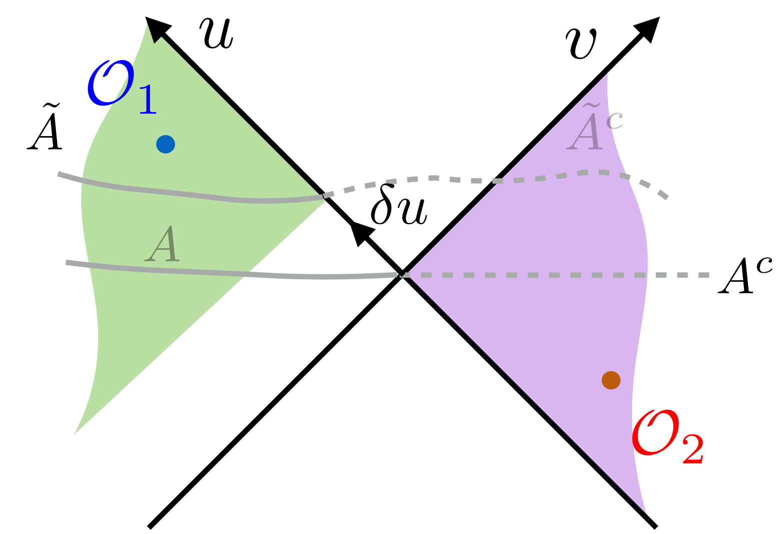

from "Heisenberg" to "Schrodinger" picture:

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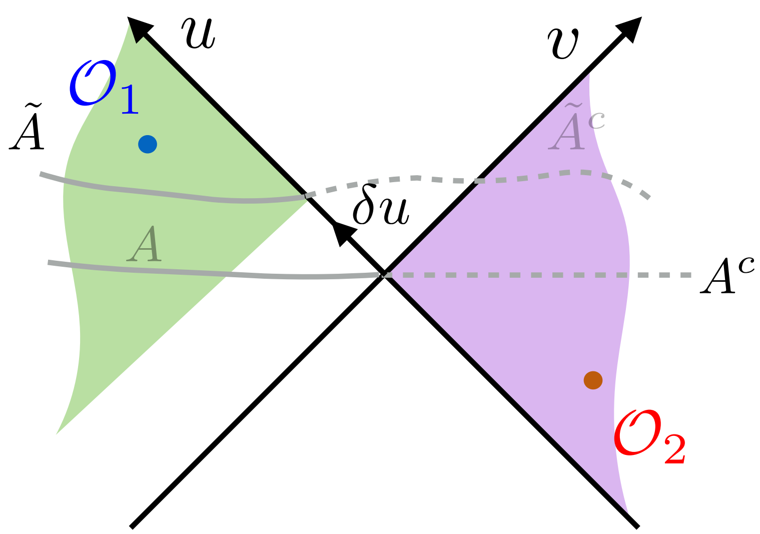

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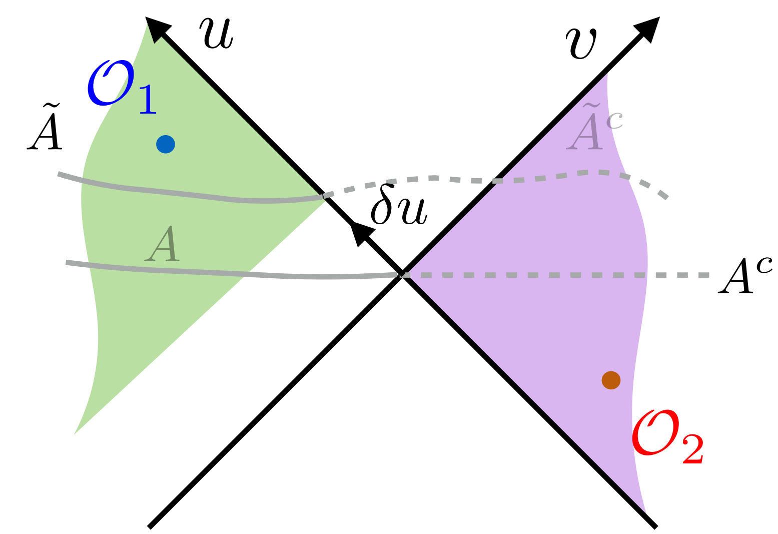

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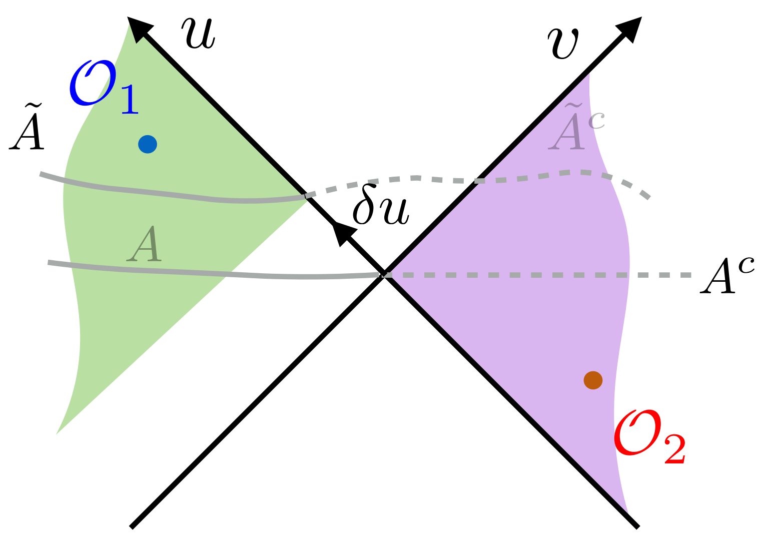

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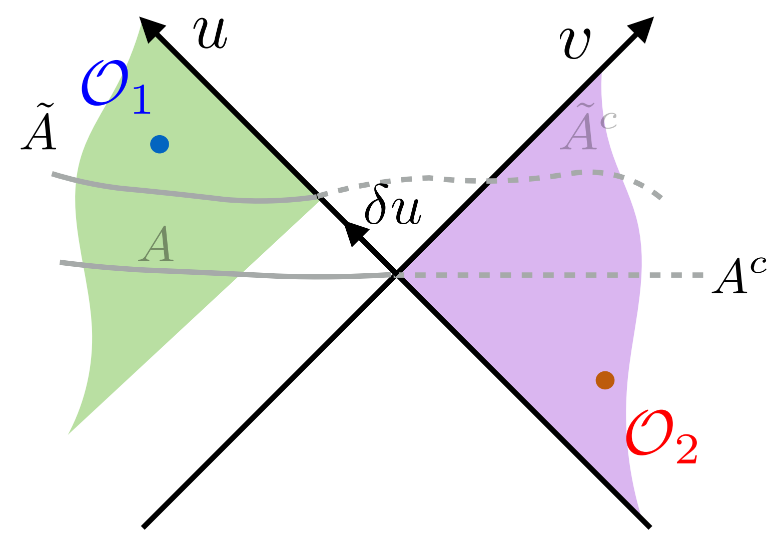

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$$= \langle\psi|e^{-isH^\psi_A}e^{isH^\psi_{\tilde{A}}}\mathcal{O}_1\;\mathcal{O}_2 e^{-isH^\psi_{\tilde{A}}}e^{isH^\psi_A}|\psi\rangle$$ recall $K^\psi_A|\psi\rangle = 0 \to H^\psi_A|\psi\rangle = H^\psi_{A^c}|\psi\rangle$ etc

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Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

So, in "Schrodinger" picture:

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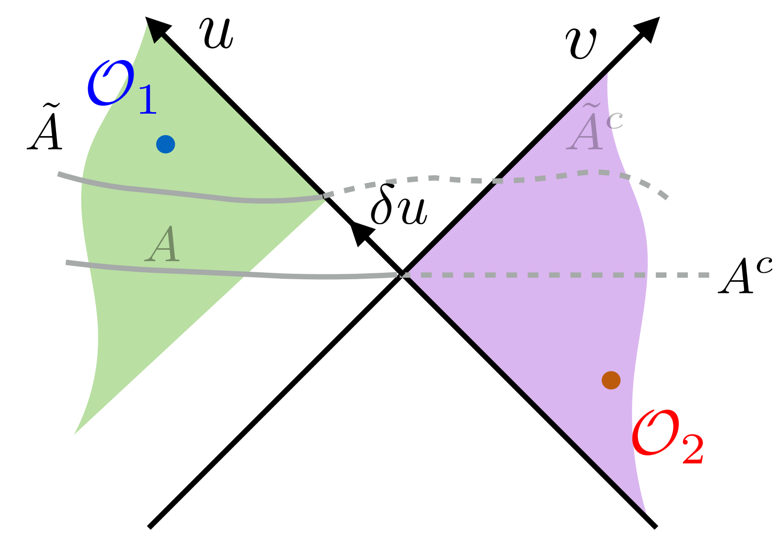

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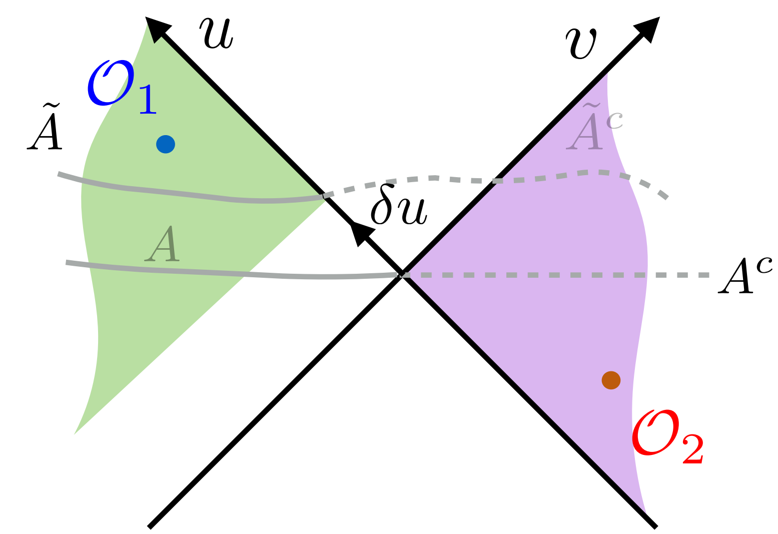

$$f(s) \propto \langle \psi_{A,\tilde{A}}(s) | \mathcal{O}_1 \mathcal{O}_2 | \psi_{A,\tilde{A}}(s) \rangle$$

where $|\psi_{A,\tilde{A}}(s)\rangle = e^{-isH^{\psi}_{\tilde{A}}} e^{isH^{\psi}_{A}} |\psi\rangle$

to use AdS/CFT, consider:

- in a holographic CFT
- bulk dual of $|\psi\rangle$ has smooth geometry
- conformal dimension Δ of $\mathcal{O}_{1,2}$: $1 \ll \Delta \ll \ell_{AdS}/\ell_{plank}$

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

So, in "Schrodinger" picture:

$$f(s) \propto \langle \psi_{A,\tilde{A}}(s) | \mathcal{O}_1 \mathcal{O}_2 | \psi_{A,\tilde{A}}(s) \rangle$$

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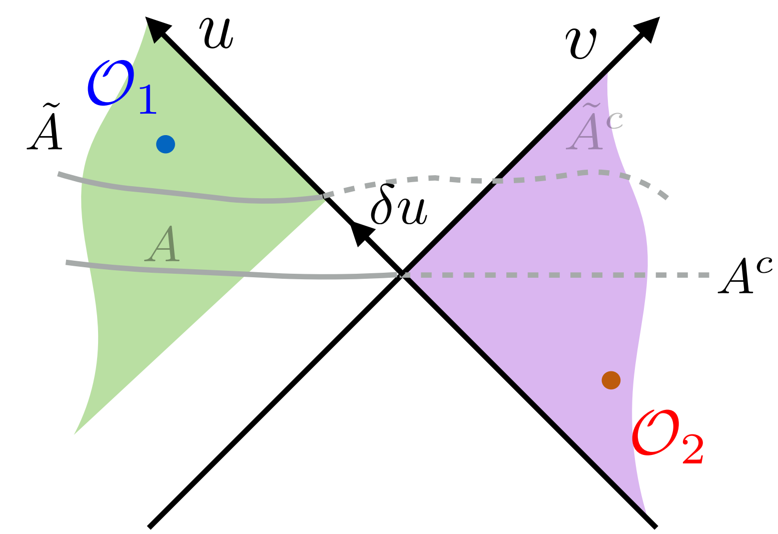
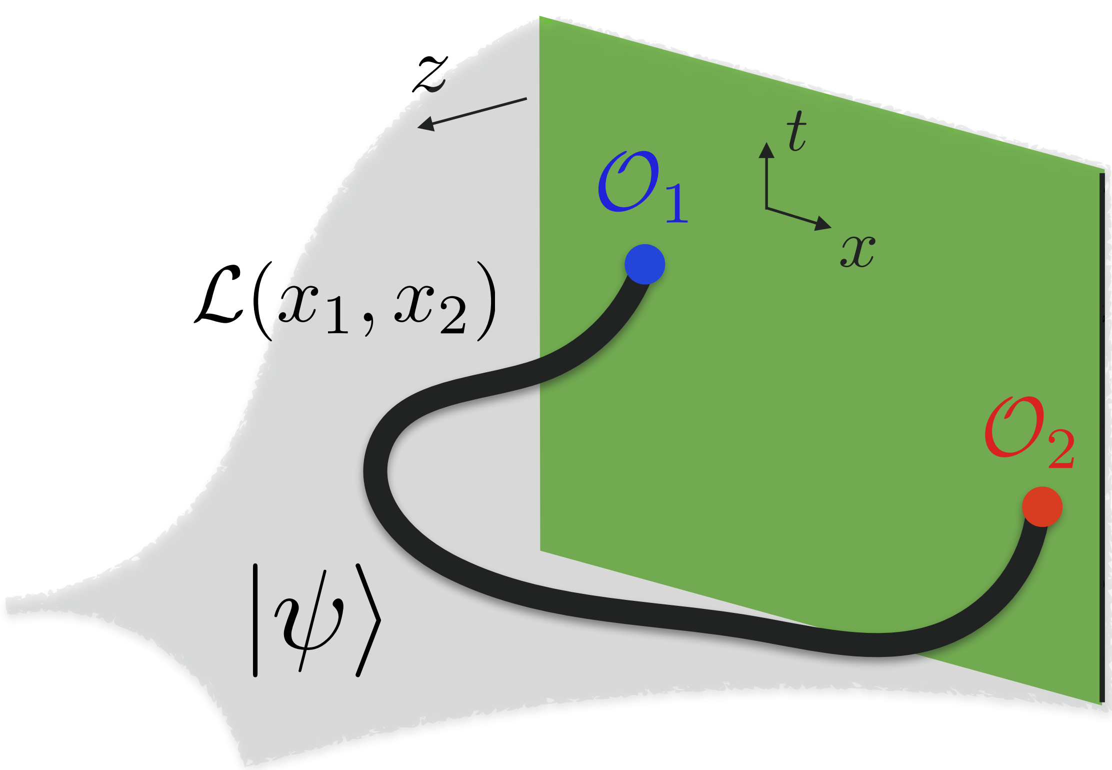


