SYK, Chaos and Higher-Spins

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Based on: JHEP 1812 (2018) 065 CP

Coupled quantum systems

Based on: arXiv:2001.03158 Alet, Hanada, Jevicki, CP

SYK, Chaos and Higher-Spins

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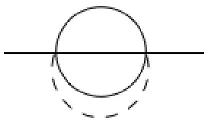
The Sachdev-Ye-Kitaev (SYK) model

(Sachdev, Ye, 1993; Parcollet, Georges, 1998; Kitaev 2015,2017, Maldacena, Stanford 2016,...)

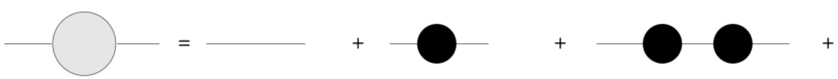
- Strongly coupled Quantum Mechanics model

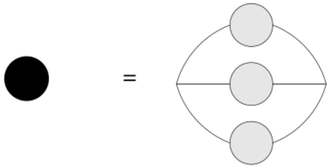
$$H = (i)^{\frac{q}{2}} \sum_{1\le i_1<i_2<\cdots<i_q\le N} j_{i_1 i_2\cdots i_q}\psi_{i_1}\psi_{i_2}\cdots\psi_{i_q}$$

$$\langle j^2_{i_1\cdots i_q}\rangle = \frac{J^2(q-1)!}{N^{q-1}} = \frac{2^{q-1}}{q}\frac{\mathcal{J}^2(q-1)!}{N^{q-1}}$$



- Perturbatively solvable in the large *N* limit



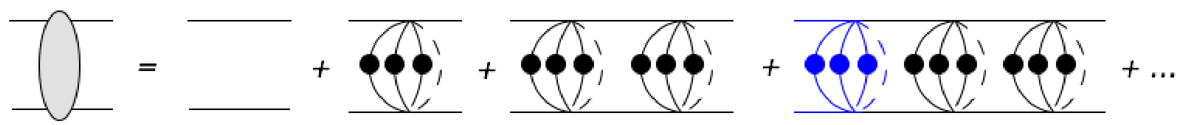


- 2-point function in the IR $\tau J \gg 1$, conformal

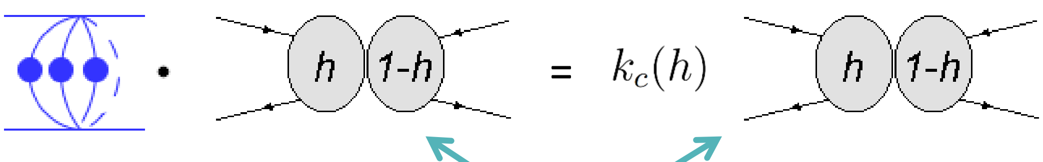
$$G_c(\tau) = \frac{b}{|\tau|^{2\Delta}}\mathrm{sgn}(\tau) \qquad J^2 b^q \pi = \left(\frac{1}{2} - \Delta\right)\tan\pi\Delta \qquad \Delta = \frac{1}{q}$$

The Sachdev-Ye-Kitaev (SYK) model

- A tower of higher-spin operators in the IR



$$\mathcal{F} = \frac{1}{1-K}\mathcal{F}_0$$



$$\mathcal{F}(\chi) = \frac{1}{1-K_c}\mathcal{F}_0 = \sum_h \Psi_h(\chi) \frac{1}{1-k_c(h)} \frac{\langle \Psi_h, \mathcal{F}_0 \rangle}{\langle \Psi_h, \Psi_h \rangle}$$

The Sachdev-Ye-Kitaev (SYK) model

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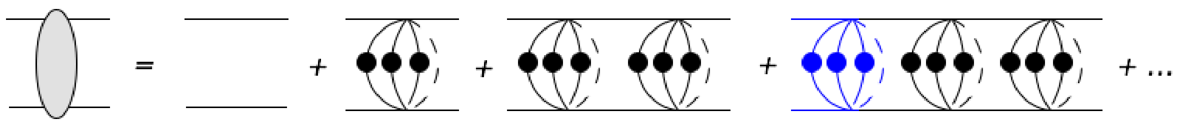

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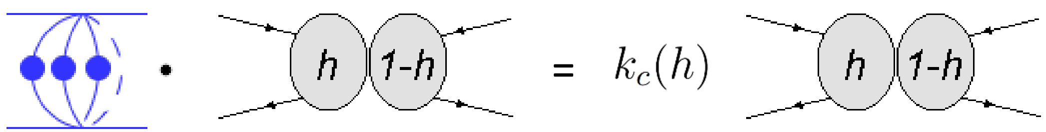
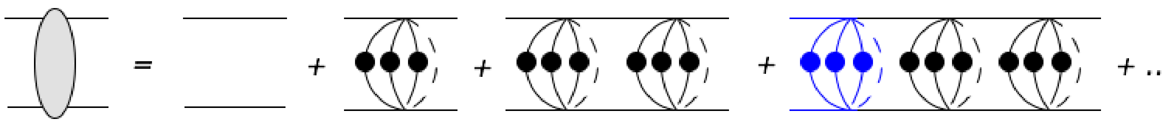


$$\mathcal{F}(\chi) = \frac{1}{1-K_c}\mathcal{F}_0 = \sum_h \Psi_h(\chi)\frac{1}{1-k_c(h)}\frac{\langle\Psi_h,\mathcal{F}_0\rangle}{\langle\Psi_h,\Psi_h\rangle}$$

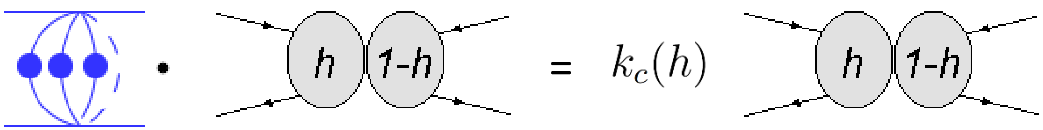
$$k_c(h) = 1 \qquad \Rightarrow \qquad h = 2\,, 3.77\,, 5.68\,, 7.63\,, \ldots$$

The Sachdev-Ye-Kitaev (SYK) model

- A tower of higher-spin operators in the IR



$$\mathcal{F} = \frac{1}{1-K}\mathcal{F}_0$$



$$\mathcal{F}(\chi) = \frac{1}{1-K_c}\mathcal{F}_0 = \sum_h \Psi_h(\chi)\frac{1}{1-k_c(h)}\frac{\langle\Psi_h,\mathcal{F}_0\rangle}{\langle\Psi_h,\Psi_h\rangle}$$

$$k_c(h) = 1 \qquad \Rightarrow \qquad h_m = 2\Delta + 1 + 2m + \epsilon_m$$

$$\mathcal{O}_m \sim \psi^i(\tau)\partial_\tau^{2m+1}\psi^i(\tau)$$

Relations to vector models ?

- An infinite tower of operators !
  (But with finite anomalous dimensions…)

- Similar models with emergent higher-spin symmetry ?
  (holographic dual to higher-spin gravity)

Why Higher-Spin ?

- What is higher-spin gravity ?
  - General relativity: graviton, spin-2
  - Higher-spin theory: graviton + higher-spin fields, spin-2,3,4,5…

    all fields are massless

- Higher-Spin theories are interesting:
  - Quantum gravity contains higher-spin fields
  - The most symmetric phase of quantum gravity

$$m^2 = \frac{1}{\alpha'}(N+a)\,, \qquad \alpha' \to \infty\,.$$

  - A special class of solvable models of the holographic principle

$$\ell_s \gg R \gg \ell_{\mathrm{Planck}}\,, \qquad \ell_s = \sqrt{\alpha'}\,,$$

$$\Rightarrow \quad \left(\frac{R}{\ell_{\mathrm{Planck}}}\right)^4 = N \gg 1\,, \qquad \frac{R^4}{\alpha'^2} = \lambda = g^2 N \ll 1$$

Relations to vector models ?

- An infinite tower of operators !

  (But with finite anomalous dimensions…)

- Similar models with emergent higher-spin symmetry ?

- One dimensional case (CP, 2017)

$$H = \frac{J}{2\,N^{3/2}}\left(\bar{\lambda}^{ab}{}_i\bar{\lambda}^{ab}{}_j\chi^i\chi^j + \lambda^{abi}\lambda^{abj}\bar{\chi}_i\bar{\chi}_j\right) + M\,\bar{\lambda}^{ab}{}_i\lambda^{abi} + \frac{u}{N^2}\bar{\lambda}^{ab}{}_i\lambda^{ebj}\phi^a\phi^e\chi^i\bar{\chi}_j + \frac{C}{2N^{3/2}}\bar{\lambda}^{ab}{}_i\bar{\lambda}^{eb}{}_j\lambda^{efi}\lambda^{afj}$$

Higher dimensions

- Why higher dimensions ?
  - Sensible notion of spins
  - Have better studied higher-spin/string models
  - Simplest example is 1+1D

Supersymmetric models

- SUSY is important to reach the SYK-like fix point

(Murugan,Stanford, Witten, 2017)

- Small number of supersymmetries for flexibility

- Look for connections with other established models

An $\mathcal{N}$=(0,2) model

(CP, 2018)

- $$S = \int d^2z d\theta d\bar{\theta} \left(-\bar{\Phi}^a \partial_{\bar{z}} \Phi^a + \frac{1}{2} \bar{\Lambda}^i \Lambda^i \right) + \int d^2z d\theta \frac{J_{ia_1 \ldots a_q}}{q!} \Lambda^i \Phi^{a_1} \ldots \Phi^{a_q}$$

  Chiral: $\Phi^a = \phi^a + \sqrt{2}\theta\psi^a + 2\theta\bar{\theta}\partial_z\phi^a \,,\qquad a = 1 \ldots N$

  Fermi: $\Lambda^i = \lambda^i - \sqrt{2}\theta G^i + 2\theta\bar{\theta}\partial_z\lambda^i \,,\qquad i = 1 \ldots M$

- $N, M \gg 1$, with $\mu = \frac{M}{N}$ fixed (but tunable)

- IR solution $G_c^I(z_1, z_2) = \dfrac{n_I}{(z_1 - z_2)^{2h_I} (\bar{z}_1 - \bar{z}_2)^{2\tilde{h}_I}} \,,\qquad I = \phi, \psi, \lambda, G$

$$h_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad h_\psi = \frac{\mu q^2 + \mu q - 2}{2\mu q^2 - 2}, \quad h_\lambda = \frac{q - 1}{2\mu q^2 - 2}, \quad h_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}$$
$$\tilde{h}_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad \tilde{h}_\psi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad \tilde{h}_\lambda = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}, \quad \tilde{h}_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2} \,.$$

Range of μ

- Convergence of the FT

$$\int r dr d\theta \, r^{2\frac{\mu q-1}{\mu q^2-1}-3} e^{i\theta} e^{ir\cos\theta}$$

