Examination of the Reserve System for Medical Resource Allocation in Pandemics

Parag A Pathak, Tayfun Sönmez, M Utku Ünver, and M Bumin Yenmez, "Fair Allocation of Vaccines, Ventilators and Antiviral Treatments:Leaving No Ethical Value Behind in Health Care Rationing"[1]

Juejue Wang[2]

juejuew@umich.edu

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[1]This paper focuses on analyzing the reserve system proposed by Pathak et al. in 2020 for pandemic rationing of medical resources

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Abstract

Regarding the medical resource shortage in the COVID-19 pandemic, several recent studies propose a general reserve system for health care rationing in seek of overcoming the limitations of the widely adopted priority system.

This thesis discusses how the reserve system deals with the challenges of the priority system and analyzes two potential benefits of the reserve system on a static basis: (1) Preventing people from exaggerating qualification through cheating, and (2) facilitating compromises among stakeholders.

The first finding is that, in the reserve system, planners can adjust the medical units assigned to reserve categories to discourage cheating, and they can achieve similar effects in the priority system by introducing supervision and punishment. Secondly, the reserve system not only simplifies the complexity of thinking by leading policymakers to create mental accounts but also promotes consensus in the negotiating process.

1 Introduction

In the COVID-19 pandemic, there are debates concerning the existing medical resource allocation guidelines, and discussions concentrate on one limitation of the currently adopted priority system about its failure to capture essential ethical values in some situations. Fair allocation requires the adoption of a multi-value ethical framework based on certain contexts. Typical ethical principles include equity, which means treating people equally with respect to benefits and burdens; instrumental valuation, which means prioritizing those who will save lives in the future or those who have saved lives in the past; and so on. Failing to represent ethical principles as designed would negatively affect the allocation efficiency(Pathak et al. 2020; Emanuel et al. 2020). Examples of the debates include deciding the prioritization of essential personnel such as the front-line healthcare workers. Some worry that if specific groups of people are prioritized through the priority system, in extreme scenarios, no units may be left for the rest of the society(Vawter et al. 2010), which may result in the exclusion of certain ethical principles.

In response to the debates, Pathak et al. summarize in their paper the challenges faced by the priority system and propose a general theory of reserve design as a solution(Pathak et al. 2020). The idea of a reserve system is not original, and the mechanism has been applied in many other contexts such as deceased donor kidney allocation(Israni et al. 2014), college admissions(Aygun and Bó 2020; Baswana et al. 2019), immigration visa allocation(Pathak, Rees-Jones, and Sönmez 2020), and so on. As is explained in their paper, a reserve system should have several other advantages. Firstly, the clear association between reserve categories and ethical principles may help stakeholders achieve compromises. Aside from this, the reserve categories are easy to interpret thus promoting transparency and fostering public trust in settings that involve community engagement. Finally, since the resources are divided into small pieces, the adjustment of the priority decision would be flexible(Pathak et al. 2020, 30).

Pathak et al. provides a lot of details about the reserve design and explains the mechanism carefully, but they do not explain the potential benefits of the reserve system rigorously. Neither does the paper provide any theoretical analysis nor conducts any case studies to illustrate the advantages. Moreover, the reserve system is not compared with the priority system in a formalized way. Having observed these problems, this thesis provides supplementary analysis for the topics that are not sufficiently discussed by Pathak et al.İn particular, I model the reserve system as various games under different settings and provide mathematical proofs for the Nash equilibrium. In addition, my analysis is static and uses vaccine allocation as examples. Vaccines can be allocated simultaneously, are fully consumed upon allocation, and there is no immediate urgency for allocation thus is an application of the proposed reserve system on a static basis(Pathak et al. 2020, 28). Vaccine allocation is also a heated topic recently as the COVID-19 vaccines are authorized and start to be allocated to the public.

This thesis is organized as follows. Section 2 provides a literature review, mainly explaining the theories and mechanisms of the reserve system and the priority system. Section 3 uses a simple example to demonstrate how the priority system and the reserve system work in a perfect world where the ethical principles have been clearly specified and people report their information truthfully. Next, Sections 4 and 5 analyze two potential advantages of the reserve system using mathematical models and relevant game theories. The two advantages are: (1) Preventing the public from misreporting their qualification, and (2) promoting compromises among different stakeholders. The report concludes in Section 6.

2 Literature Review

This thesis serves as an extension of the paper from Pathak et al, focusing on the benefits of the reserve system and comparing the reserve system with the priority system. To better understand the concepts and arguments that appear in the thesis, I will introduce relevant theories of the two allocation systems in this section.

The priority system

To implement a proper rationing system to handle a crisis, planners follow two steps to design the rationing guidelines: 1) Articulating ethical principles and 2) determining an allocation mechanism to operationalize the ethical principles. The most common allocation mechanism used in the second step is the priority system, which allocates medical units to patients based on a single priority order that captures the articulated ethical principles(Pathak et al. 2020, 6). For certain medical resources such as ventilators and ICU beds, the priority order is determined by a priority point system, a monotonic scoring function, which has two forms: single-principle point system that is based on one ethical principle(Emanuel et al. 2020), and multi-principle point system that is based on multiple ethical principles(White et al. 2009; Lee Daugherty-Biddison et al. 2017). The ethical values are evaluated by a monotonic function, and for each patient, their corresponding values will be added up to generate a single number. In general, patients with lower total point scores have higher priority than those with higher total point scores.

The reserve system

The essential idea of a reserve system is to divide the medical units into multiple segments referred to as reserve categories to represent certain ethical principles. Each category follows its specific priority order in allocating medical resources. Based on the specific priority order in each category, a patient is either eligible or is ineligible to receive medical resources from that category. A patient may be eligible for multiple categories but can only be matched with one category in which he or she eventually obtains a medical unit.

In the reserve system, a matching means allocating each patient either to a category in which he or she obtains a unit or an empty set in which the patient receives nothing. According to the general theory of the reserve system, the matching should at least follow three axioms: 1) Units are awarded only to eligible individuals, 2) no unit is wasted, and 3) the units need to be allocated based on the specific priority order in each category. These are the minimal requirements for the reserve system because one can see them being applied to real-life allocation problems either explicitly or implicitly(Pathak et al. 2020, 12).

Planners can use a subset of the Individual Proposing Deferred Acceptance Algorithm (DA)(Gale and Shapley 1962) named the Sequential Reserve Matching(SRM) to find the qualified matching that complies with the three matching axioms described above. By the SRM algorithm, the reserve categories are processed sequentially, and the category under processing would reject patients according to the corresponding priority order. The patients who are rejected from the previous categories would be allocated with units based on the priority order in the next category. This process iterates until there are no rejections. The resulting matching is referred to as the Sequential Reserve Matching.

3 Simple Case Studies

This section first uses the example of vaccine allocation to explain the mechanisms of the reserve system and two types of the priority system in perfect conditions. Perfect conditions ensure the successful implementation of the allocation systems as designed, by requiring both a comprise among stakeholders in articulating ethical values and the honesty of the public in reporting qualification information.

Pathak et al. mention three aspects that their proposed reserve system has advantages in. I will discuss the second and the third benefits in Sections 4 and 5 in detail, while this section briefly discusses the first benefit that is the ability to solve the problem of leaving out ethical values for the priority system. The reserve system always ensures the representation of every pre-specified ethical value in the allocation results. This seems to be obvious as planners can set up reserve categories for each ethical value and distribute medical units accordingly. By contrast, due to the single linear order, the priority system may not properly reflect incommensurable ethical principles and may neglect certain ethical values. However, the multi-principle point system has an advantage in emphasizing the large distinctions and undermining the small differences among individuals. In some cases, although there are unrepresented ethical values, this feature of the multi-principle point system leads to be a preferable medical rationing solution compared with that of the reserve system.

Background:

The tradition of vaccine allocation is to distribute influenza vaccines at local pharmacies or healthcare providers on a first-come-first-serve basis[1]. The problem with the first-come-first-serve allocation is that those who are less likely to be informed or those who have inadequate transportation are at a disadvantage(Kinlaw and Levine 2007). Therefore, planners can adopt a disadvantaged reserve category that prioritizes those who live far from medical centers to make sure people are treated equally.

Settings:

1. The priorities are based on the time of arrival. Those who arrive earlier have higher priority and get vaccinated earlier than those who arrive later.

There are four patients $I = \{i_1, i_2, i_3, i_4\}$ and three identical and indivisible vaccines in total to allocate. Each patient is assigned a score based on his or her priority. lower scores are associated with higher priority and higher scores are associated with lower priority.