Geodesic approximation:

$$\langle \psi | \, \mathcal{O}_1 \, \mathcal{O}_2 \, | \psi \rangle$$

$$\approx \exp\left[-m\mathcal{L}(x_1, x_2)\right]$$



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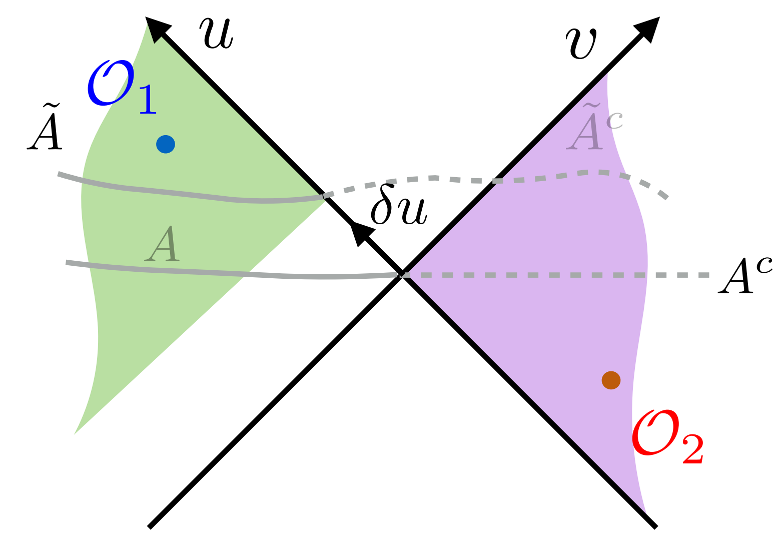

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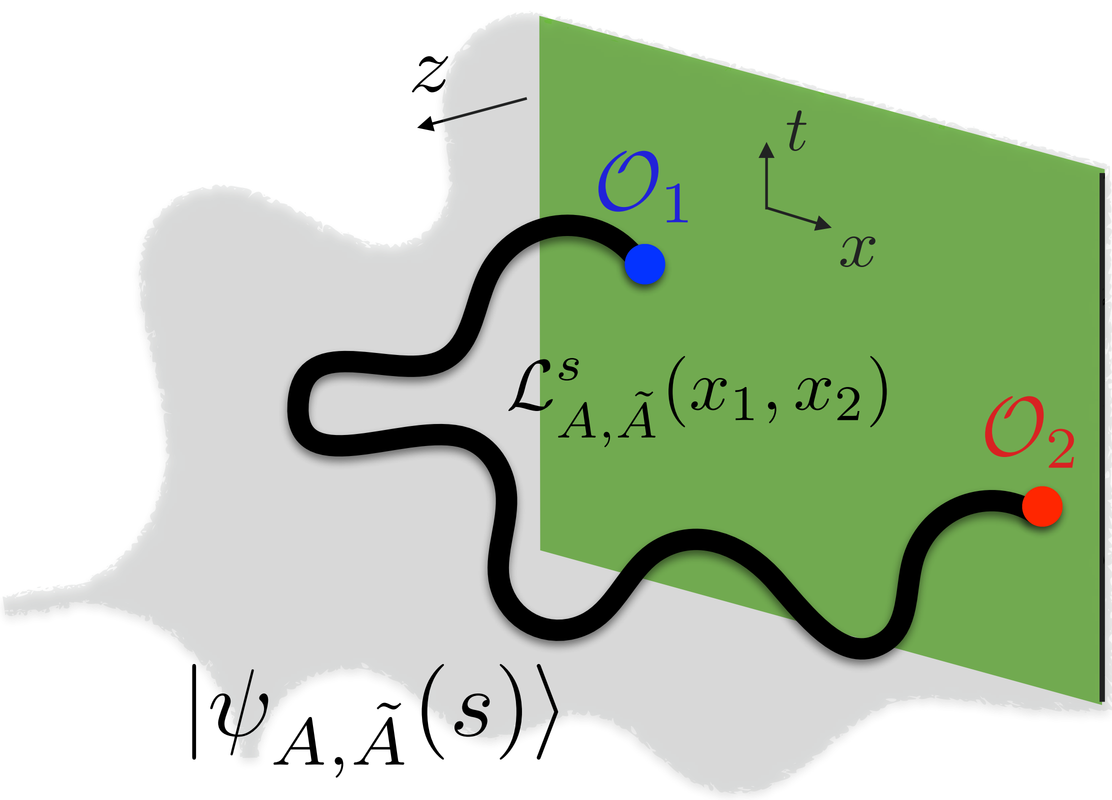
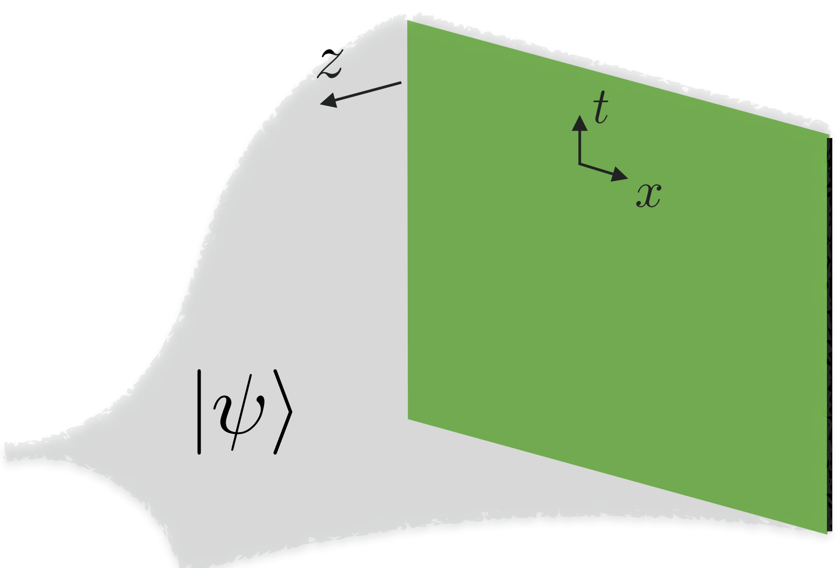


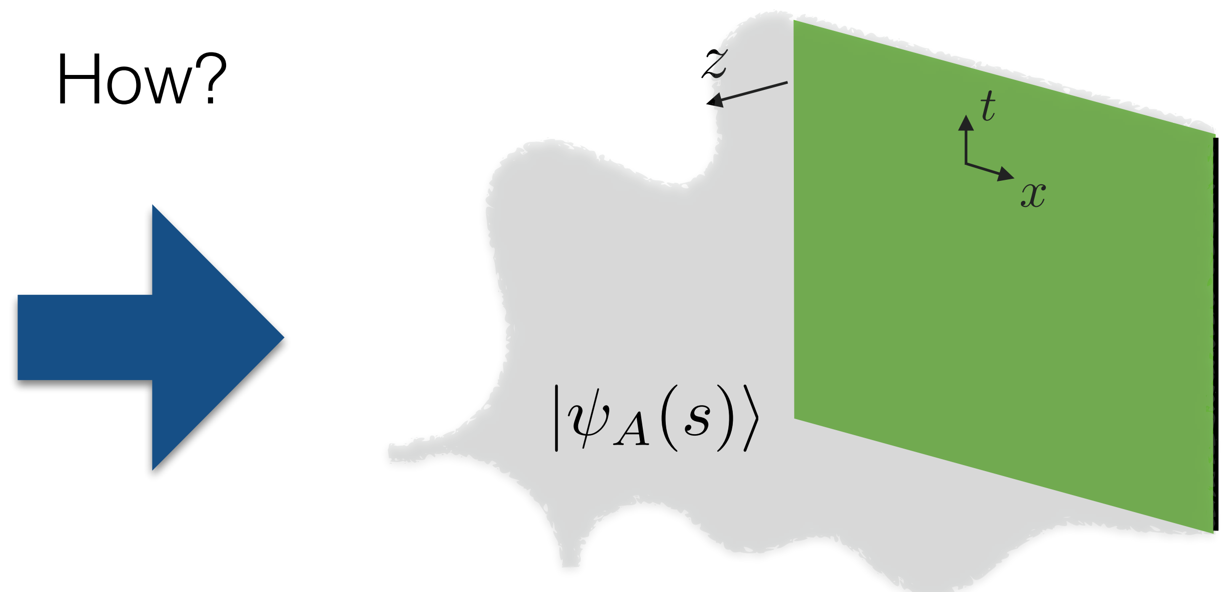
Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

consider a simpler case: $|\psi_A(s)\rangle = e^{isH_A^\psi}|\psi\rangle$ i.e. "single modular flow"



How?



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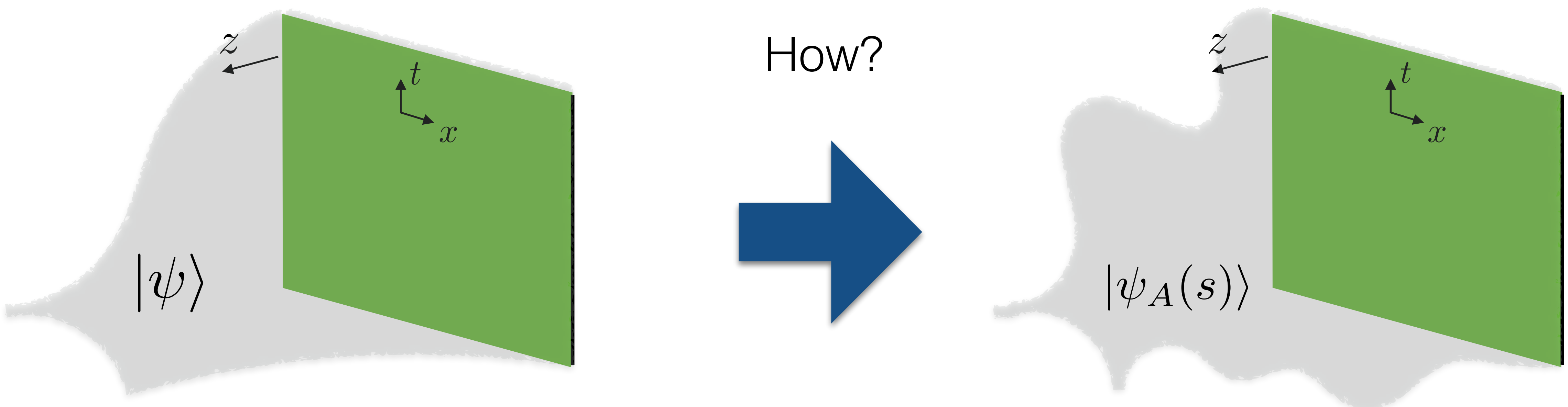

hint: for any $\mathcal{O}_A$ supported only in $D(A)$: $\langle \mathcal{O}_A \rangle_{\psi_A(s)} = \langle \mathcal{O}_A \rangle_\psi$

$$\langle \psi_A(s)|\mathcal{O}_A|\psi_A(s)\rangle = \langle\psi|e^{-isH_A^\psi}\mathcal{O}_A e^{isH_A^\psi}|\psi\rangle = \langle\psi|e^{-isH_{A^c}^\psi}\mathcal{O}_A e^{isH_{A^c}^\psi}|\psi\rangle$$

$$= \langle\psi|e^{-isH_{A^c}^\psi}e^{isH_{A^c}^\psi}\mathcal{O}_A|\psi\rangle = \langle\psi|\mathcal{O}_A|\psi\rangle$$