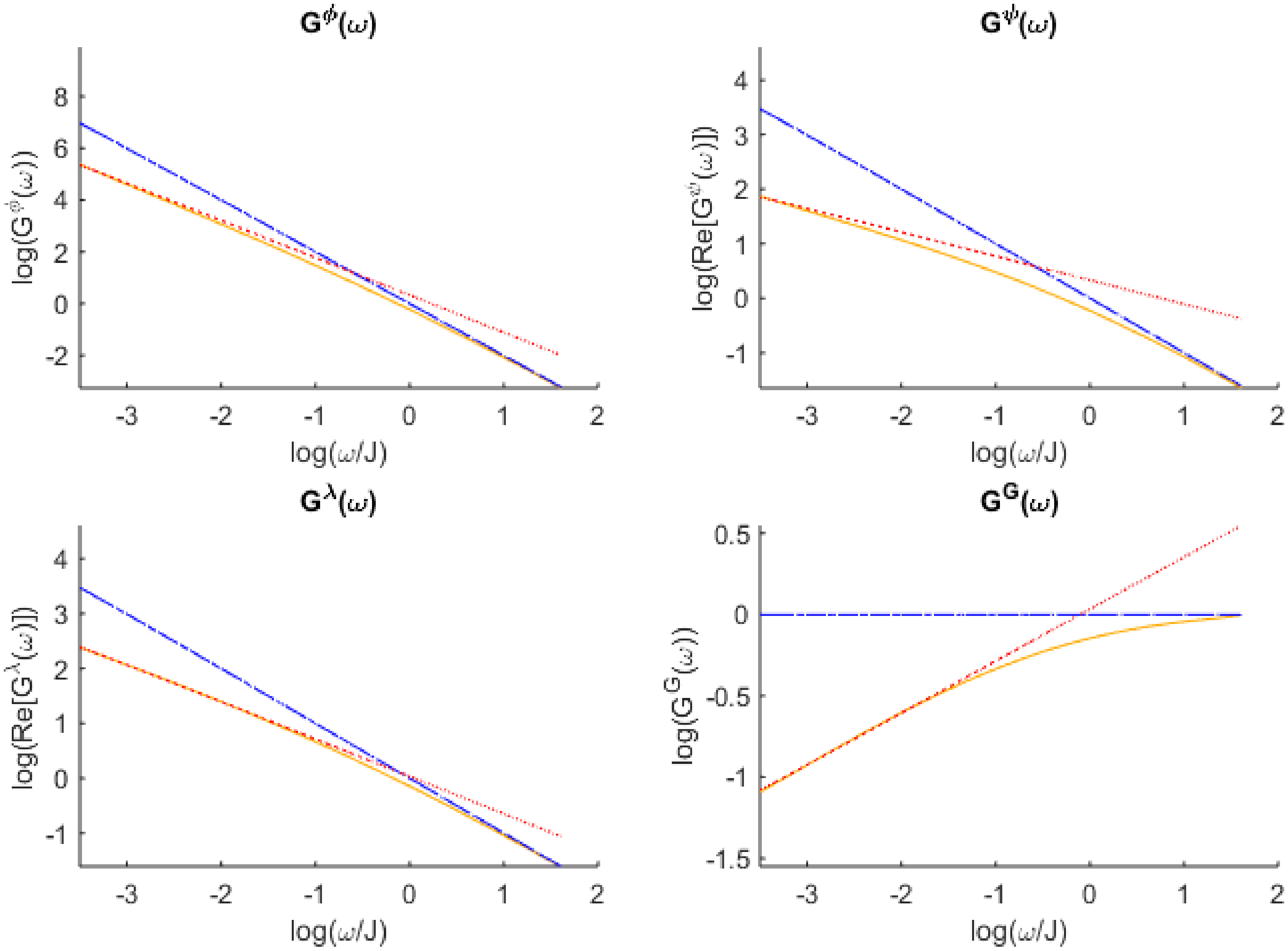
$$\Rightarrow \int dr d\theta r^{2\frac{\mu q-1}{\mu q^2-1}-2}(1+ir\cos\theta+\ldots)e^{i\theta}$$

$$\Rightarrow \int dr d\theta \, ir^{2\frac{\mu q-1}{\mu q^2-1}-1}\cos\theta e^{i\theta}$$

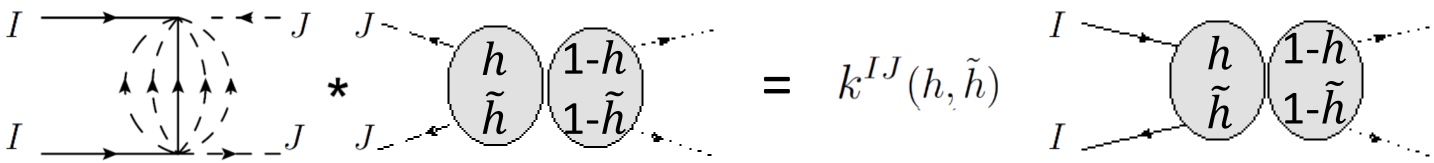
$$\boxed{\mu > \frac{1}{q}}$$

- In this range the model flows to the SYK-like fixed point

Numerical Confirmation



4-point function

- $\langle\bar{\phi}^i\phi^i\bar{\phi}^j\phi^j\rangle$, $\langle\bar{\phi}^i\phi^i\bar{\psi}^j\psi^j\rangle$, $\langle\bar{\phi}^i\phi^i\bar{\lambda}^j\lambda^j\rangle$, …
- 

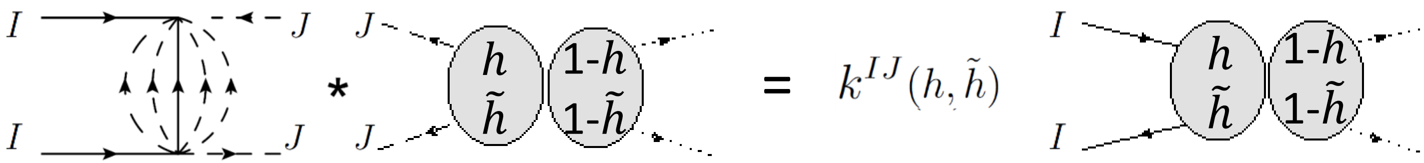
$$\Phi^I(z_1,z_2)=(z_{12})^{h-2h_I}(\bar{z}_{12})^{\tilde{h}-2\tilde{h}_I}\,,\qquad I=\phi,\psi,\lambda,G$$

$$k^{\psi\phi}=-\frac{2\mu(q-1)^2q(\mu q-1)^2\Gamma\left(\frac{(q-1)q\mu}{q^2\mu-1}\right)^2\Gamma\left(\frac{-h\mu q^2+\mu q^2+\mu q+h-2}{q^2\mu-1}\right)\Gamma\left(\tilde{h}-\frac{(q-1)q\mu}{q^2\mu-1}\right)}{(\mu q^2-1)^3\,\Gamma\left(\frac{\mu q^2+\mu q-2}{q^2\mu-1}\right)^2\Gamma\left(\frac{-h\mu q^2+(q-1)\mu q+h}{q^2\mu-1}\right)\Gamma\left(\tilde{h}+\frac{(q-1)q\mu}{q^2\mu-1}\right)}$$

…

…

4-point function

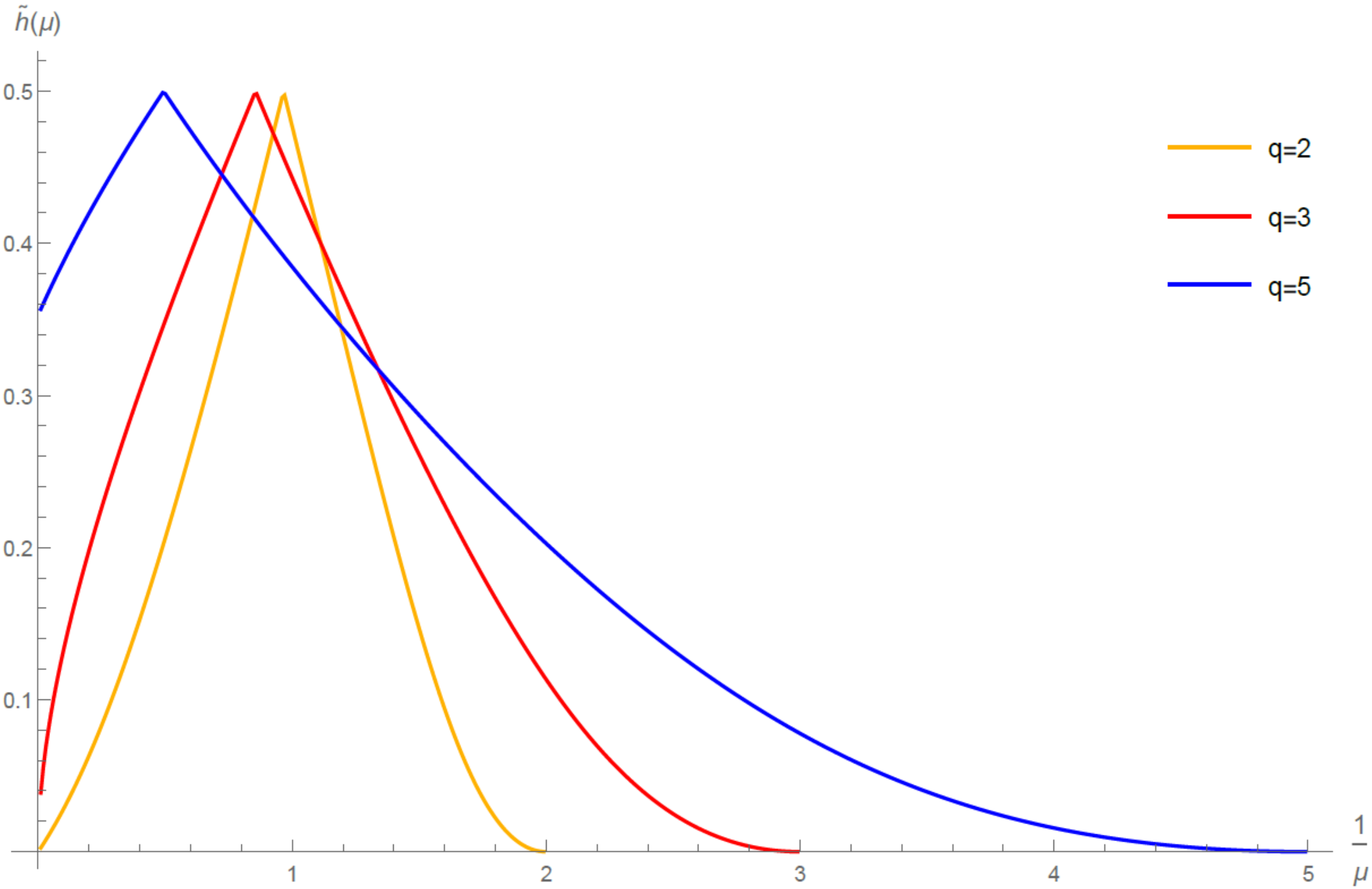
- $\langle \bar{\phi}^i \phi^i \bar{\phi}^j \phi^j \rangle$, $\langle \bar{\phi}^i \phi^i \bar{\psi}^j \psi^j \rangle$, $\langle \bar{\phi}^i \phi^i \bar{\lambda}^j \lambda^j \rangle$, ...
- 

- $\begin{pmatrix} k^{\phi\phi} & k^{\phi\psi} & k^{\phi\lambda} & k^{\phi G} \\ k^{\psi\phi} & 0 & k^{\psi\lambda} & 0 \\ k^{\lambda\phi} & k^{\lambda\psi} & 0 & 0 \\ k^{G\phi} & 0 & 0 & 0 \end{pmatrix}$ whose eigenvalue x satisfies

$$E_c(x, h, \tilde{h}, \mu, q) = x^4 - k^{\phi\phi} x^3 - \left(k^{\phi G} k^{G\phi} + k^{\phi\psi} k^{\psi\phi} + k^{\phi\lambda} k^{\lambda\phi} + k^{\psi\lambda} k^{\lambda\psi}\right) x^2 + \left(k^{\phi\phi} k^{\psi\lambda} k^{\lambda\psi} - k^{\phi\psi} k^{\psi\lambda} k^{\lambda\phi} - k^{\phi\lambda} k^{\psi\phi} k^{\lambda\psi}\right) x + k^{\phi G} k^{\psi\lambda} k^{\lambda\psi} k^{G\phi} = 0$$

- Solve x=1 to get the spectrum of $O^{\tilde{h},h}$, spin s = $|h - \tilde{h}|$.