I require the allocation results to reflect two ethical principles. One is the arrival time where the individual who comes earlier has higher priority. The other is the distance to the hospital where the individual whose house is farther to the hospital has higher priority. Since it is expected that the individuals who live closer would arrive earlier to the hospital, I make up the scores in the two categories for each patient to show this relationship, and the scores are summarized in Table 1:

Patients	C_1*(Arrival Time)*	C_2*(Distance to the Hospital)*
i_1	1	7
i_2	2	4
i_3	3	3
i_4	4	1

Table 1

Case 1: Single-principle point system

In this case, the priority system is based on one ethical value: Arrival Time. According to the corresponding scores, since patients i_1, i_2, i_3 are those with the three lowest scores in column C_1, each of them get one vaccine unit, while patient i_4 does not receive a vaccine.

There may be several types of vaccines to allocate, and each type only contains three units. If the single-principle point system is followed, patient i_4 would never have the chance to receive any type of vaccine, being the person who lives farthest from the medical center. This example demonstrates the problems lying in the single-principle point system that the system sometimes fails to represent certain ethical principles in the allocation results.

Case 2: Multi-principle point system

Patients	*Sum of Scores*
i_1	8
i_2	6
i_3	6
i_4	5

Table 2

Patients	*Sum of Scores*
i_1	8
i_2	8.5
i_3	6
i_4	5

Table 3

To implement this mechanism, the scores of both categories are added up for each patient, and those who obtain the three lowest scores receive vaccine units. In this case, patients i_2, i_3, i_4 receive the medical resources. The results are shown in Table 2.

The multi-principle point system alleviates the problem of the first-come-first-serve allocation, and patients who live far from the hospital have an opportunity to receive vaccine units. However, the first-come-first-serve principle is not reflected in the results because patient i_1 who comes the earliest does not get a vaccine.

Now, let us consider the case that the differences of the C_2 scores between patients i_1 and i_2 are smaller. As an example, holding other scores fixed, patient i_2's score in C_2 is changed from 4 to 6.5. The results are shown in Table 3. In this case, patients i_1, i_3, i_4 receive units, and the multi-principle point system accommodates both ethical principles.

The first observation is that if the eligibilities of two patients in one category are similar, deciding which patient has the higher priority is mainly dependent on other categories where the differences in eligibility are large. Secondly, although in some conditions, the multi-principle system does reflect every ethical principle, the problem of leaving out ethical values still exists.

Case 2: Reserve System with Sequencing Matching

Suppose there are two categories C_1 and C_2 that represent the ethical principles "Arrival time" and "Distance to the Hospital" respectively. Category C_1 has one unit and C_2 has two units. Each Category follows the linear priority order according to the scores in Table 1.

Subcase 1: C_1 is processed first

Steps	*Category*	*Patients*
1	C_1	i_1
2	C_2	$i_4,\ i_3$

Table 4

Step1: Since patient i_1 has the highest priority in category one, he or she receives a vaccine.

Step2: Next, category C_2 is processed. Patients i_4 and i_3 have higher priorities than others, so each of them obtains one vaccine unit. Since all three available units have been allocated, patient i_2 receives nothing. The results are shown in Table 4.

Subcase 2: C_2 is processed first

Steps	*Category*	*Patients*
1	C_2	$i_4,\ i_3$
2	C_1	i_1

Table 5

Following similar steps as for subcase 1, the allocation results are obtained and summarized in Table 5. The final results are the same as the case when C_1 is processed first, in which patients i_1, i_3, i_4 receive units but i_2 receives nothing.

Compared with the two types of the priority system, the reserve system always accommodates both ethical principles thus mitigating the problem associated with the first-come-first-serve allocation to some extent.

You can also observe that once the medical resource is scarce and the only two reserve categories have exact opposite priority orders, the processing order of reserve categories would not affect the final allocation outcome.

Discussion:

Patients	C_1(*Lifecycle principle*)	C_2(*Save the most lives principle*)
i_1	1	5
i_2	2	3
i_3	3	2
i_4	4	1

Table 6

From these examples, the reserve system with sequencing matching provides a solution to the bias caused by the first-come-first-serve allocation, and it always reflects the two ethical principles as long as the priority order in each category does not change. However, you cannot jump to a conclusion directly, after all, the multiple-principle point system shows advantages in its comprehensive consideration of individual differences.

Let us consider another example: Holding other settings as before, the two ethical principles are now changed to be the "Life-cycle Principle"(C_1) and the "Save the most lives" principle(C_2), where the "Life-cycle Principle" prioritizes those who are young, and the "Save the most lives" prioritizes those who are expected to have longer survival time. In addition, the priority scores are updated and summarized in Table 6.

To implement the reserve system, C_1 is allocated with one unit, and C_2 is allocated with two units. Moreover, since the "Save the most lives" principle is more important than the other principle, I process the "Save the most lives" principle first. Then, the reserve system with sequencing matching allocates vaccines to patients i_1, i_3, i_4 no matter which category is processed first. However, this result may not be what planners would want because when the differences between ages are small, planners would like to save more lives instead thus preferring ethically to allocate vaccines to patient i_2 rather than i_1. Since the multi-principle point system distributes vaccines to patients i_2, i_3, i_4, it provides a better allocation.

To summarize, this observation shows that there might be situations when the multi-principle point system outperforms the reserve system, which may call for careful consideration from planners before adopting the reserve system as a resolution to the priority system. Furthermore, another idea incurred from this observation is to find out ways to incorporate the benefits of the multi-principle point system into the reserve system or inversely.

At this stage, you should be clear about how to implement the reserve system and two types of priority system and understand how the reserve system reflects all ethical values in the allocation results, under perfect conditions. However, the perfect conditions are not always satisfied in reality because people may find ways to misreport their information, and the advisory committee may find it hard to reach a compromise. Section 4 and Section 5 discuss how the allocation systems deal with these two problems. Through the comparison, you can form a better understanding of whether the reserve system has advantages over the priority system with respect to these two aspects.

4 Advantage 1: Preventing people from misreporting

This section provides insights into how the reserve system and the priority system deals with information fabrication. Many states in the U.S. adopt the Honor System, a system that requires people to report their eligibility, in rationing the COVID-19 vaccines[23]. The Honor system makes cheating easy because it entirely relies on the honesty of the public. On the one hand, cheating is harmful because it violates the equity principle, gives rise to complaints, and may cause damages to the order of allocation. On the other hand, monitoring cheating is costly especially in a crisis in which it is more important to make sure people are getting vaccinated quickly than to spend time figuring out the value of each individual. Hence, it is necessary for policymakers to weigh the pros and cons of putting efforts into cheating reduction. An allocation system that prevents cheating at a relatively low cost is preferred.

4.1 Reserve system: number of units reserved for categories

In the reserve system, an individual meets either one or more categories of qualifications. In the former case, if a person believes that he is more likely to benefit from another category, he may cheat. In the latter case, people make choices on the category they would like to receive medical units from, but it is possible for them to choose a category that has no unit left for them based on the priority order. Since public decisions are closely related to the number of units in the reserve categories, decision-makers should formulate appropriate strategies to allocate units to each reserve category so as to encourage truth-telling at a low cost.

When the reserve system is implemented, patients participate in a strategic game where they make choices based on their beliefs. This part analyzes two cases of such a game. The first scenario is built on the assumption that all patients are eligible for one and only one category. The second case assumes that all patients meet the qualifications for multiple

2. How People Are Jumping The COVID-19 Vaccine Line, https://www.npr.org/sections/health-shots/2021/02/09/965841419/does-loose-enforcement-of-vaccine-eligibility-rules-encourage-line-jumping

3. Who Should Get The Coronavirus Vaccine? For Some States, The Honor System Rules, https://www.npr.org/2021/01/16/957593577/who-should-get-the-coronavirus-vaccine-for-some-states-the-honor-system-rules

categories. I will look for Nash equilibrium for each of these two cases in an effort to find out the conditions, under which players are willing to report their information truthfully. In addition, for simplicity, the general assumptions for the analysis are that (1) players are all rational, (2) players choose their strategies independently, and (3) there are two reserve categories A and B.

Based on the equilibrium I find, for the first case, my results suggest planners to allocate units proportionally to each category according to the corresponding number of eligible individuals with respect to the entire population. In the second case, any method works because people are more likely to choose categories with more units than those with fewer units, which leads to a balance that each individual has the same probability of getting medical resources.

4.1.1 Each patient is qualified for only one category

Game settings: There are N players, and H identical indivisible medical units to allocate, where $H < N$ and $N,\ H \in \mathbb{N}_+$. There are two reserve categories A and B. The number of units that are allocated to categories A and B are $\delta \cdot H$ and $(1-\delta) \cdot H$ respectively, where $\delta \in (0,1)$. Players are either type A or type B but they can decide which category they would like to report, so $S_i = \{\text{A, B}\}$, for $i \in \{1, ..., N\}$. If there are units left in the category they choose, the players receive units accordingly; otherwise, the players receive nothing.