Bulk modular flow in AdS/CFT

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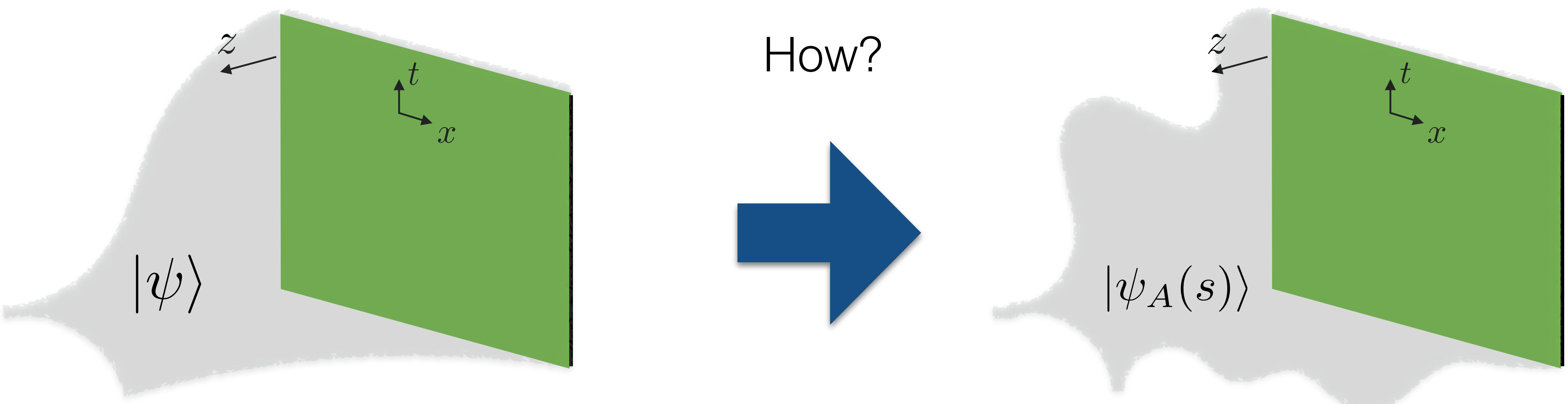


hint: for any $\mathcal{O}_A$ supported only in $D(A)$: $\langle \mathcal{O}_A \rangle_{\psi_A(s)} = \langle \mathcal{O}_A \rangle_\psi$

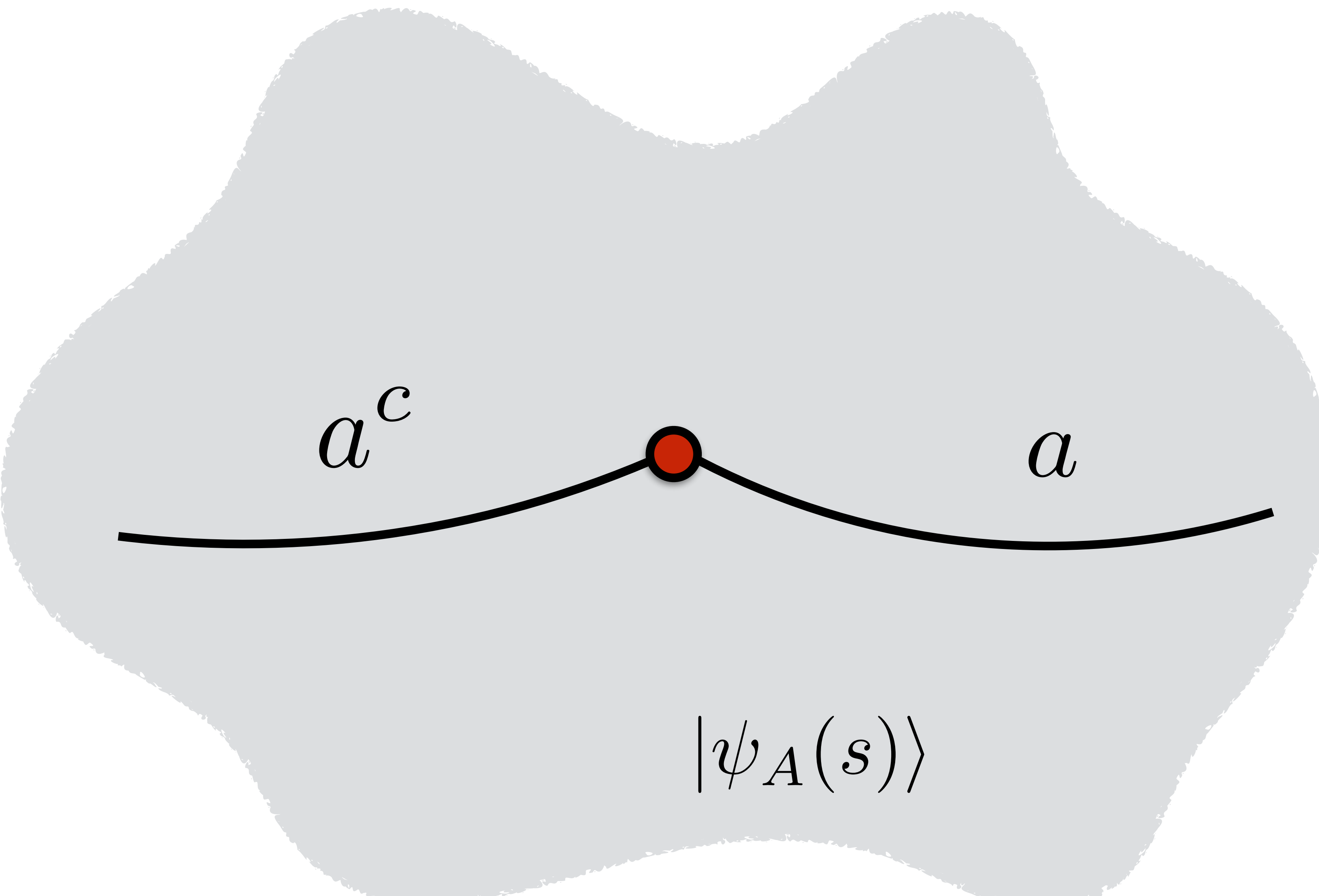
similarly,

for any $\mathcal{O}_{A^c}$ supported only in $D(A^c)$: $\langle \mathcal{O}_{A^c} \rangle_{\psi_A(s)} = \langle \mathcal{O}_{A^c} \rangle_\psi$

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

entanglement wedge reconstruction: $D(a)``\approx" D(A),\ D(a^c)``\approx" D(A^c)$



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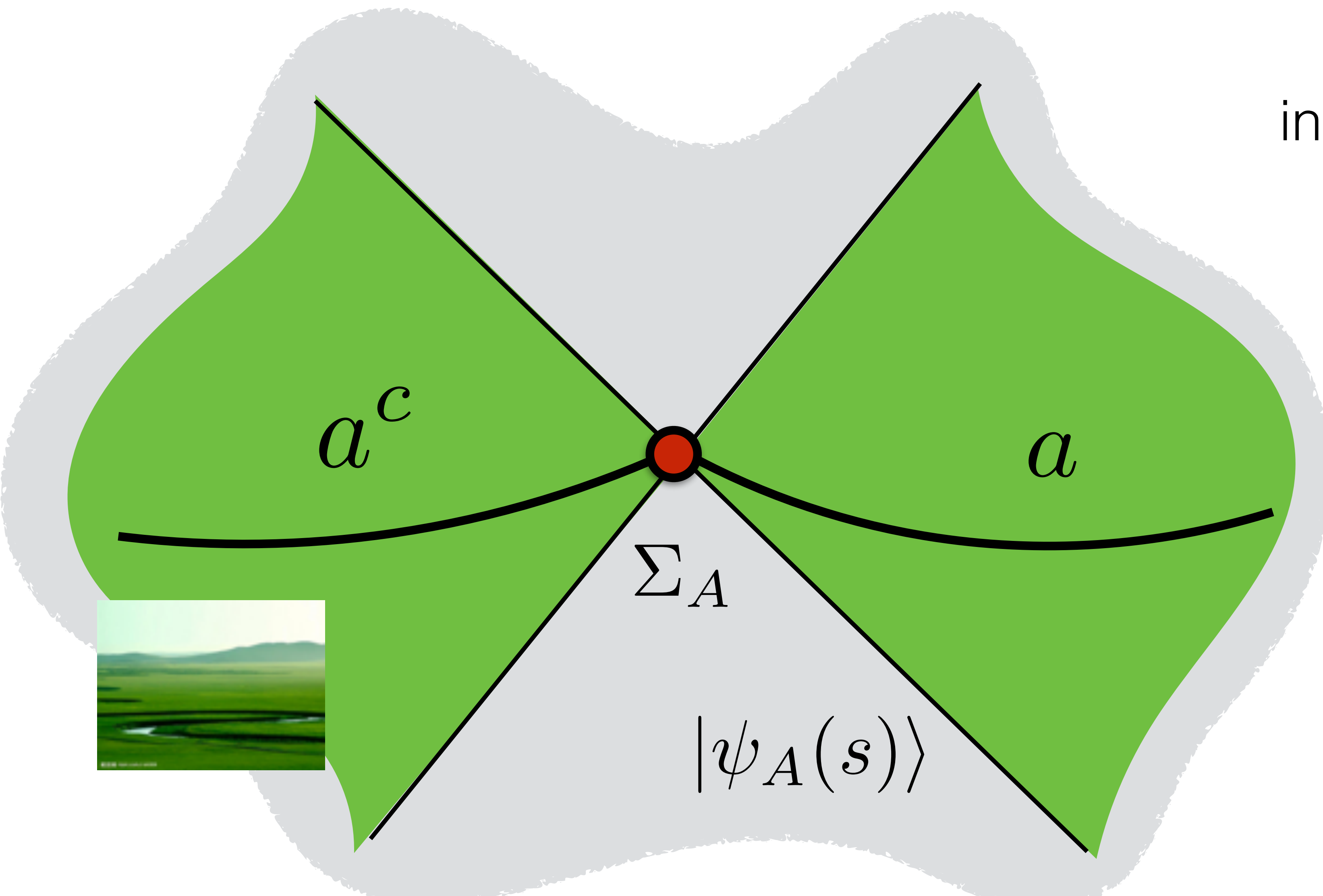

in entanglement wedges :

$$"\psi_A(s) \equiv \psi"$$

e.g.

- same metric
- same bulk fields, etc

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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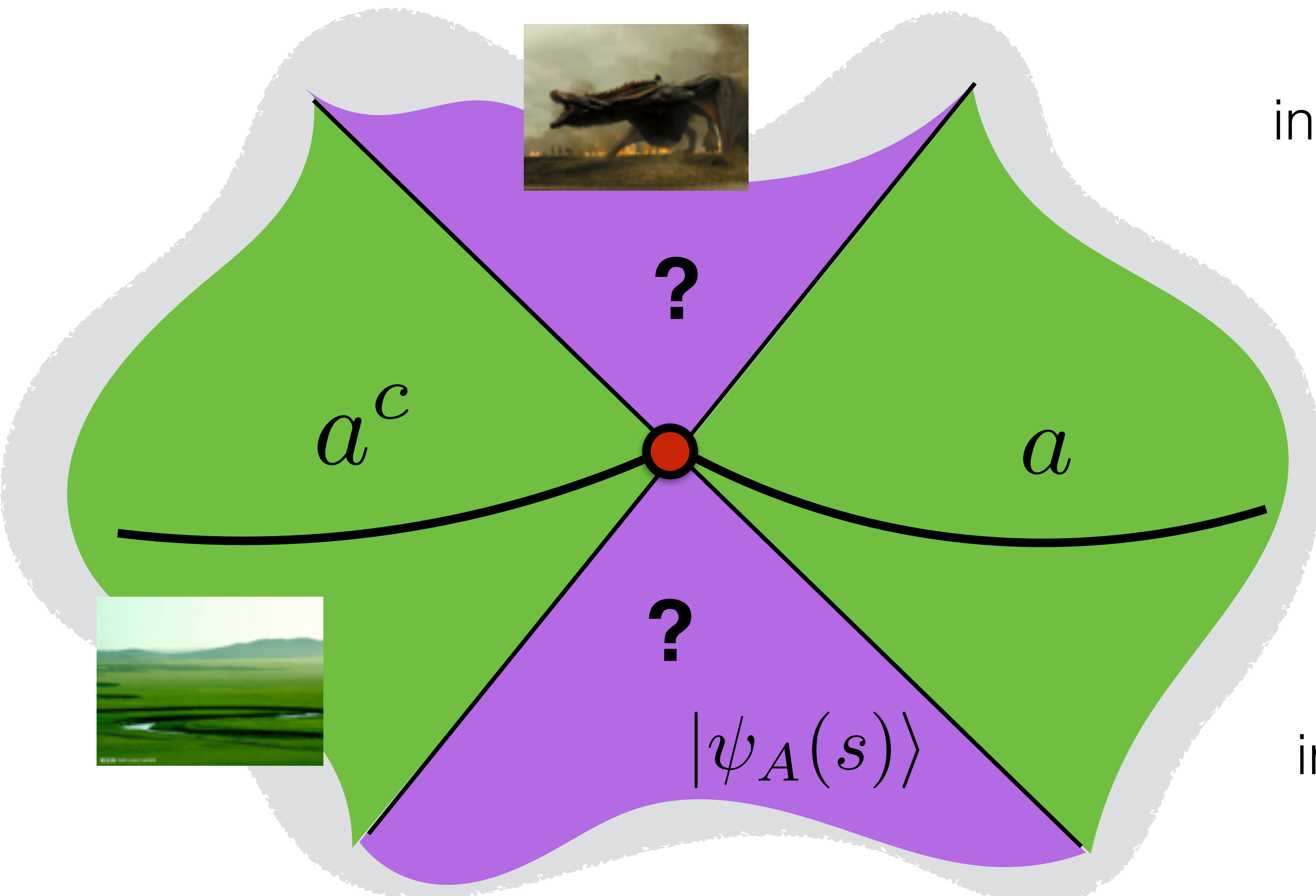

in **entanglement wedges** :

$$\text{”}\psi_A(s) \equiv \psi\text{”}$$

in **“Milne” wedges** :