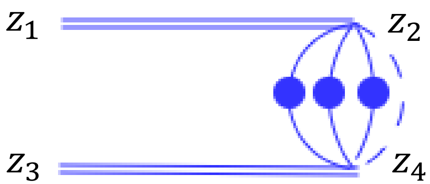
Lightest scalar operators



The Lyapunov exponent

- Out-of-Time-Ordered Correlators (Kitaev 2015, Maldacena Stanford, 2016)

$$\langle \phi^a(t+i\tau_1, x_1)\phi^b(i\tau_2, x_2)\bar{\phi}^a(t+i\tau_3, x_3)\bar{\phi}^b(i\tau_4, x_4)\rangle$$



- $K_R^{(ij)} * \Psi_R^j = k_R^{ij}\Psi_R^i$

$$\Psi_R^I(1,2) = \frac{e^{-\frac{1}{2}(h+\tilde{h})(t_1+t_2)-\frac{1}{2}(h-\tilde{h})(x_1+x_2)}}{(2\cosh\frac{x_{12}-t_{12}}{2})^{h_1+h_2-h}(2\cosh\frac{x_{12}+t_{12}}{2})^{\tilde{h}_1+\tilde{h}_2-\tilde{h}}}$$

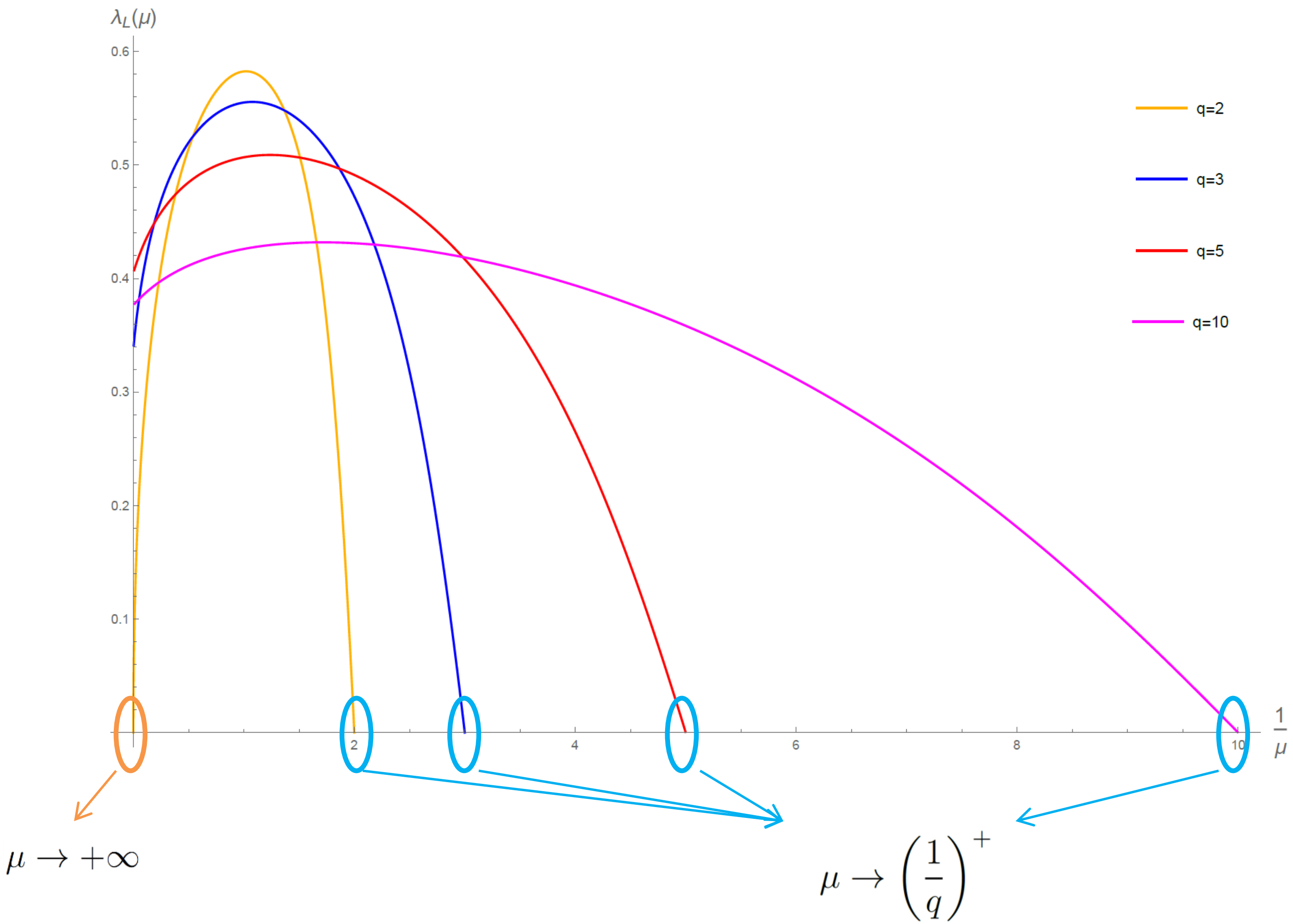
$$h = -\frac{\lambda_L}{2} + i\frac{p}{2} \qquad \tilde{h} = -\frac{\lambda_L}{2} - i\frac{p}{2}$$

(Murugan,Stanford, Witten, 2017)

- $$E_R(x,h,\tilde{h},\mu,q) = x^4 - k_R^{\phi\phi}x^3 - \left(k_R^{\phi G}k_R^{G\phi} + k_R^{\phi\psi}k_R^{\psi\phi} + k_R^{\phi\lambda}k_R^{\lambda\phi} + k_R^{\psi\lambda}k_R^{\lambda\psi}\right)x^2 + \left(k_R^{\phi\phi}k_R^{\psi\lambda}k_R^{\lambda\psi} - k_R^{\phi\psi}k_R^{\psi\lambda}k_R^{\lambda\phi} - k_R^{\phi\lambda}k_R^{\psi\phi}k_R^{\lambda\psi}\right)x + k_R^{\phi G}k_R^{\psi\lambda}k_R^{\lambda\psi}k_R^{G\phi} = 0$$

- Find λ_L by solving x=1

Two interesting limits



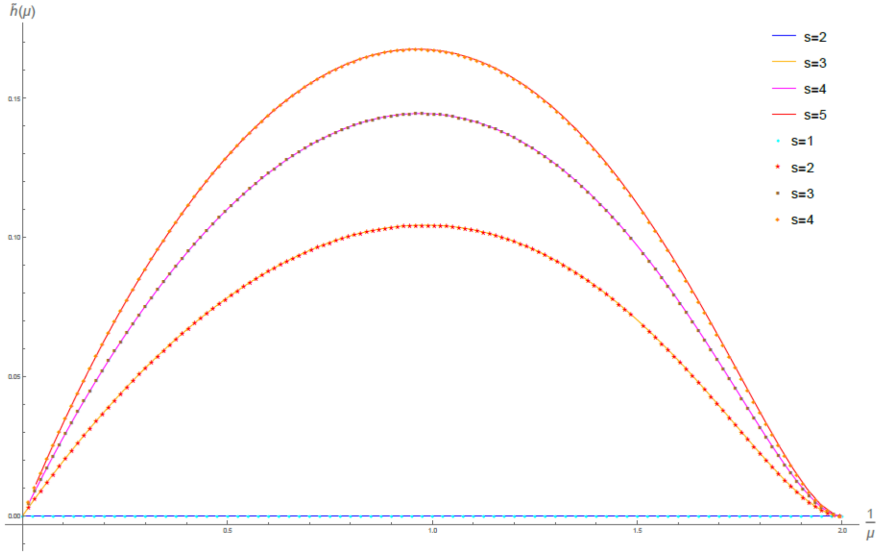
Two interesting limits

- Lyapunov exponent drops to zero
- "Integrability" takes over ?
- Large symmetries ?

$$(\tilde{h}, h) = (\gamma, \gamma + s) \text{ or } (\gamma + s, \gamma), \qquad s \in \mathbb{Z}/2$$

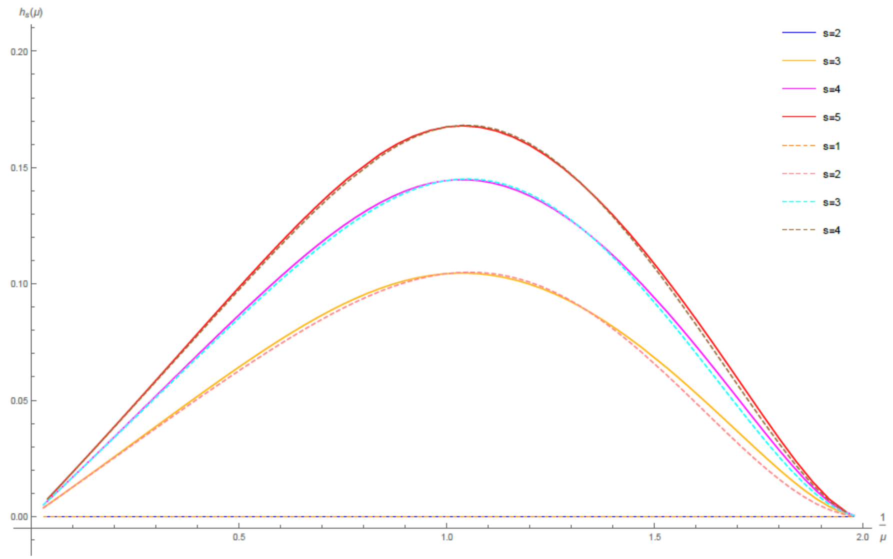
looking for the smallest γ for each μ

Lightest operators with spins



⬅ $(\gamma, \gamma + s)$

$(\gamma + s, \gamma)$ ⬇



- Emergent higher-spin conserved operators in the two limits!

- Generate large symmetry
  ➡ nonchaotic

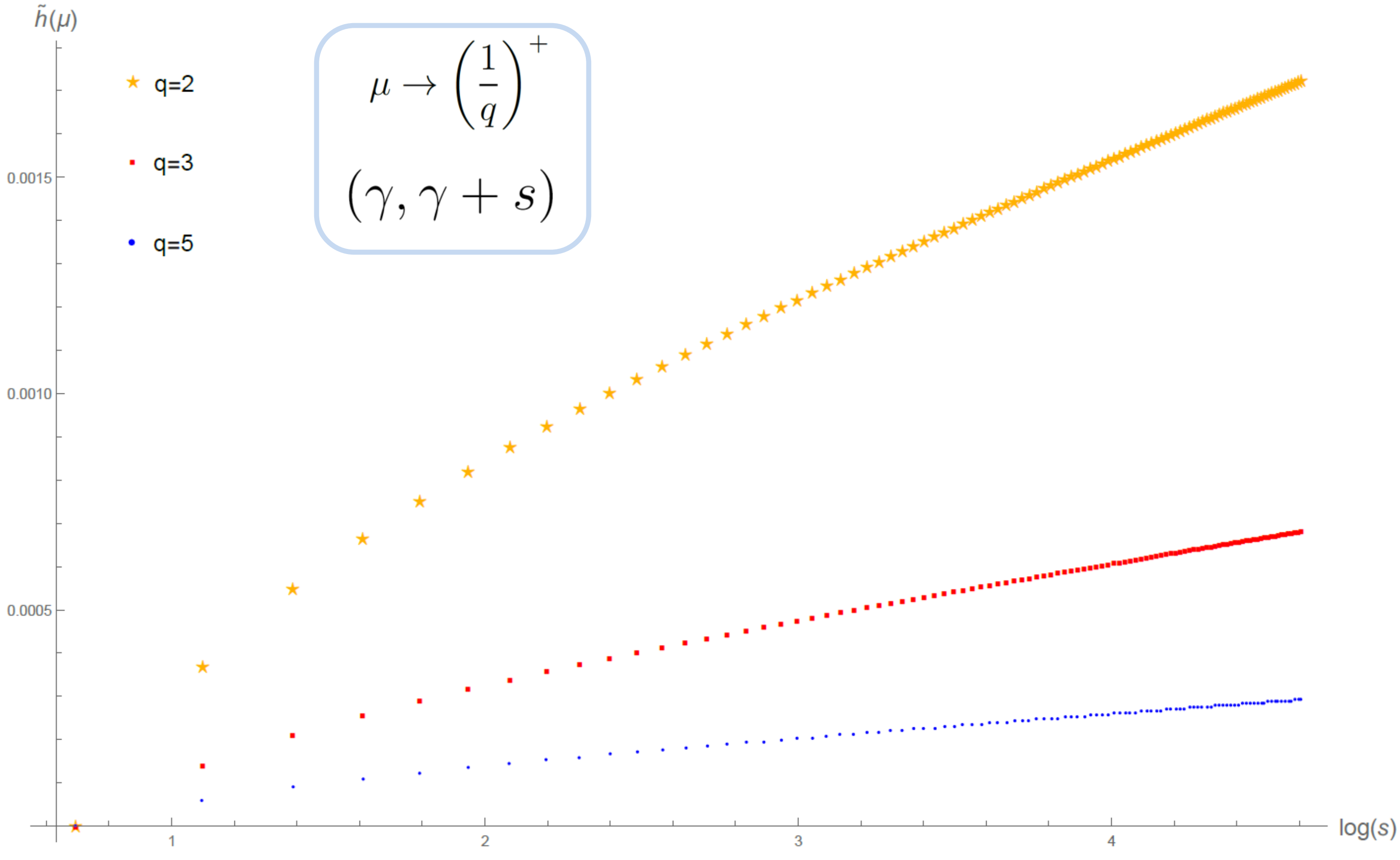
Relations with Higher-spin theories

- Reverse the direction,

- Consider small deformations away from the higher-spin point

Dispersion relation

- How does the anomalous dimension γ depend on spin s ?