For each player, the payoff of obtaining one unit is 1 and is 0 otherwise. In addition, all players know the proportion of eligible players for category A is p relative to the entire population, so the proportion for category B is $(1-p)$, where $p \in (0,1)$.

Theorem 1. *Under the settings of this game,*

- *If $\delta = p$, truth-telling is a Nash equilibrium.*
- *If $\delta > p$*
  - *If $N \leq \frac{1-\delta}{\delta-p}$, truth-telling is a Nash equilibrium.*
  - *If $N > \frac{1-\delta}{\delta-p}$, there is a pure strategy Nash equilibrium in which type A players and $(1-\delta) \cdot N$ type B players report truthfully; $(\delta - p) \cdot N$ type B players report being type A.*

- *If $\delta < p$*
  - *If $N \leq \frac{\delta}{p-\delta}$, truth-telling is a Nash equilibrium.*
  - *If $N > \frac{\delta}{p-\delta}$, there is a pure strategy Nash equilibrium in which type B players and $\delta \cdot N$ type A players report truthfully; $(p-\delta) \cdot N$ type A players report being type B.*

Note: *If truth-telling is a Nash equilibrium, all the players report their true types.*

Proof.

- Player i is a random type A player, and player j is a random type B player.
- The proof first shows that if $\delta = p$, truth-telling is a pure strategy Nash equilibrium.

  Suppose truth-telling is a pure strategy Nash equilibrium when $\delta = p$. If this assumption is correct, no one has the motivations to deviate unilaterally from being honest.

  Taking that other players are honest in reporting their types, the payoff for player i is:

$$u_i(A, s_{-i}) = \begin{cases} 1, & \text{with probability } \frac{H\cdot\delta}{N\cdot p} \\ 0, & \text{with probability } 1 - \frac{H\cdot\delta}{N\cdot p} \end{cases} \qquad u_i(B, s_{-i}) = \begin{cases} 1, & \text{with probability } \frac{H\cdot(1-\delta)}{(1-p)\cdot N+1} \\ 0, & \text{with probability } 1 - \frac{H\cdot(1-\delta)}{(1-p)\cdot N+1} \end{cases}$$

  Therefore, if $\delta = p$, the expected payoff for player i is:

$$EU_i(A) = \frac{H}{N} \qquad EU_i(B) = \frac{H\cdot(1-p)}{(1-p)\cdot N+1}$$

  Since $EU_i(A) - EU_i(B) > 0$, player i would not deviate from reporting his or her true types. The same is true for other type A players.

  Similarly, taking that other players are honest in reporting their types, the payoff for player j is:

$$u_j(A, s_{-j}) = \begin{cases} 1, & \text{with probability } \frac{H\cdot\delta}{(N\cdot p)+1} \\ 0, & \text{with probability } 1 - \frac{H\cdot\delta}{(N\cdot p)+1} \end{cases} \qquad u_j(B, s_{-j}) = \begin{cases} 1, & \text{with probability } \frac{H\cdot(1-\delta)}{N\cdot(1-p)} \\ 0, & \text{with probability } 1 - \frac{H\cdot(1-\delta)}{N\cdot(1-p)} \end{cases}$$

Therefore, if $\delta = p$, the expected payoff for player j is:

$$EU_j(A) = \frac{H \cdot p}{(N \cdot p) + 1} \qquad EU_j(B) = \frac{H}{N}$$

Since $EU_j(B) - EU_j(B) > 0$, player j would not deviate from reporting his or her true types. The same is true for other type B players.

Therefore, since no player can gain by a unilateral change of strategy if the others remain unchanged, this is a pure strategy Nash equilibrium.

- Secondly, I find the Nash equilibrium when $\delta > p$.

When $\delta > p$, the difference of player i's expected payoff is:

$$EU_i(A) - EU_i(B) = \frac{H \cdot \delta}{N \cdot p} - \frac{H \cdot (1-\delta)}{N \cdot (1-p) + 1} > \frac{H \cdot \delta}{N \cdot p} - \frac{H \cdot (1-\delta)}{N \cdot (1-p)} = \frac{H \cdot (\delta - p)}{N \cdot p \cdot (1-p)} > 0$$

Hence, type A players would not want to deviate from truth-telling. However, when $\delta > p$, the difference of player j's expected payoff is

$$EU_j(A) - EU_j(B) = \frac{H \cdot \delta}{N \cdot p + 1} - \frac{H \cdot (1-\delta)}{N \cdot (1-p)} = \frac{H \cdot (N \cdot (\delta - p) + \delta - 1)}{N \cdot (N \cdot p + 1) \cdot (1-p)}$$

$$\Rightarrow$$

$$EU_j(A) - EU_j(B) > 0 \qquad if \qquad N > \frac{1-\delta}{\delta - p}$$
$$EU_j(A) - EU_j(B) \leq 0 \qquad if \qquad N \leq \frac{1-\delta}{\delta - p}$$

The larger the difference between δ and p, and the smaller the difference between δ and 1, types B players will be more likely to deviate and to report being type A.

This finding makes sense intuitively because if $\delta - p$ is small, the increase in the expected payoff from declaring type A is small, so only if the population size is large enough, the increase in the expected payoff is not offset by the increase in the reported type A population.

Since the population size is usually big enough, $EU_j(A) - EU_j(B) > 0$ is the common case in practice, in which type B players would like to cheat. As more and more type B players report being type A, the observed p increases. When $(\delta - p) \cdot N$ types B players cheat, no player would want to deviate unilaterally, which leads to a new Nash equilibrium.

- By following the same logic, one can analyze the case when $\delta < p$. In this case,

$$EU_j(A) - EU_j(B) = \frac{H \cdot \delta}{N \cdot p + 1} - \frac{H \cdot (1-\delta)}{(1-p) \cdot N} < \frac{H \cdot \delta}{N \cdot p} - \frac{H \cdot (1-\delta)}{(1-p) \cdot N} = \frac{H \cdot (\delta - p)}{N \cdot p \cdot (1-p)} < 0$$

$$EU_i(A) - EU_i(B) = \frac{H \cdot \delta}{N \cdot p} - \frac{H \cdot (1-\delta)}{(1-p) \cdot N + 1} = \frac{H \cdot (N \cdot (\delta - p) + \delta)}{N \cdot (N \cdot (1-p) + 1) \cdot p}$$

$$\Rightarrow$$

$$EU_i(A) - EU_i(B) \geq 0 \qquad if \qquad N \leq \frac{\delta}{p - \delta}$$

$$EU_i(A) - EU_i(B) < 0 \qquad if \qquad N > \frac{\delta}{p - \delta}$$

Type B players would not deviate from truth-telling. In addition, since the population size is usually big enough, $EU_i(A) - EU_i(B) < 0$ is the common case in practice, in which type A players would like to cheat. As more and more type A players report being type B, the observed p decreases. When $(p - \delta) \cdot N$ types A players cheat, no player would want to deviate unilaterally, which leads to a new Nash equilibrium.

QED

4.1.2 Each patient is qualified for both categories

The general case

Game settings: There are N players, and H identical indivisible medical units to allocate, where $H < N$ and $N, H \in \mathbb{N}_+$. There are two reserve categories A and B. The number of units that are allocated to categories A and B are m and n respectively, where $m, n \in \mathbb{N}_+$ and $m + n = H$. Each player is eligible for both categories. Players decide which category they would like to report, so $S_i = \{\text{A, B}\}$, for $i \in \{1, ..., N\}$. If there are units left in the category they choose, the players receive units accordingly; otherwise, the players receive nothing. For each player, the payoff of obtaining one unit is 1 and is 0 otherwise. Every player knows that all the players are eligible for both groups.

Theorem 2. *Under the settings of this game, there is a symmetric mixed strategy Nash equilibrium in which each player selects category A with a probability of α^* and selects category B with a probability of $(1-\alpha^*)$, where $\alpha^* \in [0,1]$. In this case, one can always find a unique α^* that is the only one solution of the equation:*

$$\sum_{k=0}^{m-1} \binom{N-1}{k} \cdot \alpha^k \cdot (1-\alpha)^{N-1-k} = \sum_{k=0}^{n-1} \binom{N-1}{k} \cdot (1-\alpha)^k \cdot \alpha^{N-1-k}$$

Proof.