unknown, possibly with kinks/singularities

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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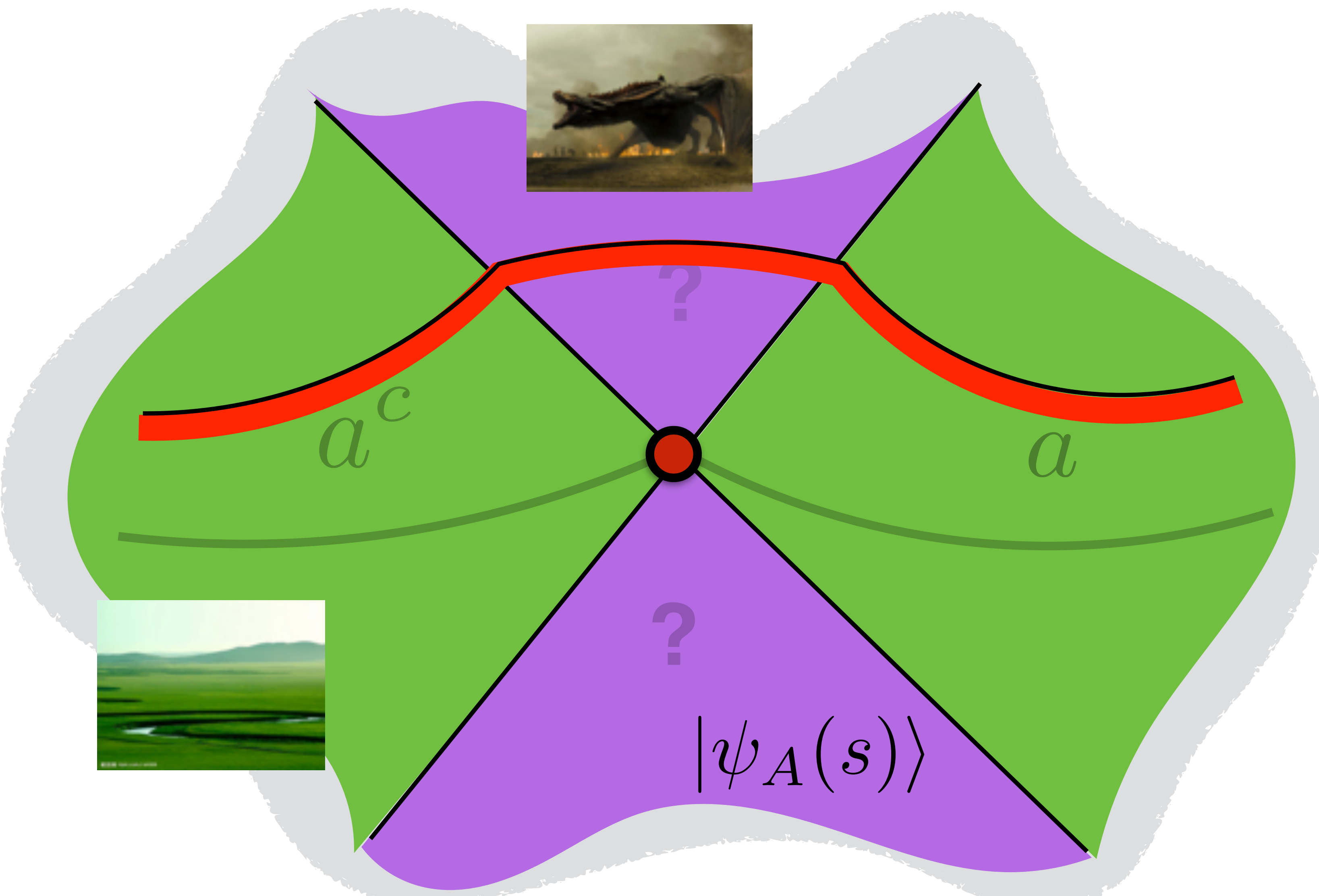

geodesic: a function of $\{x_1, x_2, s\}$

generic geodesics pass through both the entanglement and “Milne” wedges

we don’t know what to do…

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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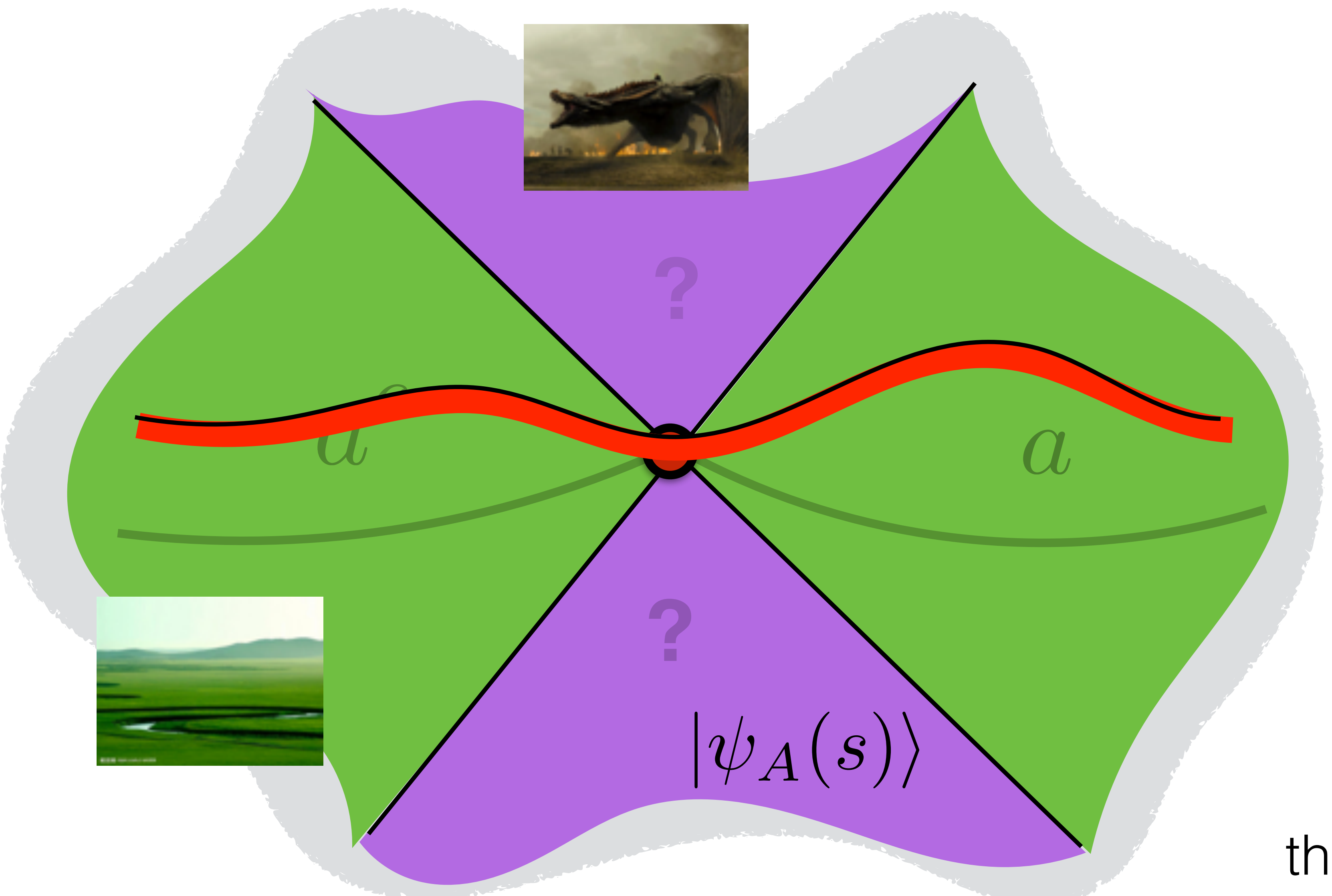


geodesic: a function of $\{x_1, x_2, s\}$

if we fine-tune one of the parameters:

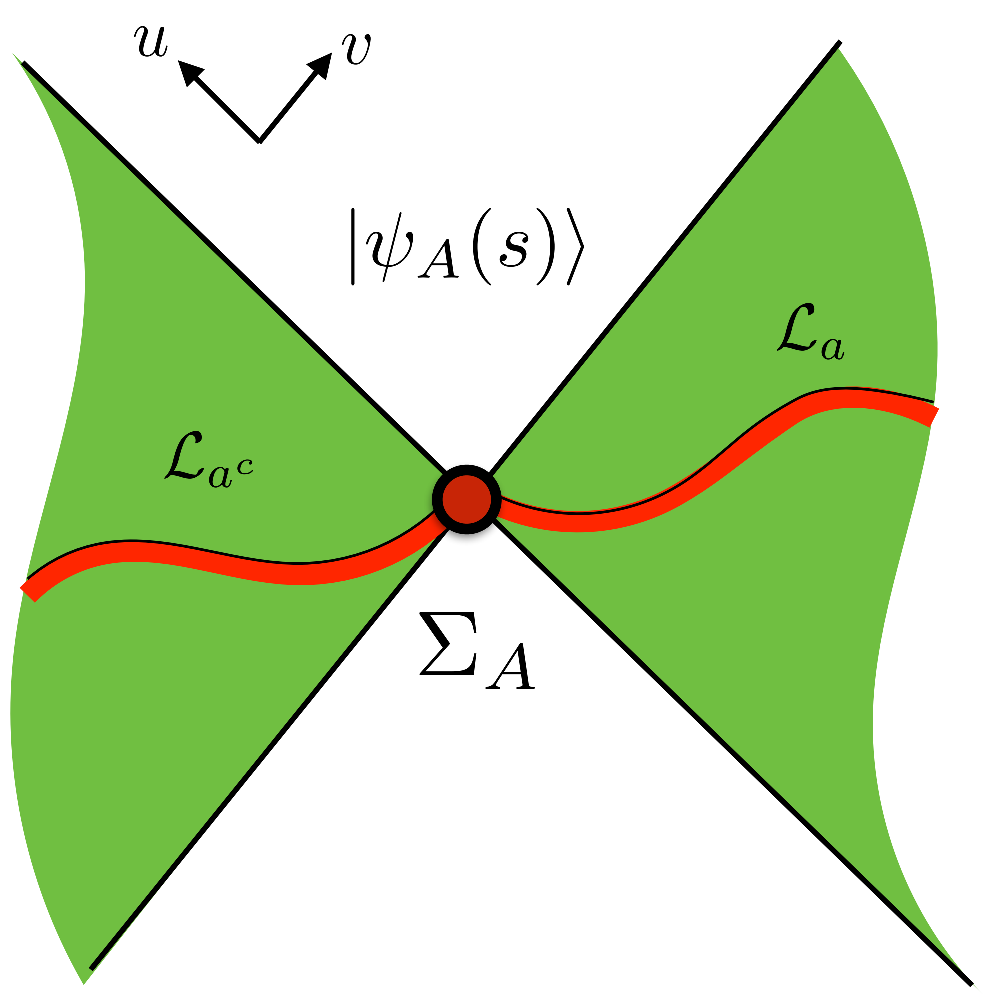
e.g. $s = s(x_1, x_2)$

the geodesic avoids the Milne wedge, passes through Σ_A

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

So, what do we know about geodesics in the entanglement wedges (EW)?



- each segment $\{\mathcal{L}_a, \mathcal{L}_{a^c}\}$ is a geodesic in the original geometry $|\psi\rangle$

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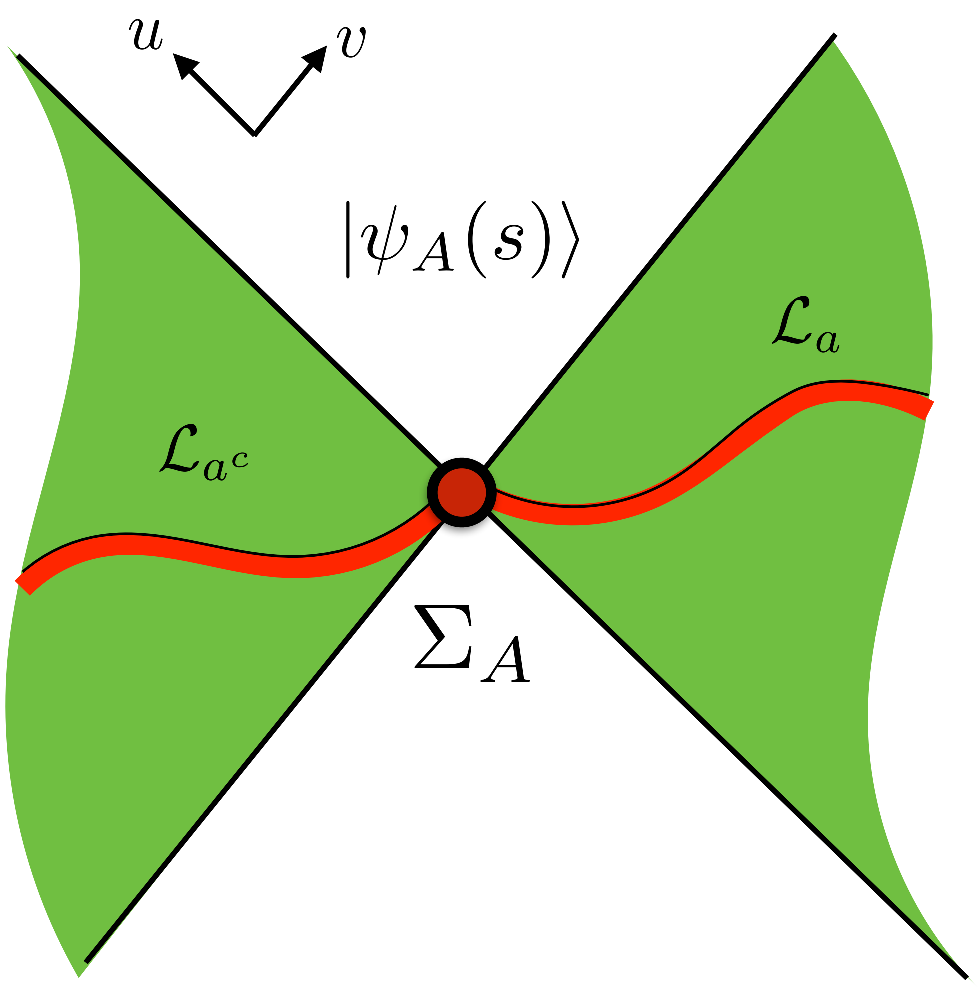

- each segment $\{\mathcal{L}_a, \mathcal{L}_{a^c}\}$ is a geodesic in the original geometry $|\psi\rangle$.
- modular flow affects the matching condition at Σ_A .