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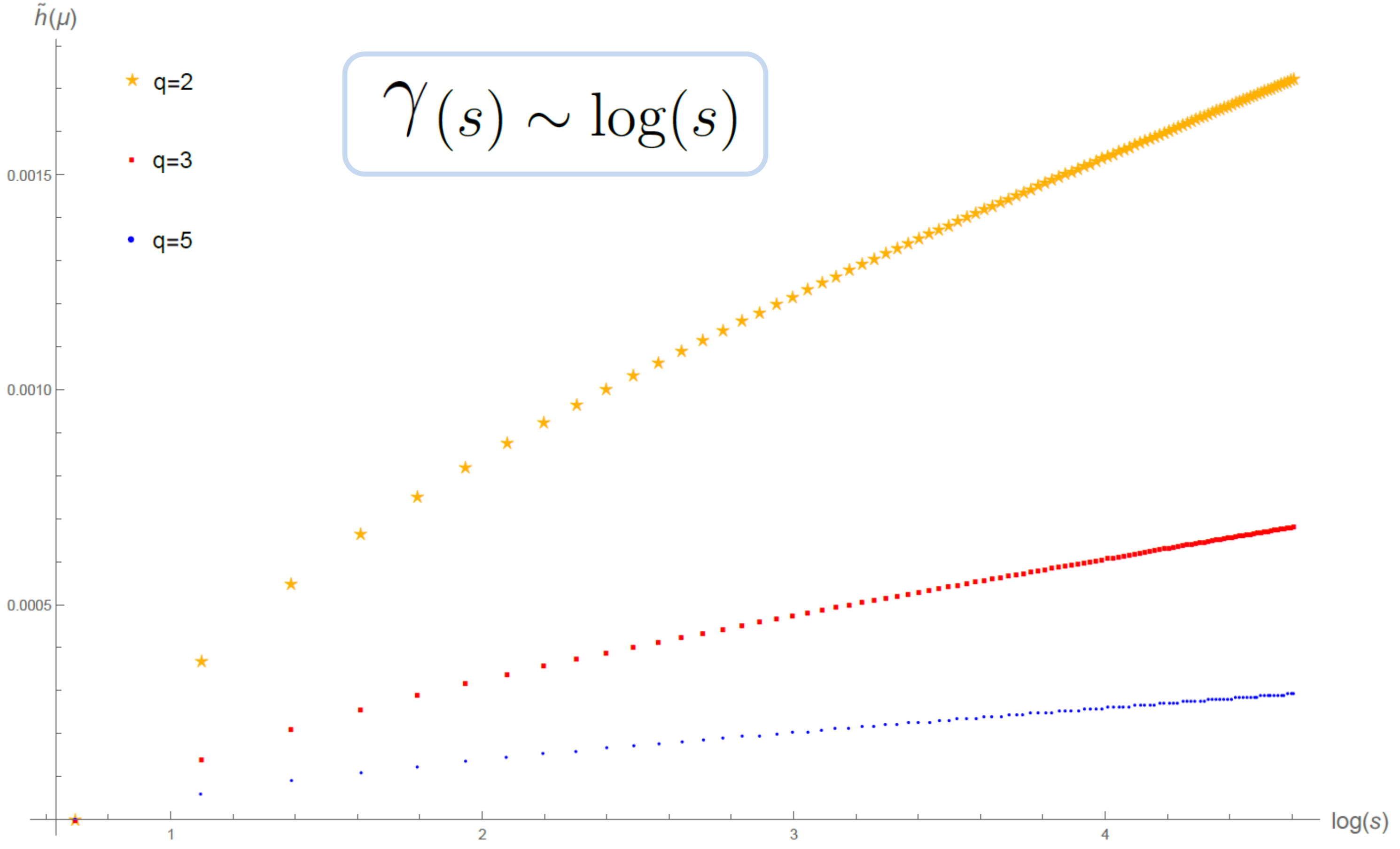
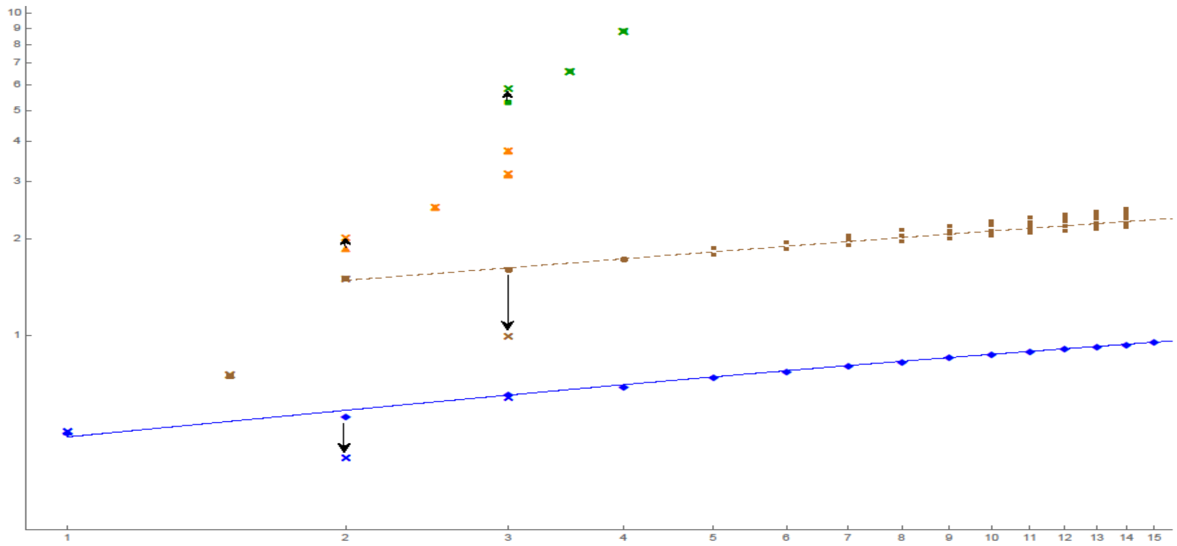


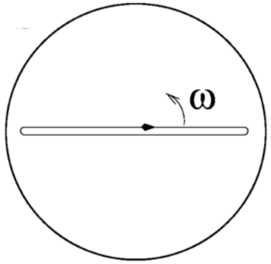
Relations with Higher-spin theories

- Higher-spin perturbation computation (Gaberdiel, CP, Zadeh, 2015)



- Rotating folded closed long string in AdS (Gubser, Klebanov, Polyakov, 2002)

$$E - S = \frac{\sqrt{\lambda}}{\pi} \ln(S/\sqrt{\lambda}) + \cdots . \qquad \lambda = g_{\rm YM}^2 N$$



logarithmic due to the AdS geometry

Relations with Higher-spin theories

- Reverse the direction,
- Consider small deformations away from the higher-spin point
- Consistent with previous results
- A toy model that mimics the process of turning off the string tension where the tuning is explicit

Comments on the higher-spin limits

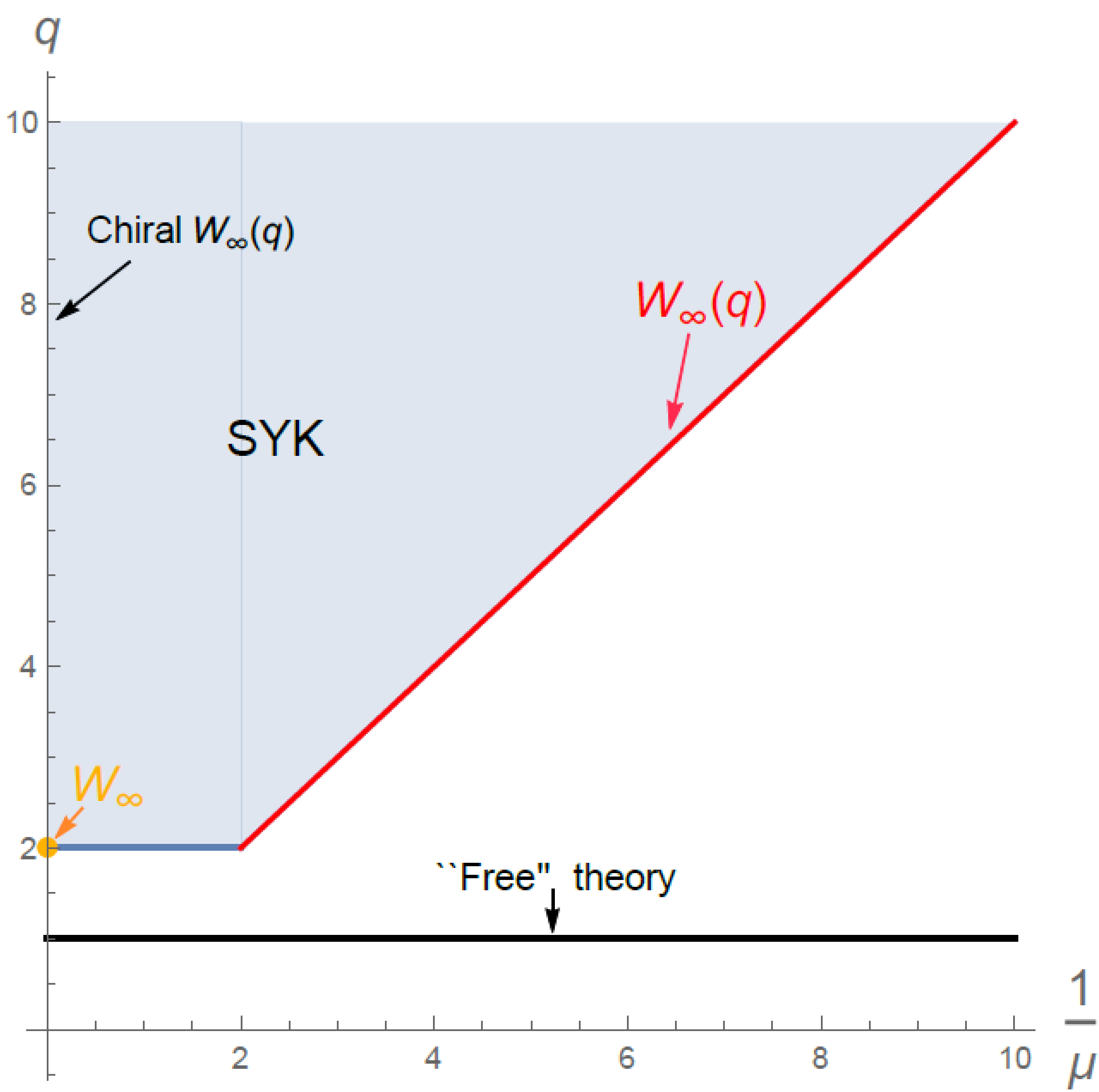
- A tower of higher-spin operators, generate a higher-spin-type algebra for each q, similar to the $W_\infty[\lambda]$ algebra in higher-spin holography

JHEP 1907 (2019) 092 Ahn, CP

- The model is not free in these limits.
  (The special property is due to delicate screening in the IR dynamics.)

