Name:________________________________

Exam Number:_______

University of Michigan Physics Department Graduate Qualifying Examination

Part I: Classical Physics

Saturday 7 May 2016 9:30 am – 2:30 pm

This is a closed book exam, but a number of useful quantities and formulas are provided in the front of the exam. (**Note that this list is more extensive than in past years.**) If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it. Answer the questions directly in this exam booklet. If you need more space than there is under the problem, continue on the back of the page or on additional blank pages that the proctor will provide. <u>Please clearly indicate if you continue your answer on another page. Label additional blank pages with your exam number, found at the upper right of this page (but not with your name). Also clearly state the problem number and "page x of y" (if there is more than one additional page for a given question).</u>

You must answer the first 8 required questions and 2 of the 4 optional questions. Indicate which of the latter you wish us to grade (e.g. by circling the question number). We will only grade the indicated optional questions. Good luck!!

Some integrals and series expansions

$$\int_{-\infty}^{\infty} \exp(-\alpha x^2)\, \mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}$$

$$\int_{-\infty}^{\infty} x^2 \exp(-\alpha x^2)\, \mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$$

$$\exp(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \cdots$$

Some Fundamental Constants

speed of light $c = 2.998 \times 10^{8}$ m/s

proton charge $e = 1.602 \times 10^{-19}$ C

Planck's constant $\hbar = 6.626 \times 10^{-34}$ J·s $= 4.136 \times 10^{-15}$ eV·s

Rydberg constant $R_{\infty} = 1.097 \times 10^{7}\,\mathrm{m}^{-1}$

Coulomb constant $k = (4\pi\varepsilon_0)^{-1} = 8.988 \times 10^{9}\ \mathrm{N{\cdot}m^2 / C^2}$

vacuum permeability $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A}$

universal gas constant $R = 8.3\ \mathrm{J / K{\cdot}mol}$

Avogadro′s number $\mathrm{N_A = 6.02 \times 10^{23}\,mol^{-1}}$

Boltzmann′s constant $\mathrm{k_B = R/N_A = 1.38 \times 10^{-23}\,J/K = 8.617 \times 10^{-5}\,eV/K}$

Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}\,\mathrm{W / m^2K^4}$

radius of the sun $R_{\mathrm{sun}} = 6.96 \times 10^{8}\,\mathrm{m}$

radius of the earth $R_{\mathrm{earth}} = 6.37 \times 10^{6}\,\mathrm{m}$

radius of the moon $R_{\mathrm{moon}} = 1.74 \times 10^{6}\,\mathrm{m}$

gravitational constant $G = 6.67 \times 10^{-11}\,\mathrm{m^3 / (kg{\cdot}s^2)}$

Required (do all of the first 8 problems)

1. (**Mechanics**) A pendulum, constructed from a bob of mass m, is suspended from a massless spring and is allowed to swing in a vertical plane under the influence of gravity.
a) What is the Hamiltonian of the system?
b) Using this result and Hamilton's equations, find the equations of motion of the pendulum.
c) Solve the equations of motion and describe the motion of the system in as much detail as you can.

2. (**Mechanics**) A cylinder of length L, radius R and mass density ρ rolls without slipping on a horizontal surface. A hole of radius $r < R$ has been drilled through the cylinder parallel to its axis at a distance $R/2$ from the center. The orientation of the cylinder is specified by an angle θ between the vertical direction and a line connecting the centers of the cylinder and the hole. If the cylinder is initially at rest with the angle $\theta \ll 1$, predict the subsequent time-dependence of θ. Are there any times where $\theta = 0$?

3. (**Mechanics**) A small point-like object is initially at rest on top of a frictionless hemisphere of radius R, but then begins to slide down the side under the influence of gravity. Measure the location of the object using the angle from vertical.

a) Find the angle at which the object loses contact with the surface.

b) When the object passes through the (extended) equatorial plane, find the angle of the velocity vector with respect to vertical.

4. (**E&M**) Suppose that under static conditions the cartesian components of the electric field in a charge-free region of space are

$$E_k = A_k + B_{kl} r_l \ ,$$

where A_k and B_{kl} are constant.
a) Show that $B_{kl} = B_{lk}$ for all k and l.
b) Show that $\sum_k B_{kk} = 0$.
c) Find the most general electrostatic potential that generates the above electric field.

5. (**E&M**) Consider an infinitely long wire with current $I = I_0\Theta(t)$ that instantaneously turns on at $t = 0$. Here $\Theta(t)$ is the unit step function defined so that $\Theta(t) = 0$ for $t < 0$ and $\Theta(t) = 1$ for $t \geq 0$.
a) Find the vector potential produced by the wire.
b) Calculate the corresponding electric and magnetic fields.
c) Show that the electric and magnetic fields approach their static values as $t \to \infty$.

6. (**E&M**) Calculate the capacitance of two concentric spherical shells of inner radius R_1 and outer radius R_2 when the space between them is filled with a dielectric that varies in polar angle as

$$\epsilon = \epsilon_0 + \epsilon_1 \cos^2 \theta \tag{1}$$

7. (**Thermodynamics**) Please answer each of the following questions as indicated.

a) Suppose that each of two identical bodies are characterized by a heat capacity at constant pressure C_p which you may assume is independent of temperature. These bodies are used as heat reservoirs for a reversible heat engine. The bodies remain at constant pressure and undergo no phase change. Their initial temperatures are T_1 and T_2 with $T_1 > T_2$ and as a result of the operation of the heat engine, they attain a common final temperature T_f. First derive an inequality relating T_f to T_1 and T_2 using entropy considerations. Then use this to find the maximum work obtainable from the engine for given initial temperatures T_1 and T_2.

b) Consider a reversible heat engine driven by two reservoirs. Suppose one can change either (but not both) of the reservoir temperatures, T_h (hot) and T_c (cold), by an amount ΔT. To increase the thermal efficiency of a reversible cycle operating between reservoirs at T_h and T_c, would it be better to increase T_h while keeping T_c constant, or decrease T_c while keeping T_h constant? Please give a quantitative discussion justifying your answer.

8. (**Optics**) Light of wavelength λ and intensity I is incident perpendicularly onto an opaque circular disk of radius R. Find the intensity of light at a point P on the line perpendicular to the disk and passing through its center. You may assume that the distance between P and the center of the disk is much larger than R.

Optional (do 2 of of the final 4 problems)

9. (**Mechanics**) A string is tied around a uniform disk of radius R and mass M. The disk is released from rest with the string vertical and its top end tied to a fixed support, so that the disk falls as the string unrolls, like a yo-yo. Find the acceleration of the disk.

10. (**E&M**) A point charge q is imbedded at the center of a sphere of linear dielectric material with susceptibility χ and radius R.
a) Find the electric field and the polarization everywhere.
b) Find all the charges. Where are they? Comment on their sum.

11. (**Thermodynamics**) A free expansion of a gas is a process where the total mean energy E remains constant. The following questions refer to this process.

a) From general thermodynamic considerations show that,

$$\left(\frac{\partial T}{\partial V}\right)_E = -\frac{T(\frac{\partial p}{\partial T})_V - p}{C_V},$$

where, C_V is the specific heat at constant volume, and p and T are respectively the pressure and temperature.

b) Using the results of part a), and assuming C_V to be temperature independent, show that a gas obeying the van der Waals equation of state is always cooled as its volume increases under a free expansion process.

12. (**Optics**) A glass cube has a refractive index of 1.5. A beam of light enters the top face obliquely and then strikes the side of the cube. Does any light emerge from the side?