Bulk modular flow in AdS/CFT

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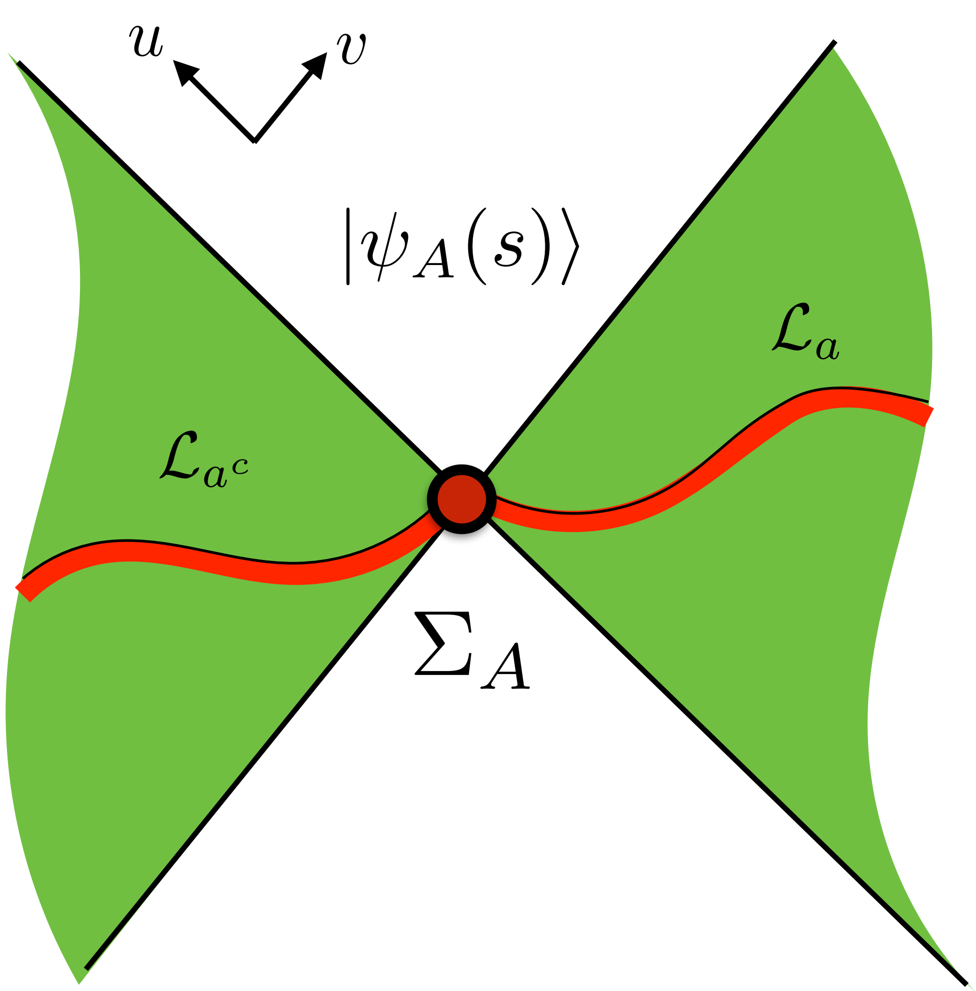

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Bulk modular flow in AdS/CFT

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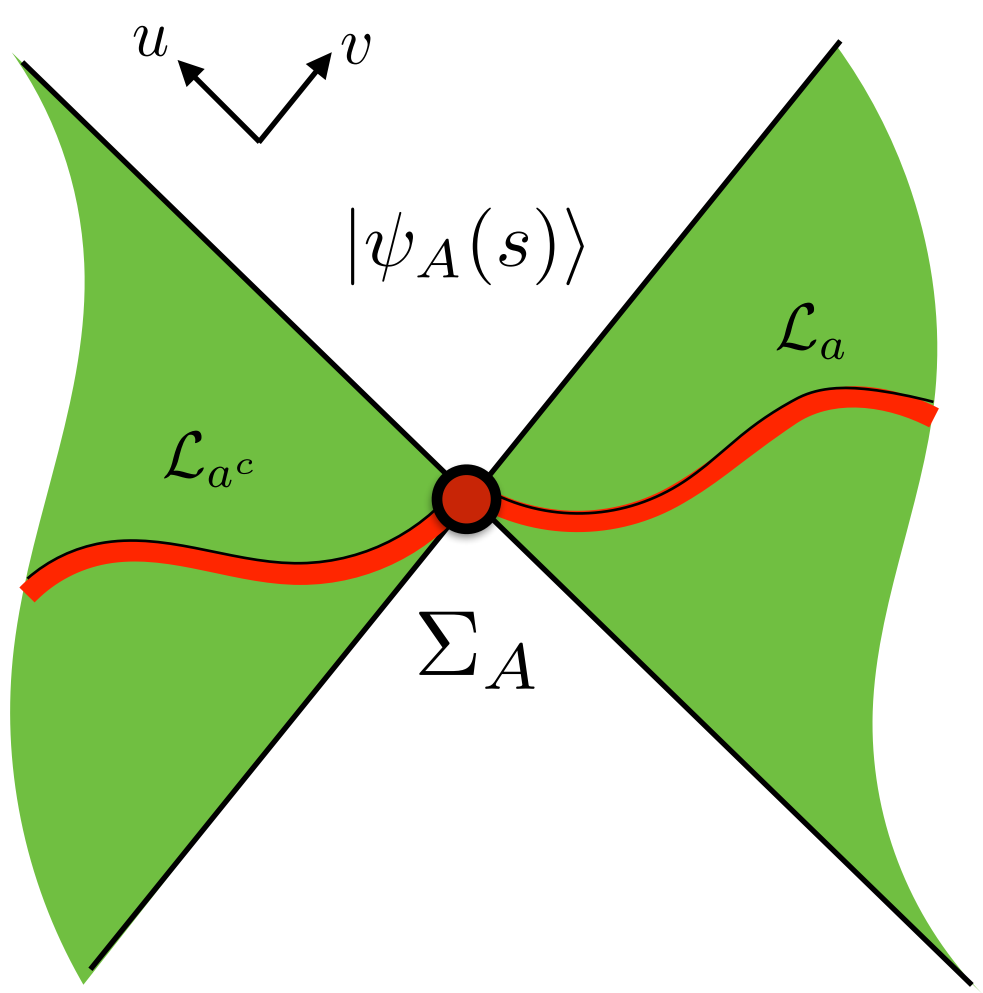

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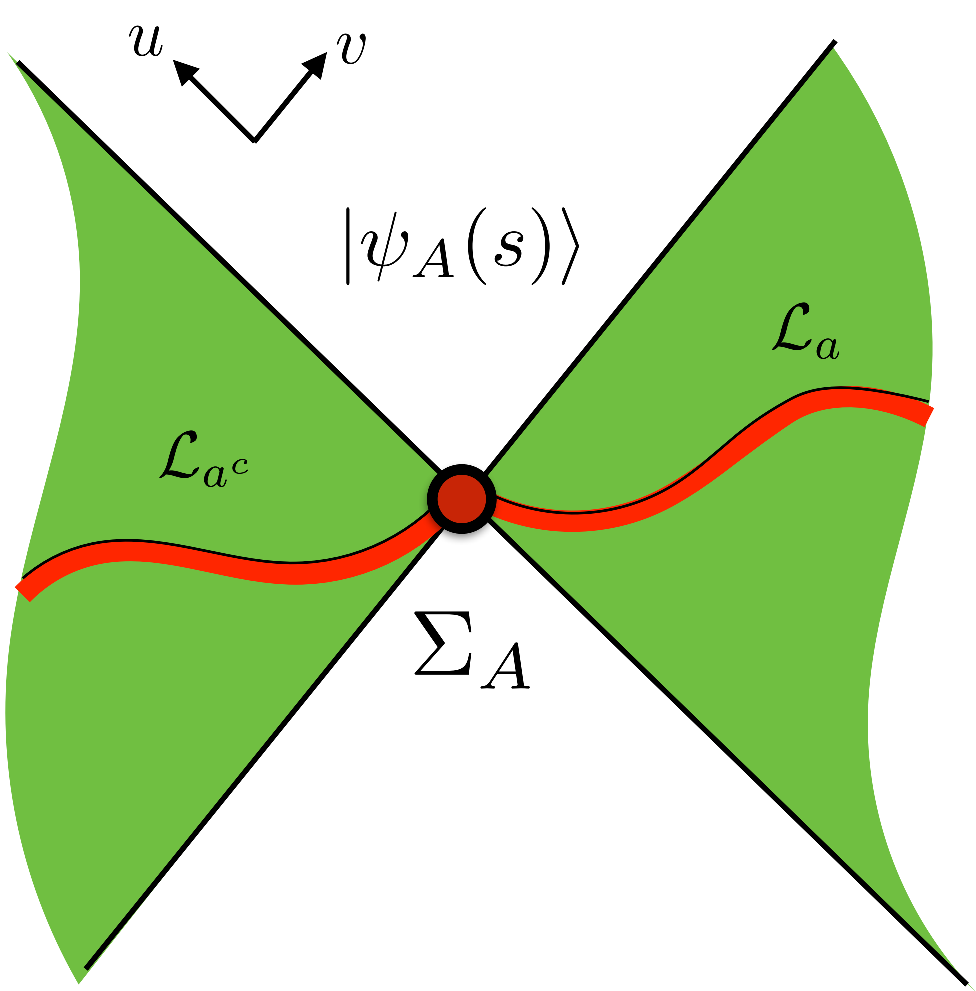

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- $\hat{A}$ is a constant in EW, $e^{isH_A^\psi(bdry)} \propto e^{isH_a^\psi(bulk)}$.
- bulk theory free (leading ordering 1/N): close to Σ_A , $H_a^\psi(bulk)$ acts like bulk Rindler Hamiltonian and generates boosts.

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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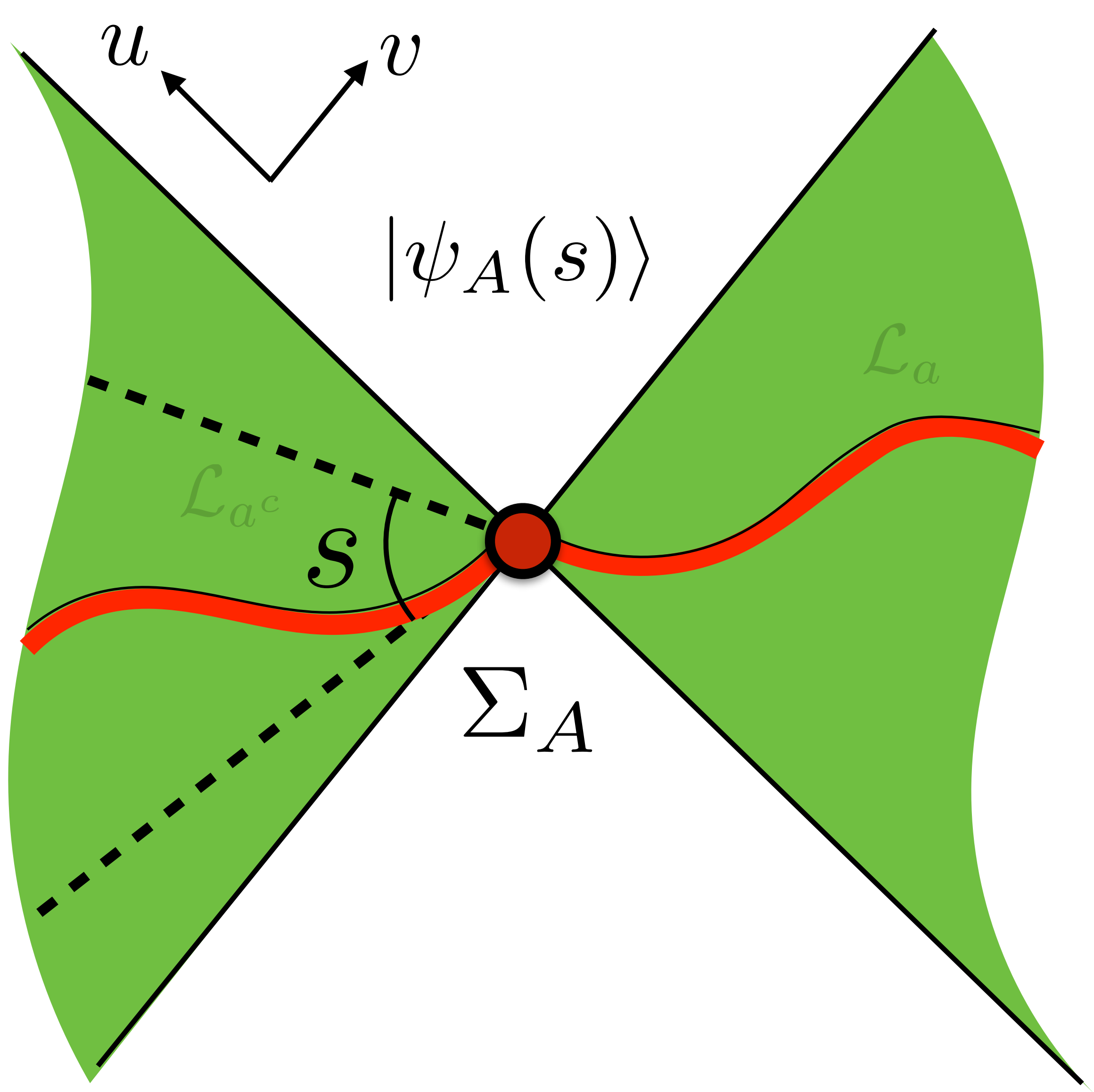

matching condition: relative boost of rapidity s across Σ_A .