- This limit can be helpful in identifying the bulk dual of SYK

- The other limit $\mu \to +\infty$ is similar, although not identical



q
10
8
6
4
2
Chiral $W_\infty(q)$
SYK
$W_\infty(q)$
W_∞
``Free'' theory
2
4
6
8
10
$\frac{1}{\mu}$

Coupled quantum systems

Based on: arXiv:2001.03158 Alet, Hanada, Jevicki, CP

Traversable wormhole

(Gao, Jafferis, Wall, 2016)

- Eternal black hole with 2 AdS boundary, turn on

$$\delta S = \int dt\ d^{d-1}x\ h(t,x)\mathcal{O}_R(t,x)\mathcal{O}_L(-t,x)$$

- The change of distance between the two sides

$$8\pi G_N \int dU T_{UU} = \frac{(d-2)}{4}\left((d-3)r_h^{-2} + (d-1)\ell^{-2}\right)\int dU h_{UU}$$

- Couplings between the two boundaries could violate the Averaged Null Energy Condition (ANEC)

$$\int_{-\infty}^{+\infty} T_{\mu\nu}k^{\mu}k^{\nu}d\lambda \geq 0$$

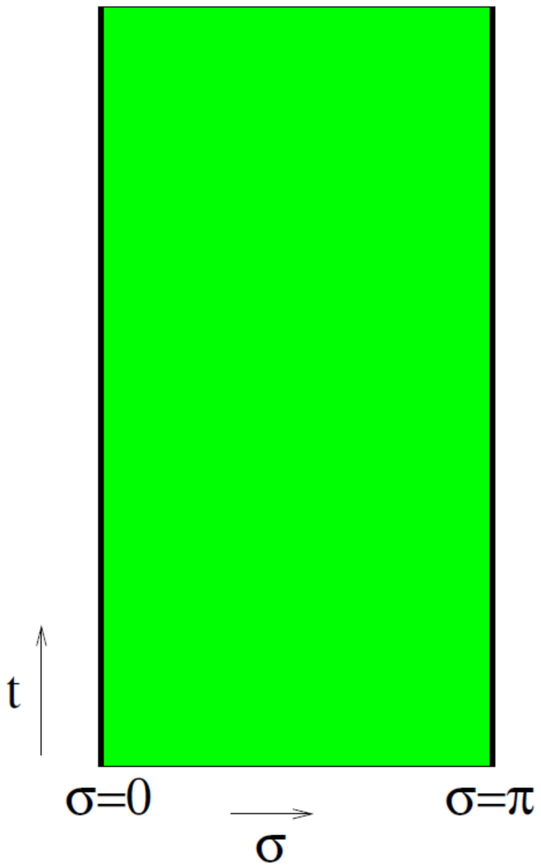
- Such a violation allows the wormhole to be traversable

A 2d Traversable wormhole

(Maldacena, Stanford, Yang 2017
Maldacena, Qi 2018)

- AdS_2 is simple

$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma} \,, \qquad \sigma \in [0, \pi]$$

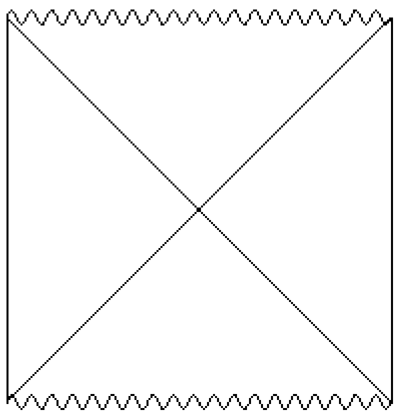


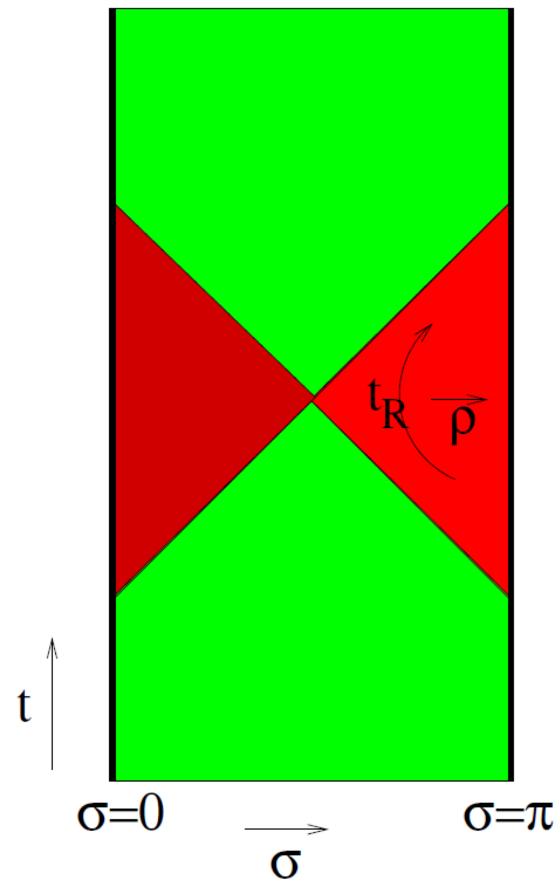
A 2d Traversable wormhole

(Maldacena, Stanford, Yang 2017
Maldacena, Qi 2018)

- AdS_2 is simple

$$ds^2 = -dt_R^2 \sinh^2 \rho + d\rho^2$$





A 2d Traversable wormhole

(Maldacena, Stanford, Yang 2017
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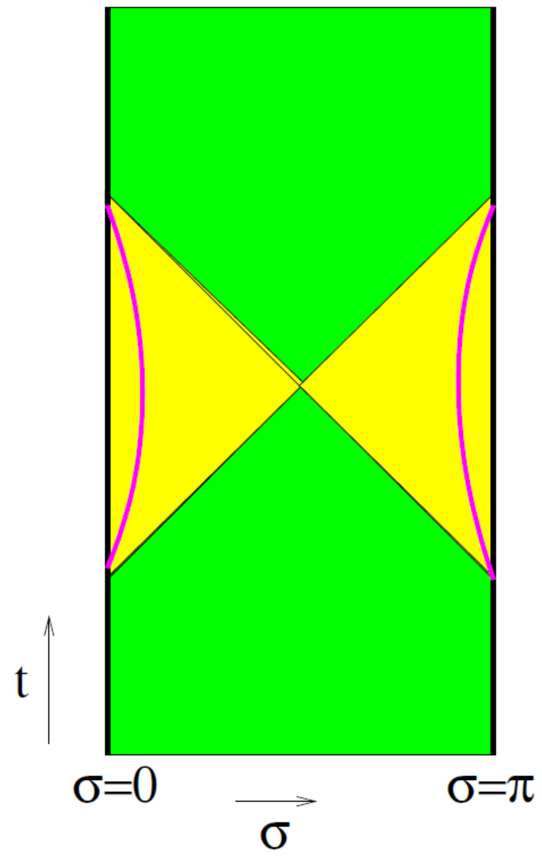
- AdS_2 is simple

$$ds^2 = -dt_R^2 \sinh^2\rho + d\rho^2$$

- Dynamics is all on the boundary

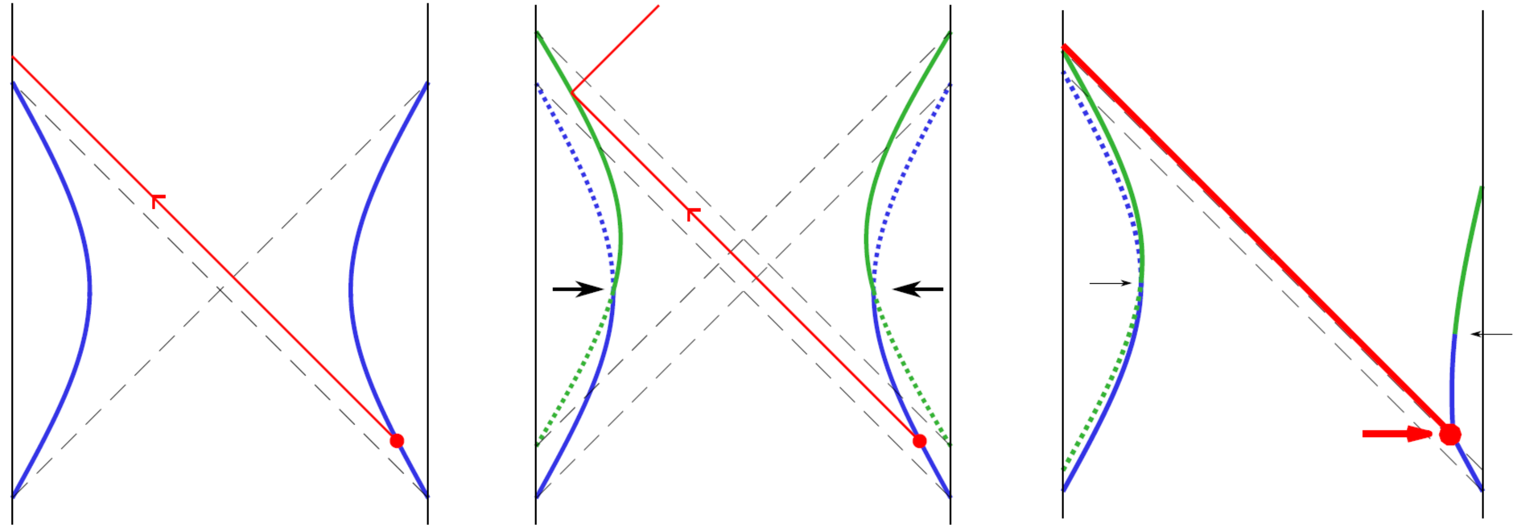


A 2d Traversable wormhole

(Maldacena, Stanford, Yang 2017
Maldacena, Qi 2018)

- AdS_2 is simple
- Dynamics is all on the boundary
- An appropriate coupling could lead to a traversable wormhole

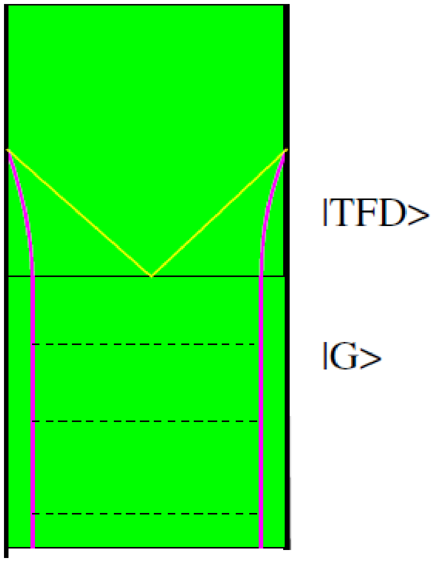
$$S_{int} = g \sum_{i=1}^{N} \int du O_L^i(u) O_R^i(u)$$

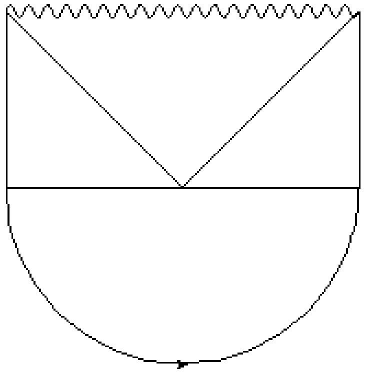


A 2d Traversable wormhole

(Maldacena, Qi 2018)

- Ground state of the coupled model is close to a thermofield double (TFD) state $|\mathrm{TFD};\beta\rangle = \sum_E e^{-\frac{1}{2}\beta E}|E\rangle_L \otimes |E\rangle_R^*$





$$\Omega = \frac{|\langle TFD|G\rangle|^2}{\langle TFD|TFD\rangle\langle G|G\rangle} \sim 1$$

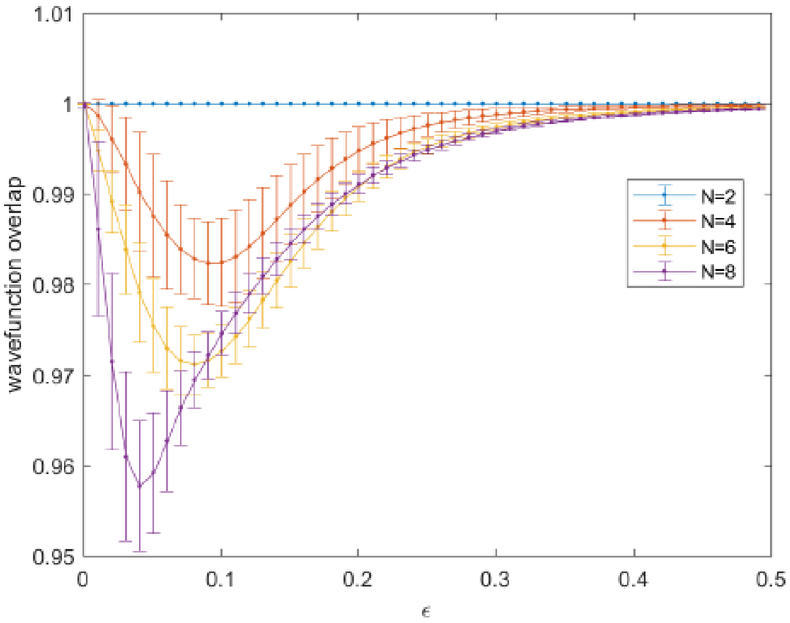
Field theory confirmation

(Maldacena, Qi 2018)