- Let us consider a random player i, taking as given other players' strategies. The payoff for player i is:

$$u_i(A, s_{-i}) = \begin{cases} 1, & \text{if } m - \sum_{j \neq i, j=1}^{N} \mathbb{1}\{s_j = A\} \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

$$u_i(B, s_{-i}) = \begin{cases} 1, & \text{if } n - \sum_{j \neq i, j=1}^{N} \mathbb{1}\{s_j = B\} \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

- The aim is to find a symmetric Nash equilibrium in which players select A with a probability of α and select B with a probability of $(1-\alpha)$, where $\alpha \in [0,1]$

  Expected payoff of choosing A:

  Let X be a random variable, and X is the number of times A occurs in $N-1$ trial. The trials are independent of each other, and the probability that A occurs in each trial should be α. Therefore, $X \sim Binomial(N-1, \alpha)$. Then,

$$P((m - \sum_{j \neq i, j=1}^{N} \mathbb{1}\{s_j = A\}) \geq 1) = P(X < m) = \sum_{k=0}^{m-1} \binom{N-1}{k} \cdot \alpha^k \cdot (1-\alpha)^{N-1-k}$$

$$EU_i(A) = \sum_{k=0}^{m-1} \binom{N-1}{k} \cdot \alpha^k \cdot (1-\alpha)^{N-1-k}$$

  Expected payoff of choosing B:

Similarly, the expected payoff of choosing B can be obtained following the same logic:

$$EU_i(B) = \sum_{k=0}^{n-1} \binom{N-1}{k} \cdot (1-\alpha)^k \cdot \alpha^{N-1-k}$$

Then, $EU_i(A) = EU_i(B)$ implies that:

$$\sum_{k=0}^{m-1} \binom{N-1}{k} \cdot \alpha^k \cdot (1-\alpha)^{N-1-k} = \sum_{k=0}^{n-1} \binom{N-1}{k} \cdot (1-\alpha)^k \cdot \alpha^{N-1-k}$$

With respect to α, the function on the left-hand side is continuous and monotone decreasing, while the function on the right-hand side is continuous and monotone increasing. The ranges of both functions are $[0, 1]$. Therefore, by the Intermediate value theorem, one can always find a unique value α^* such that the equation above holds.

- Since all other players are in the same conditions as player i, the analysis of other players is the same as that of player i. Hence, the α^* that satisfies $EU_i(A) = EU_i(B)$ is the same for $i \in \{1, ..., N\}$. Such an α^* leads to a symmetric mixed strategy Nash equilibrium in which each player selects category A with a probability of α^* and selects category B with a probability of $(1 - \alpha^*)$.

QED

A simple case as an example

The settings are the same as those for the general case in Section 4.1.2. Let $m = 2$, $n = 3$, and $N = 6$, then:

$$(1-\alpha)^5 + 5 \cdot \alpha \cdot (1-\alpha)^4 = \alpha^5 + 5 \cdot (1-\alpha) \cdot \alpha^4 + 10 \cdot (1-\alpha)^2 \cdot \alpha^3$$

$$\Leftrightarrow$$

$$\alpha \approx 0.40563$$

This means that there is a symmetric mixed strategy Nash equilibrium in which each player selects category A with a probability of 0.40563 and selects category B with a probability of 0.59437, The result makes sense intuitively because if category A is allocated with fewer units compared with category B, a general player is less likely to choose category A, given that all players are eligible for both categories.

Discussion:

In real life, there are people eligible for one category and also others who are eligible for multiple categories. I simplify the situation by assuming that there are only two reserve categories. Then, based on Theorem 1, to avoid cheating, the proportion of medical units distributed to each reserve category should be equal to the proportion of eligible players of that category with respect to the whole population size. Based on Theorem 2, those who are eligible for both categories will adjust their strategy accordingly based on the number of units reserved for each category, so they will not affect the Nash equilibrium of truth-telling found in Theorem 1.

Accordingly, if planners adopt the reserve system, there is a strategy to prevent cheating in medical rationing. The first step is to divide the people into two groups. Group one consists of those who are qualified for only one category, and group two is made up of those eligible for both categories. The second step is to allocate each reserve category with medical units according to the corresponding eligible population proportion in group one.

The implementation of this strategy requires planners to distinguish the two groups of people and to figure out the corresponding population proportion of each category in group one, which are not easy especially under emergency scenarios such as the COVID-19 pandemic. In the next section, I will analyze truth-telling for the priority system so as to compare the performances of both systems concerning the honesty incentive mechanism.

4.2 Priority system: a truth-telling analysis

In this section, I model the priority system based on the COVID-19 vaccine distribution plan recommended by the Advisory Committee on Immunization Practices (ACIP). The aim is to find the Nash equilibrium for the two strategic models with and without the introduction of punishment.

After the Food and Drug Administration (FDA) authorizes two types of COVID-19 vaccine on December 20th, 2020, ACIP updated the recommendations for which groups of individuals should be offered the vaccine first when there is a shortage in vaccine supplies.

Taking into consideration the scientific evidence regarding COVID-19 epidemiology, ethical principles, and vaccination program implementation considerations, the team divides the allocation into the following phases: Phase $1a$: health care personnel and long-term care facility residents; Phase $1b$: frontline essential workers (non–health care workers) and persons aged 75 years; Phase $1c$: persons aged 65-74 years and persons aged 16-64 years with medical conditions that increase the risk for severe COVID-19; Phase 2: all other persons aged 16 years not already recommended for vaccination in Phases $1a$, $1b$, or $1c$(Dooling 2021).

Instead of viewing the process as different phases, I model the system as placing individuals into tiers with varying priority order, and individuals in the same tier have equal priority. In this setting, each patient cannot be in more than one tier at a time. In addition, the general assumptions for the models in this section are: (1) Players are all rational, (2) players choose their strategies independently, and (3) there are two tiers A and B. My results suggest that people in tier B always have incentives to cheat unless supervision and punishment are introduced.

4.2.1 General priority system without punishment

The general case

Game settings:

There are N players and H identical indivisible medical units to allocate, where $H < N$ and $N, H \in \mathbb{N}_+$. There are two types of players and two tiers: N_1 type A players eligible for tier A, and N_2 type B players eligible for tier B, where $N_1 + N_2 = N$ and $N_1,\ N_2 \in \mathbb{N}_+$. Tier A has a higher priority than tier B, and players have equal priority within each tier.

Players decide which category to report, so $S_i = \{\text{A, B}\}$, for $i \in \{1, ..., N\}$. Players can report being in either tier but cannot report being in both. Since when $H \leq N_1$, players will always cheat and report being in tier A, I require $H > N_1$ to exclude this trivial case from the model. For each player, the payoff of obtaining one unit is 1 and is 0 otherwise. In addition, all players know the number of eligible players in each tier.

Theorem 3. *Under the settings of this game, there is a unique pure strategy Nash equilibrium in which all players report being in tier A.*

Proof.

According to my game settings in this section, whatever tier a player is eligible for, that person will be viewed as the type he or she reports, so type A players and type B players are essentially faced with the same situation. Therefore, I analyze the decision of a random player l who can be either type A or type B.

Let θ be the proportion of players other than l who declare tier A, so $\theta \in [0, 1]$.

- If $\theta \in [0, \frac{H}{N-1})$, the expected payoffs are $EU_l(A) = 1$ and and $EU_l(B) = \frac{H-\theta\cdot(N-1)}{N-\theta\cdot(N-1)}$. Since $EU_l(A) > EU_l(B)$, reporting being in tier A is more profitable than telling the truth.

- If $\theta \in [\frac{H}{N-1}, 1]$, the expected payoffs are $EU_l(A) = \frac{H}{\theta\cdot(N-1)+1}$, and $EU_l(B) = 0$. Since $EU_l(A) > EU_l(B)$, reporting being in tier A is more profitable than telling the truth.

Accordingly, for player l, since whatever strategies of other players, the expected payoff of declaring tier A is always greater than that of declaring tier B, choosing tier B is strictly dominated by choosing tier A. In fact, since players have two strategies, declaring tier A is a strictly dominant strategy for them. Hence, there is a unique pure strategy Nash equilibrium in which all the players choose to report being in tier A.

QED

4.2.2 Priority system with Punishment introduced

The general case

Game settings: The settings are the same as the settings in Section 4.2.1 in addition to one extra condition that is: If a type B player report being in tier A, he or she will be caught with a probability of ω ($\omega \in [0, 1]$) and has no chance to receive medical resources at the current point in time.