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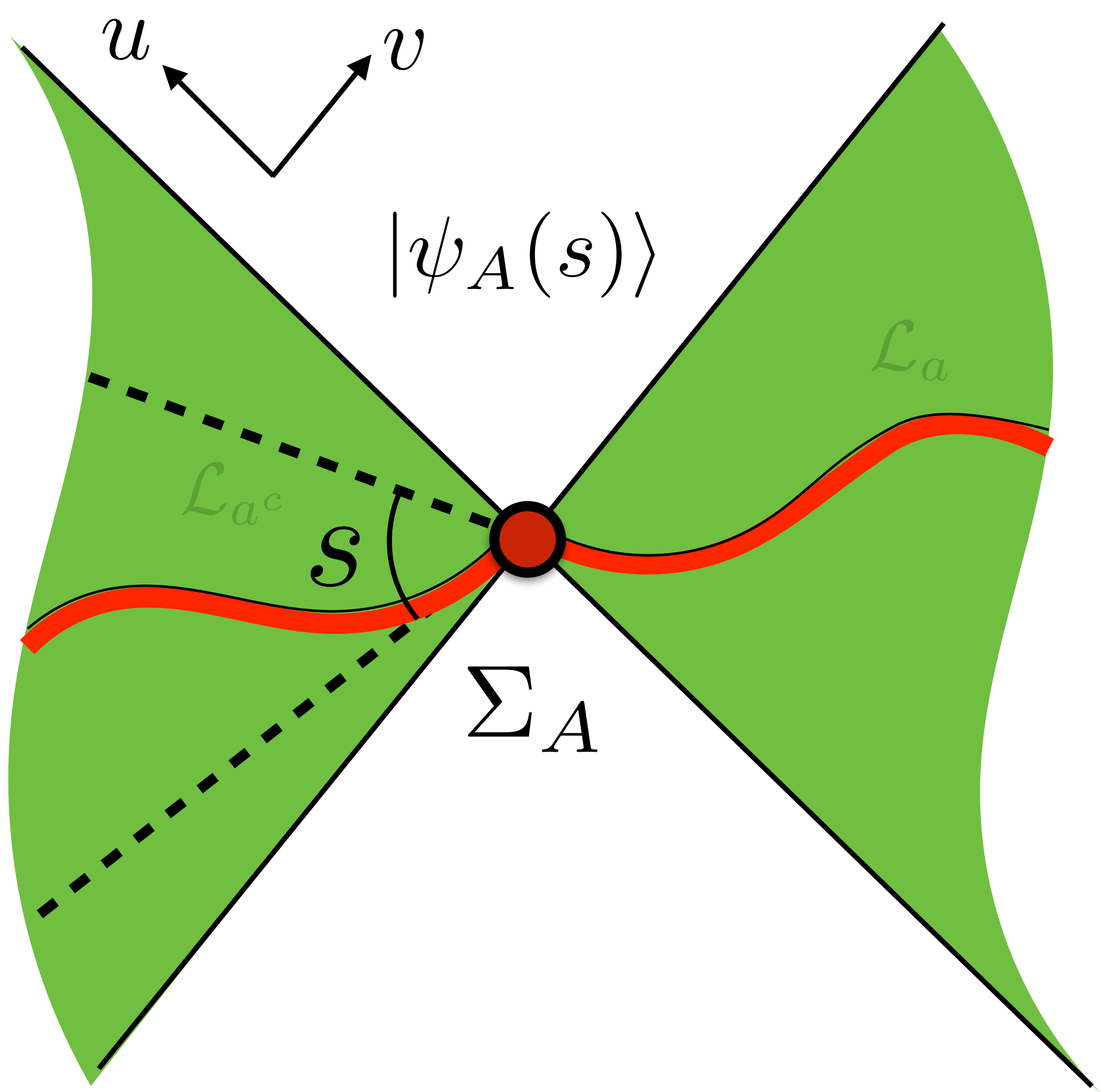

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modified notion of smoothness for curves across Σ_A in $|\psi_A(s)\rangle$.

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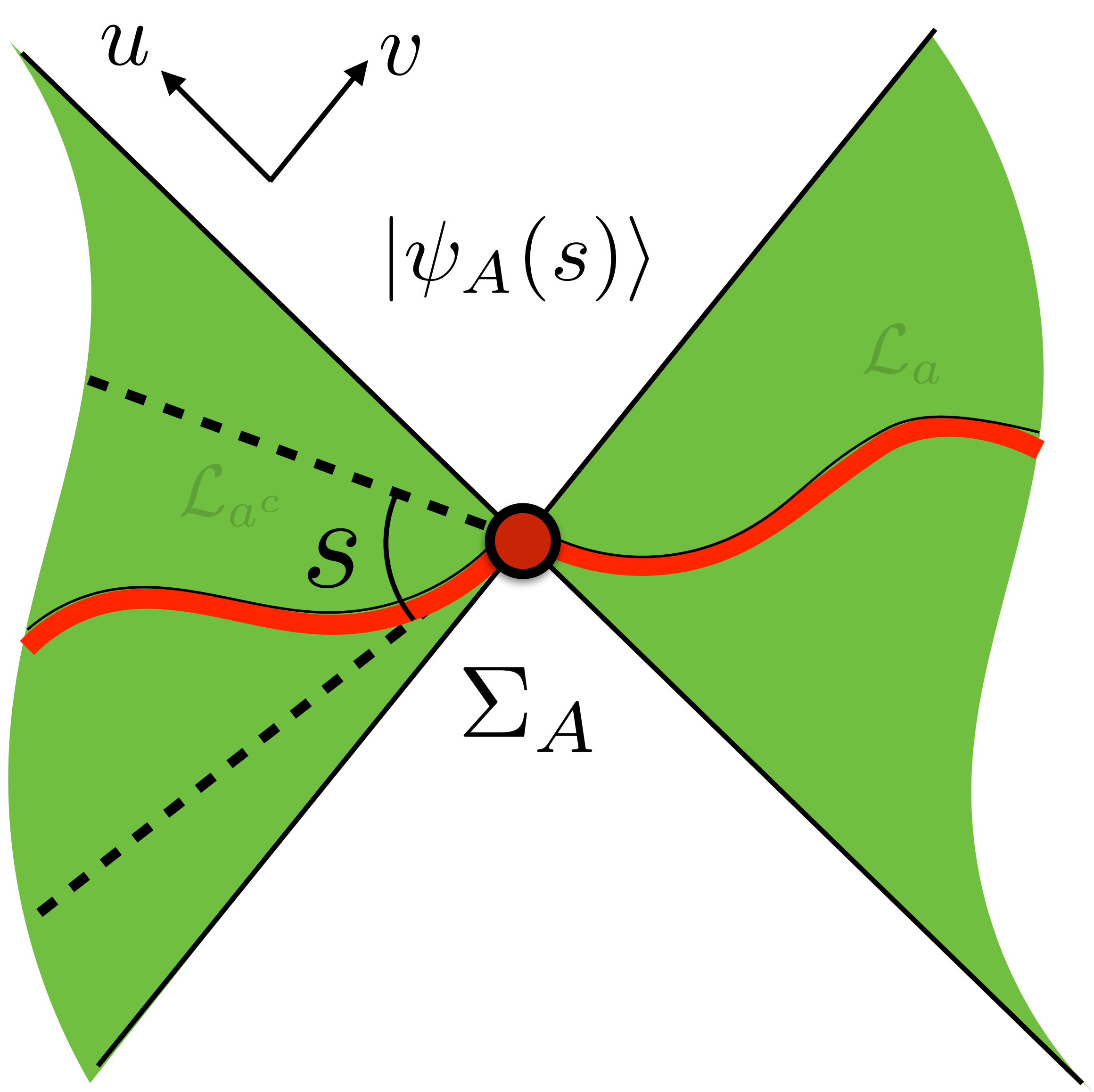

matching condition: relative boost of rapidity s across Σ_A .

modified notion of smoothness for curves across Σ_A in $|\psi_A(s)\rangle$.

fine-tuning: identify $\xi \in \Sigma_A$ s.t. at ξ

$$p_{||}\left[\mathcal{L}(\xi, x_1)\right] = p_{||}\left[\mathcal{L}(\xi, x_2)\right]$$

then

$$s(x_1, x_2) = \frac{1}{4\pi} \ln \left(\frac{p_u\left[\mathcal{L}(\xi, x_1)\right]}{p_v\left[\mathcal{L}(\xi, x_1)\right]} \right) \left(\frac{p_v\left[\mathcal{L}(\xi, x_2)\right]}{p_u\left[\mathcal{L}(\xi, x_2)\right]} \right)$$

Bulk modular flow in AdS/CFT

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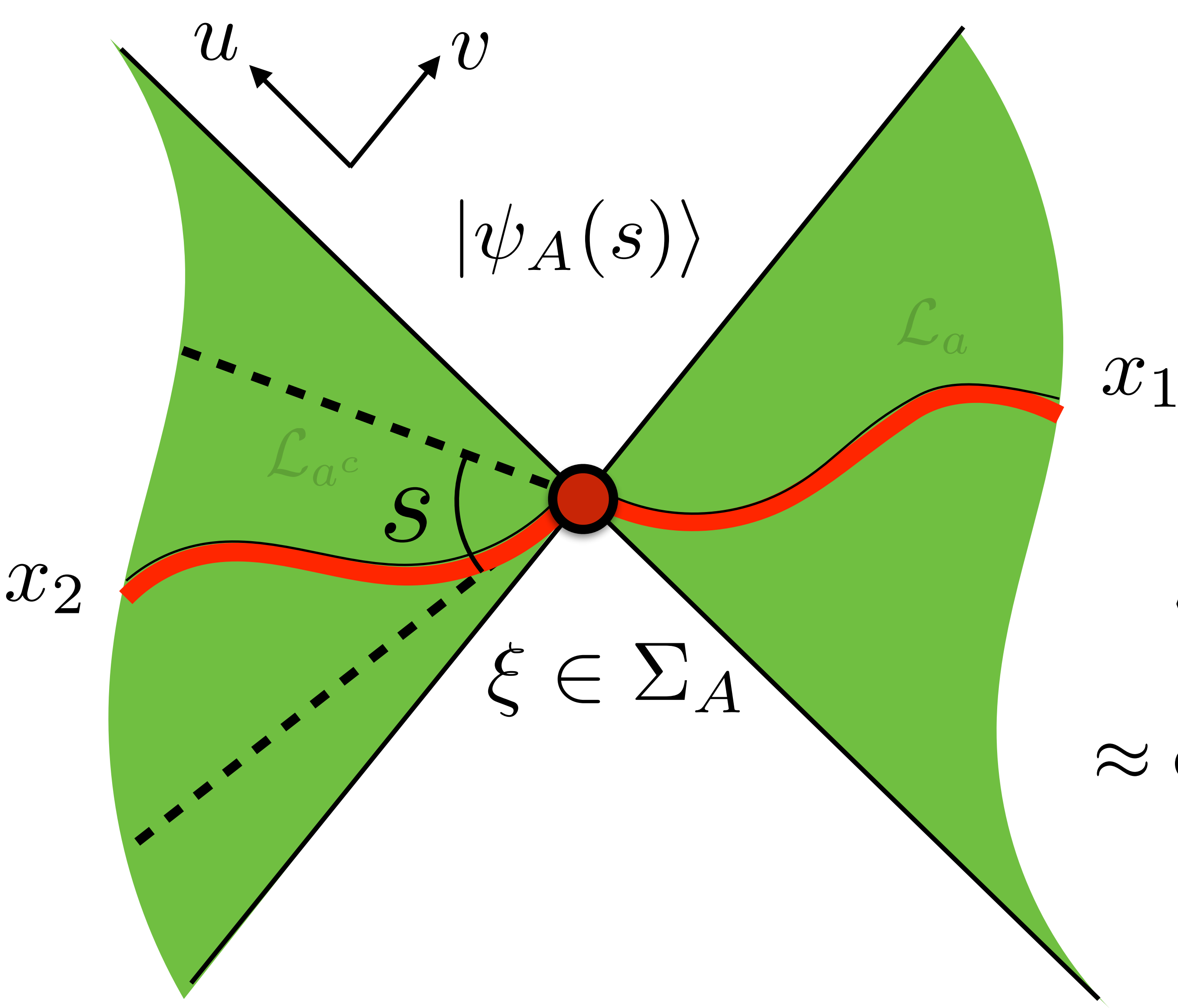