- One can test this in SYK model

$$H_{\text{int}} = i\mu \sum_j \psi_L^j \psi_R^j$$

  – Numerically



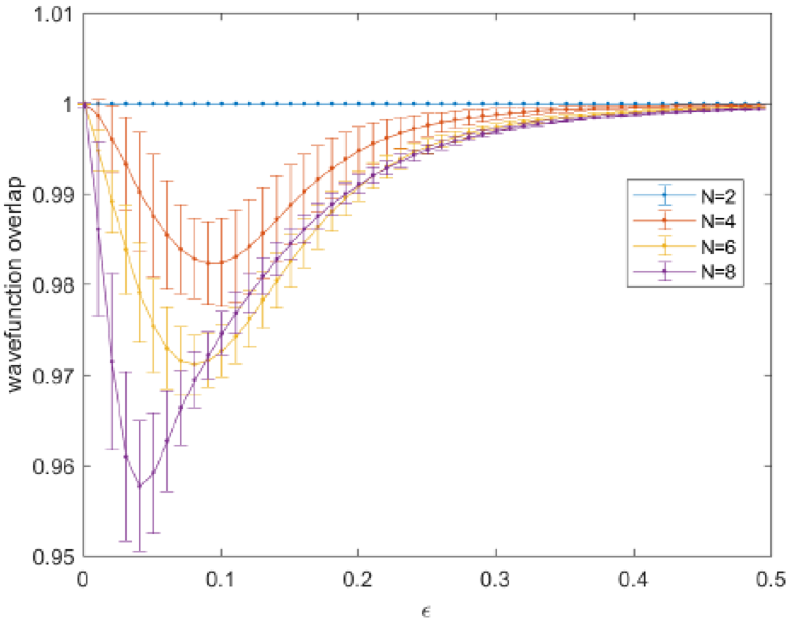
Field theory confirmation

(Maldacena, Qi 2018)

- One can test this in SYK model

$$H_{\text{int}} = i\mu \sum_j \psi_L^j \psi_R^j$$

  - Numerically
  - Large-q



$$\Omega = \frac{|\langle TFD|G\rangle|^2}{\langle TFD|TFD\rangle\langle G|G\rangle} = 1 + O\left(\frac{1}{q}\right)$$

$$\beta = \frac{2}{\alpha}\sqrt{1+\left(\frac{\alpha}{\mathcal{J}}\right)^2}\arctan\frac{\mathcal{J}}{\alpha}$$

Field theory confirmation

(Maldacena, Qi 2018)

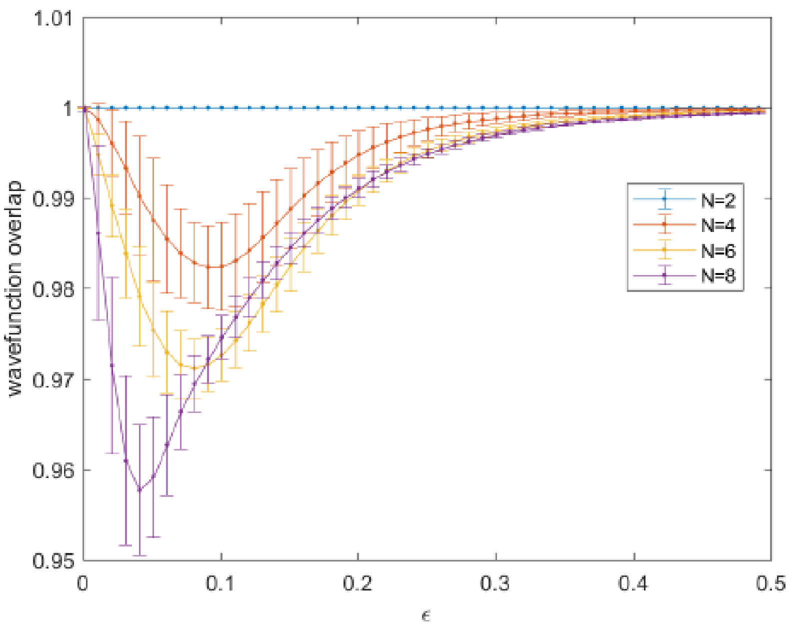
- One can test this in SYK model

$$H_{\text{int}} = i\mu \sum_j \psi_L^j \psi_R^j$$

  - Numerically
  - Large-q

$$\Omega = \frac{|\langle TFD|G\rangle|^2}{\langle TFD|TFD\rangle\langle G|G\rangle} = 1+O(\frac{1}{q})$$

$$\beta = \frac{2}{\alpha}\sqrt{1+\left(\frac{\alpha}{\mathcal{J}}\right)^2}\arctan\frac{\mathcal{J}}{\alpha}$$



A field theory question

- The previous computations are largely inspired by the intuitive gravity picture

- A question is if the result is a general feature for a wider class of quantum theories

- Not necessarily dual to pure gravity with black holes

  - Bulk dual could be more general gravitational theories
  - Might even not be holographic

General coupled models

(Alet, Hanada, Jevicki, CP 2020)

- We checked a few more models
  - Coupled SYK model
  - Coupled Spin models
  - Coupled "Free" models
- We find very similar results for all these models, regardless of the existence of a classical gravity dual

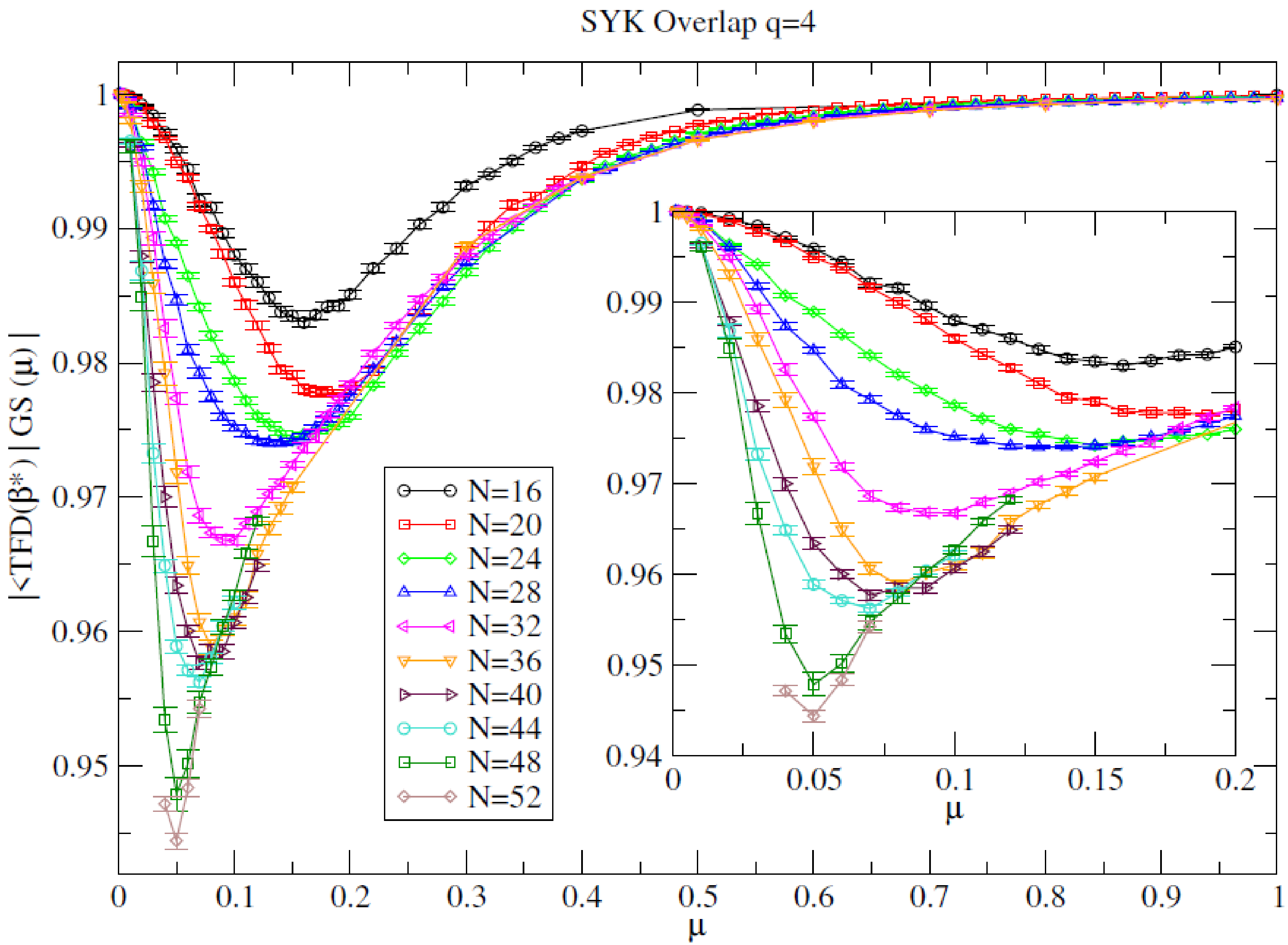
Coupled SYK model

- We push the computation towards the thermodynamic limit

- We checked the previous statements

  - Deviation of the overlap from 1 is not a numerical error

Coupled SYK model



Coupled SYK model

- We push the computation towards the thermodynamic limit

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  - Important for the overlap to be not always 1