Theorem 4. *Under the settings of this game,*

- *If* $\omega \in [1 - \frac{1}{N_2} \cdot (H - N_1), 1]$, *truth-telling is a pure strategy Nash equilibrium.*

- *If $\omega \in [0, \omega^*]$, there is a pure strategy Nash equilibrium in which all the players report being in tier A. One can always find a unique ω^* that is the only one solution of the equation:*

$$\sum_{k=0}^{N_2-1} min\{1, \frac{H}{N-k}\} \cdot \binom{N_2-1}{k} \cdot \omega^k \cdot (1-\omega)^{N_2-k} = \sum_{k=N-H}^{N_2-1} \binom{N_2-1}{k} \cdot \omega^k \cdot (1-\omega)^{N_2-1-k}$$

- *If $\omega \in [\omega^*, 1-\frac{1}{N_2} \cdot (H-N_1)]$, there are Nash Equilibria in which type A players are honest, and each type B player reports being in tier A with a probability of α^* and report being in tier B with a probability of $(1-\alpha^*)$, where $\alpha^* \in [0,1]$. In this case, one can find at least one α^* that are solutions to the equation:*

$$\cdot \sum_{k=0}^{N_2-1} \binom{N_2-1}{k} \cdot \alpha^k \cdot (1-\alpha)^{N_2-1-k} \cdot [f_A(k) \cdot (1-\omega) - f_B(k)] = 0$$

where

$$f_A(k) = \sum_{r=0}^{k} min\{1, \frac{H}{N_1+1+k-r}\} \cdot \binom{k}{r} \cdot \omega^r \cdot (1-\omega)^{k-r}$$

$$f_B(k) = \sum_{r=0}^{k} min\{1, \frac{H-N_1-(k-r)}{N_2-k}\} \cdot \mathbb{1}\{\frac{H-N_1-(k-r)}{N_2-k} \geq 0\} \cdot \binom{k}{r} \cdot \omega^r \cdot (1-\omega)^{k-r}$$

Proof.

- Firstly, I show that truth-telling is a pure strategy Nash equilibrium when $\omega \geq 1 - \frac{1}{N_2} \cdot (H-N_1)$.

  Suppose truth-telling is a Nash equilibrium. If this assumption is correct, in this scenario, players have no motivation to deviate unilaterally from telling the truth.

  - In this case, type A players would not want to deviate unilaterally because they undertake a risk if they cheat but are secured to receive medical resources by reporting the truth.
  - For a random type B player j, the expected payoff of choosing B is equal to or greater than the expected payoff of choosing A. Player j's payoff is:

$$u_j(A, s_{-j}) = \begin{cases} 1, & \text{with probability } 1-\omega \\ 0, & \text{with probability } \omega \end{cases} \qquad u_j(B, s_{-j}) = \begin{cases} 1, & \text{with probability } \frac{H-N_1}{N_2} \\ 0, & \text{with probability } \frac{N-H}{N_2} \end{cases}$$

To prevent player j from deviating from truth-telling, the condition below is required:

$$EU_j(A) - EU_j(B) = [1 \cdot (1-\omega) + 0 \cdot \omega] - [1 \cdot \frac{H-N_1}{N_2} + 0 \cdot \frac{N-H}{N_2}] \leq 0$$

$$\Leftrightarrow$$

$$\omega \geq 1 - \frac{1}{N_2} \cdot (H - N_1)$$

Therefore, if $\omega \geq 1 - \frac{1}{N_2} \cdot (H - N_1)$, truth-telling is a Nash equilibrium. Otherwise, reporting tier A is profitable for type B players, so they have the motivations to deviate unilaterally.

This result makes sense intuitively. Holding other variables constant, when the number of type B players (N_2) increases, more supervision is required to avoid cheating. In addition, holding everything else fixed, when the production of medical resources (H) increases, the required supervision decreases.

- Secondly, I prove that when ω is too small, the Nash equilibrium strategy profile is that all the players report being in tier A.

  Assume there is such a pure strategy Nash equilibrium. If this assumption is correct:

  - For a random type A player i,

$$EU_i(A) - EU_i(B) =$$

$$\sum_{k=0}^{N_2} min\{1, \frac{H}{N-k}\} \cdot \binom{N_2}{k} \cdot \omega^k \cdot (1-\omega)^{N_2-k} - \sum_{k=N-H}^{N_2} \binom{N_2}{k} \cdot \omega^k \cdot (1-\omega)^{N_2-k+1} =$$

$$\sum_{k=0}^{N-H-1} \frac{H}{N-k} \cdot \binom{N_2}{k} \cdot \omega^k \cdot (1-\omega)^{N_2-k} + \sum_{k=N-H}^{N_2} \binom{N_2}{k} \cdot \omega^{k+1} \cdot (1-\omega)^{N_2-k} > 0$$

  Therefore, type A players have no incentive to deviate unilaterally from truth-telling.

  - For a random type B player j, the expected payoff of choosing A is equal to or greater than the expected payoff of choosing B.

Therefore, taking as given other players' strategies, to prevent player j from deviating from choosing A, the condition below is required:

$$EU_j(A) - EU_j(B) =$$

$$\sum_{k=0}^{N_2-1} min\{1, \frac{H}{N-k}\} \cdot \binom{N_2-1}{k} \cdot \omega^k \cdot (1-\omega)^{N_2-k} -$$

$$\sum_{k=N-H}^{N_2-1} \binom{N_2-1}{k} \cdot \omega^k \cdot (1-\omega)^{N_2-1-k} \geq 0$$

$$\Leftrightarrow$$

$$\omega \leq \omega^*$$

$$where\ \omega^*\ is\ the\ unique\ solution\ of\ the\ equation:$$

$$\sum_{k=0}^{N_2-1} min\{1, \frac{H}{N-k}\} \cdot \binom{N_2-1}{k} \cdot \omega^k \cdot (1-\omega)^{N_2-k} = \sum_{k=N-H}^{N_2-1} \binom{N_2-1}{k} \cdot \omega^k \cdot (1-\omega)^{N_2-1-k}$$

With respect to ω, $EU_j(A)$ is continuous and monotone decreasing, and $EU_j(B)$ is continuous and monotone increasing. Additionally, when $\omega = 0$, $EU_j(A) \geq 0 = EU_j(B)$, and when $\omega = 1$, $EU_j(A) \leq 1 = EU_j(B)$. Therefore, by the Intermediate value theorem, one can always find a unique value of ω^* such that $EU_j(A) = EU_j(B)$. In addition, when $\omega \leq \omega^*$, $EU_j(A) \geq EU_j(B)$; when $\omega \geq \omega^*$, $EU_j(A) \leq EU_j(B)$.

Therefore, if $\omega \leq \omega^*$, the Nash equilibrium is the strategy profile that all players report being in tier A. This result makes sense intuitively. When the probability of being caught is too small, type B players can still benefit from misreporting.

- Thirdly, if $\omega \in [\omega^*, 1 - \frac{1}{N_2} \cdot (H - N_1)]$:

  - Assuming that all type A players are honest, I claim that there is a symmetric mixed strategy Nash equilibrium for type B players in which each player cheat with a probability of α, where $\alpha \in [0, 1]$.

    To begin with, a symmetric mixed strategy Nash equilibrium always exists in my game settings. According to the classic paper of Nash, every finite symmetric game has a symmetric Nash equilibrium(Nash 1951). My game is a finite symmetric game. On the one hand, since all medical resources are ultimately distributed to players, this game is finite. On the other hand, the payoff functions are the same for all players, so it is also symmetric. Hence, there is always a symmetric Nash equilibrium of my game. Additionally, from the proof above, I have shown that when $\omega \in [\omega^*, 1 - \frac{1}{N_2} \cdot (H - N_1)]$, there is no symmetric pure strategy Nash equilibrium so a mixed strategy Nash equilibrium must exist.

In such a symmetric mixed strategy Nash equilibrium, no type B players would want to deviate unilaterally, so the condition below needs to be satisfied for a random type B player j:

$$EU_j(A) - EU_j(B) = 0$$

$$\Leftrightarrow$$

$$\cdot \sum_{k=0}^{N_2-1} \binom{N_2-1}{k} \cdot \alpha^k \cdot (1-\alpha)^{N_2-1-k} \cdot [f_A(k) \cdot (1-\omega) - f_B(k)] = 0$$

$$where$$

$$f_A(k) = \sum_{r=0}^{k} min\{1, \frac{H}{N_1+1+k-r}\} \cdot \binom{k}{r} \cdot \omega^r \cdot (1-\omega)^{k-r}$$

$$f_B(k) = \sum_{r=0}^{k} min\{1, \frac{H-N_1-(k-r)}{N_2-k}\} \cdot \mathbb{1}\{\frac{H-N_1-(k-r)}{N_2-k} \geq 0\} \cdot \binom{k}{r} \cdot \omega^r \cdot (1-\omega)^{k-r}$$

Since a symmetric mixed strategy Nash equilibrium always exist, given certain values of N_1, N_2, H, and ω ($\omega \in [\omega^*, 1 - \frac{1}{N_2} \cdot (H - N_1)]$), there is at least one α^* ($\alpha^* \in [0,1]$) that satisfies the equation above.