Therefore, for $s^* = s(x_1, x_2)$

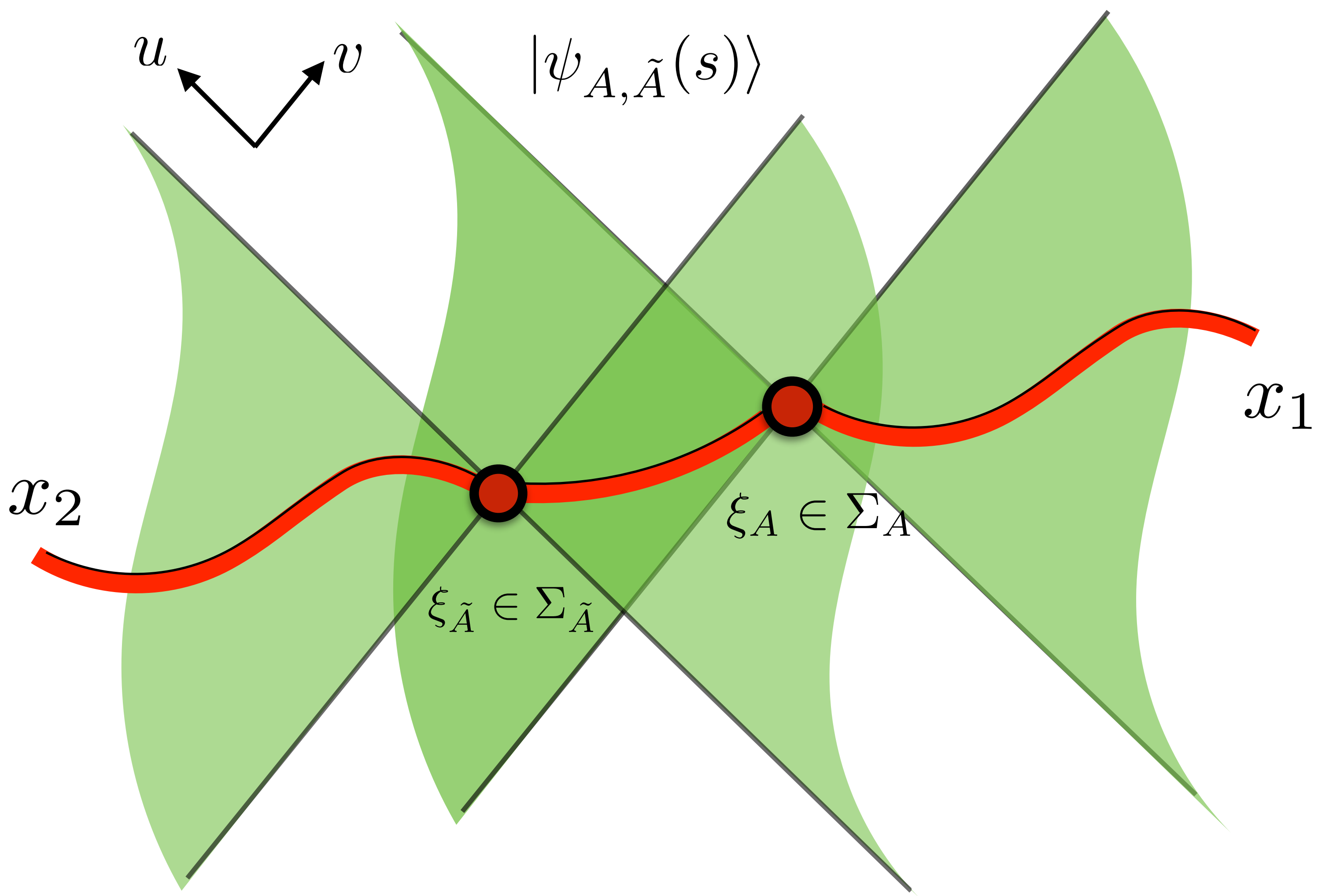
$$\langle \mathcal{O}_1 \mathcal{O}_2^A(s^*) \rangle_\psi = \langle \mathcal{O}_1 \mathcal{O}_2 \rangle_{\psi_A(s^*)}$$

$$\approx \exp\left[-m\mathcal{L}(\xi, x_1) - m\mathcal{L}(\xi, x_2)\right]$$

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

We can extend this to the "double modular flow": $|\psi_{A,\tilde{A}}(s)\rangle = e^{-isH^{\psi}_{\tilde{A}}} e^{isH^{\psi}_{A}} |\psi\rangle$



matching conditions at

$$\xi_A \in \Sigma_A, \; \xi_{\tilde{A}} \in \Sigma_{\tilde{A}}$$

select $s^* = s(x_1, x_2)$

$$\langle \mathcal{O}_1^A(s^*) \mathcal{O}_2^{\tilde{A}}(s^*) \rangle_\psi = \langle \mathcal{O}_1 \mathcal{O}_2 \rangle_{\psi_{A,\tilde{A}}(s^*)}$$

$$\approx \exp\left[-m\left(\mathcal{L}(\xi_A, x_1) + \mathcal{L}(\xi_{\tilde{A}}, \xi_A) + \mathcal{L}(\xi_{\tilde{A}}, x_2)\right)\right]$$

Bulk modular flow in AdS/CFT

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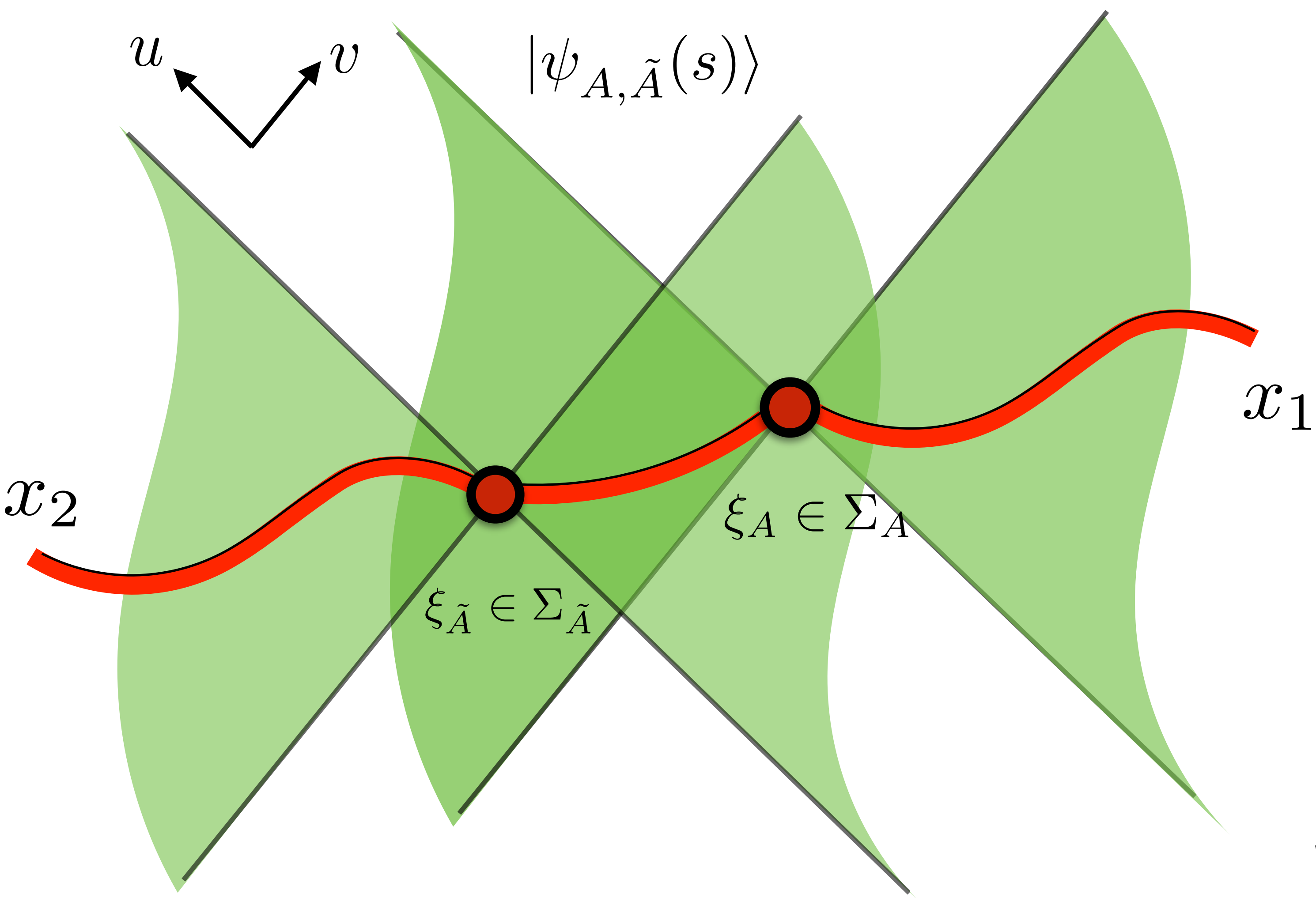


matching conditions at

$$\xi_A \in \Sigma_A,\ \xi_{\tilde{A}} \in \Sigma_{\tilde{A}}$$

select $s^* = s(x_1, x_2)$

in the near boundary limit $z \to 0$, successfully reproduced the CFT result in the light-cone limit $z \propto uv$

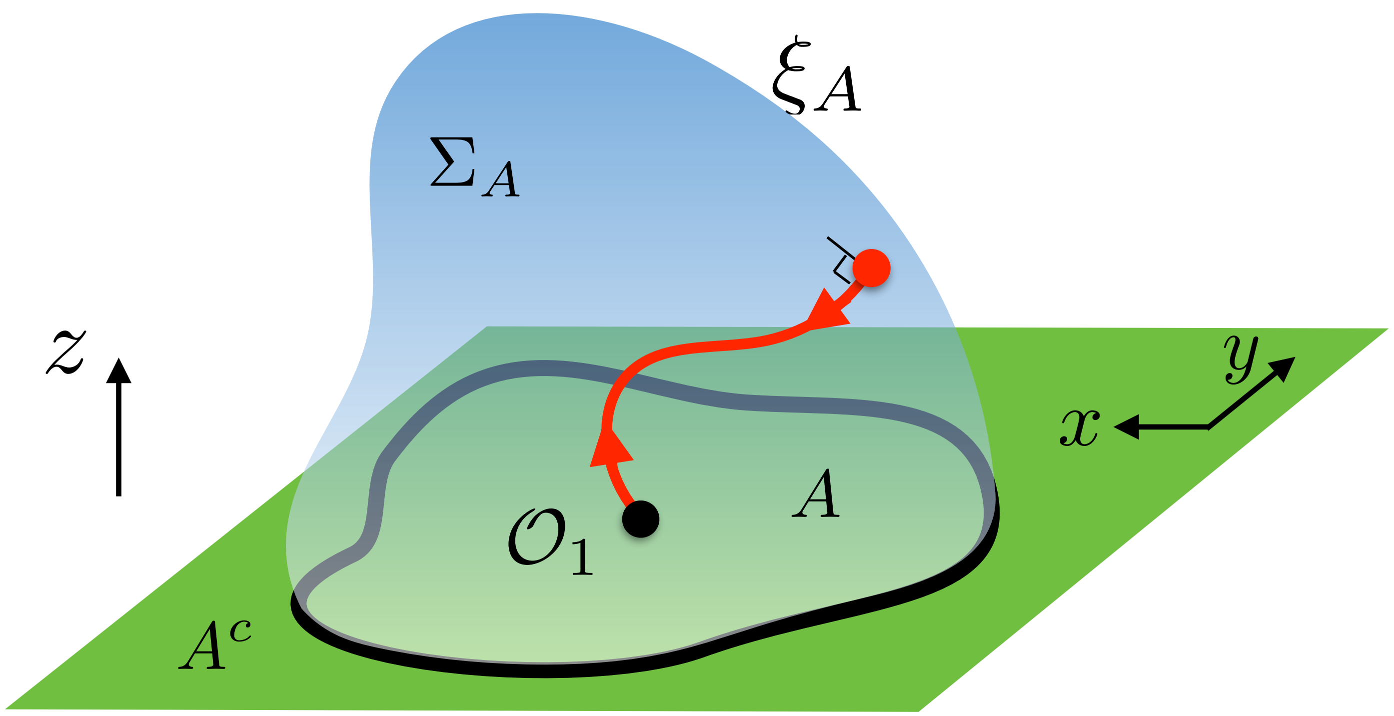
Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

Applications:

Mirror conjugation: $\mathcal{O}^J = e^{\pi K \psi_A} \mathcal{O} e^{-\pi K \psi_A} = \mathcal{O}^A(i\pi)$ K. Papadodimas, S. Raju, 2014

$f_\pi \propto \langle \mathcal{O}_1^A(i\pi)\mathcal{O}_1\rangle_\psi$ "single modular flow" with $s = i\pi$



$i\pi$ boost = reflection

$$\langle \mathcal{O}_1^J \mathcal{O}_1\rangle_\psi \approx \exp\left[-2m\mathcal{L}(\xi_A, x_1)\right]$$