Coupled SYK model

- We push the computation towards the thermodynamic limit

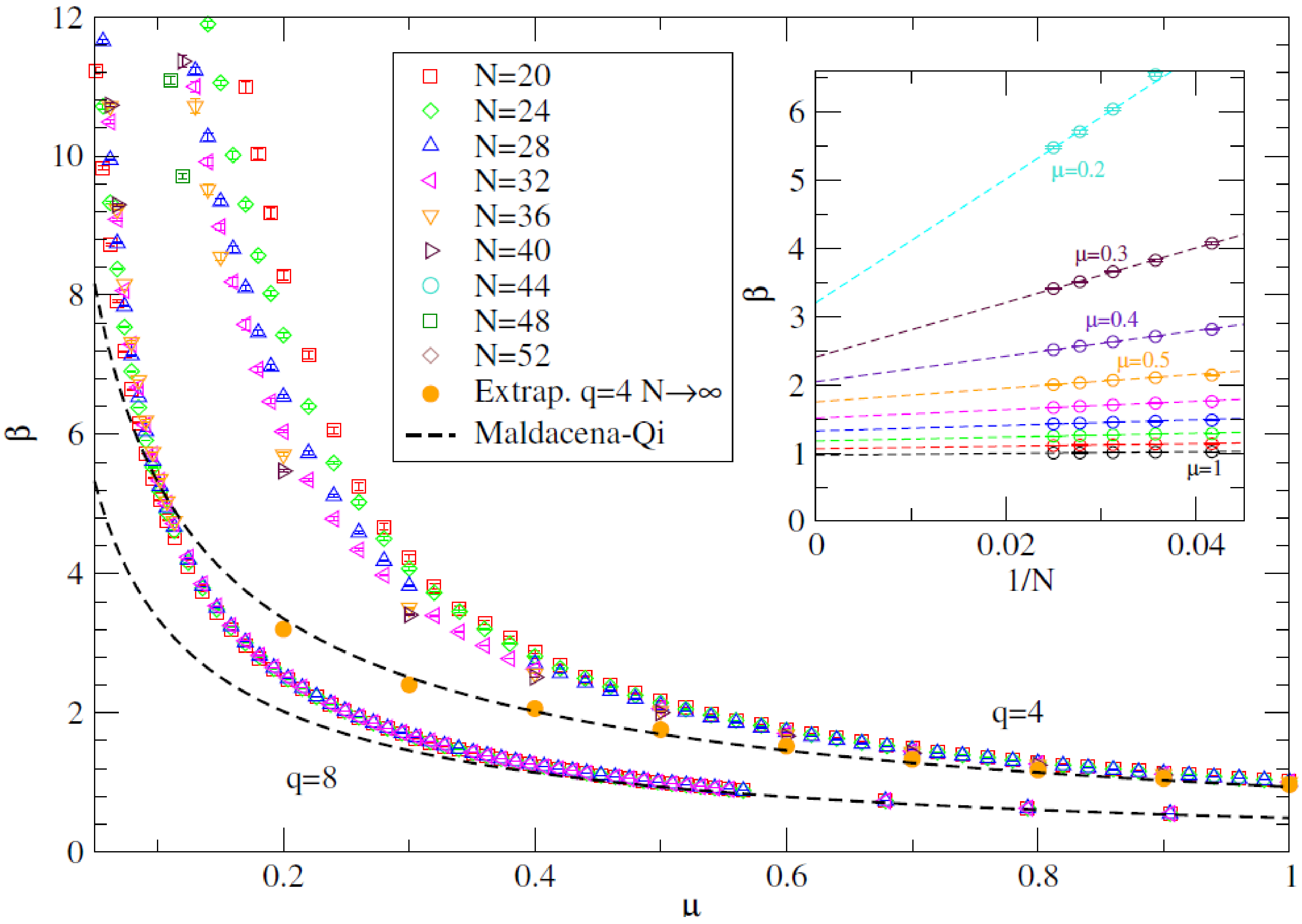
- We checked the previous statements

  - Deviation of the overlap from 1 is not a numerical error

  - Important for the overlap to be not always 1

  - The effective temperature of the TFD

Coupled SYK model



Coupled spin model

- $$\hat{H}_\alpha = \sum_{i=1}^{L/2} \left(\frac{1}{4} \vec{\sigma}_{i,\alpha} \vec{\sigma}_{i+1,\alpha} + \frac{\vec{w}_{i,\alpha}}{2} \vec{\sigma}_{i,\alpha} \right), \qquad \alpha = L, R$$

- $\vec{w}_{i,\alpha}$ is randomly chosen from the interval $[-W, +W]$

- Different phases for different W

  - $W = 0$, Integrable
  - $0 \leq W < W_c$, Ergodic ($W_c \sim 3.7$ in this setting)
  - $W_c < W$, Many-body localization

- No clear gravity dual

Coupled spin model

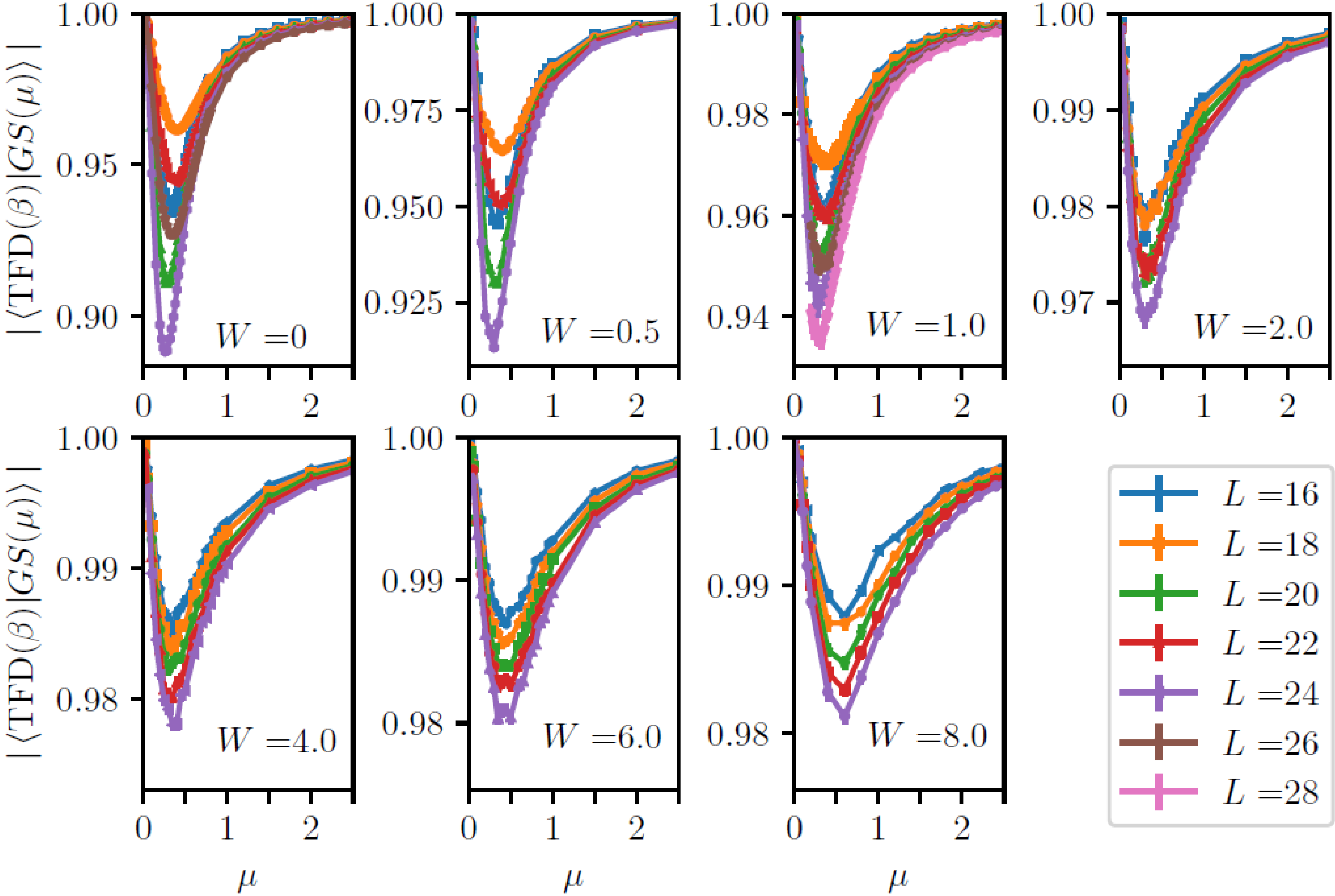
- $$\hat{H}_\alpha = \sum_{i=1}^{L/2} \left(\frac{1}{4} \vec{\sigma}_{i,\alpha} \vec{\sigma}_{i+1,\alpha} + \frac{\vec{w}_{i,\alpha}}{2} \vec{\sigma}_{i,\alpha} \right), \qquad \alpha = L, R$$
- Turn on a suitably chosen coupling

$$\hat{H}_{\text{int}} = \mu \sum_{i=1}^{L/2} \left(\hat{\Sigma}_i^\dagger \hat{\Sigma}_i + \hat{\Sigma}_i \hat{\Sigma}_i^\dagger \right)$$

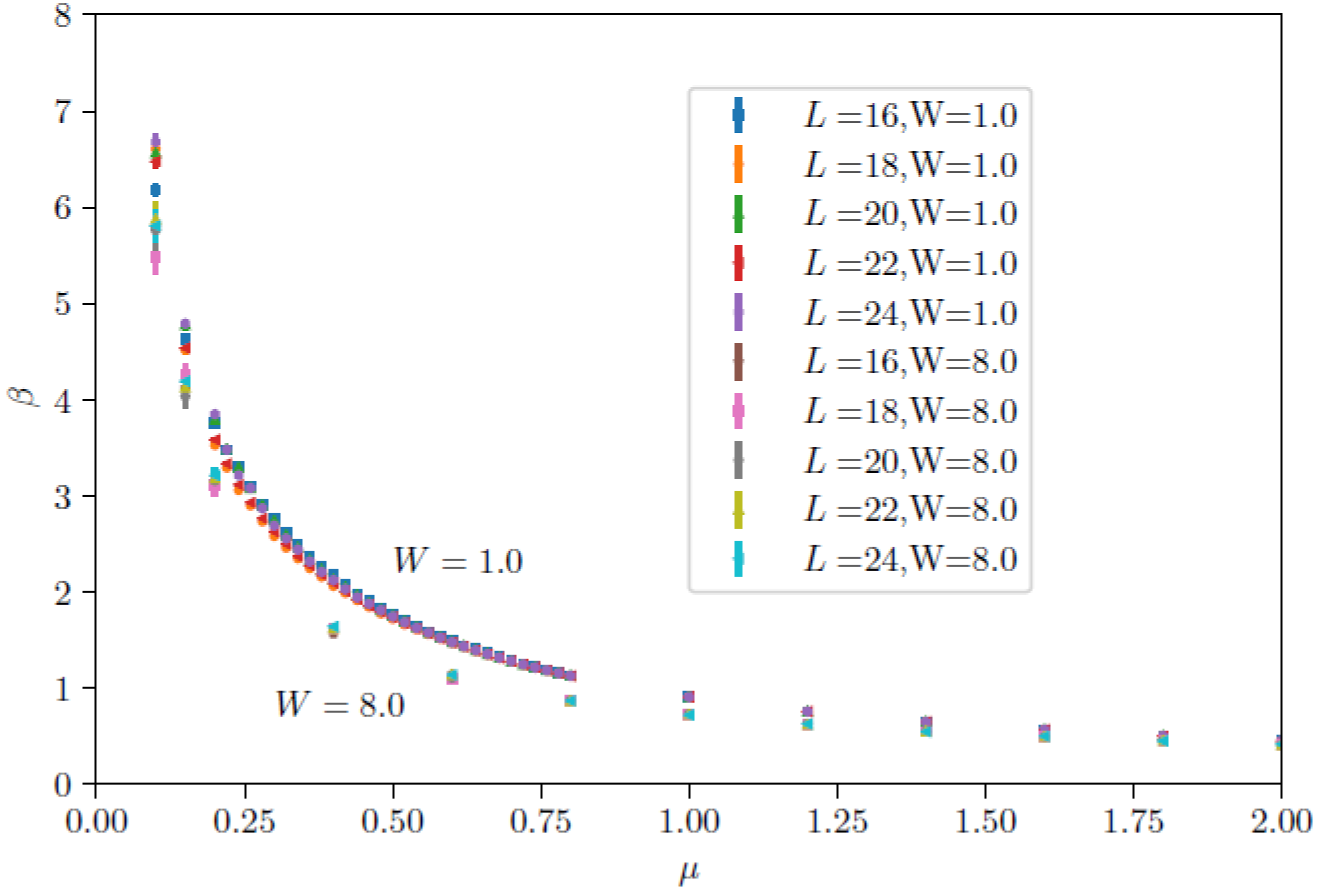
$$\hat{\Sigma}_i = \sigma_{iL}^+ - (\sigma_{iR}^-)^*, \qquad \hat{\Sigma}_i^\dagger = \sigma_{iL}^- - (\sigma_{iR}^+)^* \qquad \sigma_{i\alpha}^\pm = \frac{\sigma_{i\alpha}^x \pm \sqrt{-1}\sigma_{i\alpha}^y}{2}$$

- Study the overlap between the ground state of this coupled model and the TFD at some temperature

Coupled spin model



Coupled spin model



Coupled harmonic oscillators

- $\hat{H} = \frac{\hat{p}_{\mathrm{L}}^2}{2} + \frac{\omega^2 \hat{x}_{\mathrm{L}}^2}{2} + \frac{\hat{p}_{\mathrm{R}}^2}{2} + \frac{\omega^2 \hat{x}_{\mathrm{R}}^2}{2} + \frac{C_+}{2} \left(\hat{x}_{\mathrm{L}} + \hat{x}_{\mathrm{R}}\right)^2 - \frac{C_-}{2} \left(\hat{x}_{\mathrm{L}} - \hat{x}_{\mathrm{R}}\right)^2$

- Can solve this problem analytically (No need for large-N and strong coupling, hence not necessarily holographic)