However, it is not a trivial task to discuss the monotonicity of "$EU_j(A) - EU_j(B)$" and to study how α^* changes as ω varies. In particular, I find it difficult to determine the sign of "$f_A(k) \cdot (1-\omega) - f_B(k)$". Therefore, further research is needed to rigorously analyze the Nash equlibrium. In this section, I use a simple example to study these questions numerically through simulation by fixing values for parameters N_1, N_2, and H.

- Next, given that each type B player cheats with a probability of α^*, I will show that there is no motivation for type A players to deviate unilaterally from telling the truth.

If each type B player cheats with a probability of α^*, the probability that q ($q \in \mathbb{Z}_+$ and $q \in [0, N_2]$) type B players cheat is a function of α^*: $p_q(\alpha^*) = \binom{N_2}{q} \cdot (\alpha^*)^q \cdot (1-\alpha^*)^{N_2-q}$. When q type B players cheat, given that other type A players are honest, I denote the corresponding expected payoffs of a random type A player i by $EU_i^q(A)$ and $EU_i^q(B)$. Accordingly,

$$EU_i(A) - EU_i(B) = \sum_{q=0}^{N_2} p_q(\alpha^*) \cdot (EU_i^q(A) - EU_i^q(B))$$

* If $q \in [0, H - N_1)$, $EU_i^q(A) - EU_i^q(B) = 1 - (1-\omega) = \omega > 0$
* If $q \in [H - N_1, N_2]$,

$$\begin{aligned} &EU_i^q(A) - EU_i^q(B) = \\ &\sum_{k=0}^{N_1+q-H-1} \frac{H}{N_1+q-k} \cdot \binom{q}{k} \cdot \omega^k \cdot (1-\omega)^{q-k} + \\ &\sum_{k=N_1+q-H}^{q} min\{1, \frac{k-(N_1+q-H-1)}{N_2-q+1}\} \cdot \binom{q}{k} \cdot \omega^{k+1} \cdot (1-\omega)^{q-k} + \\ &\sum_{k=N_1+q-H}^{q} (1 - min\{1, \frac{k-(N_1+q-H-1)}{N_2-q+1}\}) \cdot \binom{q}{k} \cdot \omega^k \cdot (1-\omega)^{q-k} \geq 0 \end{aligned}$$

  Therefore, $\forall\ q \in \mathbb{Z}_+$ and $q \in [0, N_2]$, $EU_i^q(A) - EU_i^q(B) \geq 0$. Hence, $EU_i(A) - EU_i(B) \geq 0$. In this case, type A players have no incentive to deviate unilaterally from truth-telling.

- In summary, when $\omega \in [\omega^*, 1 - \frac{1}{N_2} \cdot (H - N_1)]$, there are Nash Equilibria in which type A players are honest, and each type B player reports being in tier A with a probability of α^*, where $\alpha^* \in [0, 1]$.

QED

The simplest case as an example

Game settings: The settings are the same as those for the general case in Section 4.2.2. Let $N = 3$, $H = 2$, $N_1 = 1$, and $N_2 = 2$, so $S_i = \{\text{A, B}\}$, for $i \in \{1, 2, 3\}$.

Proposition 4.1. *Under the settings of this game,*

- *If* $\omega \in [0.5, 1]$*, truth-telling is a pure strategy Nash equilibrium*
- *If* $\omega \in [0, 0.4495]$*, there is a pure strategy Nash equilibrium in which all the type B players report being in tier A.*
- *If* $\omega \in [0.4495, 0.5]$*, there is a Nash equilibrium in which type A players are honest, and each type B player reports being in tier A with a probability of* α^**, where* $\alpha^* \in [0, 1]$*. In this case, one can find a unique* α^* *that is the solution to the equation:*

$$\alpha^*(\omega) = \frac{\omega - \frac{1}{2}}{\frac{1}{3} \cdot (\omega + 2) \cdot (1 - \omega) - \frac{1}{2}}$$

Proof.

- For the type A player, declaring tier A is a weakly dominant strategy because the expected payoff of choosing tier A is always equal to or higher than that of choosing tier B regardless of the strategies of other players. Specifically:

(1) Type B players are honest: $EU_A(A) - EU_A(B) = 1 - \frac{2}{3} \cdot (1 - \omega) > 0$

(2) One type B player cheat, and the other one is honest:

  * If 0 type B player is caught: $EU_A(A) - EU_A(B) = \frac{1}{2} - \frac{1}{2} \cdot (1 - \omega) \geq 0$
  * If 1 type B player is caught: $EU_A(A) - EU_A(B) = 1 - (1 - \omega) \geq 0$

(3) Both type B players cheat:

  * If 0 type B player is caught: $EU_A(A) - EU_A(B) = \frac{2}{3} - 0 > 0$
  * If 1 type B player is caught: $EU_A(A) - EU_A(B) = 1 - (1 - \omega) \geq 0$
  * If 2 type B players are caught: $EU_A(A) - EU_A(B) = 1 - (1 - \omega) \geq 0$

Therefore, it is plausible to assume that the type A player is always honest.

		Player B_2	
		A	B
Player B_1	A	$(\omega \cdot (1-\omega) + \frac{2}{3} \cdot (1-\omega)^2,\ \omega \cdot (1-\omega) + \frac{2}{3} \cdot (1-\omega)^2)$	$(1-\omega,\ \omega)$
	B	$(\omega,\ 1-\omega)$	$(\frac{1}{2}, \frac{1}{2})$

ω	Pure Strategy Nash Equilibria
$[0, 0.4495)$	(A, A)
0.4495	(A, A), (A, B), (B, A)
$(0.4495, 0.5)$	(A, B), (B, A)
0.5	(A, B), (B, A), (B, B)
$(0.5, 1]$	(B, B)

- For type B players, given that the type A player is honest, I generate a matrix representation for the game between the two type B players:

  In addition, using α to denote the probability of misreporting for player B_2, to find the symmetric mixed strategy Nash equilibrium when $\omega \in [0.4495, 0.5]$, I require:

$$EU_{B1}(A) - EU_{B1}(B) = 0$$

$$\Leftrightarrow$$

$$\alpha \cdot [\omega \cdot (1-\omega) + \frac{2}{3} \cdot (1-\omega)^2] + (1-\alpha) \cdot (1-\omega) - [\omega \cdot \alpha + \frac{1}{2} \cdot (1-\alpha)] = 0$$

$$\Leftrightarrow$$

$$\alpha^*(\omega) = \frac{\omega - \frac{1}{2}}{\frac{1}{3} \cdot (\omega + 2) \cdot (1-\omega) - \frac{1}{2}}$$

  There is a mixed strategy Nash equilibrium in which each player cheat with a probability of $\alpha^*(\omega)$. Moreover, $\alpha^*(\omega)$ is monotone decreasing when $\omega \in [0.4495, 0.5]$(see details in Figure 1).

  In addition, Figure 2 shows how "$EU_{B1}(A) - EU_{B1}(B)$" changes as α varies when setting ω in different ranges. You can observe from the plots that:

  (1) If $\omega \in [0.5, 1]$, $EU_{B1}(A) - EU_{B1}(B) \leq 0$.

  (2) If $\omega \in [0, 0.4495]$, $EU_{B1}(A) - EU_{B1}(B) \geq 0$.

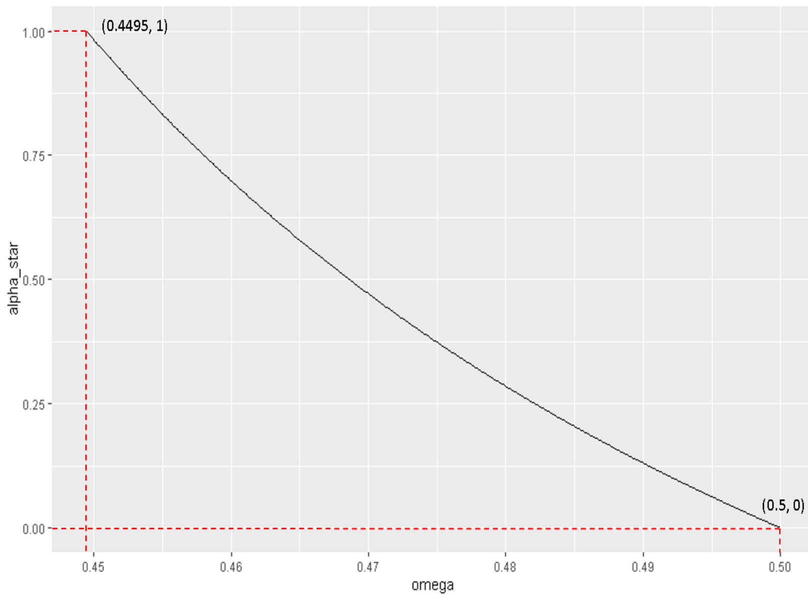

Figure 1: The variation of α^* over different values of ω

(3) If $\omega \in [0.4495, 0.5]$, there is always one unique $\alpha^*(\alpha^* \in [0, 1])$ such that $EU_{B1}(A) - EU_{B1}(B) = 0$

(4) Holding α constant, as ω increases, "$EU_{B1}(A) - EU_{B1}(B)$" decreases.