Bulk modular flow in AdS/CFT

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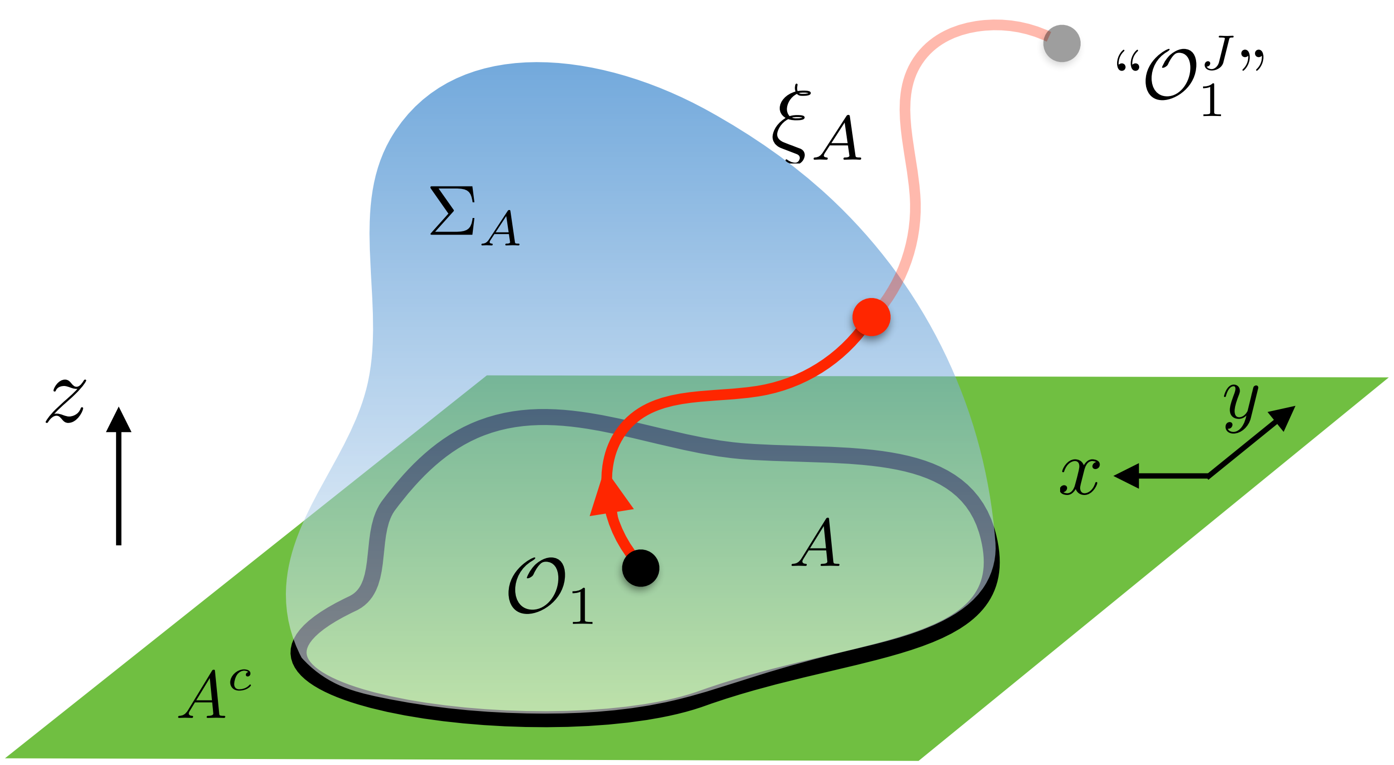


$i\pi$ boost = reflection

$\langle \mathcal{O}_1^J \mathcal{O}_1\rangle_\psi \approx \exp\left[-2m\mathcal{L}(\xi_A, x_1)\right]$

RT surface serves as a mirror for implementing conjugation

Bulk modular flow in AdS/CFT

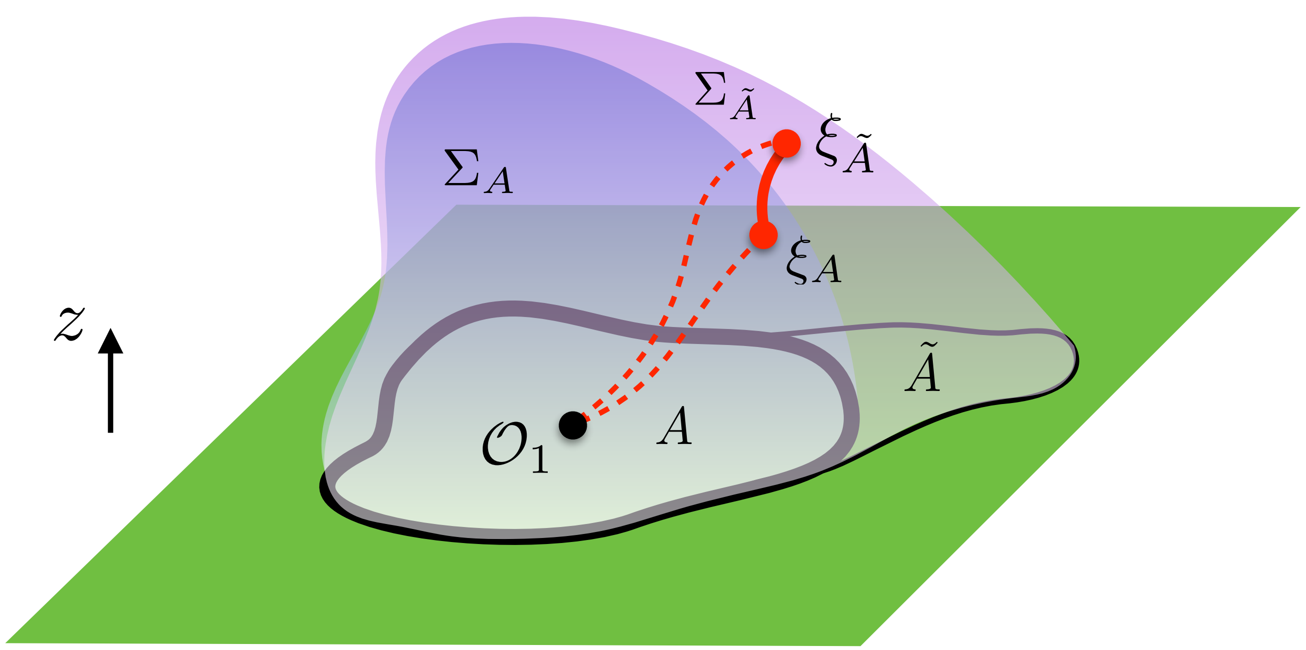
T. Faulkner, M. Li, H. Wang, 2018

Applications:

entanglement wedge nesting (EWN)

consider: $f(s) \propto \langle \mathcal{O}_1^{\tilde{A}}(s+i\pi)\mathcal{O}_1^{A}(s)\rangle_\psi$, $\tilde{A} = A + \delta A$

for $\delta A \to 0$, $f(s) = \langle \mathcal{O}_1^{J}\mathcal{O}_1\rangle_\psi \approx \exp\left[-2m\mathcal{L}(\xi_A, x_1)\right]$ for all s



$$-m^{-1}\ln\left[\frac{\langle \mathcal{O}_1^{\tilde{A}}(s+i\pi)\mathcal{O}_1^{A}(s)\rangle_\psi}{\langle \mathcal{O}_1^{A}(i\pi)\mathcal{O}_1\rangle_\psi}\right]$$

$$\approx \mathcal{L}\left(\xi_{\tilde{A}}, \xi_A\right) + \mathcal{O}(\delta A^2)$$

Bulk modular flow in AdS/CFT

T. Faulkner, M. Li, H. Wang, 2018

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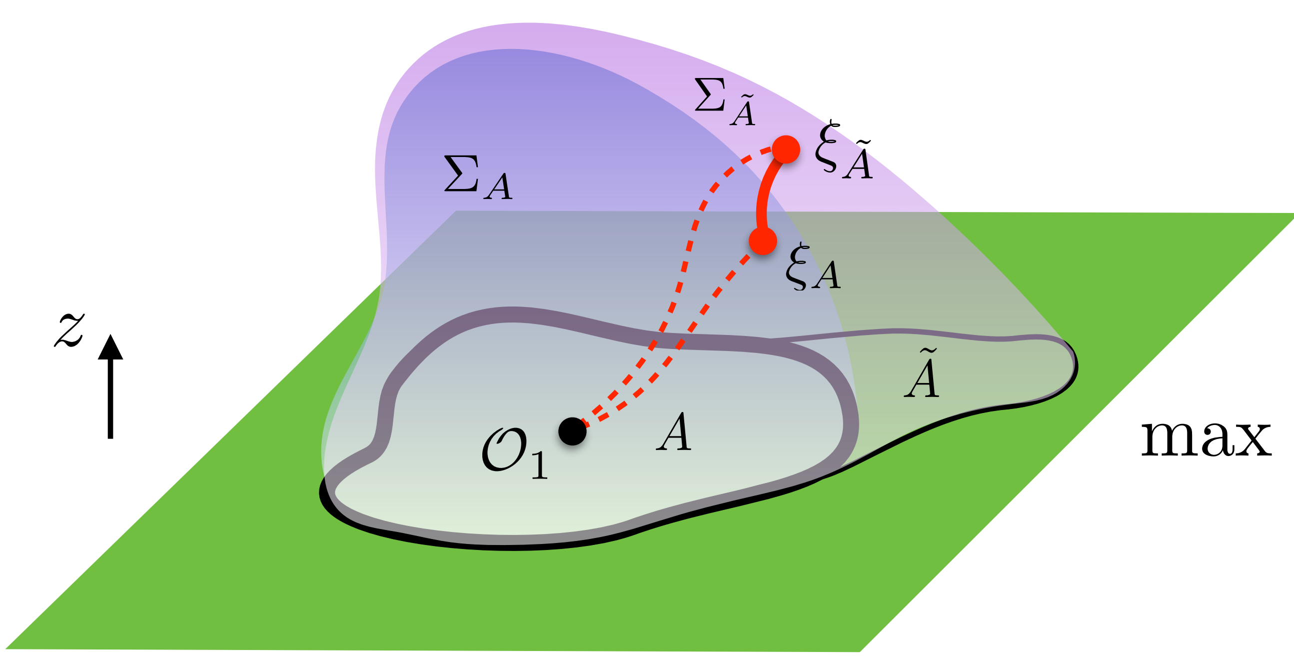

EWN in CFT (for $|\delta A| \ll |A|$)

$$\max\left\{ \ln\left[\frac{\langle \mathcal{O}_1^{\tilde{A}}(s+i\pi)\mathcal{O}_1^{A}(s)\rangle_\psi}{\langle \mathcal{O}_1^{A}(i\pi)\mathcal{O}_1\rangle_\psi}\right], s \in \mathbb{R} \right\} \leq 0$$

for space-like δA

Bulk modular flow in AdS/CFT

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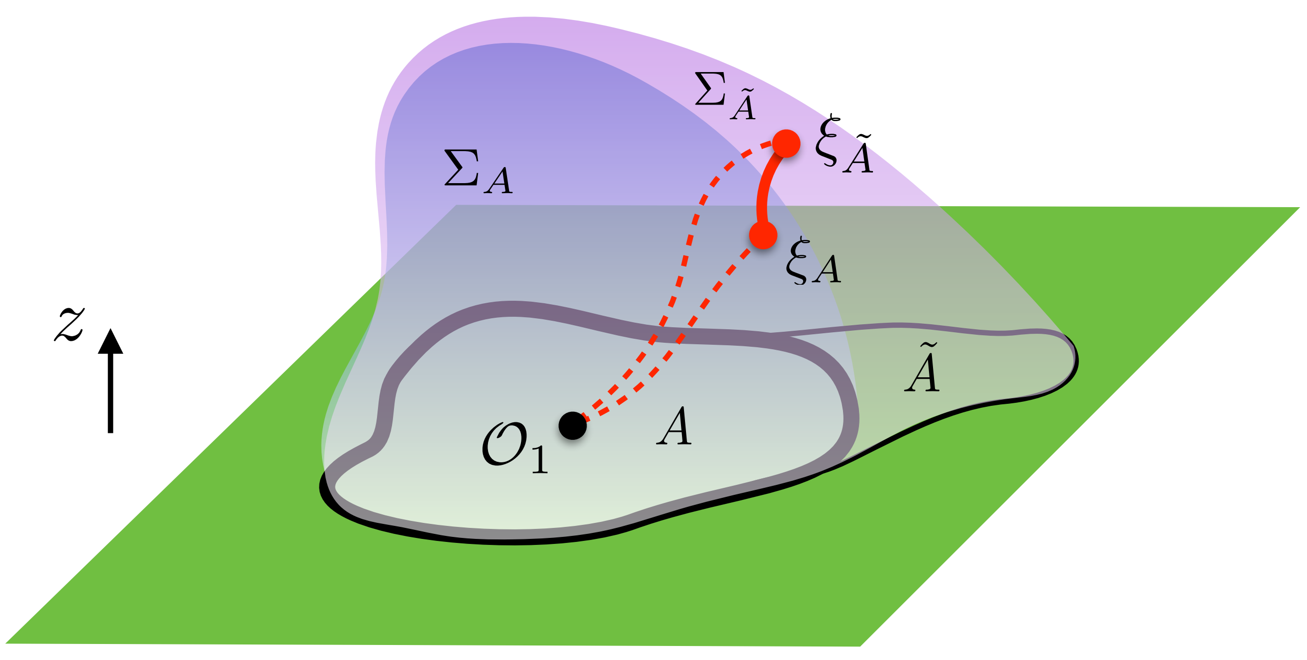

in Tomita-Takaseki theory:

can be derived from

$$|U(t)| \leq 1,\ U(t) = e^{-iK^{\psi}_{\tilde{A}}t}e^{iK^{\psi}_{A}t}$$

Conclusion/Outlook

- general proofs of energy conditions in QFTs
- physical picture encoded in the entanglement structures (modular flow)
- holographic proof of QNEC using EWN: RT surface dynamics
- boundary modular flow "knows" about these…
- prescription for (fine-tuned classes of) modular flows in AdS/CFT

Conclusion/Outlook

Future directions:

- what happens in the "Milne wedges"?
- 1/N corrections to the prescription
- other bulk constraints from boundary modular flow, e.g. quantum focusing conjecture (QFC)?

Thank you!