- $\hat{H} = \frac{\hat{p}_+^2}{2} + \frac{\omega_+^2 \hat{x}_+^2}{2} + \frac{\hat{p}_-^2}{2} + \frac{\omega_-^2 \hat{x}_-^2}{2}$

$$\omega_+ = \sqrt{\omega^2 + 2C_+}, \qquad \omega_- = \sqrt{\omega^2 - 2C_-}, \qquad \hat{x}_\pm = \frac{\hat{x}_{\mathrm{L}} \pm \hat{x}_{\mathrm{R}}}{\sqrt{2}}$$

Coupled harmonic oscillators

- Ground state

$$|0\rangle_{\text{coupled}} = \mathcal{N}^{-1/2} e^{A_1(\hat{a}_{\text{L}}^{\dagger 2}+\hat{a}_{\text{R}}^{\dagger 2})} e^{A_2 \hat{a}_{\text{L}}^{\dagger}\hat{a}_{\text{R}}^{\dagger}} |0\rangle_{\text{L}} |0\rangle_{\text{R}}$$

$$A_1 = \frac{1}{4}\frac{r_+ - r_+^{-1}}{r_+ + r_+^{-1}} + \frac{1}{4}\frac{r_- - r_-^{-1}}{r_- + r_-^{-1}}, \qquad A_2 = \frac{1}{2}\frac{r_+ - r_+^{-1}}{r_+ + r_+^{-1}} - \frac{1}{2}\frac{r_- - r_-^{-1}}{r_- + r_-^{-1}} \qquad r_\pm = \sqrt{\frac{\omega_\pm}{\omega}}$$

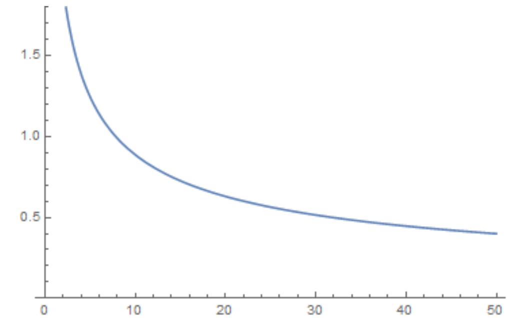
- The ground state is identical to a TFD when

$$A_1 = 0 \quad \Longleftrightarrow \quad \sqrt{1+2C_+/\omega^2}\sqrt{1-2C_-/\omega^2} = 1$$

- At this value we can determine the effective temperature

$$|0\rangle_{\text{coupled}} = \mathcal{N}^{-1/2} |A_2|^{-1/2} \sum e^{-E_n/2T_{\text{eff}}} |n\rangle_{\text{L}} |n\rangle_{\text{R}}$$

$$T_{\text{eff}} = -\frac{\omega}{2\log\left(\left|\frac{r_+ - r_+^{-1}}{r_+ + r_+^{-1}}\right|\right)}$$



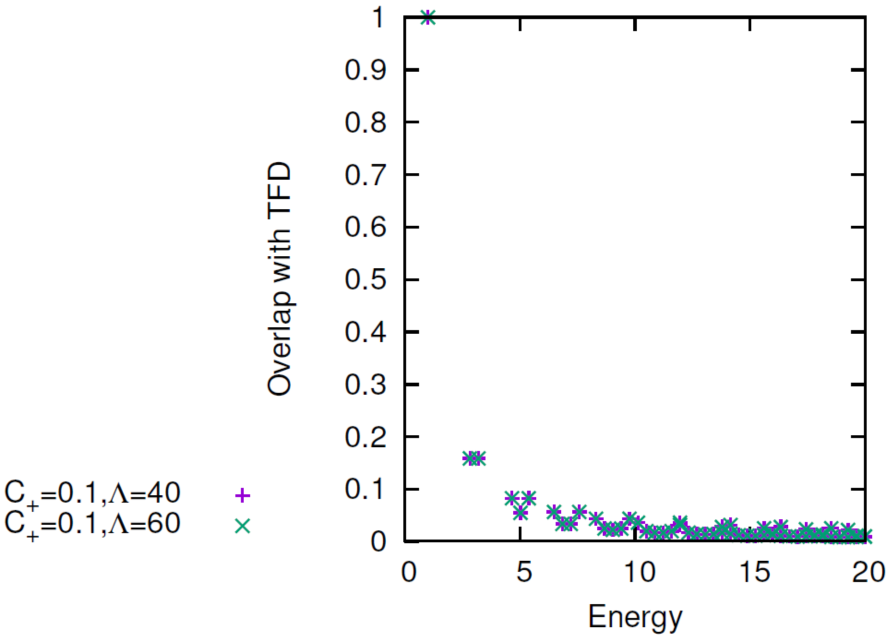
Coupled harmonic oscillators

- We can further study how excitation in the coupled model affect the comparison with TFD

$$|\langle \mathrm{TFD}(\beta)|n_+, n_-\rangle_{\mathrm{coupled}}|^2$$

- The overlap decays fast as energy injected

- This is as expected as if there is a gravitational dual picture, even though it is not believed to be the case



Coupled harmonic oscillators

- We can further compute the entanglement between the two sides as temperature increases

- We consider the coupled model in a thermal state with

$$\rho(\beta) = \sum_n e^{-\beta E_n} |n\rangle\langle n|$$

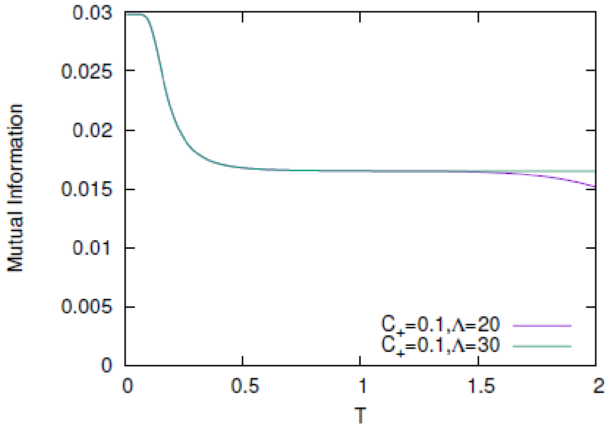
where $|n\rangle$ is the energy eigenstates of the coupled Hamiltonian

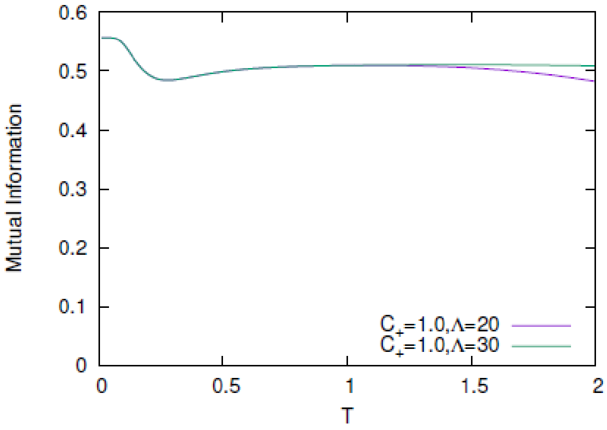
- Consider the simplest quantity: the L-R mutual information

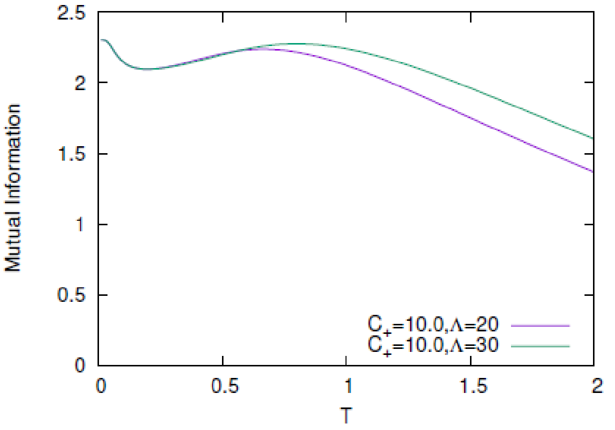
$$S_{\mathrm{EE,L}} + S_{\mathrm{EE,R}} - S_{\mathrm{therm}}$$

We expect it to decrease as temperature increases.

Coupled harmonic oscillators







- Strange behavior (not monotonic decreasing)

- Conjecture: it is due to non-vanishing classical correlations

Coupled harmonic oscillators

- Considered a different quantity

$$S_{\mathrm{EE,L}} + S_{\mathrm{EE,R}} - S_{\mathrm{diag}} = S_{\mathrm{EE,L}} + S_{\mathrm{EE,R}} - S_{\mathrm{thermal}} - (S_{\mathrm{diag}} - S_{\mathrm{thermal}})$$

where the "diagonal entropy" is computed by

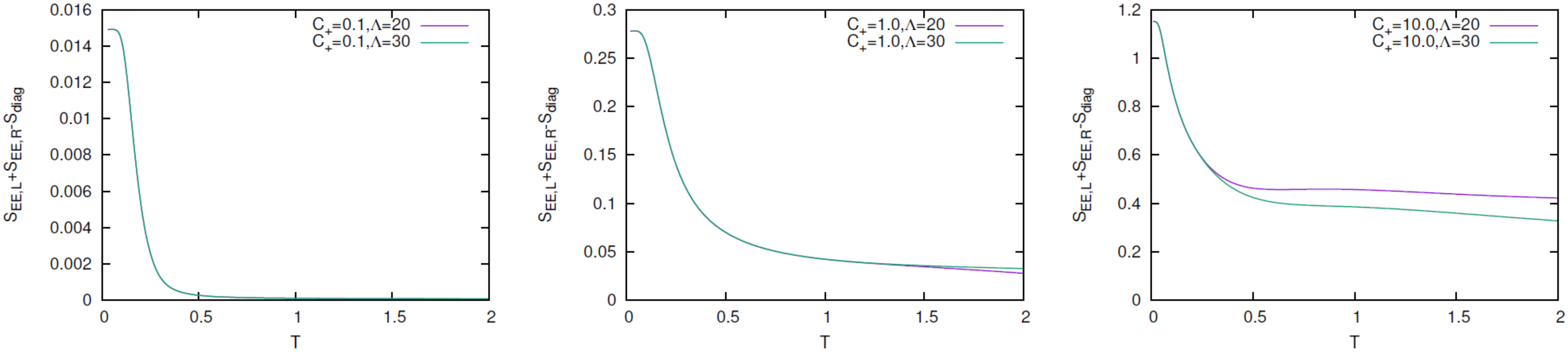
$$\hat{\rho}_{\mathrm{diag}} = \sum_{n_{\mathrm{L}}, n_{\mathrm{R}}} \rho_{n_{\mathrm{L}}, n_{\mathrm{R}}; n_{\mathrm{L}}, n_{\mathrm{R}}} \left(|n_{\mathrm{L}}\rangle\langle n_{\mathrm{L}}|\right) \left(|n_{\mathrm{R}}\rangle\langle n_{\mathrm{R}}|\right)$$

if the thermal density matrix is

$$\hat{\rho} = \sum_{n_{\mathrm{L}}, n_{\mathrm{R}}, n'_{\mathrm{L}}, n'_{\mathrm{R}}} \rho_{n_{\mathrm{L}}, n_{\mathrm{R}}; n'_{\mathrm{L}}, n'_{\mathrm{R}}} \left(|n_{\mathrm{L}}\rangle\langle n'_{\mathrm{L}}|\right) \left(|n_{\mathrm{R}}\rangle\langle n'_{\mathrm{R}}|\right)$$

- The last factor is meant to capture the classical L-R correlations

Coupled harmonic oscillators



- Find better behaviors

- A more thorough understanding is still needed.

Coupled free matrix/vector models

- The analysis of the harmonic oscillators essentially carry through to the cases of free matrix/vector models
- i.e. Ground state ~ TFD
- But due to non-trivial gauge invariance constraints, thermodynamic behaviors are different

General coupled theories

(Alet, Hanada, Jevicki, CP 2020)

- We checked a few models
  - Coupled SYK model
  - Coupled Spin models
  - Coupled "Free" models
- We find very unexpected similar results for all these models, regardless of the existence of a classical gravity dual
- From the field theory point of view, the coupled models could provide a simple way to prepare thermofield double states

  (if the property is truly universal, maybe even in labs with simple systems)

Thank you !