These observations imply that when $\omega \in [0.4495, 0.5]$, as supervision becomes more strict (i.e. as ω increases), with the same strategy, the profits of cheating become smaller and smaller, so type B players are less likely to cheat(i.e. α^* decreases). When the supervision is strict enough(i.e. $\omega \in [0.5, 1]$), for type B players, being honest is always more profitable than cheating. Conversely, when the supervision is loose enough(i.e. $\omega \in [0, 0.4495]$), for type B players, cheating is always more profitable than being honest.

These results are consistent with Theorem 4.

QED

Discussion:

The assumption of two tiers is applicable in real life. The COVID-19 vaccine allocation can be used as an example. At a point in time, the four tiers can be further categorized into two groups, which are viewed as the two big tiers as assumed in my model. The first group contains tiers with higher priority, and individuals who are eligible in these tiers are secured

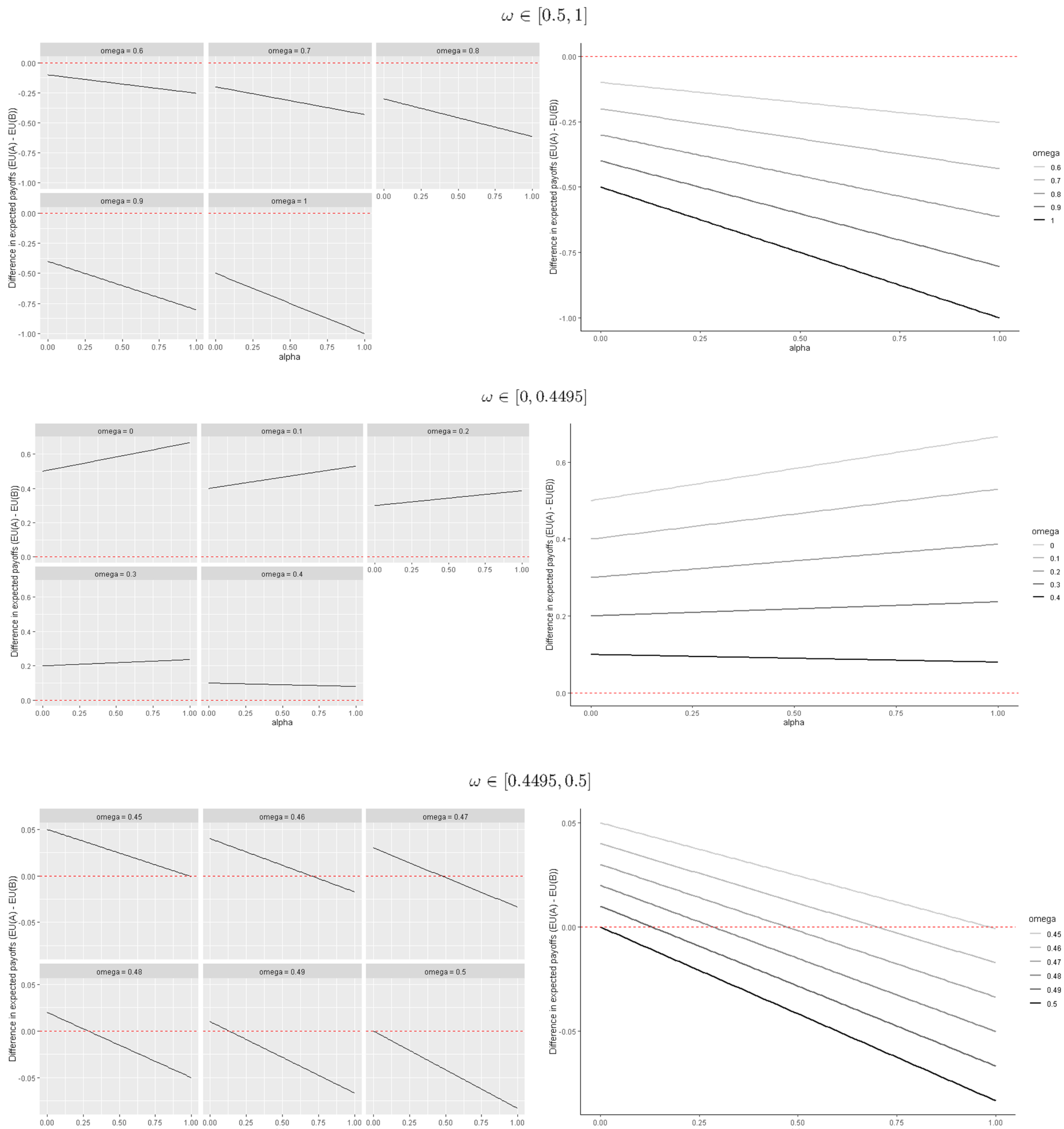

Figure 2: The variation of the difference in expected payoffs over different values of α, given ω in certain ranges

to receive a vaccine unit. The rest of the tiers are in the second group in which people have a chance but are not secured to obtain vaccines. In the early stage of vaccine allocation when there is a severe shortage, the first group is tier $1a$, and the second group contains the other three tiers. In a later stage when the production of COVID-19 vaccine increases, tier $1a$, $1b$, $1c$ are in the first group, and tier 2 is in group two.

According to Theorem 3 and 4, the priority system prevents cheating only with adequate supervision, which costs time and money. Moreover, planners can control the amount of cheating by adjusting the level of supervision. Therefore, implementing the priority system with punishment tends to be more flexible than adopting the cheating-preventing strategy for the reserve system. Furthermore, it is also possible that simply announcing the adoption of a punishment without strict implementation would reduce cheating because the public is not always fully rational, though this idea still needs to be rigorously justified.

5 Advantage 2: Helping people to reach an agreement

This section analyzes the decision-making process of policymakers by dividing the process into two phases: personal decision making and negotiation. I will compare the reserve system and the priority system to figure out which mechanism has the lower cost in reaching a compromise.

For medical rationing, there are two main entities: Policymakers who specify ethical values and design distribution guidelines, and the public who participate in the distribution and report their qualifications. Previous sections analyze the behaviors of the public, examining the conditions in which misreporting can be prevented. However, the analysis does not consider the behaviors of the policymakers, and it assumes implicitly that there is an agreement among stakeholders in articulating the priority order of ethical values.

In fact, figuring out and sorting the ethical values to represent in allocation results is tricky, and a comprise is difficult to reach. Although in most cases, policymakers are able to respond in time and provide advice for rationing, their recommendations are not always optimal. For example, there are debates on whether to prioritize essential personnel. Those who against this idea worry that the prioritization may harm the rights of other groups(Vawter et al. 2010)[4]. Some states choose not to implement it, and others give vague instruction for the prioritization[5]. Essentially, policymakers make satisfactory decisions but not optimal ones in an emergency.

5.1 Individual decision-making

I first consider how a member of the advisory committee makes decisions, and I take the paper written by Kőszegi and Matějka as a reference. In this recently published paper, the authors formulate a theory for the idea that mental budgeting simplifies the complex decision process in multi-product consumption(Kőszegi and Matějka 2020).

Mental budgeting, or mental accounting, is named by Richard H. Thaler in 1999, which describes the process that people budget money into mental accounts for different categories of expenditures(Thaler 1999). According to Kőszegi and Matějka, there are two main explanations for mental accounting. The more developed explanation states that people make mental budgets for self-control purposes such as mitigating over-consumption(Shefrin and Thaler 1988; Koch and Nafziger 2016). Another explanation is that mental budgeting makes it easier for people to make choices when optimization is not feasible(Zhang and Sussman 2018). The paper from Kőszegi and Matějka is in accordance with the latter explanation and formalizes the categorization and simplification mechanisms of mental budgeting in the context of consumption, assuming a person's attention is costly as well as flexible.

The decision-making process of distributing medical units to people based on ethical values is analogous to spending money on different products based on preferences, if the medical units are viewed as money, and the patients are viewed as products with distinct characteristics. The relative preference of products can be treated as the priority order of

4. New York State Department of Health and New York State Task Force on Life and the Law Update Ventilator Allocation Guidelines, https://www.health.ny.gov/press/releases/2015/2015-11-25_ventilator_allocation_guidelines.htm

5. COVID-19 Addendum: Allocation of Scarce Resources in Acute Care Facilities Recommended for Approval by State Disaster Medical Advisory Committee (SDMAC), http://azpha.wildapricot.org/resources/covid-19-addendum.pdf

ethical principles. Therefore, the theories and findings developed in the literature for mental accounting can be mapped to the medical resource allocation problem.

The Reserve System:

The reserve system simplifies the thinking process of an individual policymaker by guiding him (or her) to create mental accounts for medical resources. To demonstrate this idea, the first step is to show that the mechanism of medical rationing matches the settings and conditions of the models introduced in Kőszegi and Matějka's paper. According to proposition 3 of the paper, in the context that goods belonging to the same category are better substitutes with each other than for goods in other categories, an agent's optimal information acquisition is to have a fixed mental budget for each category, given the attention cost is sufficiently high(Kőszegi and Matějka 2020, 15).

Before discussing optimal information acquisition, it is essential to understand what kind of information can be acquired. In the multi-product consumption case, people observe information about their taste for one good or the relative or total taste for several goods. Analogously, in medical rationing, advisory committee members acquire information about their beliefs concerning the priority for one ethical value or the relative priority for several ethical values. When the attention cost is high such as in a crisis situation, it becomes impossible for planners to acquire all possible information, so they need to think about what type of information is the most important and what they should choose to optimally think about.

Firstly, the model developed in the literature assumes diminishing marginal utility in consumption, which is also a proper assumption in the case of medical rationing. Here, a group of people who share similar features is viewed as a product in consumption, so the entire population is divided into multiple groups. Two reasons may explain why there is a declining utility. To begin with, as more medical units are distributed to a certain group of people, people in other groups are more likely to cheat and pretend to be in the preferable group because the distribution is unbalanced, which is similar to the case that has been proven in Section 4.1. The misreporting may harm the fairness of the distribution which is not what policymakers want to see. Another reason is that when a lot of resources are allocated to a certain group, fewer resources are left for the rest of the population, which violates the preference for fairness of the society and affects the reflection of other ethical principles. Therefore, the marginal utility of policymakers decreases as more units are allocated to a certain group of people. When the number of medical units exceeds the number of people in a group, the utility of policymakers will start to decrease.

Secondly, the reserve system classifies patients into reserve categories based on ethical values, and an individual is a better substitute for other individuals in its category than for individuals in a different reserve category. In the reserve system, each reserve category follows its own priority order, so individuals in the same reserve category share more similar features and are more like substitutes than people in other categories. Moreover, with the goal of representing certain ethical values in allocation results, policymakers may treat patients in the same reserve category as substitutes because they reflect the same set of ethical principles.

Thirdly, the attention cost for a planner is expected to be sufficiently high in an emergency situation such as the COVID-19 pandemic because policymakers need to make quick responses with limited information. The cognitive ability of an individual may also be restricted, which makes it nearly impossible to make optimal choices to maximize one's utility.

Since the main conditions have been satisfied as discussed above, it is plausible to apply proposition 3 of Kőszegi and Matějka's paper to the reserve system in health care rationing. Hence, the optimal information acquisition strategy for a policymaker is distributing a fixed amount of medical units to each category. When making budgets, a policymaker can leave his or her plans incomplete and can make decisions separately for different categories. This strategy allows the policymaker to preserve most of his or her expected utility when it is impossible to consider all choices.

In addition, corollary 1 derived from proposition 3 is also applicable, which states that (1) when there are shocks towards a certain category, the budgets for other categories are independent of the shock and will not be affected. (2) The shock will not change the total allocation for that category but may only change the relative allocation levels of different groups of patients within that category(Kőszegi and Matějka 2020, 15). These statements essentially mean that the priority orders for different categories are independent of each other. When the utility of allocating medical units to certain groups of patients changes, policymakers adjust the priority orders in each category.

Therefore, the mechanism of the reserve system exactly matches with this optimal information acquisition strategy introduced in the paper by Kőszegi and Matějka, so it simplifies the decision-making process for an individual policymaker in an optimal way by leading them to make mental budgets.

The Priority System:

By contrast, for the priority system, policymakers usually need to figure out a priority point system to determine the priority order of each patient. This process requires the policymakers to consider all the ethical values at the same time. Although the expected utility for a policymaker can be maximized when there is enough time to figure out the exact priority order, the thinking process is too complex and is nearly impossible especially in an emergency. Therefore, policymakers may find it difficult to make their own decisions even before the negotiation when the problem becomes even more sophisticated.

In terms of the priority system, there are situations when the policymakers may make budget accounts unconsciously. For example, when they give recommendations for the COVID-19 vaccine allocation(Dooling 2021). In this example, there is a severe shortage of vaccine supplies at the early stage of the distribution, and planners only need to think about the groups of people with the highest priorities. On December 1st, 2020, ACIP recommended allocating vaccines to the health care personnel and long-term care facility residents first without specifying the priority orders for the rest of the population. ACIP updated its recommendation On December 20th, 2020 and introduced the second and third tier.

This may be taken as an extreme case of proposition 3 from Kőszegi and Matějka in which the attention cost is extremely high, there are only two mental budgets for two categories, and the agent processes little information. Essentially, the advisory committee members sacrifice more utility in seek of more reduction in attention cost.

In the individual decision-making process, the reserve system simulates the budget accounting process thus simplifying the complexity of thinking for policymakers, which is one of its advantages over the general priority system.

5.2 Negotiation

After each member of the advisory committee forms their own decisions, they need to negotiate so as to reach an agreement in the final recommendation for the rationing guidelines. During this phase, they should also take into consideration the opinions of the public. In recommending the allocation for COVID-19 vaccines, the CDC does conduct surveys to get general opinions of different population groups(Dooling 2021)[6]. Since different people have different decisions or criteria, the optimal strategy would be the one that compromises all possible considerations. In addition, since there can be a lot of criteria, it is possible that people holding completely opposite views.

Compared with the priority system, the reserve system is more flexible and helps to implement a compromise in the negotiating process. This claim has been discussed in a paper published recently(Sönmez et al. 2020). The paper demonstrates this idea by providing the example of prioritizing essential personnel in the allocation of scarce therapeutic remdesivir. In their example, if the essential personnel is not prioritized, courses of remdesivir are allocated proportionally by the patient types in the population. If the essential personnel is prioritized by the priority system, all the essential personnel receive treatment but there are not many resources left for the rest of the population. Policy-makers may be confined to these extreme options, and those opposing the idea of giving prioritization to essential personnel would not be willing to compromise.

The reserve system permits middle options by allowing policymakers to specify the reserve categories, the criteria for allocation in each category, the number of medical resources assigned to each category, and the processing order of the categories. In this way, the

6. Details can be found on the website: https://www.cdc.gov/vaccines/hcp/acip-recs/vacc-specific/covid-19/evidence-table-phase-1b-1c.html

reserve system offers wider choices for policy-makers thus promoting the achievement of the agreement(Sönmez et al. 2020, 2-3).

6 Conclusion

This paper compares the reserve system and the priority system on a static basis with respect to ethical value representation, cheating mitigation, and decision-making simplification. The underlying mechanisms of the reserve system allow planners to prevent deliberate information fabrication by allocating medical resources proportionally to reserve categories based on the corresponding eligible population size, while this strategy is costly. By contrast, the mechanisms of the priority system encourage cheating, but the problem can be solved through the incorporation of supervision and punishment, which also has a cost but provides a more flexible solution for decision-makers. Moreover, by mapping medical rationing to multi-product consumption and applying relevant theories, I conclude that the reserve system relieves the sophisticated thinking process of an individual decision-maker through mental budgets. With its promotion of compromise in negotiation, the reserve system shows its advantage in decision-making simplification.

These results suggest that the reserve system and the priority system have strengths and weaknesses. In practice, decision-makers need to consider the specific context and the needs in selecting allocation systems for healthcare rationing. Decision-makers can also consider combining the two allocation systems to give full play to the advantages of both systems.

My current work has many limitations which inspire further research. Firstly, my analysis of the reserve system in Section 4.1 assumes only two reserve categories. A more rigorous argument about cheating mitigation can be developed by allowing more reserve categories. Secondly, Theorem 4 still requires advanced mathematical and computational techniques to study the uniqueness and the comparative statics of the Nash equilibrium so that the effects of punishment on cheating reduction can be better measured. Thirdly, researchers can consider developing game-theoretic models of bargaining to study the negotiating process in recommending medical rationing guidelines, which should provide more insights from the mathematical perspective.

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